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ASSESSING MODEL UNCERTAINTY IN THE CONCEPTUAL DESIGN OF A MONOPROPELLANT PROPULSION SYSTEM

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An assessment of model uncertainty via probabilistic methods is described. An important question that arises in conceptual design is how accurate do models have to be to be useful? That is to say, when do other uncertainties in a higher fidelity model counteract its accuracy when compared to a lower fidelity model faced with these same uncertainties? A detailed definition of model uncertainty is provided. As an example, the model uncertainty in the conceptual design of a monopropellant blowdown hydrazine propulsion system is investigated. A simple model that estimates the mass and performance of the propulsion system is summarized. A description of a higher fidelity model follows. The two models are compared for three recent missions. For the propulsion examples investigated, the high-fidelity model provides a significant benefit over the simple model in estimating the propulsion system dry mass. However, for estimating the propellant mass required, the high-fidelity and simple models are comparable. The analysis discussed in this paper is an integral part of ongoing research into propagating and mitigating the effect of all types of uncertainty in the design and development of complex multidisciplinary engineering systems.

Nomenclature

A, B, C	= viscosity constants
A_0, a, B_0, b, c_0	= Beattie-Bridgeman constants
A_x	= cross-sectional area, m^2
c	= exhaust velocity, m/sec
E	= random variable
F	= thrust, N
f	= friction factor
f_{burst}	= burst factor
$f_{vol_hold\text{-}up}$	= hold-up/residual volume fraction
$g(E)$	= probability density function
k_1, k_2	= thrust constants for a given engine
k_3, k_4	= exhaust velocity constants for a given engine
l	= length, m
$(l/\phi)_{equiv}$	= equivalent length to diameter ratio
M	= molecular mass (molecular weight), kg/kmol
m	= mass, kg
$\dot{m}$	= mass flow rate, kg/sec
n	= total number of component types
p	= pressure, Pa
q	= quantity or number
R	= gas constant of a given gas, J/kg-K
$\hat{R}$	= set of all real numbers
Re	= Reynolds number
T	= temperature, K
t	= thickness, m
V	= internal volume, m^3
v	= flow speed, m/sec
y	= dependent variable
z	= total number of ΔV maneuvers
$\alpha_1, \alpha_2, \alpha_3$	= constants used in calculating density
β, γ, δ	= alternate Beattie-Bridgeman "constants"
Δp	= pressure drop, Pa
ΔV	= change in velocity, m/sec
η	= expulsion efficiency
κ_1, κ_2	= pressure drop constants for a given component
μ	= viscosity, kg/m-sec

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v	= specific volume, m^3/kmol
ρ	= density, kg/m^3
ϕ	= diameter, m
ψ	= percent value
g_0	= earth's gravitational acceleration, 9.80665 m/sec^2
$\Re$	= universal gas constant, 8314.51 J/kmol-K

Subscripts

bends	= tubing bends
comp	= all components
engines_op	= engines operating
hold-up	= hold-up/residual
hold-up_tank	= hold-up/residual in the propellant tank
hold-up_tubing	= hold-up/residual in the tubing
inlet	= thruster inlet
inlet_s	= thruster inlet at the start of a ΔV maneuver
inlet_p	= thruster inlet at the end of a ΔV maneuver
MEOP	= maximum expected operating pressure
per_tank	= per propellant tank
pres	= pressurant
prop	= propellant
prop_ACS	= attitude control propellant
prop_hold-up	= hold-up/residual propellant
prop_per_ΔV	= propellant expended during a ΔV maneuver
prop_usable	= total usable propellant
prop_ΔV	= total usable propellant for ΔV maneuvers
s/c	= spacecraft after a ΔV maneuver
s/c_f	= spacecraft after the final ΔV maneuver
s/c_s	= spacecraft prior to first ΔV maneuver
tank	= propellant tank
total_prop	= total loaded propellant
total_tanks	= total of all tanks
wall	= tubing wall

Superscripts

i	= component number
j	= ΔV maneuver number
t	= true value

Introduction

Uncertainty has been defined and classified in various ways in different fields. A classification of uncertainty for the design and development of complex multidisciplinary systems is provided in a companion paper.[1] The following section begins with a detailed definition of model uncertainty. An explanation of how uncertainty impacts model validity follows.

Model Uncertainty

Model uncertainty is the accuracy of a mathematical model to describe an actual physical system of interest. Model uncertainty, also known as model-form, structural, or prediction-error uncertainty, is a form of epistemic uncertainty. That is to say, model uncertainty is often simply due to a lack of knowledge. Model uncertainty is associated with the use of one or more simplified relationships between the basic variables used in representing the 'real' relationship or phenomenon of interest.[2] In its simplest form, model uncertainty concerns the uncertainty in representation of physical behavior.

Approximation Errors

For physical processes that are relatively well understood, deficiencies in certain models are often called *approximation errors* rather than model uncertainty. For example, in the modeling of the specific volume of a gas, the models can be ordered in terms of increasing accuracy (decreasing model uncertainty) as follows: ideal-gas law, van der Waals equation, Beattie-Bridgeman equation, and Benedict-Webb-Rubin (BWR) equation. The ideal gas law neglects intermolecular forces between molecules and uses only one constant. The van der Waals equation uses two constants to allow for interaction and volume effects. The Beattie-Bridgeman equation uses five constants and is accurate over a much larger range. The BWR equation uses eight constants and is even more versatile. In general, this ordering is appropriate, but for individual gases there is no guarantee that any one model will be more accurate than any other because even the ideal gas law can be very accurate for specific conditions such as low pressures and high temperatures.

Numerical and Programming Errors

Model uncertainty also arises from numerical and programming error. *Numerical error* can arise due to finite precision arithmetic and can be reduced by using higher precision computers and software. *Programming error* occurs during development of the model due to blunders or mistakes by the programmer. Although there is no straightforward method for estimating programming errors, they can be detected by the person who committed it, resolved by better communication, or discovered by redundant

organizational and operational procedures and protocols.[3] This paper investigates only approximation error of model uncertainty.

Uncertainty impact on model validity

Model uncertainty can be reduced with effort, research, and increased availability of data. Johannes van der Waals improved upon the ideal gas law in 1873 as part of his doctoral thesis.[4] Similarly, Beattie and Bridgeman improved upon the van der Waals equation in 1928 and Benedict, Webb, and Rubin improved upon the Beattie-Bridgeman equation in 1940. All of these researchers were aided with increased and improved data of various gases. The four models for the specific volume example discussed are simple compared to models for more complicated systems. All four models are single equations that determine the specific volume (dependent variable) through two independent variables: temperature and pressure. A complex multidisciplinary system, such as a spacecraft, automobile, or submarine, may use many mathematical submodels, each with possibly dozens of equations. The complexity of the models depends on the physical complexity of each phenomenon being considered, the number of physical phenomena being considered, and the level of coupling of different types of physics.[3] Simple models for complex systems are often used since the time and cost required to develop a high-fidelity model are prohibitive from a schedule and/or resource perspective. Even when high-fidelity models have been developed, they are often not chosen because of higher computational costs associated with their solution. The ratio of computational cost for a higher fidelity model to a lower fidelity model is commonly high, sometimes exceeding a factor of a 100.[3]

Probabilistic Representation

Model uncertainty in a dependent variable can be represented as a random variable and related to the true value:[5]

$$y' = y - E \tag{1}$$

A probability density function can be determined for E if sufficient true values are available to compare to the values determined by a model. Two properties must hold for a continuous probability density function, PDF, to be valid:[6]

$$\begin{aligned} &f(E) \geq 0 \quad E \in \hat{R} \\ &\int_{\Re} f(E)\,dE = 1 \end{aligned} \tag{2}$$

If E is a discrete random variable, a discrete PDF can be used and the integral in equation (2) is replaced by a sum. The advantage of using a PDF to represent model uncertainty is that it can be easily convolved with other uncertainties that are represented probabilistically.

Thermodynamic Example

Returning to the thermodynamic example discussed earlier, the model error of the specific volume can be represented by a PDF. For a range of temperatures and pressures the difference between each of the four equations can be compared with actual measured data.[7] These differences can be sorted into bins and transformed to a PDF as shown in Fig. 1 for nitrogen.

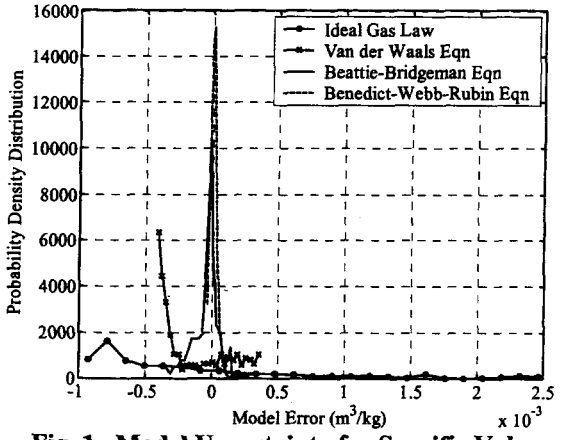

Fig. 1 Model Uncertainty for Specific Volume.

Fig. 1 is useful in assessing model uncertainty in the context of uncertainties in both temperature and pressure. Almost 400 values of the specific volume were used in creating Fig. 1 for temperatures ranging from 160 to 650 K (-113 to 377 °C) and pressures ranging from 0.1 to 101.3 MPa (1 to 1000 atm). The more values used, the smoother the resulting PDFs. Fig. 1 illustrates that the ordering of the four equations is indeed valid. The fact that the error grows at tails of distributions for the ideal gas law and the van der Waals equation indicate that neither truly represent the thermodynamics of the problem over the given range of the independent variables (temperature and pressure). The ideal gas law, van der Waals equation, and Beattie-Bridgeman equation all underestimate the specific volume. These two models are inappropriate since they unduly distort the actual thermodynamic behavior. However, the Beattie-Bridgeman equation does represent the thermodynamics of the problem (as does the BWR equation) since the errors drop off nearly symmetrically either side of their peaks. A normal distribution can be fit to both PDFs. The Beattie-

Bridgeman PDF fit has a mean and standard deviation of approximately $-4.3(10)^{-5}$ and $9.22(10)^{-5}$ m^3/kg, respectively. The BWR PDF fit has a mean and standard deviation of approximately $2.1(10)^{-5}$ and $4.87(10)^{-5}$ m^3/kg, respectively. The much narrower PDF of the BWR fit and its smaller standard deviation do indeed confirm that the BWR equation is a better model for determining the specific volume of nitrogen than the Beattie-Bridgeman equation over the given range of the independent variables.

The remainder of this paper documents model uncertainty applied to a monopropellant propulsion system. First, the two models investigated are introduced. Next, the models are analyzed and compared through three recent space missions. The paper ends with concluding remarks.

Modeling of a Monopropellant Propulsion System

A monopropellant blowdown propulsion system provides an excellent engineering example to investigate model uncertainty. This section begins with a brief introduction to monopropellant propulsion systems and their history. Two models of a blowdown monopropellant propulsion system are presented. The first is a simple model. The second is a high-fidelity model. The objective of both models is to ascertain the dry mass and propellant mass of the system. This section provides a detailed discussion of the assumptions and governing equations that define each model.

Monopropellant Propulsion History & Types

As the name suggests, a monopropellant system uses a single liquid, which reacts alone by chemical decomposition. The use of liquid monopropellants instead of bipropellants simplifies the design of the propulsion system by reducing the number of tanks and components required, but a large penalty is paid in performance. Engineers developed the first monopropellant systems (using principally hydrogen peroxide) after World War II mainly as gas generators for turbo machinery. In 1949, the Jet Propulsion Laboratory funded both hydrazine engine and catalyst development, and has been credited with the start of the industry. With the advent of a good room-temperature catalyst, developed by the Shell Oil Company (Shell 405) in 1964, hydrazine soon replaced hydrogen peroxide as the primary monopropellant due to its superior stability and reduced hazard in production, storage, and handling.[8] The first civilian application of catalytic monopropellant hydrazine propulsion in space was with the Applications Technology Satellite-3 (ATS-3) in 1967 and the hydrazine engine and component industry was firmly established soon thereafter.[8] Monopropellant hydrazine propulsion systems have since been used on dozens of spacecraft, both Earth-orbiting and interplanetary. Notable spacecraft such as Viking 1 & 2 Landers (1975); Voyager 1 & 2 (1977); Magellan (1989); and Mars Pathfinder (1996) used monopropellant hydrazine as either the primary or secondary (auxiliary) propulsion system. Work has been ongoing at various levels since the 1950's to discover and develop a higher performing monopropellant than hydrazine. New monopropellants such as Hydroxyl Ammonium Nitrate (HAN) and HAN blends are currently under development.[9]

Simple model

The simple model of a monopropellant propulsion system determines the total loaded propellant and the total dry mass.

Propellant and pressurant

The mass of the spacecraft changes as ΔV maneuvers are performed. The total change in velocity is simply

$$\Delta V_{total} = \sum_{j=1}^{z} \Delta V^{j} \tag{3}$$

These ΔVs along with a requirement on the amount of attitude control propellant are typically the driving requirements placed on the propulsion system. The performance of the engines used for maneuvers is constant, regardless of the engine type, tank pressure, or pressure drop through the propulsion system. The spacecraft mass after the final change in velocity maneuver is

$$m_{s/c_f} = m_{s/c_s} \cdot e^{-\Delta V_{total}/c} \tag{4}$$

The usable propellant for ΔV is

$$m_{prop_\Delta V} = m_{s/c_s} - m_{s/c_f} \tag{5}$$

The total usable propellant is

$$m_{prop_usable} = m_{prop_\Delta V} + m_{prop_ACS} \tag{6}$$

Hold-up/residual propellant occurs in a monopropellant propulsion system due to propellant remaining in the propellant tank and tubing. Hold-up/residual propellant is unusable but must be accounted for and loaded prior

to launch. The total amount of hold-up/residual propellant is

$$m_{prop_hold-up} = \left(\frac{\psi_{hold-up}\%}{1-\psi_{hold-up}\%} \right) \cdot m_{prop_usable} \tag{7}$$

The hold-up/residual percent is typically ~1% for monopropellant blowdown hydrazine systems.[10] The total propellant loaded is therefore

$$m_{total_prop} = m_{prop_usable} + m_{prop_hold-up} \tag{8}$$

It should be noted that the total propellant loaded is almost always greater, sometimes significantly greater than the value obtained by equation (8). The difference between the actual total propellant loaded and the estimated total propellant loaded represents a propellant margin (fill margin). The topic of margins is discussed more fully in a companion paper.[1]

The volume of total propellant is

$$V_{total_prop} = \frac{m_{total_prop}}{\rho_{prop}} \tag{9}$$

The total internal volume of the propellant tanks is a function of the percent ullage:

$$V_{total_tanks} = \frac{V_{total_prop}}{1-\psi_{ullage}\%} \tag{10}$$

The internal volume per propellant tank is

$$V_{per_tank} = \frac{V_{total_tanks}}{q_{tanks}} \tag{11}$$

The density of the pressurant is found from the equation of state:

$$\rho_{pres} = \frac{p_{MEOP}}{R \cdot T} \tag{12}$$

The volume of pressurant is

$$V_{pres} = V_{total_tanks} - V_{total_prop} \tag{13}$$

The mass of the pressurant is

$$m_{pres} = \rho_{pres} \cdot V_{pres} \tag{14}$$

Components

Generic masses are assumed for the various components of the propulsion system. The list of these components and their generic masses are summarized in Table 1.

Table 1 Mass Summary of Typical Propulsion Components

Component	Mass (kg)
Engine (0.9 N or 4.5 N)	0.33
Filter	0.15
Latch valve	0.50
Orifice	0.02
Pressure transducer	0.27
Service valve (gas)	0.23
Service valve (liquid)	0.23
Tubing & fittings	1.50

The total mass of the components is

$$m_{comp} = \sum_{i=1}^{n} m^i \cdot q^i \tag{15}$$

It should be emphasized that Table 1 lists the mass of tubing and fittings as 1.5 kg regardless of the length, diameter, or thickness of the tubing or the number or type of fittings. Furthermore, temperature transducers, although seen on propulsion schematics later in the paper, are not included in the analysis and assumed to be book kept with the thermal control subsystem. The tank mass is calculated via the pressure-volume/weight (*PV/W*) figure of merit:

$$m_{per_tank} = \frac{f_{burst} \cdot p_{MEOP} \cdot V_{per_tank}}{(PV/W) \cdot g_0} \tag{16}$$

The *PV/W* figure of merit includes the mass and volume impact of a propellant management device required to hold propellant over outlet ports of tank in micro gravity. *PV/W* values are typically quoted in units of inches. However, in this paper *PV/W* values are provided in the SI unit of meter for consistency. Hence, the total tank mass in the propulsion system is

$$m_{total_tanks} = m_{per_tank} \cdot q_{tanks} \tag{17}$$

Mass totals

The total "dry" mass of the propulsion system is

$$m_{total_dry} = m_{pres} + m_{comp} + m_{total_tanks} \tag{18}$$

Finally, the total wet mass of the propulsion system is

$$m_{total_wet} = m_{total_dry} + m_{total_prop} \tag{19}$$

It should be noted that the "dry" mass includes the pressurant mass but excludes the hold-up/residual propellant mass. The hold-up/residual is included in the total propellant mass in equation (19).

High-fidelity model

Depending on a given system, elements of the simple model could be inaccurate. The high-fidelity model of a monopropellant propulsion system addresses many of the shortcomings of the simple model. Components are selected from a database; tubing and fitting masses are calculated; propellant, pressurant, and flow properties are determined; pressure drop in the system is calculated, and the various masses are totaled. This process for the high-fidelity model is iterative.

Components

Components for the propulsion system are selected from a database. The types of components are identical to those of the simple model (e.g., the types summarized in Table 1) with the addition of tanks. For each component type there are several relevant parameters that uniquely define a given component from others. Values for the component parameters such as mass, tubing diameter, and pressure drop that populated this database were obtained from company product catalogs, company websites, and/or company technical reports. It should be noted that for tanks, the mass and volume returned by the database include the mass and volume of the propellant management device, respectively. The database for all components is not exhaustive. It primarily includes components that have been used on recent missions. It should be stressed that the components listed in this database do not represent a vendor preference on behalf of the authors. The calculation of the total mass of the components, m_{comp}, remains equation (15) with n equal to the total number of component types including tanks but excluding tubing and fittings.

Engines (thrusters) are also selected from a database. Since the propellant tank pressure in a blowdown propulsion system decreases as maneuvers are performed, the performance of the engines will also decrease. Engine thrust and exhaust velocity are not constant but in fact functions of the inlet pressure to the engine:

$$F = k_1 \cdot p_{inlet} + k_2 \tag{20}$$

$$c = k_3 \cdot p_{inlet}^{\ k_4} \tag{21}$$

The constants k_1, k_2, k_3, and k_4 are unique to the individual engine type (manufacturer model). For example, if the thrust, exhaust velocity, and inlet pressure are in N, m/sec, and Pa, respectively, the coefficients k_1, k_2, k_3, and k_4 for the Aerojet MR-111C are $3.871(10)^{-7}$, $5.075(10)^{-3}$, 1080.1, and $4.796(10)^{-2}$, respectively.[11] Equations (20) and (21) are plotted in Fig. 2 for the Aerojet MR-111C.

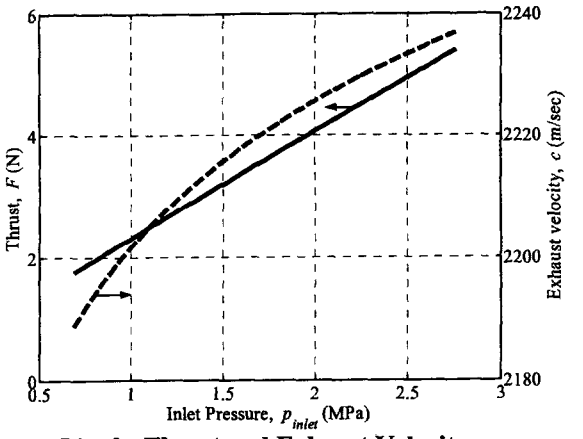

Fig. 2 Thrust and Exhaust Velocity as a Function of Inlet Pressure.

These coefficients are empirically determined and are included in the aforementioned database. It should be noted that the thrust and exhaust velocity are also functions of the propellant temperature. The data plotted in Fig. 2 assumes a propellant temperature of 20 °C (68 °F).

Tubing and fittings

In an analogous manner to the components, tubing is selected from a discrete set of possibilities. Typically in preliminary design, the tubing outer diameter and wall thickness are specified while the length of tubing is estimated. The tubing inner diameter is simply

$$\phi_{inner} = \phi_{outer} - 2 \cdot t_{wall} \tag{22}$$

The cross-sectional area is therefore

$$A_x = \frac{\pi}{4} \cdot \phi_{inner}^{\ 2} \tag{23}$$

The volume of the tubing is

$$V_{tubing} = \frac{\pi}{4} \cdot l_{tubing} \cdot \left(\phi_{outer}{}^2 - \phi_{inner}{}^2\right) \tag{24}$$

Finally, the mass of the tubing is

$$m_{tubing} = V_{tubing} \cdot \rho_{tubing} \tag{25}$$

The mass of fittings is typically estimated and not calculated during preliminary design.

Propellant properties

The density of the propellant is not a constant but in fact a quadratic function of the propellant temperature:

$$\rho_{prop} = \alpha_1 \cdot T^2 + \alpha_2 \cdot T + \alpha_3 \tag{26}$$

The constants α_1, α_2, and α_3 for hydrazine are -4.5284(10)$^{-4}$ kg/K^2-m^3, -0.62668 kg/K-m^3, and 1230.78 kg/m^3, respectively.[8] The viscosity of the propellant is also a function of its temperature:

$$\mu_{prop} = 0.001 \cdot 10^{A \cdot T^2 + B \cdot T + C} \tag{27}$$

The constants A, B, and C for hydrazine are 1.5395(10)$^{-5}$ K^{-2}, -0.015384 K^{-1}, and 3.1788, respectively.[8]

Since the performance of the engines decreases as the propulsion system pressure decreases, the challenge is determining the pressure in the system after each maneuver is performed. The mass of the spacecraft after each maneuver is

$$m_{s/c}^{j} = m_{s/c}^{j-1} \cdot e^{-\Delta V^j / c^j} \tag{28}$$

In this case, c_j is the average engine exhaust velocity during maneuver j and $m_{s/c}^{0}$ is the initial total wet mass of the spacecraft (m_{s/c_s}). This average engine exhaust velocity during maneuver j is determined by integrating equation (21):

$$c^j = \frac{\int_{p_{inlet_f}^j}^{p_{inlet_s}^j} \left[k_3 \cdot \left(p_{inlet}^j\right)^{k_4}\right] dp_{inlet}^j}{p_{inlet_s}^j - p_{inlet_f}^j} \tag{29}$$

Likewise, the average engine thrust is determined by integrating equation (20):

$$F^j = \frac{\int_{p_{inlet_f}^j}^{p_{inlet_s}^j} \left[k_1 \cdot p_{inlet}^j + k_2\right] dp_{inlet}^j}{p_{inlet_s}^j - p_{inlet_f}^j} \tag{30}$$

The propellant expended during a maneuver is a modified version of equation (5):

$$m_{prop_per_\Delta V}^{j} = m_{s/c}^{j} - m_{s/c}^{j-1} \tag{31}$$

where $m_{s/c}^{z}$ is the spacecraft mass after the final change in velocity maneuver. The total usable propellant for ΔV is therefore

$$m_{prop_\Delta V} = \sum_{j=1}^{z} m_{prop_per_\Delta V}^{j} \tag{32}$$

The total usable propellant remains equation (6). Propellant remains in the tank if the tank expulsion efficiency is less than 100%. The volume fraction of hold-up/residual in a propellant tank is

$$f_{vol_hold-up} = 1 - \eta_{tank}\% \tag{33}$$

The mass of hold-up/residual in the tank is

$$m_{hold-up_tank} = \left(\frac{f_{vol_hold-up}}{1 - f_{vol_hold-up}}\right) \cdot m_{prop_usable} \tag{34}$$

Propellant also remains in the tubing since there is no longer a pressure to force the propellant to the thrusters. The volume of hold-up/residual in the tubing is

$$V_{hold-up_tubing} = A_x \cdot l_{tubing} \tag{35}$$

The mass of hold-up/residual in the tubing is therefore

$$m_{hold-up_tubing} = V_{hold-up_tubing} \cdot \rho_{prop}$$

Finally, the total mass of hold-up/residual propellant is

$$m_{prop_hold-up} = m_{hold-up_tank} + m_{hold-up_tubing} \tag{36}$$

The total propellant loaded and volume of total propellant remain equations (8) and (9), respectively. Equation (9) can also be modified to determine the volume of propellant expended during a maneuver:

$$V^{j}_{prop_per_\Delta V} = \frac{m^{j}_{prop_per_\Delta V}}{\rho_{prop}} \tag{37}$$

Since the database provides the internal volume per tank, equation (11) can be used to determine the total internal volume available (V_{total_tanks}).

Pressurant properties

The equation of state provided in equation (12) is based on the ideal gas law. It becomes increasingly inaccurate at high pressures. A more accurate method to determine the density of the pressurant is via the Beattie-Bridgeman equation of state:

$$p = \frac{\Re \cdot T}{v^2}\left(1 - \frac{c_0}{v \cdot T^3}\right)\left(v + B_0 - \frac{B_0 \cdot b}{v}\right) - \frac{v \cdot A_0 - A_0 \cdot a}{v^3} \tag{38}$$

The constants A_0, a, B_0, b, and c_0 for helium are 2188.62 kg-m^5/kmol^2-sec^2, 0.05984 m^3/kmol, 0.014 m^3/kmol, 0 m^3/kmol, and 40 m^3-K^3/kmol, respectively.[12] Coefficients for other gases are found in reference 12. Equation (38) can be rewritten as a quartic function of the specific volume:

$$p \cdot v^4 - \Re \cdot T \cdot v^3 - \beta \cdot v^2 - \gamma \cdot v - \delta = 0 \tag{39}$$

The "constants" β, γ, and δ in equation (39) are functions of the temperature and defined:

$$\beta = \Re \cdot T \cdot B_0 - A_0 - \frac{\Re \cdot c_0}{T^2}$$

$$\gamma = -\Re \cdot T \cdot B_0 \cdot b + a \cdot A_0 - \frac{\Re \cdot B_0 \cdot c_0}{T^2}$$

$$\delta = \frac{\Re \cdot B_0 \cdot b \cdot c_0}{T^2}$$

For a given pressure and temperature, equation (39) yields four roots: two imaginary numbers, a negative real number, and a positive real number. The density of the pressurant in kg/m^3 is therefore

$$\rho_{pres} = \frac{M}{v^*} \tag{40}$$

The molecular mass of helium is 4.003 kg/kmol and v^* is the positive real root of equation (39) (the only root that makes physical sense). Using equations (39) and (40) with the maximum expected operating pressure and the temperature yield the initial pressurant density (ρ^0_{pres}). The initial volume of pressurant (V^0_{pres}) remains equation (13). However, a modified version of equation (13) can be used to determine the volume of pressurant in the propellant tanks after each maneuver:

$$V^{j}_{pres} = V_{total_tanks} - V_{total_prop} + \sum_{j=1}^{z} V^{j}_{prop_per_\Delta V} \tag{41}$$

The mass of pressurant is determined via equation (14).

Flow properties

The mass flow rate is a function of the number of engines operating, the thrust, and the exhaust velocity:

$$\dot{m} = \frac{q_{engines_op} \cdot F}{c} \tag{42}$$

The flow speed of the propellant is

$$v_{prop} = \frac{\dot{m}}{\rho_{prop} \cdot A_x} \tag{43}$$

The Reynolds number (based on the diameter) of the fully-developed flow through the tubing is

$$Re_{\phi} = \frac{\rho_{prop} \cdot v_{prop} \cdot \phi_{inner}}{\mu_{prop}} \tag{44}$$

For laminar flow where Re_{ϕ} remains below ~2000, the friction factor is simply[13]

$$f = \frac{64}{Re_{\phi}} \quad (laminar) \tag{45}$$

Since the thrust in equation (42) changes as maneuvers are performed, the mass flow rate, flow speed, Reynolds number, and friction factor also change.

Pressure drop

The performance of a blowdown propulsion system is reduced due to a combination of decreasing tank pressure and a pressure drop that occurs between the propellant tank(s) and the engine (thruster) combustion chamber(s). The pressure in tank after a maneuver:

$$p_{tank}^{j} = \frac{V_{pres}^{0} \cdot p_{MEOP}}{V_{pres}^{j}} \tag{46}$$

A pressure drop occurs in components, tubing, and tubing bends that the propellant must pass through. The pressure drop through a given component is a function of the propellant mass flow rate:

$$\Delta p^{i} = \kappa_1^{i} \cdot \dot{m} + \kappa_2^{i} \tag{47}$$

The constants κ_1 and κ_2 unique to the individual component type (such as filter, latch valve, or thruster valve). These coefficients are empirically determined and are included in the aforementioned database. Hence, the pressure drop through all components in the propulsion system is simply

$$\Delta p_{comp} = \sum_{i=1}^{n} \Delta p_{comp}^{i} \cdot q_i \tag{48}$$

The pressure drop through the propulsion tubing of a fully-developed flow is[14]

$$\Delta p_{tubing} = \frac{f \cdot \rho_{prop} \cdot v_{prop}^{2} \cdot l_{tubing}}{2 \cdot \phi_{inner}} \tag{49}$$

The pressure drop through the bends in the propulsion tubing is[14]

$$\Delta p_{bends} = \frac{1}{2} q_{bends} \cdot f \cdot \rho_{prop} \cdot v_{prop}^{2} \cdot (l / \phi)_{equiv} \tag{50}$$

Propulsion systems typically use several 90° standard elbows in routing the tubing between the tank(s) and the engine(s). The total pressure drop through the propulsion system is therefore

$$\Delta p_{total} = \Delta p_{comp} + \Delta p_{tubing} + \Delta p_{bends} \tag{51}$$

Hence, the inlet pressure to an engine is

$$p_{inlet} = p_{tank} - \Delta p_{total} \tag{52}$$

Since the flow properties are changing as maneuvers are performed, the total pressure drop also changes.

Mass totals

The previous seven subsections indicate the need to iteratively solve for the inlet pressure to the engines for each maneuver (equations (20) and (52) and many in between). Once this iteration converges for the total propellant mass, the total dry mass of the propulsion system is

$$m_{total_dry} = m_{comp} + m_{tubing} + m_{fittings} + m_{pressurant} \tag{53}$$

The total wet mass of the propulsion system remains equation (19). It should be noted that the high fidelity model assumes that the spacecraft remains at a constant prescribed temperature and that all burns are isothermal. This assumption may be false if the thermal inertia of the propulsion system is small and long ΔV maneuvers are required. The model also does not include an estimate for the amount of pressurant that dissolves into the propellant. Certain pressurants, such as nitrogen, dissolve significantly into the propellant resulting in a pressure drop in the propellant tanks of a percent or more. Additional routines could be added to include these phenomena to further improve the model fidelity but were not done due to time and resource constraints.

Model Analysis and Comparison

The two models previously described were applied to three recent space missions that used a monopropellant hydrazine blowdown propulsion system: Deep Space 1 (DS1), Mars Exploration Rover (MER) project, and the Solar Terrestrial Relations Observatory (STEREO). Although the three missions share a common propulsion system design, each has different requirements. Moreover, each of the three missions faces different aleatory uncertainties. Aleatory uncertainty is inherent variation associated with a physical system or environment under consideration and is described in more detail in a companion paper.[1] It should be noted that the three missions are at very different phases in the design life: DS1 was successfully operated from 1998 to 2001, MER is in final preparation for launch, while STEREO is still in detailed design. This section provides a brief overview, major propulsion system requirements, resulting propulsion system design, and modeling values assumed in the analysis of each mission. The section ends with a comparison of two models.

DS1

DS1 launched from Cape Canaveral on October 24, 1998 aboard a single Boeing Delta II 7326 launch vehicle. DS1 was the first mission of NASA's New Millennium program, chartered to validate in space new technologies important for future space and Earth

science programs. During its primary mission, 12 of these advanced technologies (solar electric propulsion; solar concentrator arrays; autonomous on-board navigation and other autonomous systems; several telecommunications and microelectronics devices; and two low-mass integrated science instrument packages) were tested.[15] DS1 was 3-axis stabilized spacecraft with a launch mass – including propellant – of 474 kg (1,045 lbm).[16] In the extended mission, it encountered comet Borrelly and returned unique images and other science data. During its hyperextended mission, it conducted further technology tests. The spacecraft was retired on December 18, 2001.[17]

Propulsion requirements and design

DS1 was the first deep-space spacecraft to use electric propulsion as the primary propulsion of the spacecraft. The monopropellant blowdown system was used as a secondary propulsion system. It had four major requirements: small ΔV maneuvers; one-axis spacecraft control when the ion engine was engaged; three-axis spacecraft control when the ion engine was not engaged; and turn and point capability. These requirements are summarized in Table 2.[18]
DS1 used helium as the pressurant. The propulsion schematic is shown in Fig. 3. The propulsion subsystem mass list is provided in Table 3.

Model Values Assumed

The aleatory uncertainties assumed in the DS1 analysis for the simple and high-fidelity models are provided in Table 4 and Table 5, respectively. The simple model

Table 2 DS1 Propellant Budget

Activity	Mass (kg)
Launch vehicle injection errors	3.7
Reorient for launch vehicle injection burns	0.2
Optical navigation reorientations	0.2
Downlink reorientations	0.1
Attitude control & disturbance torques	
Optical navigation deadbanding	5.7
Downlink deadbanding	0.1
Solar radiation center of pressure/ gravity offset	1.0
Cruise deadbanding	6.9
Encounter maneuvers	6.9
Encounter reorientations	~0.0
TOTAL	24.8

assumes a burst factor of 4. The masses of individual components in the simple analysis were assumed to have a normal distribution with a mean equal to the value listed in the second column of Table 1 and a standard deviation equal to a tenth of the mean. The masses of individual components in the high-fidelity model were obtained from a database as described earlier in the paper. The high-fidelity model assumes a custom distribution for the number of tubing bends since the exact number was not known: 13 (10%), 14 (20%), 15 (50%), and 16 (20%). The uncertainty in the 24.8 kg propellant total of Table 2 was estimated at +/- 2.4 kg (20%) yielding a total propellant requirement of 27.1 kg. Since little formal documentation was found to further understand these requirements, the total

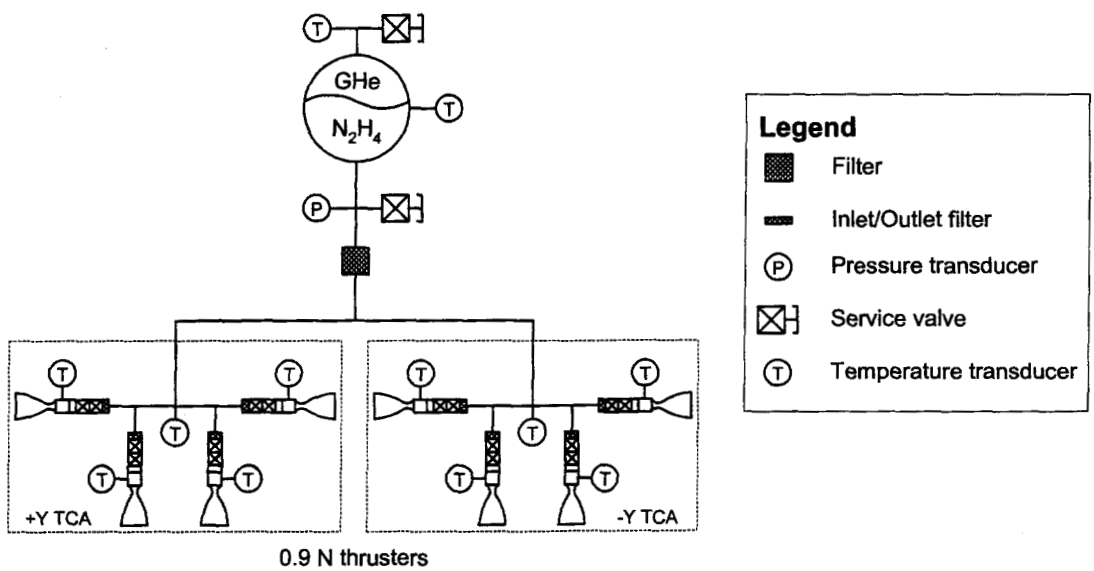

Fig. 3 DS1 Propulsion System.

Table 3 DS1 Propulsion Mass List

Component Type	Manufacturer/Model Number	Qty	Unit Mass (kg)	Total Mass (kg)
Tank (propellant)	PSI/80275-1	1	5.71	5.71
Filter	VACCO/203950-4	1	0.13	0.13
Pressure transducer	Gulton-Statham/PA4089-450	1	0.26	0.26
Service valve (gas)	VACCO/V1E10483-01 FDV	1	0.11	0.11
Service valve (liq)	VACCO/V1E10483-02 FDV	1	0.10	0.10
Engines (0.9 N)	Aerojet/MR-103C	8	0.28	2.24
Tubing	¼" 0.028" wall tubing			0.81
Fittings				0.19
Pressurant	GHe			0.01
TOTAL DRY				9.6
Propellant loaded	N_2H_4			31.1
TOTAL WET				40.7

Table 4 Uncertainties Assumed in the Simple Model

	DS1		MER		STEREO	
Variable	Dist.	Parameters	Dist.	Parameters	Dist.	Parameters
c	Normal	2108.4, 49.0	Normal	2108.4, 49.0	Normal	2108.4, 49.0
m_{prop_ACS}	Normal	24.8, 1.0	Normal	3.85, 0.0812	Normal	2, 0.2
m_{s/c_s}	Normal	474, 4.74	Normal	1072, 10.72	Normal	620, 62
P_{MEOP} [a]	Normal	2551, 25	Normal	2944, 28	Normal	2206, 22
PV/W	Normal	7620, 762	Normal	7620, 762	Normal	7620, 762
T_{prop}	Normal	293, 2	Normal	293, 2	Normal	293, 2
ΔV_{total}	Custom	0, 1	Lognorm	3.09, 0.39	Lognorm	4.76, 0.10
ρ_{prop}	Normal	1004, 10.04	Normal	1004, 10.04	Normal	1004, 10.04
$\psi_{prop_hold\text{-}up}$	Normal	1.0, 0.1	Normal	1.0, 0.1	Normal	1.0, 0.1
ψ_{ullage}	Normal	20, 2	Normal	31, 3	Normal	34.4, 3.44

[a]kPa

attitude control propellant mass was assumed to be equal to the total propellant requirement of Table 2. This requirement was modeled in both the simple and high-fidelity models as a normal distribution with a mean and standard deviation of 24.8 and 1.0 kg, respectively. No ΔV maneuvers were explicitly assumed. The mass list (Table 3) and the schematic (Fig. 3) provide both the type and quantities of components assumed in the analysis.

MER

NASA will launch the next generation of robotic explorers to the planet Mars in the summer of 2003. Two identical rovers will be delivered to the surface of Mars to remotely conduct geologic and atmospheric investigations. The two self-sufficient mobile science laboratories will be able to traverse large distances during their three month surface missions while performing in-situ analysis of a number of rock and soil targets which may hold clues to past water activity.

The MER project will conduct fundamentally new observations of Mars geology, including the fist micro-scale study of rock samples, as well as a detailed study of surface environments for the purpose of calibrating and validating orbital remote sensing data.[19] The MER flight system consists of four major components: an Earth-Mars cruise stage; an atmospheric entry, descent, and landing system or aeroshell (consisting of a heatshield and backshell); a lander; and a mobile science rover with an integrated instrument package. During the interplanetary transfer to Mars, the cruise stage will provide most of the traditional spacecraft subsystem functionality (such as propulsion, power, communications, thermal, and attitude control).[19]

Table 5 Uncertainties Assumed in the High-Fidelity Model

	DS1		MER		STEREO	
Variable	Dist.	Parameters	Dist.	Parameters	Dist.	Parameters
l_{tubing}	Normal	8.2, 0.082	Normal	7.62, 0.0762	Normal	12.19, 0.12
$(l/\phi)_{equiv}$	Normal	30, 3	Normal	30, 3	Normal	30, 3
$m_{fittings}$	Normal	0.2, 0.02	Normal	0.1, 0.05	Normal	0.2, 0.05
m_{prop_ACS}	Normal	24.8, 1.0	Normal	3.85, 0.0812	Normal	2, 0.2
m_{s/c_s}	Normal	474, 4.74	Normal	1072, 10.72	Normal	620, 62
P_{MEOP} [b]	Normal	2551, 25	Normal	2944, 28	Normal	2206, 22
T_{prop}	Normal	293, 2	Normal	298, 2	Normal	293, 2
η_{tank}	Beta	330, 1	Beta	330, 1	Beta	200, 1
ρ_{tubing}	Normal	7750, 7.75	Normal	7750, 7.75	Normal	7750, 7.75
ψ_{ullage}	Normal	20, 2	Normal	31, 3	Normal	34.4, 3.44

[b] kPa

Propulsion requirements and design

The MER propulsion system is used for spin-rate control, attitude control, and five trajectory correction maneuvers (TCMs). A detailed breakout of the attitude control requirement and the PDFs of the ideal ΔVs for the five TCMs are provided in a companion paper.[1] The MER propulsion system is equipped with two spherical titanium propellant tanks and two diametrically opposed thruster clusters, each containing four 4.5 N thrusters. MER uses helium as the pressurant. The propulsion schematic is shown in Fig. 4. The propulsion subsystem mass list is provided in Table 6.

Model Values Assumed

The aleatory uncertainties assumed in the MER analysis for the simple and high-fidelity models are provided in Table 4 and Table 5, respectively. The simple model assumes a burst factor of 4. The mass of individual components in the simple and high-fidelity models follows the description provided with the DS1 example. The high-fidelity model assumes a custom distribution for the number of tubing bends since the exact number was not known: 4 (20%), 5 (30%), 6 (30%), 7 (10%), and 8 (10%). The PDFs of the ideal ΔVs for the five TCMs previously described are numerically convolved to obtain a total ideal ΔV distribution. The numerical convolution consists of four steps: taking the Fourier transform of each distribution, multiplying these five

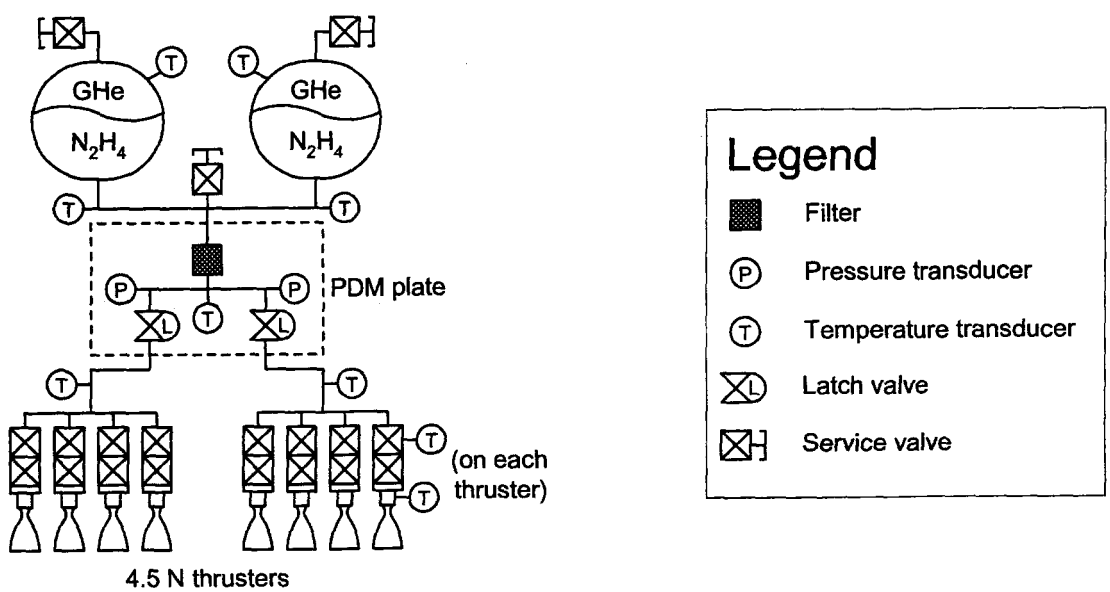

Fig. 4 MER Propulsion System.

Table 6 MER Propulsion Mass List

Component Type	Manufacturer/ Model Number	Qty	Unit Mass (kg)	Total Mass (kg)
Tank (propellant)	PSI/80275-1	2	5.76	11.52
Filter	VACCO/F0D10672-01	1	0.13	0.13
Latch valve	VACCO/V1E10864-A	2	0.34	0.68
Pressure transducer	Tavis Corp./30001-0500	2	0.22	0.44
Service valve (gas)	VACCO/V1E10483-01 FDV	2	0.11	0.22
Service valve (liq)	VACCO/V1E10483-02 FDV	1	0.11	0.11
Engines (4.5 N)	Aerojet/MR-111C	8	0.33	2.64
Tubing	¼" 0.035" wall tubing			0.20
Fittings				0.15
Pressurant	GHe			0.3
TOTAL DRY				16.4
Propellant loaded	N_2H_4			47.0
TOTAL WET				63.4

transforms together, inverting the result, and finally normalizing the resulting distribution. This numerical convolution is shown in Fig. 5. Also shown in Fig. 5 is a lognormal distribution that was fit to this convolved distribution. This distribution (with parameters 3.09 and 0.39) was used for the total ideal ΔV in the simple model. The high fidelity-model assumed the actual five individual PDFs. The ideal ΔVs in both models were multiplied by 1.5 to obtain the implemented (mass equivalent) ΔVs (e.g, the ΔV that a propulsion system needs to provide). As seen in Table 4 and Table 5, the attitude control propellant requirement was modeled in both the simple and high-fidelity models as a normal distribution with a mean and standard deviation of 3.85 and 0.0812 kg, respectively.

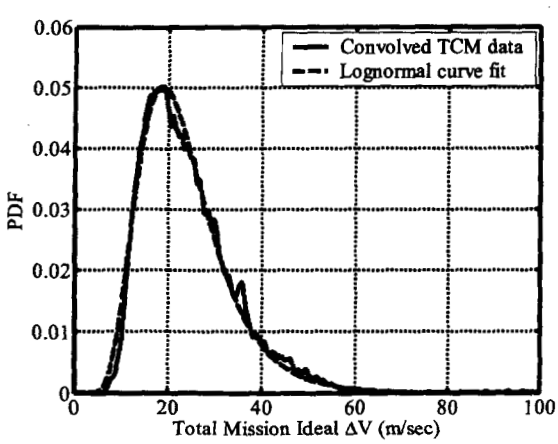

Fig. 5 Ideal ΔV Totals for MER.

STEREO Overview

STEREO is the third mission in NASA's Solar Terrestrial Probes program, scheduled to launch in November 2005 aboard a single Boeing Delta II 7925 launch vehicle. This two-year mission will employ two nearly identical space-based observatories to provide the first-ever, 3-D stereoscopic images to study the nature of coronal mass ejections (CMEs). The STEREO mission will provide a new perspective on solar eruptions by imaging CMEs and background events from two observatories simultaneously. To obtain 3-D images of the sun, the twin observatories must be placed into an orbit where they will be offset from one another. One observatory will be placed "ahead" of the Earth in its orbit and the other, "behind" using a series of lunar swingbys. The two STEREO observatories will be nearly identical with selective redundancy. Each consists of two solar-powered, 3-axis-stabilized spacecraft, each with a launch mass – including propellant – of approximately 620 kg (1,364 lbm). The spacecraft bus will be built by the Johns Hopkins University Applied Physics Laboratory (APL), in Laurel, Md., with NASA Goddard Space Flight Center procuring the instruments. Each observatory and its instruments will be integrated at APL. The twin STEREO observatories will be launched together. Within the third stage of the Delta II, one observatory will sit atop the other via a specially designed payload adapter.[20]

Propulsion requirements and design

The STEREO propulsion system is used for attitude control and nine trajectory correction maneuvers (TCMs). The attitude control requirement was

estimated at 2.5 kg and the PDFs of the implemented ΔVs for the first four TCMs are provided in Fig. 6. The final five TCMs were deterministic maneuvers with implemented ΔVs of 4, 1, 8, 2, and 20 m/sec. The STEREO propulsion system is equipped with two titanium propellant tanks and eight 4.5 N thrusters. STEREO uses nitrogen as the pressurant. The propulsion system schematic is shown in Fig. 7. The propulsion subsystem mass list is provided in Table 7.

Model Values Assumed

The aleatory uncertainties assumed in the STEREO analysis for the simple and high-fidelity models are provided in Table 4 and Table 5, respectively. The simple model assumes a burst factor of 2. The mass of individual components in the simple and high-fidelity models follows the description provided with the DS1 example. The high-fidelity model assumes a custom distribution for the number of tubing bends since the exact number was not known: 7 (25%), 8 (25%), 9 (25%), and 8 (25%). The PDFs of the implemented ΔVs for the four TCMs previously described are numerically convolved to obtain a total implemented ΔV distribution. This numerical convolution is shown in Fig. 8. Also shown in Fig. 8 is a lognormal distribution that was fit to this convolved distribution. This distribution (with parameters 4.76 and 0.10) was used for the total implemented ΔV in the simple model. The high fidelity-model assumed the actual four individual PDFs. Both models assumed the final five TCM deterministic ΔVs of 4, 1, 8, 2, and 20 m/sec

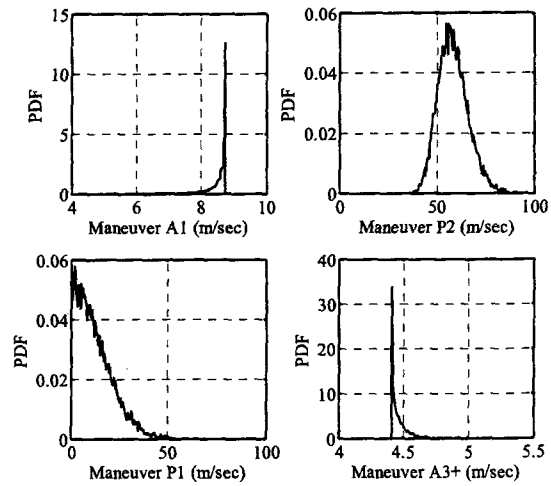

Fig. 6 STEREO Implemented Statistical TCM ΔVs.

described earlier. As seen in Table 4 and Table 5, the attitude control propellant requirement was modeled in both the simple and high-fidelity models as a normal distribution with a mean and standard deviation of 2.0 and 0.2 kg, respectively. This distribution yields a 99% value of 2.5 kg (the actual STEREO attitude control propellant requirement).

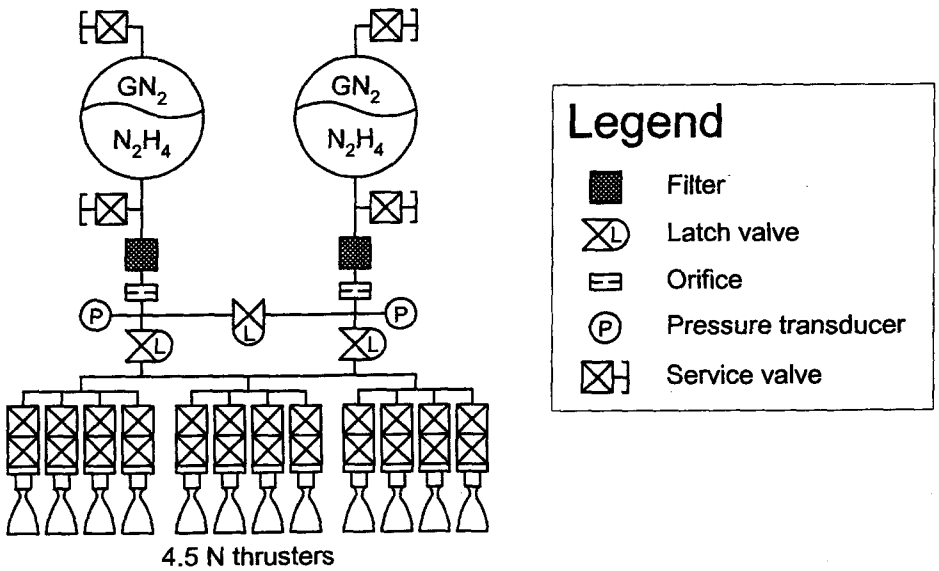

Fig. 7 STEREO Propulsion System.

Table 7 STEREO Propulsion Mass List

Component Type	Manufacturer/ Model Number	Qty	Unit Mass (kg)	Total Mass (kg)
Tank (propellant)	PSI/80455-1	2	5.64	11.28
Filter	VACCO/F1D10767-01	2	0.16	0.32
Latch valve	VACCO/V1E10747-01	2	0.34	0.68
Pressure transducer	Paine/213-76-260-02	2	0.22	0.44
Service valve (gas)	VACCO/V1E10572-02	2	0.11	0.22
Service valve (liq)	VACCO/V1E10572-01	2	0.11	0.22
Engines (4.5 N)	Aerojet/MR-111C	8	0.43	3.44
Orifice		2	0.02	0.04
Tubing & fittings	¼" 0.028" wall tubing			3.12
Pressurant	GN_2			0.85
TOTAL DRY				20.6
Propellant loaded	N_2H_4			60.5
TOTAL WET				81.1

Model Comparison

A Monte Carlo simulation using 5000 samples was performed for both the simple and high-fidelity models for all three missions. The two primary parameters of interest that the simulation yields are the total propulsion dry mass and the total propellant required. The actual values of the total propulsion dry mass and the total propellant required from Table 3, Table 6, and Table 7 are then subtracted from the simulation values to obtain an approximation error. The results for the propulsion system dry mass are shown in Fig. 9. It is apparent from Fig. 9 that the high-fidelity model provides a significant benefit over the simple model in estimating the propulsion system dry mass. With the exception of DS1, the means of the high-fidelity model are closer to zero. In all three cases the variance of the high-fidelity model is significantly less than that of the simple model. It should be noted that dry mass comparisons are often difficult for monopropellant systems since many mass lists often includes components that are might be book kept with other subsystems such as structures or thermal control. The distinction between the various subsystems for certain "propulsion" components is difficult. Every effort was made to be consistent in this respect as seen by the mass lists.

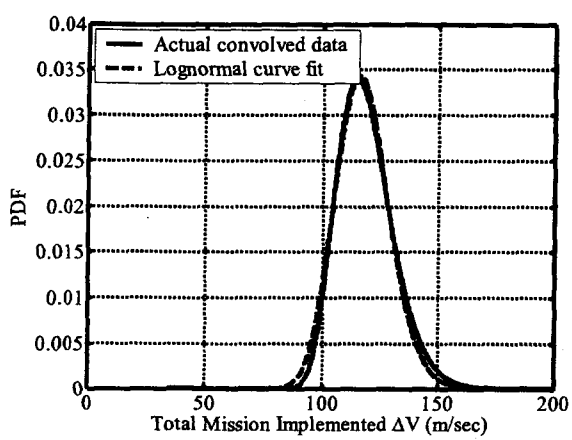

Fig. 8 Implemented ΔV Totals for STEREO.

The results for the total propellant required are shown in Fig. 10. It is apparent from Fig. 10 that in the case of total propellant required, the high-fidelity and simple models are comparable. The distributions are nearly coincident indicating it is not worth the added computational expense to use the high-fidelity model to estimate total propellant required. Fig. 10 also seems to indicate that both MER and STEREO significantly underestimate the total propellant required. This is not really true. Mass comparisons are even more difficult for the propellant required than the dry mass since there is no true (measured) value for propellant required until the mission is completed and flight telemetry analyzed. A decision is made during conceptual design as to what statistical value to assume for the various TCMs. Typically a value of 99% is chosen and the corresponding propulsion analysis is completed based on that statistical value. Actual missions use less, sometimes a lot less than the propellant required. Furthermore, propellant tanks are often filled to capacity at the launch pad, exceeding the calculated propellant required. In short, the underestimating of propellant required for both models should not be taken too seriously since a full statistical analysis of the TCMs was completed and not a deterministic value based on a 99% confidence number.

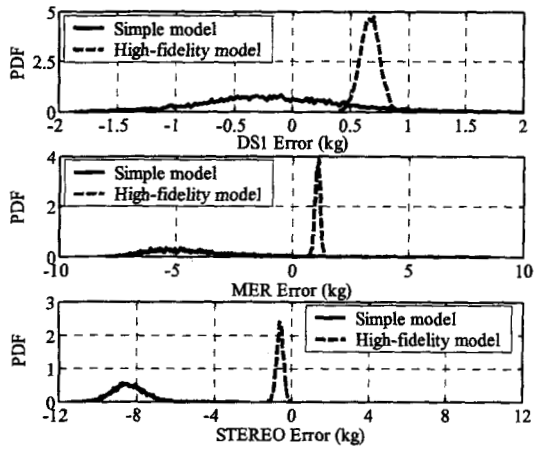

Fig. 9 Dry Mass Comparison of Results

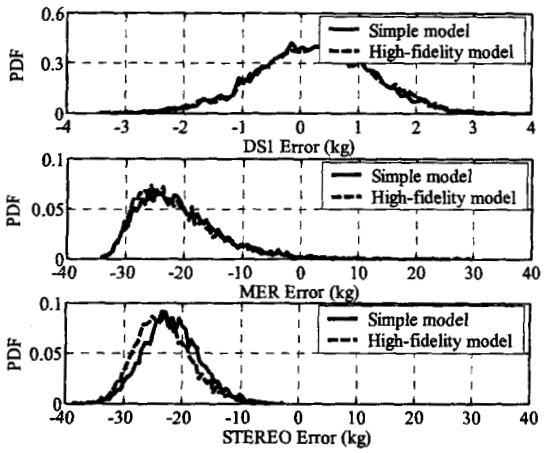

Fig. 10 Propellant Mass Comparison of Results

Conclusions

This paper addressed an important question that arises in conceptual design: how accurate do models have to be to be useful? That is to say, when do other uncertainties in a higher fidelity model counteract its low model uncertainty when compared to a lower fidelity model faced with these same uncertainties? As an example, the model uncertainty in the conceptual design of a monopropellant blowdown hydrazine propulsion system was investigated. For the example investigated, the high-fidelity model provides a significant benefit over the simple model in estimating the propulsion system dry mass. However, for estimating the propellant mass required, the high-fidelity and simple models are comparable. The common belief that the higher the fidelity of the physics-based model, the better the results will always be was found to be false. Furthermore, constraints on human resources for model development and computer resources for simulation can obviate the usefulness of a higher-fidelity model regardless of its comparative benefit to that of a simple model.

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