Faraday Rotation and Interferometric/Polarimetric SAR

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Abstract – The potentially measurable effects of Faraday rotation on linearly polarized interferometric or polarimetric SAR measurements from space are addressed. Backscatter measurements subject to Faraday rotation are first modeled, and then the impacts are assessed using actual SAR data. Data characteristics found to be most sensitive to a small Faraday rotation (< 20 degrees) are the cross-pol backscatter [σ^o(HV)] and the like-to-cross-pol correlation [e.g. ρ(HHHV*)]. For a diverse, but representative, set of natural terrain the level of distortion across a range of backscatter measures is shown to be acceptable (i.e. minimal) for Faraday rotations of less than 5 degrees, and 3 degrees if the radiometric uncertainty in the HV backscatter is specified to be less than 0.5 dB.

Next a step-by-step procedure is outlined for correction (or calibration) of fully polarimetric data subject to Faraday rotation, to recover the true scattering matrix. The final steps in the procedure involve a novel strategy for estimation and correction of Faraday rotation. Sensitivity analyses are presented which show that at least one algorithm can be used to estimate Ω to within ± 3 or 5 degrees, with reasonable levels of residual cross-talk, noise floor and channel amplitude and phase imbalance. This approach is relevant for future L-band spaceborne SARs and removes one key obstacle to the deployment of even longer wavelength SARs (e.g. a UHF or P-Band SAR) in Earth orbit.

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Introduction and Scope

- Faraday rotation is a problem that needs to be taken into consideration for longer wavelength SAR's
- Worst-case predictions for Faraday rotation for three common wavebands:

	Ω (degrees)
C-Band (6 cm)	2.5°
L-Band (24 cm)	40°
P-Band (68 cm)	321°

- In this presentation we will:

  a. *Determine what level of Faraday rotation is acceptable for a reasonable set of allowable calibration errors*

  b. *Show how correction for Faraday rotation can be included in a calibration procedure for polarimetric SARs*

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Effects on Polarimetric Measurements

- General problem:

$$\begin{pmatrix} M_{hh} & M_{vh} \\ M_{hv} & M_{vv} \end{pmatrix} = A(r,\theta)\, e^{j\phi} \begin{pmatrix} 1 & \delta_2 \\ \delta_1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & f_1 \end{pmatrix} \begin{pmatrix} \cos\Omega & \sin\Omega \\ -\sin\Omega & \cos\Omega \end{pmatrix} \begin{pmatrix} S_{hh} & S_{vh} \\ S_{hv} & S_{vv} \end{pmatrix} \begin{pmatrix} \cos\Omega & \sin\Omega \\ -\sin\Omega & \cos\Omega \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & f_2 \end{pmatrix} \begin{pmatrix} 1 & \delta_3 \\ \delta_4 & 1 \end{pmatrix} + \begin{pmatrix} N_{hh} & N_{vh} \\ N_{hv} & N_{vv} \end{pmatrix}$$

- For H and V polarization measurements, and ignoring other effects, the measured scattering matrix, **M**, can be written as **M** = **RSR**, i.e.

$$\begin{bmatrix} M_{hh} & M_{vh} \\ M_{hv} & M_{vv} \end{bmatrix} = \begin{bmatrix} \cos\Omega & \sin\Omega \\ -\sin\Omega & \cos\Omega \end{bmatrix} \begin{bmatrix} S_{hh} & S_{vh} \\ S_{hv} & S_{vv} \end{bmatrix} \begin{bmatrix} \cos\Omega & \sin\Omega \\ -\sin\Omega & \cos\Omega \end{bmatrix}$$

i.e.

$$M_{hh} = S_{hh}\cos^2\Omega - S_{vv}\sin^2\Omega + \left(S_{hv} - S_{vh}\right)\sin\Omega\cos\Omega$$
$$M_{vh} = S_{vh}\cos^2\Omega + S_{hv}\sin^2\Omega + \left(S_{hh} + S_{vv}\right)\sin\Omega\cos\Omega$$
$$M_{hv} = S_{hv}\cos^2\Omega + S_{vh}\sin^2\Omega - \left(S_{hh} + S_{vv}\right)\sin\Omega\cos\Omega$$
$$M_{vv} = S_{vv}\cos^2\Omega - S_{hh}\sin^2\Omega + \left(S_{hv} - S_{vh}\right)\sin\Omega\cos\Omega$$

- This is non-reciprocal for $\Omega \neq 0$, (i.e. $M_{hv} \neq M_{vh}$, even though $S_{hv} = S_{vh}$)

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Effects on Interferometric Measurements

- For an HH-polarization measurement (e.g. JERS-1) the expected value of the radar cross section in the presence of Faraday rotation is:

$$\left\langle M_{hh} M_{hh}^{*} \right\rangle = S_{hh} S_{hh}^{*} \cos^{4}\Omega - 2\mathrm{Re}\left(S_{hh} S_{vv}^{*} \right) \sin^{2}\Omega \cos^{2}\Omega + S_{vv} S_{vv}^{*} \sin^{4}\Omega$$

assuming $\left\langle S_{hh} S_{hv}^{*} \right\rangle = \left\langle S_{hv} S_{vv}^{*} \right\rangle = 0$ (azimuthal symmetry)

- For repeat-pass data, the decorrelation due to Faraday rotation will be:

$$\rho_{\mathrm{Faraday}} = \frac{\left| \left(S_{hh} S_{hh}^{*} \cos^{2}\Omega - S_{hh} S_{vv}^{*} \sin^{2}\Omega \right) \right|}{\sqrt{\left(S_{hh} S_{hh}^{*} \right) \left(S_{hh} S_{hh}^{*} \cos^{4}\Omega - 2\mathrm{Re}\left(S_{hh} S_{vv}^{*} \right) \sin^{2}\Omega \cos^{2}\Omega + S_{vv} S_{vv}^{*} \sin^{4}\Omega \right)}}$$

- Either of these two measures will depend on both the Faraday rotation angle, Ω, and the polarization signature of the terrain

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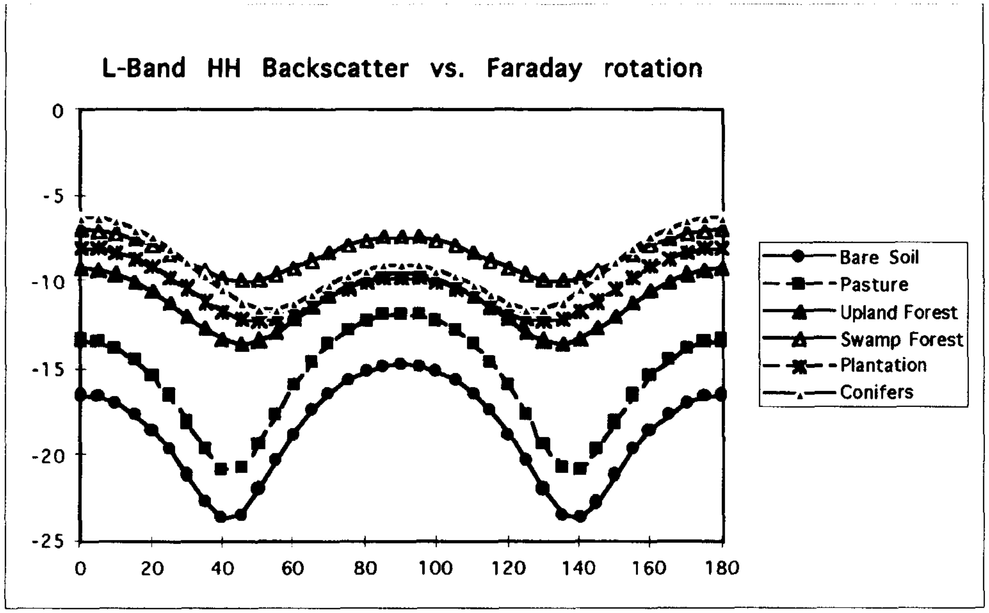
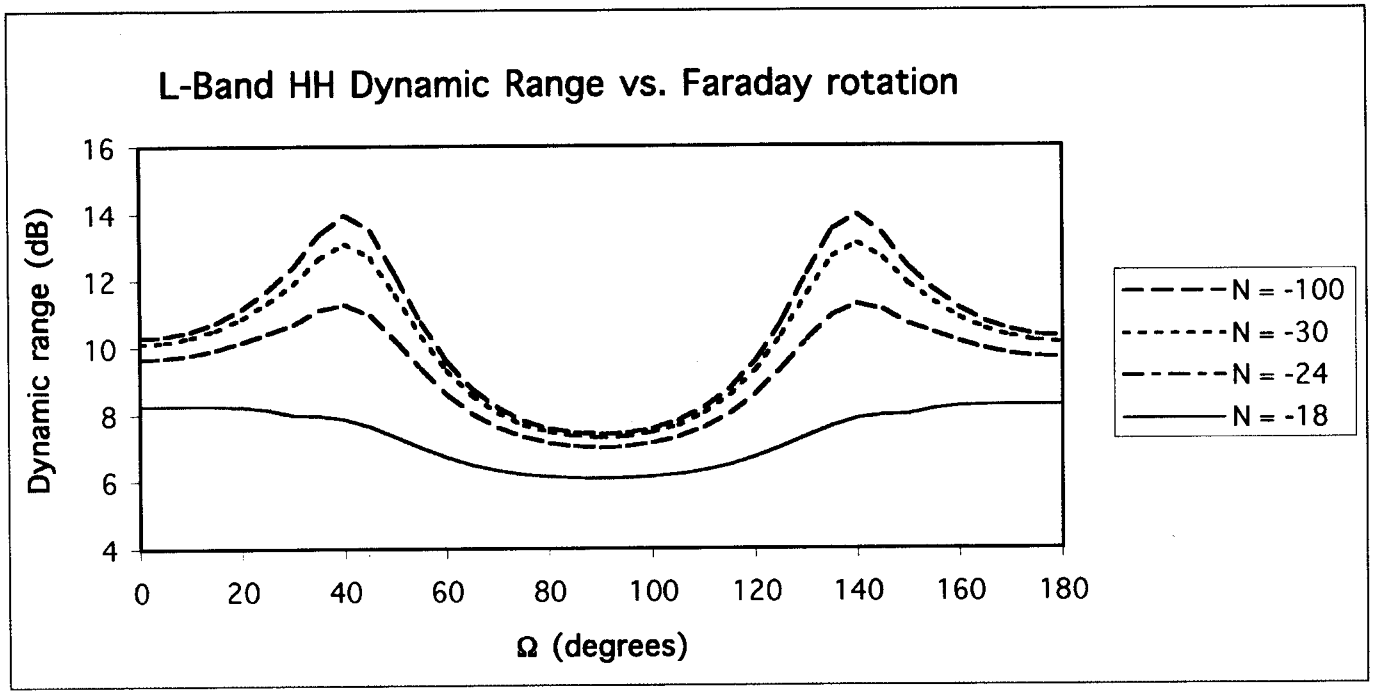
- Modeled L-Band 'HH' backscatter vs. Faraday rotation angle:



- Backscatter drops off to a null at Ω = 45 degrees
- Depth of null depends on polarization signature - but effects would probably be masked by noise (at -17 dB) in JERS-1 data, for example
- P-Band results are similar in behavior

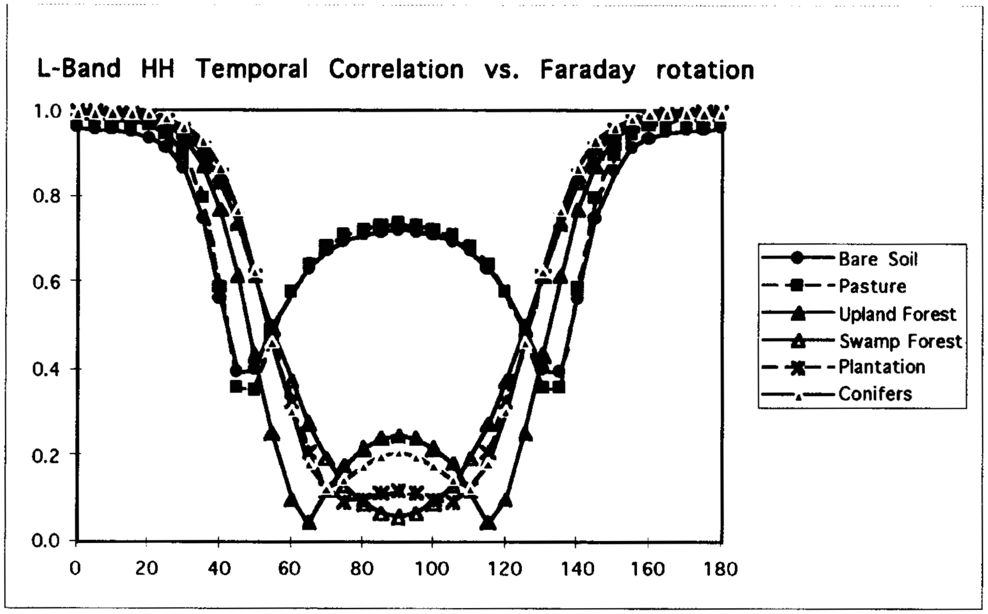
Introducing Additive Noise

- Some effects due to Faraday rotation may be masked by additive noise



Effects on Estimates of Decorrelation

- Modeled L-Band 'HH' backscatter decorrelation due to Faraday rotation:



- Little noticeable decorrelation up to $\Omega = 30$ degrees

- At $\Omega = 90$ degrees, correlation is the same as the HH-VV correlation for that scatterer type (e.g. 0.25 for Upland Forest)

Summary of Model Results

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- Spread of relative errors introduced into backscatter measurements across a wide range of measures for a diverse set of scatterer types
- Effects considered negligible (i.e. less than desired calibration uncertainty*) are shaded

Measure	$\Omega = 3$	$\Omega = 5$	$\Omega = 10$	$\Omega = 20$	$\Omega = 40$	$\Omega = 90$
$\Delta\sigma^o$(HH) - dB	0	-0.1	-0.2 -> -0.5	-0.9 -> -1.9	-2.7 -> -7.2	-2.7 -> +1.7
$\Delta\sigma^o$(VV) - dB	0	-0.1	-0.2 -> -0.5	-0.9 -> -1.8	-1.8 -> -7.3	-1.7 -> +2.7
$\Delta\sigma^o$(HV) - dB	+0.1 -> +0.5	+0.3 -> +0.7	+1.0 -> +3.7	+2.6 -> +7.6	+4.6 -> +10.8	0
$\Delta\rho(HH_1HH_2^*)$	0	0	0 -> -0.01	0 -> -0.03	-0.15 -> -0.42	-0.24 -> -0.87
$\Delta\rho$(HHHV*)	+0.11 -> +0.27	+0.18 -> +0.42	+0.32 -> +0.64	+0.43 -> +0.75	+0.17 -> +0.39	0
$\Delta\rho$(HHVV*)	0 -> -0.01	-0.02 -> +0.01	-0.06 -> +0.06	-0.21 -> +0.25	-0.13 -> +0.87	0
ϕ (HV-VH) - deg	0	0	0 or 180	0 or 180	180	0
ϕ (HH-VV) - deg	-0.2 -> +2.2	-0.5 -> +6.4	-2.1 -> +31.6	-11.0 -> +102.4	-143 -> +171.2	$= -\phi(\text{HH-VV*})\|_{\Omega=0}$

*Radiometric uncertainty - 0.5 dB
Phase error - 10 degrees
Correlation error - 6%

- A Noise-equivalent sigma-naught of - 30dB is assumed

Estimating the Faraday Rotation Angle, Ω

1. (Freeman, 2003) Since speckle and additive noise may be present in the backscatter signatures, **Ω** may be estimated from averaged 2nd-order statistics:

$$Z_{hv} = 0.5\left(M_{vh} - M_{hv}\right)$$

then estimating **Ω** from:

$$\Omega = \frac{1}{2}\tan^{-1}\sqrt{\frac{\left\langle Z_{hv} Z_{hv}^{*}\right\rangle}{\left(\left\langle M_{hh} M_{hh}^{*}\right\rangle + \left\langle M_{vv} M_{vv}^{*}\right\rangle + 2\,\mathrm{Re}\left\{\left\langle M_{hh} M_{vv}^{*}\right\rangle\right\}\right)}}$$

2. (Bickel and Bates, 1967) - for fully polarimetric (linear polarized) spaceborne SARs, it is straightforward to estimate the Faraday rotation angle, **Ω**, via:

$$\Omega = \frac{1}{2}\tan^{-1}\left[\frac{\left(M_{vh} - M_{hv}\right)}{\left(M_{hh} + M_{vv}\right)}\right]$$

for any type of scatterer

Estimating the Faraday Rotation Angle, Ω

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- Sensitivity to Residual System Calibration Errors (shaded cells represent errors in Ω > 3 degrees)

NOISE

a) NE σ^0	-100 dB	-50 dB	-30 dB	-24dB	-18 dB
$\Delta\Omega_1$ (deg)	0	1.2	11	18	23.6
$\Delta\Omega_2$ (deg)	0	0	1.3	5.4	31.1

CHANNEL AMPLITUDE IMBALANCE

b) $\lvert f_1 \rvert^2$	0.0 dB	0.1 dB	0.2 dB	0.3 dB	0.4 dB	0.5 dB	1.0 dB
$\Delta\Omega_1$ (deg)	0	0.4	0.9	1.3	1.8	2.2	4.4
$\Delta\Omega_2$ (deg)	0	0.1	0.3	0.4	0.6	0.7	1.4

CHANNEL PHASE IMBALANCE

c) Arg (f_1)	0 deg	2 deg	5 deg	10 deg	20 deg
$\Delta\Omega_1$ (deg)	0	1.3	3.4	6.6	12.4
$\Delta\Omega_2$ (deg)	0	0.4	1.0	2.1	5.1

CROSS-TALK

d) $\lvert\delta\rvert^2$	-50 dB	-30 dB	-25 dB	-20dB	-15 dB
$\Delta\Omega_1$ (deg)	0	0.1	0.2	0.7	2.1
$\Delta\Omega_2$ (deg)	0.3	2.6	4.7	8.2	15.4

Estimating the Faraday Rotation Angle, Ω

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- Combining effects for a 'typical' set of system errors, we see that a cross-talk level < -30 dB is necessary to keep the error in Ω < 3 degrees using measure (2)

a) P-Band	$\lvert\delta\rvert^2$= -30 dB	$\lvert\delta\rvert^2$= -25 dB
$\Delta\Omega_1$ (deg)	10.5	10.5
$\Delta\Omega_2$ (deg)	3.2	5.1

b) L-Band	$\lvert\delta\rvert^2$= -30 dB	$\lvert\delta\rvert^2$= -25 dB
$\Delta\Omega_1$ (deg)	10.6	10.5
$\Delta\Omega_2$ (deg)	2.9	5.2

For Measure (1) error is dominated by additive noise

- P-Band case has channel amplitude imbalance of 0.5 dB, phase imbalance of 10 degrees and NE σ^o = -30 dB
- L-Band case has channel amplitude imbalance of 0.5 dB, phase imbalance of 10 degrees and NE σ^o = -24 dB

Correcting for Faraday Rotation

- To correct for a Faraday rotation of **Ω,** use:

$$\tilde{\mathbf{S}} = \mathbf{R}^{t}\mathbf{M}\mathbf{R}^{t}$$

where $\mathbf{R}^{t} \equiv \mathbf{R}^{-1}$. This can be written:

$$\begin{bmatrix} \tilde{S}_{hh} & \tilde{S}_{vh} \\ \tilde{S}_{hv} & \tilde{S}_{vv} \end{bmatrix} = \begin{bmatrix} \cos\Omega & -\sin\Omega \\ \sin\Omega & \cos\Omega \end{bmatrix} \begin{bmatrix} M_{hh} & M_{vh} \\ M_{hv} & M_{vv} \end{bmatrix} \begin{bmatrix} \cos\Omega & -\sin\Omega \\ \sin\Omega & \cos\Omega \end{bmatrix}$$

- Since values of $\tan^{-1}$ are between $\pm\pi/2$, values of **Ω** will be between $\pm\pi/4$, which means that **Ω** can only be estimated modulo $\pi/2$

- This problem can be identified from the cross-pol terms by comparing measurements before correction and after, i.e.:

$$\left\langle \tilde{S}_{hv}\tilde{S}^{*}_{vh} \right\rangle = -\left\langle M'_{hv}M'^{*}_{hv} \right\rangle$$

and

$$0.25\left\langle \left(\tilde{S}_{hv} + \tilde{S}_{vh}\right)\left(\tilde{S}_{hv} + \tilde{S}_{vh}\right)^{*} \right\rangle \quad \left\langle M'_{hv}M'^{*}_{hv} \right\rangle$$

- This test should readily reveal the presence of a $\pi/2$ error in **Ω**, provided that the cross-pol backscatter S_{hv} (or S_{vh}) is not identically zero

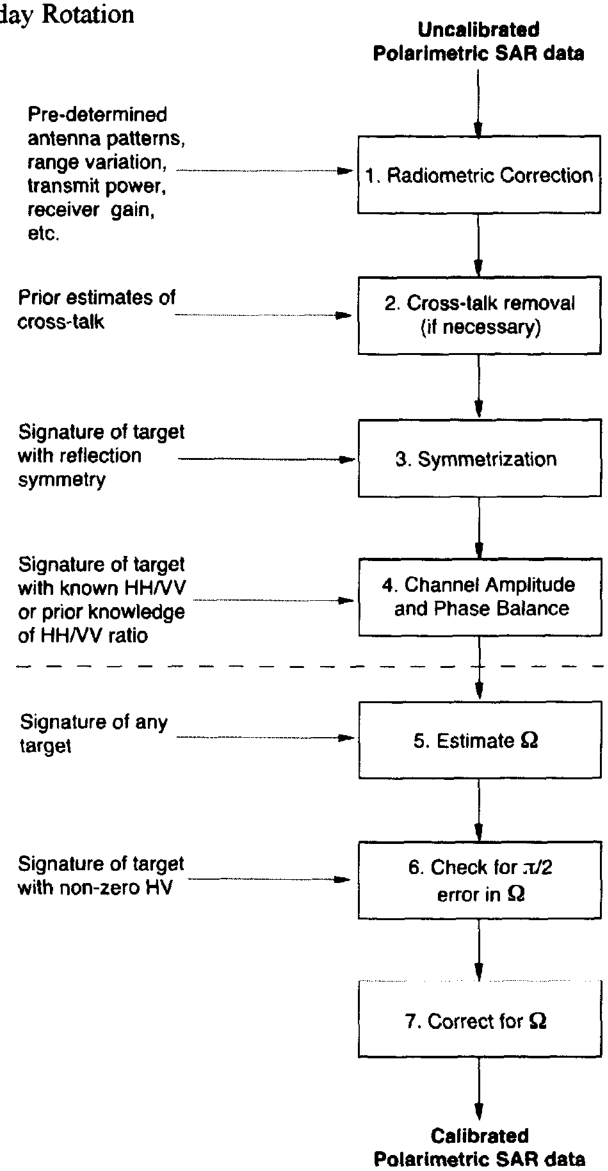
- Calibration Procedure for Polarimetric SAR data

    (Cannot estimate cross-talk from data)

    (Use any target with reflection symmetry to 'symmetrize' data)

    (Trihedral signature or known channel imbalance)

- Taking Faraday rotation <u>and</u> 'typical' system errors into account



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- **SUMMARY**
  - Chief noticeable effects of significant Faraday rotation on single-pol data should be:
    1. A few dB drop in measured backscatter level (σ^o)
    2. A drop in correlation in repeat-pass data
  - For $\Omega < 3$ degrees, calibration errors are acceptable for most measures
  - An exception is the like-to-cross pol correlation coefficient, which is always severely distorted by a small Faraday rotation.
  - A simple approach has been described to estimate and correct Faraday rotation found in fully polarimetric data
  - This has been embedded in a fully polarimetric calibration scheme
  - This removes one key obstacle in the path of a future Earth-orbiting spaceborne P-Band SAR