30 Years of Testing Relativistic Gravity: Where do we go from here?

Slava G. Turyshev

The talk will cover the theoretical framework, history, technology and recent results of the search for violation of general relativity in experiments performed in the solar system (i.e. LLR, spacecraft and planetary Doppler and range observations; equivalence principle, PPN β and γ etc). There are a number of theoretical reasons to question the validity of general relativity. Despite the success of modern gauge field theories in describing the electromagnetic, weak, and strong interactions, it is still not understood how gravity should be described at the quantum level. In theories that attempt to include gravity, new long-range forces can arise in addition to the Newtonian inverse-square law. Even at the purely classical level, and assuming the validity of the Equivalence Principle (EP), Einstein's theory does not provide the most general way to generate the space-time metric. Regardless of whether the cosmological constant should be included, there are also important reasons to consider additional fields, especially scalar fields.

I will focus on the techniques, methods and improvements in the LLR tests of G-dot and other PPN parameters enabled by the new APOLLO instrument being developed in New Mexico. Accurate analysis of precision ranges to the Moon has provided several tests of gravitational theory: the equivalence principle, geodetic precession, PPN parameters β and γ, and the constancy of the gravitational constant G. Other possible tests include the inverse square law at 20,000 km length scales and the PPN parameter α_1. The uncertainties of these tests have decreased as data accuracies have improved and data time span has lengthened. Currently we are exploring the modeling improvements necessary to proceed from cm to mm range accuracies. Looking to future exploration, what characteristics are desired for the next generation of ranging devices, what fundamental questions can be investigated, and what are the challenges for modeling and data analysis?

I will also discuss a mission concept for the Laser Astrometric Test of Relativity (LATOR) that will enable tests of the second order gravitational deflection effects in the solar gravity. Recent technological advances allow us to carry out direct tests in a weak gravitational field to measure effects of second order in the field strength. Although it does not follow that the strong-field behavior of relativistic gravitation would be determined by establishing the second-order terms, it must be true that if violations are observed the strong-field solutions would have to be modified. It also follows from recent progress in gravity quantization that there is a need for modification of the field equations of general relativity. The determination of the gravitational bending of light by the Sun to the second order would yield both new estimates of γ and the second-order term in the metric responsible for spatial curvature. The Laser Astrometric Test of Relativity (LATOR) is a revolutionary concept designed to directly address these issues.

30 Years of Testing Relativistic Gravity:
Where do we go from here?

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Outline:

- The PPN Formalism:
  - PPN Parameters
  - Recent Results & Methods Used
  - Potential Improvements
- New Frontiers for Testing Relativity:
  - Motivation & Goals for next 5-10 years
  - Possible Experiments & Required Technologies
- The LATOR Mission:
  - Mission Concepts
  - Addressable Fundamental Physics
  - A Code U Baby?

The Incumbent Model:
General Theory of Relativity

- Field Equations $$R_{mn} - \frac{1}{2} g_{mn} R = \frac{8\pi G}{c^4} T_{mn}$$
  - Non-linear partial differential equations of 2-nd order with respect to metric tensor g_{mn}: $\sim \partial^2 g_{mn} / \partial x_k^2$
- Techniques to find solutions:

Dynamics ~1/c	Field Strength, ~G: Strong	Field Strength, ~G: Weak
Non-Stationary	Numerical	Semi-Analytical
Quasi-Stationary	Numerical/Analytical	Analytical

- Gravitational experiments in the Solar System are well described by the Quasi-Stationary-Weak-Field Approximation of the field equations
- Equations are expanded in series with respect to a small parameter (v/c), in fact, virial theorem holds $(v/c)^2 \sim GM/c^2r$

Quasi-Stationary-Weak-Field Approximation: The PPN Framework

- 10-parameter PPN metric (Nordtvedt '68), Will & Nordtvedt '72):

$$
\begin{aligned}
g_{00} = & \, 1-2U+2\beta U^2+2\xi\Phi_w-(2\gamma+2+\alpha_3+\zeta_1-2\xi)\Phi_1- \\
& -2(3\gamma-2\beta+1+\zeta_2+\xi)\Phi_2-2(1+\zeta_3)\Phi_3-2(3\gamma+3\zeta_4-2\xi)\Phi_4+ \\
& +(\zeta_1-2\xi)A+(\alpha_1-\alpha_2-\alpha_3)w^2U+\alpha_2 w^i w^j U_{ij}-(2\alpha_3-\alpha_1)w^i V_i+O(\varepsilon^3), \\
g_{0i} = & -\tfrac{1}{2}(4\gamma+3+\alpha_1-\alpha_2+\zeta_1-2\xi)V_i-\tfrac{1}{2}(1+\alpha_2-\zeta_1+2\xi)W_i- \\
& -\tfrac{1}{2}(\alpha_1-2\alpha_2)w_i U-\alpha_2 w^j U_{ij}+O(\varepsilon^{5/2}), \\
g_{ij} = & \, \gamma_{ij}[1+2\gamma U+O(\varepsilon^3)]
\end{aligned}
$$

PPN Formalism is an excellent tool to test:

- Equivalence Principle,
- Variation in gravitational constant G,
- Alternative theories of Gravity

PPN Parameters and Their Significance

Parameter	What it measures relative to GR	Value in GR	Value in semi-conservative theories	Value in fully conservative theories
γ	How much space-curvature produced by unit mass	1	γ	γ
β	How much "non-linearity" in the superposition law for gravity	1	β	β
ζ	Preferred location effects?	0	ζ	ζ
α_1	Preferred frame effects?	0	α_1	0
α_2	–	0	α_2	0
α_3	–	0	0	0
α_3	Violation of conservation of total momentum	0	0	0
ζ_1		0	0	0
ζ_2	–	0	0	0
ζ_3	–	0	0	0
ζ_4	–	0	0	0

C. Will, 1998

PPN parameters quantifying fundamental properties of space–time and are responsible for certain symmetries.

Scalar-Tensor Theories of Gravity

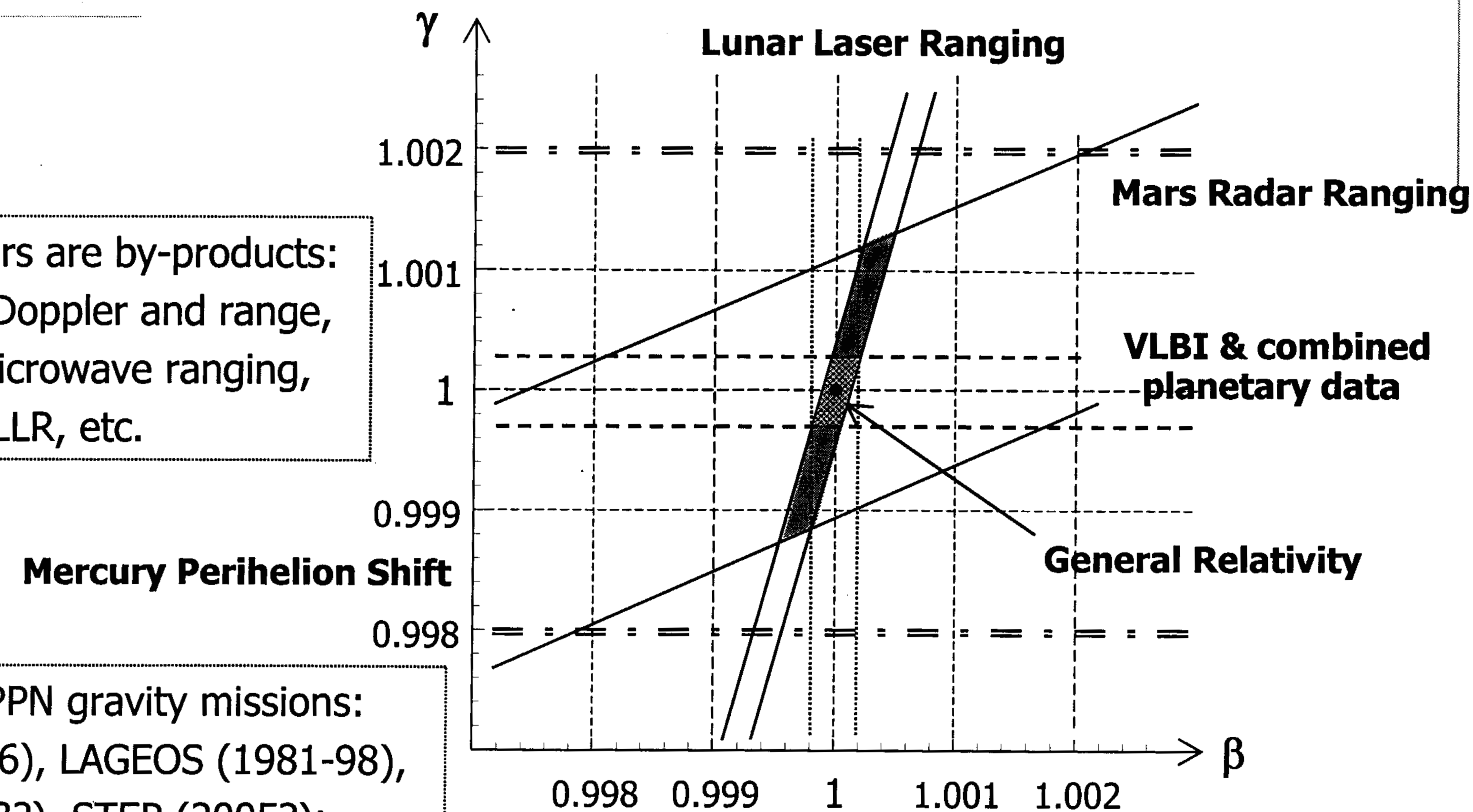
Theory	Arbitrary Functions or Constants	Cosimc Matching in GR	PPN Parameters				
			γ	β	ζ	α_1	α_2
General Relativity	None	None	1	1	0	0	0
Scalar-Tensor: Brans-Dicke	ω	ϕ_0	$\frac{1+\omega}{2+\omega}$	1	0	0	0
Generic ST	$A(\varphi),\ V(\varphi)$	φ	$\frac{1+\omega}{2+\omega}$	$1+\Lambda$	0	0	0
Rosen's Bimetric	None	c_0, c_1	1	1	0	0	$\frac{c_0}{c_1}-1$

A number of theories, alternative to general relativity, are still viable.

Fundamental Physics Laboratory – Our Solar System

PPN parameters are by-products:

- Spacecraft Doppler and range, planetary microwave ranging,
- VLBI, SLR, LLR, etc.



Designated PPN gravity missions:

- GP-A (1976), LAGEOS (1981-98), GP-B (2003?), STEP (2005?);
- LLR (on-going)

Recent technology advances enable even more accurate tests.

Current Limits on PPN Parameters

Parameter	Effect	Limit	Comments	Reference
$\gamma - 1$	Time delay Light Deflection	2×10^{-3} 3×10^{-4}	Viking ranging VLBI	Reasenberg et al. (1976) Eubanks et al. (1998)
$\beta - 1$	Perihelion Shift Nordtvedt Effect & Time Delay & VLBI	3×10^{-3} $< 5 \times 10^{-4}$	$J_2 = 10^{-7} \Leftarrow$ helioseismolgy $\eta = 4\beta - \gamma - 3$ assumed & γ from VLBI, planetary	Shapiro (1990) Anderson & Williams (2001) (grand-fit)
η	Nordtvedt Effect* Nordtvedt Effect	1.3×10^{-3} $(2 \pm 8) \times 10^{-4}$	Lunar Laser Ranging $\eta = 4\beta - \gamma - 3$ assumed	Baessler et al. (1999) Anderson & Williams (2001)
ξ	Earth Tides	10^{-3}	gravimetry data	
α_1	Orbital polarization	10^{-4}	Lunar Laser Ranging PSR J2317+1439	Müller et al. (1996) Bell, Camilo & Damour (1996)
α_2	Solar spin precession	4×10^{-7}	Solar alignment with ecliptic	Will (2001)
α_3	Pulsar acceleration	2×10^{-20}	Pulsar $\dot{P}$ statistic	Bell & Damour (1996)
ζ_1		2×10^{-3}	Combined PPN bounds	
ζ_2	Binary self-acceleration	4×10^{-5}	$\ddot{P}$ for PSR 1913+16	Will (1992)
ζ_3	Newton's 3rd law	10^{-8}	Lunar acceleration	Bartlett & van Buren (1986)
ζ_4	(Non-independent)		$6\zeta_4 = 3\alpha_3 + 2\zeta_1 - 3\zeta_3$	Will (1976)

Here parameter $\eta = 4\beta - \gamma - 3 - \frac{10}{3}\xi - \alpha_1 - \frac{2}{3}\alpha_2 - \frac{2}{3}\zeta_1 - \frac{1}{3}\zeta_2$

New Discipline: Applied General Relativity

- Last 30 years navigation, geodesy, time synchronization, etc.
  - Astrometry made GR an applied engineering discipline:
  - GPS, spacecraft is the most demanding discipline for accurate gravity modeling: SIM (~3-10 μas), GAIA (~1 μas)

QSWF approximation is well mapped to the 1PPN order.

Experiments testing the 2PPN order are needed!

<u>What to expect in 3-5 years?</u>

- VLBI ($\sim 5\times10^{-5}$):
  - Number of observations (1.5 M to 15 M → factor of 3)
  - Higher frequencies (need a "user" for DSN)
- LLR ($\sim 3\times10^{-5}$):
  - mm accuracies (thermal effects, new ranging station, etc)
- Microwave ranging ($\sim 3\times10^{-5}$): a designated mission !!!?

Theoretical Challenges to General Relativity

- Fundamental Physics Challenges:
  - Appearance of space-time singularities;
  - GR description breaks down in the regions with large curvature;
  - Cosmology: accelerating Universe, the nature of 'dark energy'.

- Alternative Theories of Gravity motivated by the need for Gravity Quantization and by Observational Cosmology:
  - Inflationary cosmologies, 'quintessence', superstrings models;
  - Many-dimensional Kaluza-Klein models;
  - Scalar-tensor theories – dilaton fields.

- These deviations from GR lead to:
  - Violation of the EEP; Non-universality of the free fall;
  - Modification of large-scale gravitational phenomena, and
  - Cast doubt upon the constancy of the "constants."

Experimental Motivations for Precision Tests of General Relativity

- Scalar-tensor theories – dilaton fields
  - Damour & Nordtvedt (1993): $\gamma - 1 \sim 10^{-5}\text{-}10^{-6}$
- Time variation in the fine structure constant:
  - Murphy et al. (2001): $\dot{\alpha}/\alpha H_0 \sim 10^{-5}$
  - Time dependency in the gravitational constant?

$$\dot{G}/GH_0 = \eta = (4\gamma - \beta - 3) \sim 10^{-5} - 10^{-6}$$

- COBE - local thermal anisotropy: $\delta T/T \sim 10^{-5}$
- Accelerating Universe (SN Ia type):
  - Is there local evidence for 'dark energy'?
- Pioneer anomaly (Newton's Law on a large scale?):

$$a_p = -(8.63 \pm 1.37) \times 10^{-8} \ \text{cm/s}^2$$

Existing & Previously Proposed Concepts:

Goal σ_γ	Project/Method	Comments
$(5-10)\times10^{-5}$	Cassini/solar conjunction	Two experiments in 2002 and 2003. Ka and X band. Anderson et al (2002)
$(3-7)\times10^{-5}$	Gravity Probe B Geodetic and Lense-Thirring precisions	Launch in 2003? Buchmann and Everitt (1994)
3×10^{-5} 1×10^{-6}	Mercury Relativity Satellite Mercury Relativity Orbiter	Ashby et al (1995): $\sigma_\beta = 2.3\times10^{-4}$ $\sigma(J_{2\odot}) = 2.5\times10^{-8}$, $\delta[\dot{G}/G] = 9\times10^{-14}$ yr^{-1} Bender et al. (1994): above & laser transponder
4×10^{-6}	Mars Laser Ranging time delay, SEP, 'grand fit'	Murphy et al. (2002): range $\sim\pm1$ cm, long-range forces, $\delta[\dot{G}/G] = 3\times10^{-15}$, Mars missions
$\sim1\times10^{-6}$	Asteroid Laser Ranging time delay, perehileon precession	Icarus, ...?? Range $\sim\pm1$ cm, JPL experize, long-range forces, β, $\dot{G}/G$, designated mission
3×10^{-6}	GAIA/μas astrometry	ESA, Launch $\sim$ 2014. Objects $\sim5\times10^7$
1×10^{-6}	POINTS/μas astrometry light deflection	NASA: work stopped. Reasenberg & Chandler (1989)
1×10^{-7}	Solar Orbit Relativity Test light deflection	ESA: not chosen. Veillet and Stanford (1994) Laser transponder/receiver
$\sim10^{-7}-10^{-9}$	LATOR I: optical interferometery 2PPN order light deflection to $\sim0.2\mu$as	MIDEX Proposal: Shao et al. (1994), JPL experts Optical interferometry & Laser transponder/receiver

The LATOR Mission:
Laser Astrometric Test of Relativity

The Primary Objective of the LATOR Mission:

- To discover and explore Fundamental Physical Laws governing mater, space, and time via testing relativistic gravity with 3-5 orders of magnitude improvement.

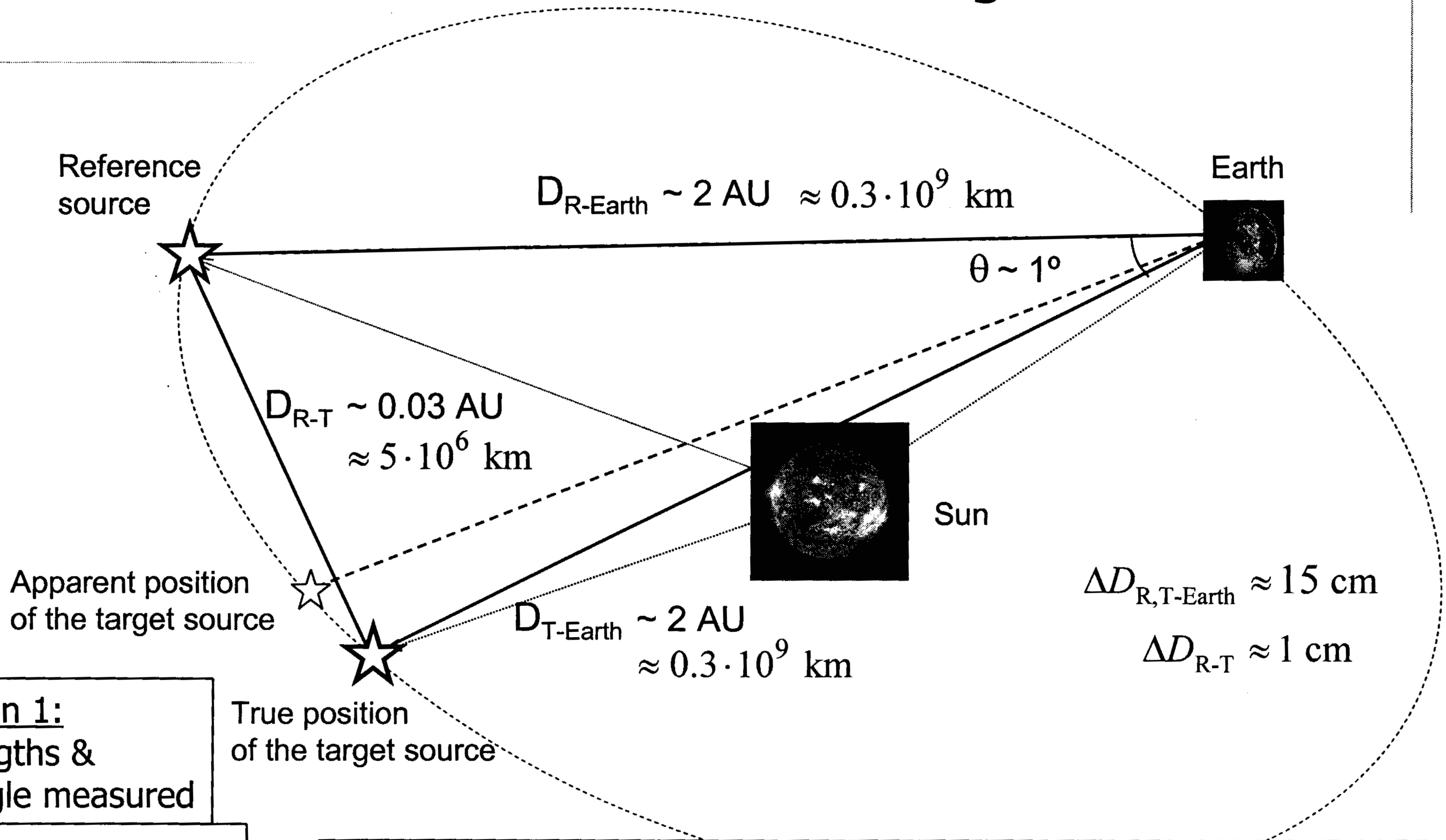
A MIDEX-class LATOR experiment uses:

- Two spacecraft @ 1AU, heliocentric orbit & laser transponders;
- Optical interferometer to measure angles between the spacecraft.

1. Laser transponders for pointing and range at 2AU:
   - Interplanetary Laser transponders with accuracy ~1 cm;
   - Target acquisition with solar background.
2. Angle Measurement Accuracy:
   - Laser Interferometer on ISS with ~100m baseline;
   - Accuracy ~0.02 μas (needed 1% of $(M/R)^2$ term ~3.5 μas);
   - 0.02 μas => 0.1 picorad ~10pm.

SIM demonstrated laser metrology repeatability <10pm (~0.03 Hz)

The LATOR Mission:
Relativistic Deflection of Light



Option 1:
3 lengths &
1 angle measured

Option 2:
2 drag free S/C
Angle vs. time

Geometric redundancy allows for accurate measurement of relativistic gravitational light deflection to 1 part in 10^9.

Magnitudes of the Effects

	Analytic Form	Value (μas)	Value (pm)
First Order	$2(1+\gamma)\frac{M}{R}$	1.75×10^6	1.537×10^{21}
Finite Distance to Earth	$-\frac{1}{2}(1+\gamma)\frac{M}{R}\frac{R^2}{r_E^2}$	-9.5	
Frame-Dragging	$\pm\frac{1}{2}(7\Delta_1+\Delta_2)\frac{J}{R^2}$	± 0.7	
Solar Quadrupole	$2(1+\gamma)J_2\frac{M}{R}$	0.2	
Second Order	$([2(1+\gamma)-\beta+\frac{3}{4}\Lambda]\pi-2(1+\gamma)^2)\frac{M^2}{R^2}$	3.5	65.8

SIM demonstrated laser metrology repeatability <10pm

Optical Technologies – metrology, transponders, interferometers – are the Strategic Components of the progress in Fundamental Physics Research

All key technologies are already available, as a result of SIM, Starlight, LISA

Why is LATOR Potentially Orders of Magnitude more sensitive?

- **Optical vs. Microwave:**
  - Solar plasma effects decrease as λ^2: from 10cm (3GHz) to 1 μm 300 THz is a 10^{10} reduction in solar plasma optical path fluctuations.
- **Orbit determination:**
  - Drag-free satellites are needed by LISA;
  - Redundant optical truss is an alternative to ultra-precise orbit determination. LATOR is insensitive to S/C buffeting from solar wind and solar radiation pressure (compare to Cassini/LLR/GP-B)
- **Potential for a low cost experiment:**
  - Optical SNR is very high (~1700), ~0.1 W lasers with freq stability & lifetime already developed for telecom / flight qualified for SIM.
  - Optical apertures in the 5cm to 10cm range sufficient;
  - Options exist for no motorized moving parts.

JPL Team X study demonstrates feasibility of LATOR

Fundamental Physics with LATOR

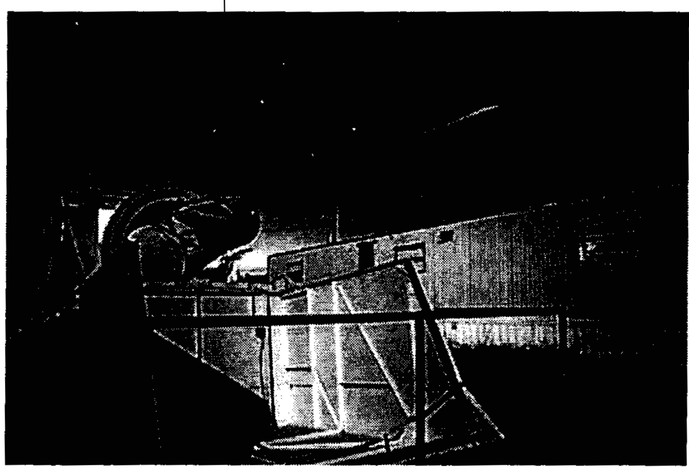
- LATOR will achieve accuracy 3-5 orders better then currently available by measuring the 2PPN order relativistic deflection of light
  - PPN parameter γ: $\sim 10^{-7} - 10^{-9}$;
  - Direct observation of PPN parameter β to 1% accuracy;
  - A number of theories of gravity will be tested in a new regime.

- Other interesting contributions:
  - Relativistic frame-dragging effect;
  - Solar physics: solar J2; mass, atmosphere;
  - Solar System bodies: masses, distances;
  - VLF Gravitational Waves?
  - Extension of the PPN Formalism; Relativistic Reference Frames;
  - Models for light propagation in a non-stationary gravity field;
  - New insights on the Equivalence Principle.

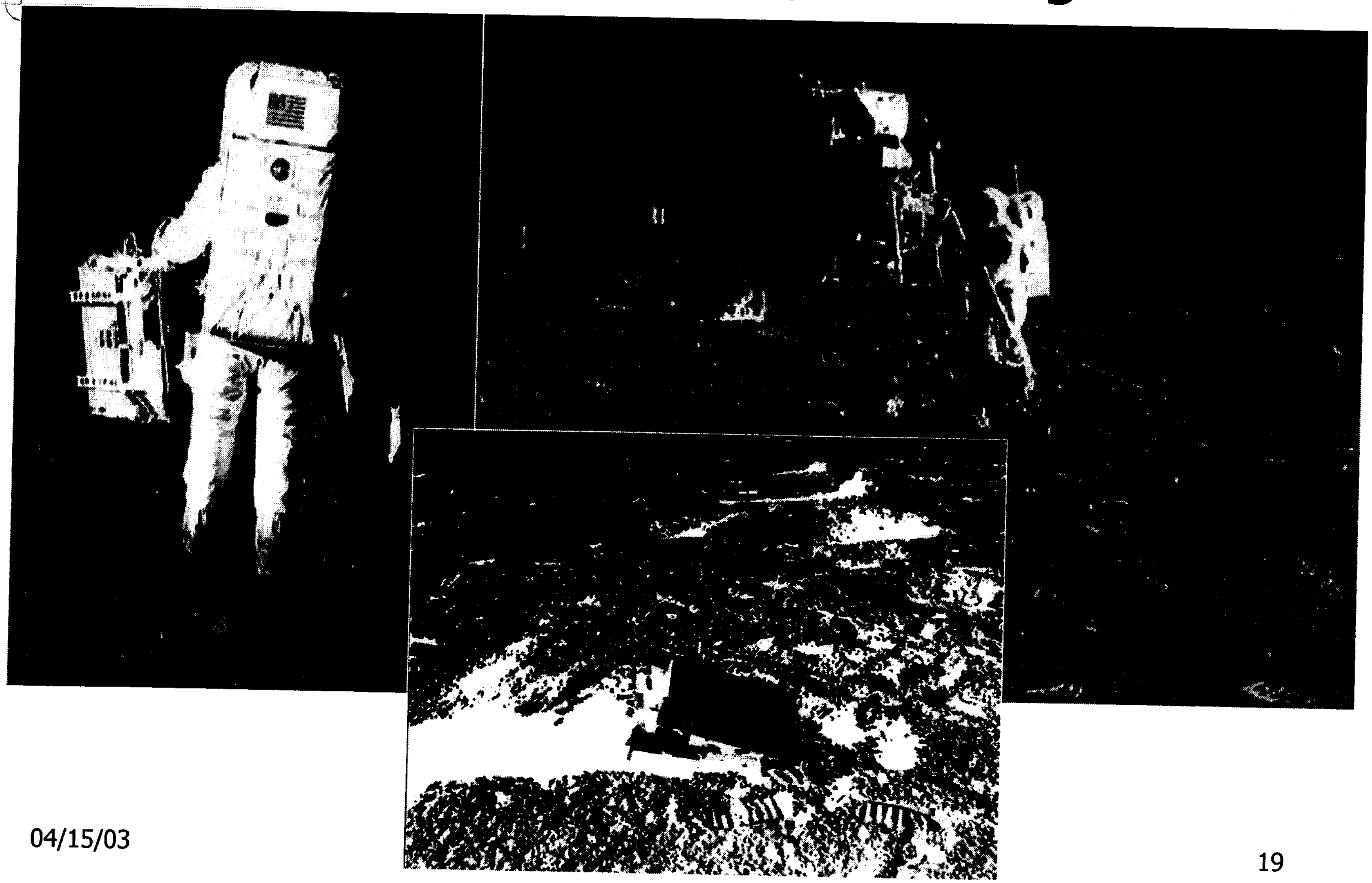
The expected results are important - the LATOR Mission should be done.

Improving LLR Tests of Gravitational Theory:

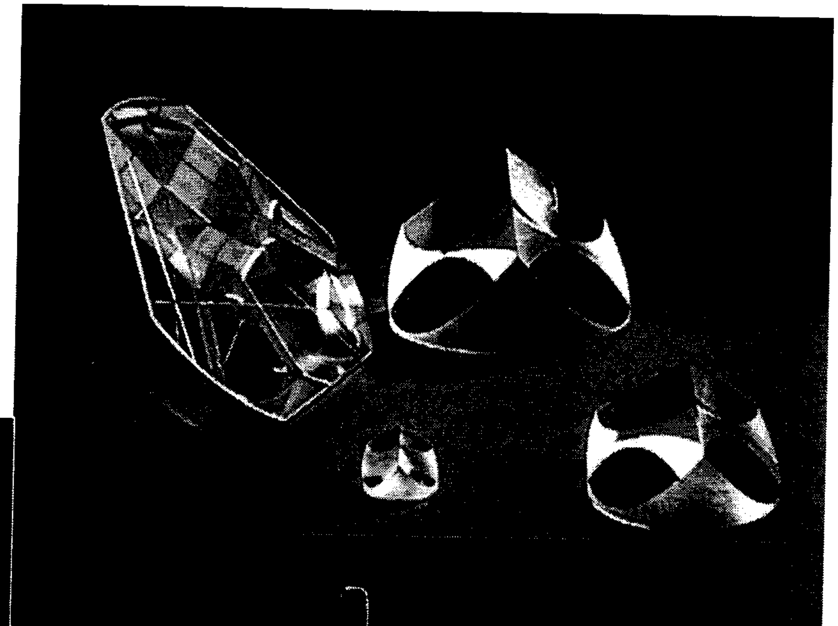
- Lunar Laser Ranging:
  - Excellent Legacy of the Apollo Program
  - Relativistic Research with LLR
  - Recent Results
- The New LLR Challenge:
  - Apache Point Ranging Station
  - Expected Accuracy
  - Addressable Fundamental Physics
- The Modeling Effort:
  - Effects and significance

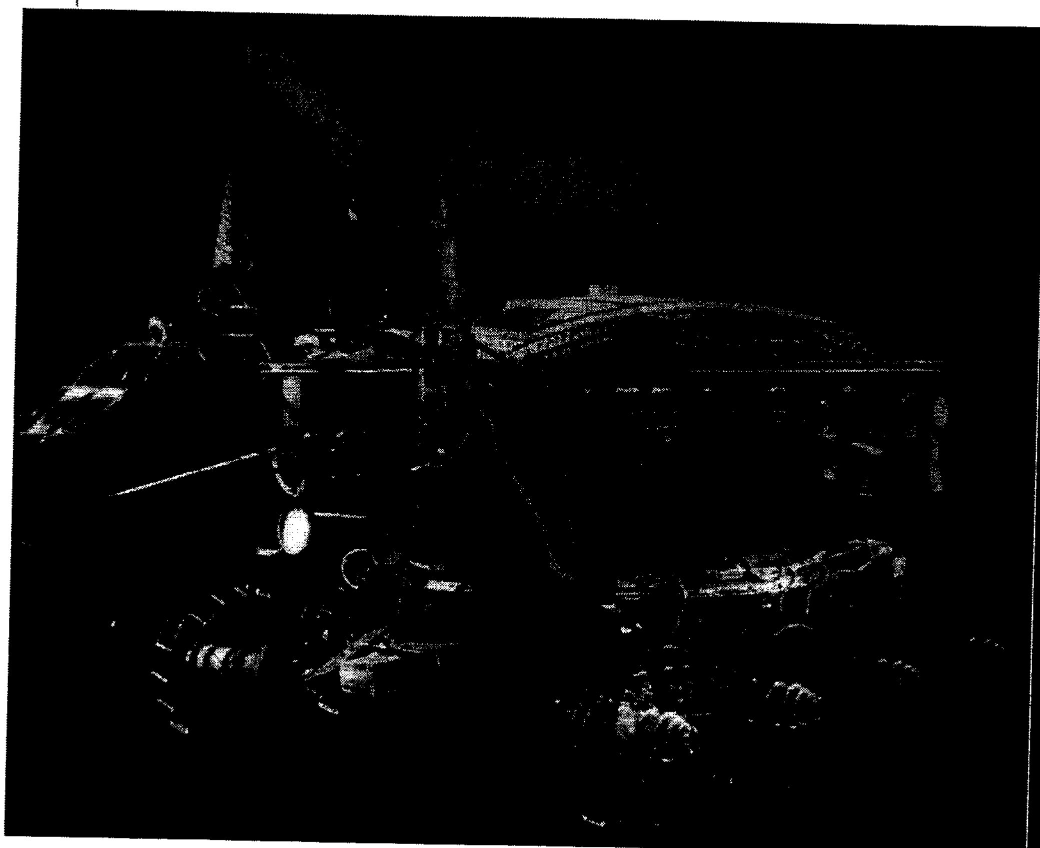


Excellent Legacy of the Apollo Program

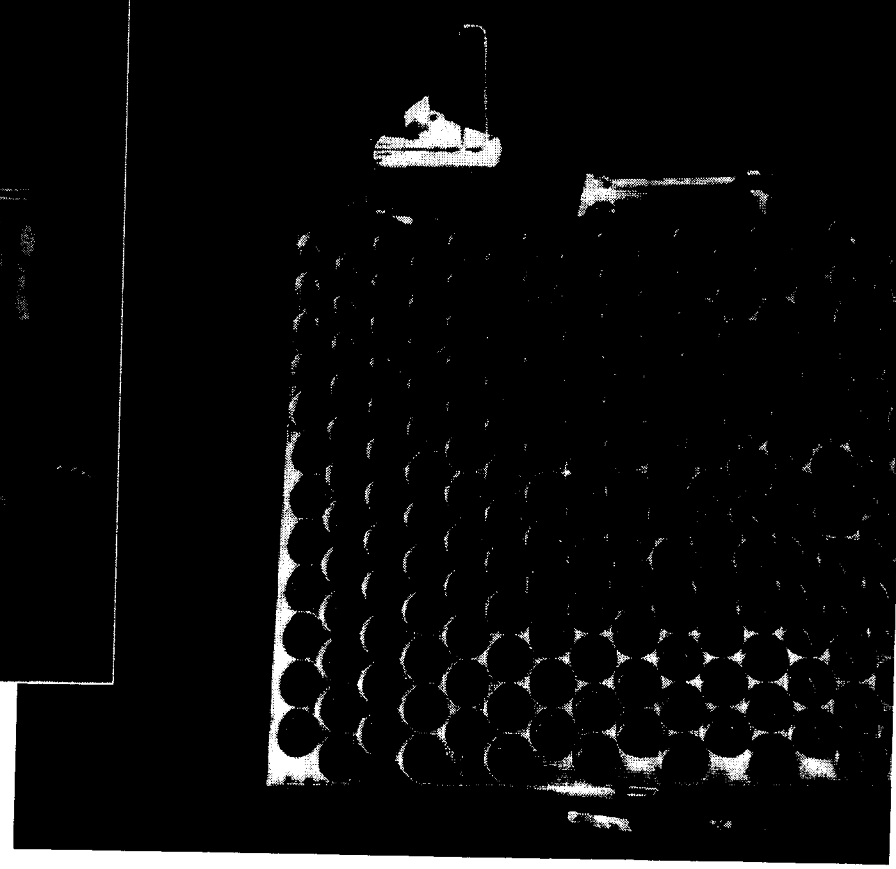


Lunar Retroreflectors





Lunakhod Rover



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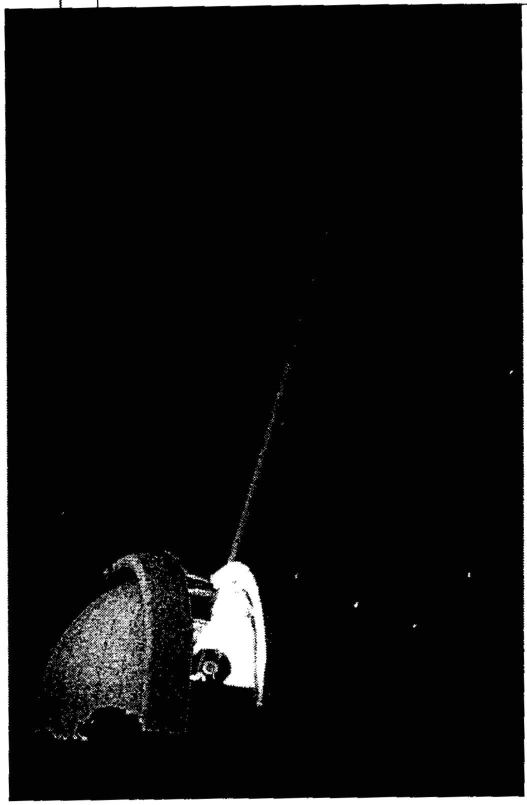
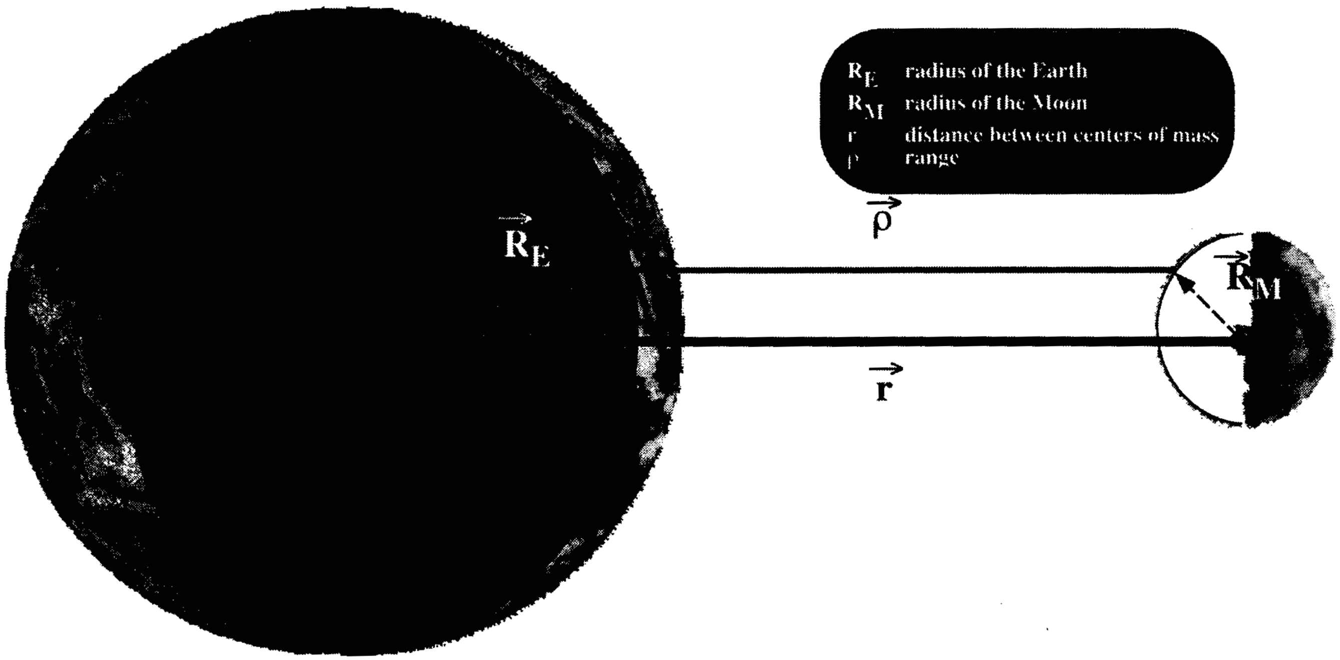
History and Present day of LLR:

- Laser Ranges between observatories on the Earth and retroreflectors on the Moon started in 1969 and continue to the present.



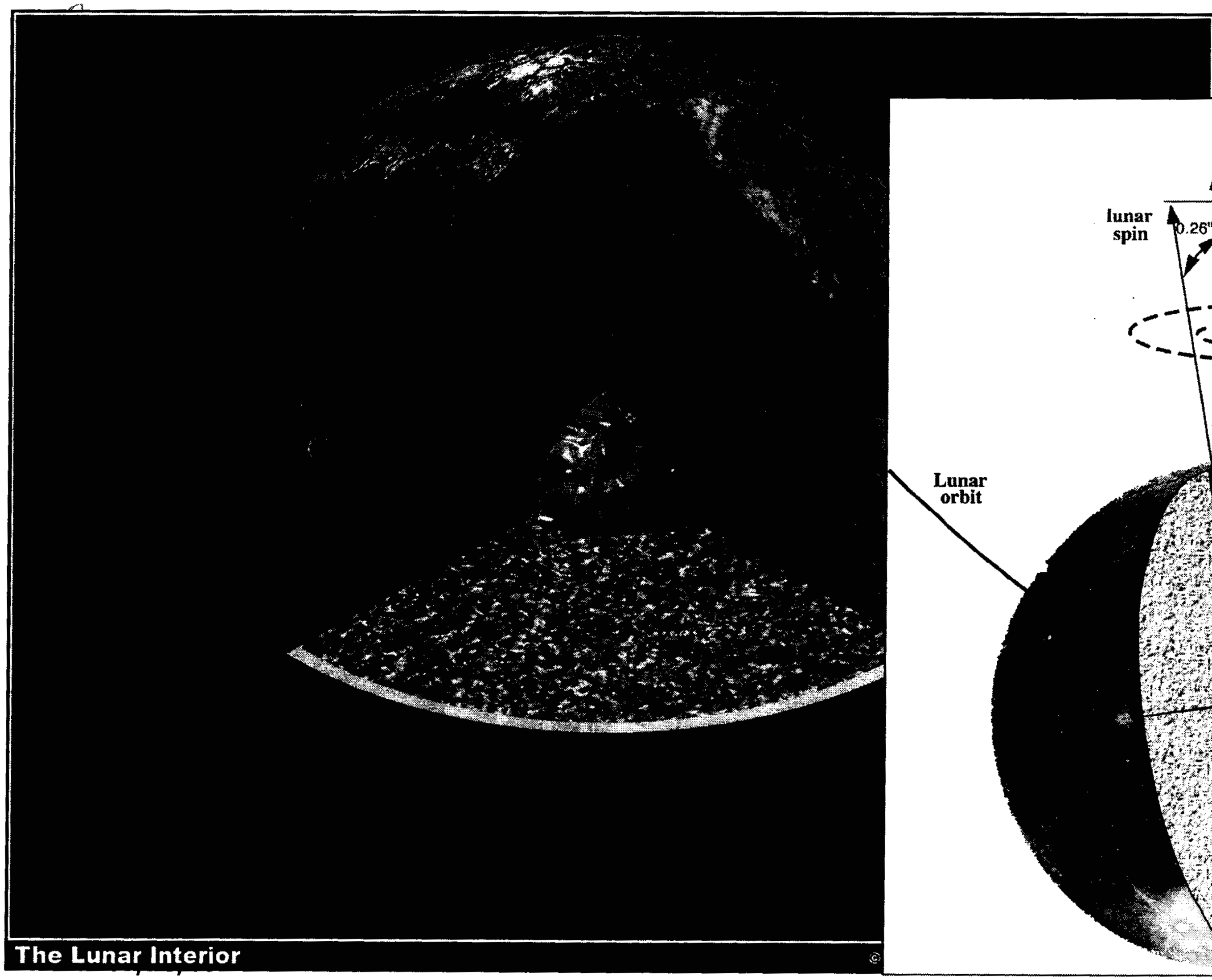
- 4 retroreflectors are ranged:
  - Apollo 11, 14, & 15 sites and
  - Lunakhod 2 Rover

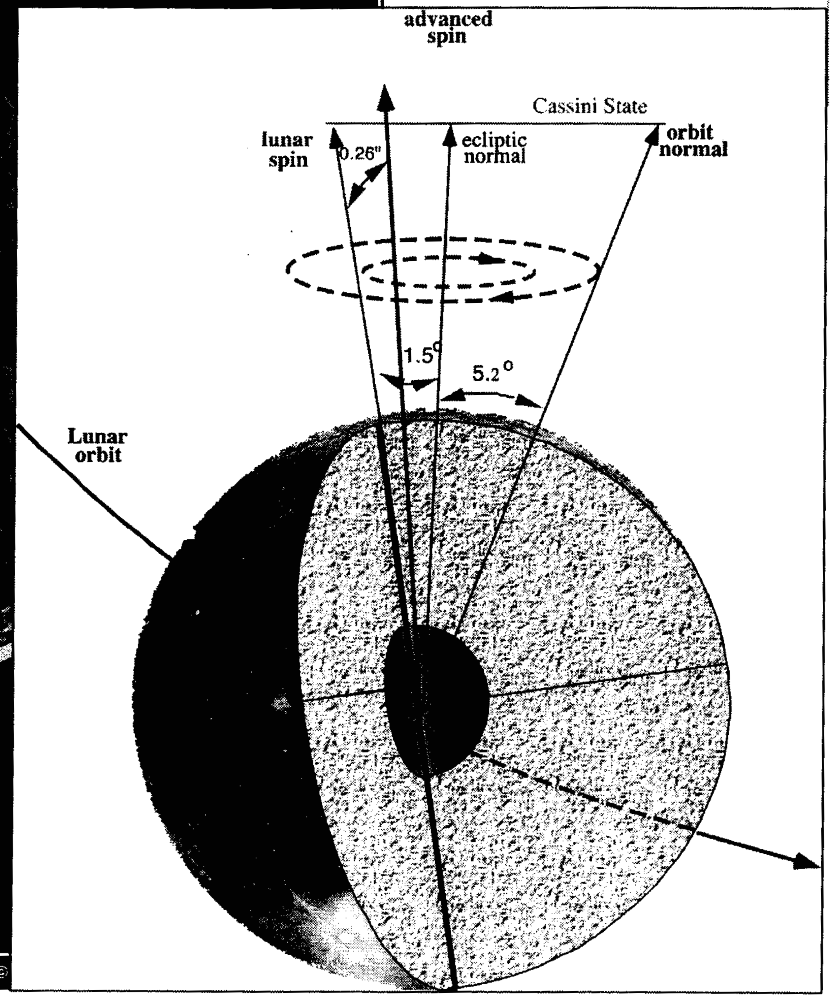
- LLR Ranges conducted primarily from 3 observatories:
  - McDonald (Texas)
  - OCA (Grasse)
  - Haleakala (Hawaii)



RE radius of the Earth
RM radius of the Moon
r distance between centers of mass
ρ range
ρ
RE
RM
r

LLR: Probing the Moon's Interior from the Earth





Quasi-Stationary-Weak-Field Approximation: The PPN Framework

- 10-parameter PPN metric (Nordtvedt '68), Will & Nordtvedt '72):

$$g_{00} = 1 - 2U + 2\beta U^2 + 2\xi\Phi_w - (2\gamma + 2 + \alpha_3 + \xi_1 - 2\xi)\Phi_1 -$$
$$-2(3\gamma - 2\beta + 1 + \xi_2 + \xi)\Phi_2 - 2(1 + \varsigma_3)\Phi_3 - 2(3\gamma + 3\xi_4 - 2\xi)\Phi_4 +$$
$$+(\xi_1 - 2\xi)A + (\alpha_1 - \alpha_2 - \alpha_3)w^2U + \alpha_2 w^i w^j U_{ij} - (2\alpha_2 - \alpha_1)w^i V_i + O(\varepsilon^3),$$
$$g_{0i} = -\tfrac{1}{2}(4\gamma + 3 + \alpha_1 - \alpha_2 + \xi_1 - 2\xi)V_i - \tfrac{1}{2}(1 + \alpha_2 - \xi_1 + 2\xi)W_i -$$
$$-\tfrac{1}{2}(\alpha_1 - 2\alpha_2)w_i U - \alpha_2 w^j U_{ij} + O(\varepsilon^{5/2}),$$
$$g_{ij} = \gamma_{ij}[1 + 2\gamma U + O(\varepsilon^3)]$$

$$\frac{\Delta a}{a} \equiv \frac{2(a_1 - a_2)}{(a_1 + a_2)} = \left(\frac{M_G}{M_I}\right)_1 - \left(\frac{M_G}{M_I}\right)_2, \qquad \frac{M_G}{M_I} = 1 + \eta\frac{U}{Mc^2}$$

$$\frac{\Delta a}{a} = \eta\left(\frac{U_e}{M_e c^2} - \frac{U_m}{M_m c^2}\right) = -4.45\times 10^{-10}, \qquad \eta \equiv 4\beta - \gamma - 3.$$

If η=1, this effect would produce a 13 m displacement

LLR Today and The Challenge

- Present range accuracy: < 1.5 cm
- Fundamental Physics Experiments:
  - Equivalence Principle: $\eta=(4\beta-\gamma-3) \sim 1\times10^{-4}$ or $\delta a/a \sim 10^{-13}$
  - Time variation in Grav. constant: Gdot $\sim 1.1\text{x}10^{-12}$ 1/yr

- Apache Point Laser Ranging Station:
  - August 2003
  - Better distribution in a month (schedule)
  - Should also be able to range during full Moon
- Integrates well with current NASA Strategic Missions:
  - Mars Missions, such as Mars Telesat 2009, etc

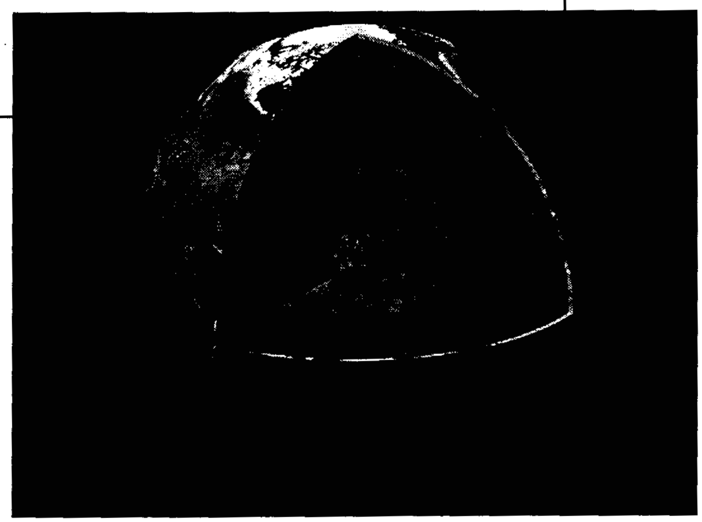
The LLR Challenge: to model range to a ~ mm accuracy
- Dynamical effects; thermal effects; atmosphereric and etc.

Expected results:
- Principle of Equivalence $\eta=(4\beta-\gamma-3)\sim 1\times10^{-5}$ or $\delta a/a \sim 10^{-14}$
- Time variation in Grav. constant: Gdot $\sim <1\text{x}10^{-13}$ 1/yr

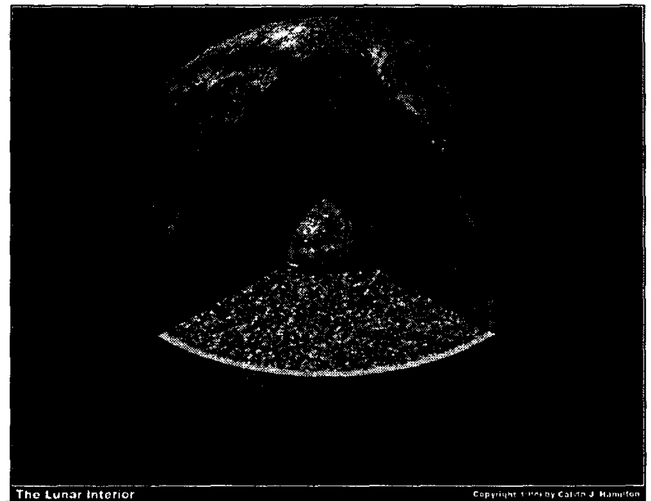
Well-understood effects with analytical formulations or are straightforward, but are not yet implemented:

Earth:

1. Several periodic tidal effects on the Earth are noteworthy
2. The Earth's surface distorts elastically due to atmospheric pressure variations.
3. An annual relativity effect on station radius with 1 mm amplitude.
4. A new algorithm for mapping atmospheric delay vs elevation.
5. The dynamical effect of the Earth's J22 harmonic is about 0.6 mm with a 12.5 hr period.
6. The Earth's J2 is slowly decreasing.



Well-understood effects (2):



Moon:

1. An annual periodic term of 8 mm amplitude at the equator, due to the time transformation, which projects into ~3 mm in range.
2. Another relativistic effect on the rotation is geodetic precession.
3. Solar tides on the Moon cause a 2 mm periodic displacement with 1/2 synodic month period.
4. Solar tides also influence the rotation.
5. From the lunar rotation it is known that the Moon has a sizable tidal dissipation with a bulk monthly tidal Q of 37. This Q should cause a shift of the tidal displacement of about 2 mm.
6. The time delay due to refraction in the corner cubes exceeds 1 cm, but is mostly constant.

Effects to be investigated:

1. Torque due to the flow at an oblate boundary between a fluid core and a solid mantle gives a torque.
2. If the Moon has an inner solid core, there can be gravitational torques between the inner core and the mantle.
3. Monthly thermal expansion of retroreflector heights are 1-2 mm for the Apollo, but is ~5 mm for the Lunokhod reflectors.
4. The dynamical sensitivity to the higher degree gravity harmonics of the Earth and Moon should be reconsidered.
5. The relativistic transformation effects, particularly the time transformation.
6. Temperature effects on the telescope must be considered.
7. The Earth's atmosphere tilts with respect to the surface.

Effects poorly understood:

1. Solutions for the orbital eccentricity rate give an anomalous value after accounting for tidal dissipation on Earth and Moon.
2. Changes in local ground water cause small motions of the surface at the ranging site. An on-site gravimeter should help.
3. The atmospheric delay model assumes a static atmosphere, but the atmosphere is not static and there are horizontal pressure gradients. An extended array of pressure guages.
4. Part of atmospheric loading that depends on pressure surrounding the site. An on-site gravimeter should help.
5. Other effects: solar radiation pressure, lunar thermal expansion, solar tides on the moon, etc.

Both size and signature will determine the modeling priority

Conclusions:

1. LLR contributes significantly to astrometry, geodesy, geophysics, lunar planetology, and gravitational physics
2. Most of the terms $1/c^2$ order, or the Post-Newtonian order contribute to the measured details of the lunar orbit, therefore, LLR achieves near-completeness as a gravity experiment and probe
3. Expected improvement of the range accuracy will bring more interesting results, especially for the Fundamental physics.
4. LLR experience will provide important milestone for manned and robotic future space exploration: optical communication on the solar system scale.

LLR modeling effort will significantly improve accuracy of this unique Laboratory for the Fundamental Physics and Relativistic Gravity Tests.