RELATIVE TIME AND FREQUENCY ALIGNMENT BETWEEN TWO LOW EARTH ORBITERS, GRACE

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Abstract: The two GRACE (Gravity Recovery and Climate Experiment) spacecraft were launched into a near polar circular orbit around the earth in March of 2002. The two spacecraft serve as test masses to measure the Earth's gravitational field. Both spacecraft carry ultra-stable oscillators (USO) with an Allan Variance of about $10^{-8}$. The USO's drive both the microwave links and GPS receivers. To cancel out long term errors on the USO's a linear combination of the 1-way microwave links is used (dual-one-way). In order to form the dual-one-way measurement and cancel our long term USO error, time must be synchronized between the two spacecraft to about 150 picoseconds. This synchronization is accomplished using the GPS data. For each spacecraft, the GPS data are used to solve for the orbital positions and the difference between the onboard clock and a ground reference clock every 5 minutes. The relative clock is determined by the difference of these two solutions.

Validation of the relative clock accuracy includes the difference of the clock solutions on the 6 hour overlaps, typically less than the 150 picosecond goal and unique combination of the one-way microwave links that allows independent comparison of the GPS determine relative frequency of the USO's to a measurement made by the microwave link.

I. Introduction

Two GRACE satellites were launched on board a single ROCKOT launch vehicle on March 17, 2002, from Plesetsk (62.7° N, 40.3° E), Russia. They are in a near polar orbit at about 500 km in altitude separated by about 200 km. Its primary mission is to recover both the static and time varying nature of the earth's mass distribution [Watkins et al., 1995; Watkins et al., 2000].

Fig. 1 shows the main components of the GRACE mission system. There are two GRACE spacecraft, referred to as GRACEA and GRACEB. Each spacecraft carries a codeless dual-frequency GPS receiver, a K/Ka band ranging instrument (KBR) [Dunn et al., 2002], an ultra-stable oscillator (USO), an accelerometer and two star trackers [Jorgensen et al., 1997]. The accelerometer is used to remove the non-gravitational effects from the spacecraft positions. K/Ka band measurements aided by GPS measurements of the residual effects are used to determine the gravitational forces due to the earth's mass distribution.
the light time between the two spacecraft) clock
errors cancel and first order ionosphere effects
are eliminated. The combination that eliminates
long-term clock error is referred to as dual-one-
way range [MacArthur et al., 1985, Thomas,
1999] and can be explain briefly as follows, let

$$
\phi_A = C_A(t_r) - C_B(t_t) = R + C'_A(t_r) - C'_B(t_t)
$$

be the measurement of phase at spacecraft A,
which is the difference of the clock(USO) at
GRACEA at receive time and the clock at
GRACEB at transmit time including any clock
errors (and relativistic effects). This clock
difference can further be expanded into the
actual range, $R$, and a difference of clock error
terms represented by the superscript e-terms
above. Similarly for the phase measurement at
GRACEB:

$$
\phi_B = C_B(t_r) - C_A(t_t) = R + C'_B(t_r) - C'_A(t_t)
$$

Adding these two equations together, we see that
if the clock errors were constant over the light
time (difference between transmit and receive
times) the errors cancel in the sum.

$$
\phi_A + \phi_B = 2R + C'_A(t_r) - C'_A(t_t) + C'_B(t_r) - C'_B(t_t)
$$

In the above argument, we are assuming near
simultaneous sampling of the phase at both
GRACEA and GRACEB. To achieve this near
simultaneous sampling, we use GPS to align
time between the two spacecraft to better than
0.15 nano-seconds (ns). Since the USO drives
both the GPS receiver and the KBR instrument,
precision orbit determination (POD) can be
performed to determine the absolute time tag of
KBR measurements and the spacecraft position
[Bertiger et al., 2002] for details of this solution process.
Here we concentrate on the relative clock error
between the two GRACE spacecraft and the
validation of its accuracy.

In the GPS solution process no relativistic model
of the clock behavior is included. Thus the
solution for any deviation from a fixed frequency
will appear in the clock solution. The solutions
for the spacecraft position and clock are
performed with data arcs that are 30 hours in
length centered on noon of each day, thus there
are six hours of common data from one arc to the
next. In these six-hour overlaps, the difference in
solutions gives a measure of the solution
precision and accuracy. The measure of accuracy
is inferred from a long history of position
overlaps compared to independent measures of
positional accuracy such as satellite laser
ranging.

The on-board USO is used to generate the local
model of the phase of the GPS signal and 1-Hz
samples of this phase measurement are
decimated to 5-minute samples and processed in
the orbit determination and clock determination
process. The GPS code measurements of absolute range are sampled every 5-minutes and smoothed against the phase measurements. USO stability is given in terms of Allan Variance, with the measured values shown in Table 1. The range of values cover a range of temperature and pressure regimes.

Table 1, USO Stability on GRACE, Precursor Measurement

<table>
<thead>
<tr>
<th>Tau (sec)</th>
<th>GRACE A (X10^{-13})</th>
<th>GRACE B (X10^{-13})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>1.1-1.3</td>
<td>1.3-1.6</td>
</tr>
<tr>
<td>2</td>
<td>1.3-1.4</td>
<td>1.7-2</td>
</tr>
<tr>
<td>10</td>
<td>1.2-1.3</td>
<td>1.3-1.8</td>
</tr>
<tr>
<td>100</td>
<td>1.2-1.4</td>
<td>1.3-1.8</td>
</tr>
<tr>
<td>1000</td>
<td>1.1-3.2</td>
<td>1.8-3.5</td>
</tr>
</tbody>
</table>

Figures 2 and 3 show a representative sample of the clock errors (really, both error and relativistic effects) after the removal of a linear trend. Notice the distinct difference in the two plots. The periodic nature of the plot for GRACE B is consistent with the periodic effect from general relativity with the earth as a point mass and the clock in an eccentric orbit about that mass. The change in amplitude, $2\sqrt{GM*\mu}e/c$, where $a$ is the semi-major axis, $e$ is the eccentricity of the orbit, GM is the gravitational constant times the mass of the Earth, and $c$ is the speed of light, is consistent with the changes in eccentricity. For GRACE A, the periodic relativistic effect is dwarfed by the other errors in the clock. Of course, as noted above, the GRACE mission is only dependent on very short-term clock stability.

Since the GPS clock solutions are performed on 30-hour data arcs centered on noon of each day, we can look at the difference in the clock solutions during the 6-hour overlap period from 21:00 on one day to 03:00 on the next day. This difference is a measure of the clock solution precision and similar tests with position overlaps indicate that it is a close measure of accuracy [Bertiger et al., 2002]. To eliminate edge effects in the solution process we delete an hour on each
side of the overlapping period and look at differences from 22:00 to 02:00 on the next day. For relative clock precision and accuracy, we look at the difference of GRACE A clock in the overlap – the difference in GRACE B clock in the overlap. This difference removes any effects of the reference clock, since the reference clock is common to both GRACE spacecraft but may switch from 30-hour arc to 30-hour arc. Fig. 4 shows an histogram of the RMS of the overlap differences for almost one year, from April 1, 2002 to March 16, 2003. The median RMS overlap is 68 ps, well within the bounds of the 150 ps mission requirements.

Figure 4, Histogram of Clock Overlaps, Median: 68 ps

IV Clock Rate KBR Compared to GPS Measurement

As a final validation of the GPS clock solution, we can use the KBR data itself as an independent measure of the clock rate (frequency) since both data streams are driven by the same set of USO’s. A few equations are necessary to explain the comparison and the relevant combination of KBR data.

Let

\[ \phi_2(t) = \text{K or Ka phase up to a bias measured at spacecraft 1} \] (we switch to using 1 and 2 for A and B in the equations)

at true time \( t \)

\[ (1) \quad \phi_2^1(t) = \phi^1(t) - \phi^2(t - \tau_2^1) + I \]

where, \( \phi^1(t) \) is the phase generated at true receive time \( t \) at spacecraft 1 and \( \tau_2^1 \) is the travel time from spacecraft 2 to 1 including light time and other delays, \( I \) is an ionosphere induced phase shift, and \( \phi^2(t - \tau_2^1) \) is the phase generated at spacecraft 2 at true time \( t - \tau_2^1 \).

Local time, at true-time \( t \), at the receiver, is defined by \( \tilde{t} = \frac{\phi^1(t)}{f_i} \) where \( f_i \) is the assumed nominal rf frequency for spacecraft \( i \), \( i = 1, 2 \) at K or KA band. An arbitrary constant, synchronizing the epochs of true time and local time is omitted.

The corresponding equation for the phase measured at spacecraft 2 is obtained by just interchanging the numbers 1 and 2 in equation for the measured phase at spacecraft 1.

Taking the difference of the measured phases at spacecrafts 1 and 2 and dividing by the sum of the frequencies at the two spacecraft we obtain after dropping the ionospheric phase shift for now:

\[ (2) \quad \frac{\phi_2^1(t) - \phi_2^2(t)}{f_1 + f_2} = \frac{\tilde{t}^1(t)}{1 + \frac{f_2}{f_1}} - \frac{\tilde{t}^2(t)}{1 + \frac{f_1}{f_2}} = \frac{\tilde{t}^1(t - \tau_2^1)}{1 + \frac{f_2}{f_1}} - \frac{\tilde{t}^2(t - \tau_1^1)}{1 + \frac{f_1}{f_2}} \]

Writing local time, \( \tilde{t} \), as \( t + \epsilon(t) \), and differentiating equation (2) with respect to true time \( t \),
Let $Af = f_2 - f_1$ be the difference in the rf frequencies at the two spacecraft, about 500 KHz. Substituting $2 + \Delta f/f_1$ for $1 + f_2/f_1$ and similarly for the terms with 1 and 2 reversed after expanding to first order in $\Delta f/f$ (recall the rf frequencies are either around 24 or 32 Ghz)

\[
\frac{\phi(t) - \phi^2(t)}{f_1 + f_2} = \frac{2}{1 + f_2/f_1} - \frac{2}{1 + f_1/f_2} + \frac{-t_1^2}{1 + f_2/f_1} - \frac{-t_2^2}{1 + f_1/f_2} + \frac{\dot{\phi}(t)}{1 + f_2/f_1} - \frac{\dot{\phi}^2(t)}{1 + f_1/f_2} + \frac{\dot{\phi}^2(t - \tau_1^2)}{1 + f_2/f_1} - \frac{\dot{\phi}^2(t - \tau_2^2)}{1 + f_1/f_2}
\]

The first term on the right hand side is a large known bias, which is removed in the comparison plots below. The second two terms are basically the difference in light travel times between the two spacecraft at true time $t$. It is periodic with a very small bias and can be removed to high accuracy with our knowledge of the spacecraft position. The time delay between reception and transmission, $\tau$, is typically less than a millisecond with the spacecraft separation of about 200 km and the second derivative of the clock error is typically less than $10^{-17}$ s/s². Thus the fourth and fifth terms are the same to about $10^{-17}$ s/s and the sum of these two terms gives a measure of the relative local clock rate at true-time $t$. It is the sum of the fourth and fifth terms that will be compared to the clock solution obtained with GPS. The argument that $\tau$ has a small effect on the value of the rate of change of time can be applied to show that even though we have implicitly assumed in (3) that the data are sampled at each spacecraft at the same true time $t$, this simultaneous sampling need only be good to the millisecond level to compare rates to the $10^{-17}$ s/s level. The last 4 terms of equation (4), will yield values that are close to constant. For March 2-7, 2003, the maximum deviation of the sum of the last four terms from their mean on each day was less than $1.8 \times 10^{-16}$. Also note that all the terms in equation (4) are the same for both the K and Ka frequencies since they either are independent of the nominal frequency or have a ratio of frequency ($K = (3/4)KA$ for each spacecraft).

Since the ionosphere free combination exactly sums to 1, all the arguments for the terms on the right hand side are unaffected by eliminating the small differential ionosphere term that was dropped in forming the difference in equation (2).

Figure 5 shows the relative clock rates as determined by GPS and KBR on a typical day, March 2, 2003. At the scale of the drift in rate, there is almost no difference in the two measures of relative clock rate. Figure 6, displays the difference in the two determinations of relative clock rate. There is an over all mean difference of $-0.065$ ps/s which we cannot currently explain. This rate difference would mean clock difference of 5.6 ns in a day, too large for an error in GPS system. The RMS about the mean is $0.059$ ps/s. The periodic variation in Fig. 6 is consistent with periodic errors in position, $0.06$ ps/s corresponds to about $1.8$ microns/s (0.06*speed of light), about the magnitude of rate of orbital along track positional errors. Orbit errors, since they are dynamic in nature, tend to have periods the same as the orbital period, about 90 minutes. Thus the periodic errors are probably due to GPS clock rate determination errors and
could be reduced if the clock errors were treated as some constrained correlated process noise instead of unconstrained white-noise.

The relative clock between the two orbiting GRACE spacecraft can be determined to better than 150 ps. A new method of determining relative clock rate using inter-satellite phase measurements gives agreement to the GPS determined values consistent with errors in the GPS system. GPS clock determination could be improved by taking further advantage of the clock stability.

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References


