

# **An Alternative Ionospheric Correction Algorithm for Satellite-Based Augmentation Systems in Low-Latitude Region**

Attila Komjathy, Lawrence Sparks, Tony Mannucci and Xiaoqing Pi

*Jet Propulsion Laboratory/ California Institute of Technology  
M/S 238-600, 4800 Oak Grove Drive, Pasadena, CA 91109*

## **ABSTRACT**

The ionospheric correction algorithms have been characterized extensively for the mid-latitude region of the ionosphere where benign conditions usually exist. The United States Federal Aviation Administration's (FAA) Wide Area Augmentation System (WAAS) for civil aircraft navigation is focused primarily on the Conterminous United States (CONUS). Other Satellite-Based Augmentation Systems (SBAS) include the European Geostationary Navigation Overlay Service (EGNOS) and the Japanese Global Navigation Satellite System (MSAS). Researchers are facing a more formidable challenge in addressing the ionospheric impact on navigation using SBAS in other parts of the world such as the South American region or India. At equatorial latitudes, geophysical conditions lead to the so-called Appleton-Hartree (equatorial) anomaly phenomenon, which results in significantly larger ionospheric range delays and range delay spatial gradients than is observed in the CONUS or European sectors.

In this paper, we use data from the South American region to perform a quantitative assessment of WAAS-type ionospheric correction algorithms in this region. For the study, we accessed a worldwide network of 400+ dual-frequency GPS receivers. The network includes: 1) the Continuously Operating Reference Sites (CORS) in the United States; 2) stations in and near South America as part of the Brazilian Network of Continuous Monitoring of GPS (RBMC), operated by the Brazilian Institute of Geography and Statistics (IGBE); and (3) sites included in the International GPS Service (IGS) global network. Data sets have been selected to include both quiet and storm days. To provide ground-truth and calibrate GPS receiver and transmitter inter-frequency biases, we processed the GPS data using Global Ionospheric Mapping (GIM) software developed at the Jet Propulsion Laboratory to compute calibrated high resolution observations of ionospheric total electron content (TEC).

In this study we investigated the performance of planar and quadratic fit models to augment or replace the WAAS algorithm in Brazil. We found that for quiet day, the WAAS planar fit residuals are less than 13 meters (1.42 meter RMS). When using a quadratic approach, we obtained residuals less than 9 meters with 1.29 meter RMS. After investigating a number of storm and quiet days, we concluded that the quadratic approach resulted on average in a 20 percent improvement in accuracy over the planar fit approach.

## INTRODUCTION

The Wide-Area Augmentation System (WAAS) developed for the Conterminous United States (CONUS) is only one of the several Space-Based Augmentation Systems (SBAS) under consideration worldwide. Other SBAS developments are under way in Europe, Japan, India and Brazil.

Relatively benign ionospheric conditions in the mid-latitude CONUS region are compatible with accurate ionospheric range corrections for WAAS. Providing ionospheric corrections for Brazil is significantly more challenging, since ionospheric range delays and range delay spatial gradients are among the largest in the world in the absence of ionospheric storms (during infrequent ionospheric storms, even mid-latitude regions present challenging conditions). In summary, the ionosphere in the Brazilian sector shows significantly different behavior from that of the mid-latitude sector.

The ionosphere has been extensively studied to support WAAS at the CONUS sector. The published literature discussing ionospheric corrections for WAAS in the CONUS is extensive; see e.g. Enge et al., [1996], WAAS MOPS [1999], Walter et al., [2000] and Sparks et al., [2002]. Various alternative ionospheric correction algorithms have been presented by e.g., Hansen et al., [1997], Sparks et al., [2000] and Blanch et al., [2002]. A potential application of WAAS algorithms to Brazil has recently been investigated by Klobuchar et al., [2002] using simulated data. The temporal and spatial variability of the low-latitude ionosphere was studied in the context of ionospheric storms by e.g., Dehel and Corbelli [2002] and Fedrizzi et al., [2001] using a network of dual-frequency GPS receivers in Brazil. Investigating the possible application of the current WAAS algorithm using actual GPS data is the natural progression of the previous studies and therefore the main focus of this paper.

In Komjathy et al., [2002b], for the first time, we assessed the WAAS's planar fit algorithm in the equatorial region where the spatial gradients and the absolute slant TEC are known to be the highest in the world. We found that in Brazil the dominant error source for the WAAS planar fit algorithm is the inherent spatial variability of the equatorial ionosphere with ionospheric slant range delay residuals as high as 15 meters and root-mean square residuals for the quiet day of 1.9 meters. This compares to a maximum residual of 2 meters in CONUS, and 0.5 meter RMS. We revealed that ionospheric gradients in Brazil are at the 2 meter over 100 km level. Contrary to results obtained for CONUS, we discovered that a major ionospheric storm had a small impact on the planar fit residuals in Brazil.

In this research, we first review the estimation method used to solve for inter-frequency biases (nuisance parameters) in the GPS satellites and receivers using a global network of 230 GPS sites in order to provide "ground truth" data for the analysis. Subsequently, we describe the WAAS planar fit algorithm used to estimate the vertical ionospheric range delay at fixed latitude and longitude locations known as ionospheric grid points (IGPs). This will be followed by examining the application of using a higher order, quadratic fit as a WAAS algorithm and augment or replace the WAAS algorithm in Brazil and compare ionospheric range residuals using planar and quadratic fits in the Brazilian sector.

## GIM BIAS ESTIMATION STRATEGY

To provide ground-truth, we used the Global Ionospheric Mapping (GIM) software developed at the Jet Propulsion Laboratory [Mannucci et al., 1998] to compute high precision slant ionospheric delay by removing the satellite and receiver differential biases from the ionospheric observables, generated from carrier-phase data adjusted to match the ionospheric delay based on dual-frequency pseudoranges. The estimation of the satellite and receiver biases is described here briefly.

Ionospheric measurements from a GPS receiver can be modeled with the well-known single-shell ionospheric model using the following observation equation [see e.g. Mannucci et al., 1999 and Komjathy et al., 2002a]:

$$TEC = M(h, E) \sum_i C_i B_i(lat, lon) + b_r + b_s, \quad (1)$$

where

*TEC* is the slant Total Electron Content measured by the linear combination of the GPS dual-frequency carrier phase and pseudorange ionospheric observables, typically expressed in TEC units. One TEC Unit ( $10^{16}$  electron/m<sup>2</sup>) corresponds to about 0.163 meter ionospheric delay at the L1 frequency,

*M(h, E)* is the thin-shell mapping function for ionospheric shell height *h* and satellite elevation angle *E* (for the definition of the thin-shell geometric mapping function see e.g. Mannucci et al., [1998] or Komjathy [1997]),

*B<sub>i</sub>(lat, lon)* are horizontal basis functions (based on, for example, bicubic splines or bilinear interpolants) evaluated at the ionospheric pierce point (IPP) – the intersection of the ray path of a signal propagating from the satellite to the receiver with a thin spherical shell – located at latitude *lat* and longitude *lon* on the thin shell,

*C<sub>i</sub>* are basis function coefficients (real numbers),

*b<sub>r</sub>, b<sub>s</sub>* are the satellite and receiver differential biases, assumed constant over periods of 24 hours or more.

The dependence of vertical TEC on latitude and longitude is parameterized as a linear combination of the two-dimensional basis functions *B<sub>i</sub>* which are functions of solar-geomagnetic longitude and latitude [Mannucci et al., 1998] (We note that the summation in Equation 1 is over all basis functions *B<sub>i</sub>*). Using the carrier phase-leveled ionospheric GPS observables, a Kalman filter simultaneously solves for the instrumental biases and the coefficients *C<sub>i</sub>* which are allowed to vary in time as a random walk stochastic process [Iijima et al., 1999]. The basis functions currently used are based on a bicubic spline technique developed at JPL [Lawson, 1984].

Although the main focus of this research is the investigation of the Brazilian sector, we decided to use a global network of some 230 stations to solve for high precision satellite and receiver differential biases that are used to correct the measurements. Research has shown that the most reliable satellite bias estimates can be achieved when using the data strength of a global network of GPS receivers instead of regional GPS networks [Komjathy, 1997]. We note that the WAAS system itself uses a similar estimation scheme for biases applied over the regional WAAS network.

## WAAS PLANAR AND QUADRATIC FIT IONOSPHERIC MODELS

In the currently implemented WAAS ionospheric real-time correction algorithm, the vertical ionospheric delay is estimated at each ionospheric grid point (IGP) by constructing a planar fit to a set of (bias-corrected) slant measurements projected to vertical:

$$TEC = M(h, E)[a_0 + a_1d_E + a_2d_N] , \quad (2)$$

where

$a_0, a_1, a_2$  are the planar fit parameters,  
 $d_E, d_N$  are the distances from the IGP to the IPP in the eastern and northern directions, respectively.

Each least squares fit includes all IPPs that lie within a minimum fit radius surrounding the IGP. If the number of IPPs within this minimum radius is less than  $N_{min}$ , the fit radius  $R_{fit}$  is extended until it encompasses  $N_{min}$  points. In this study we do not tabulate data when the fit radius is increased to its maximum value of  $R_{max}$  without having reached  $N_{min}$  points.  $R_{max}$  was chosen to be 2100 km which is the value in the current WAAS implementation.

In our modified WAAS ionospheric real-time correction algorithm, we estimated the vertical ionospheric delay at each IGP by using a quadratic fit to the slant ionospheric measurements projected to the vertical:

$$TEC = M(h, E)[a_0 + a_1d_E + a_2d_N + a_3d_E^2 + a_4d_Ed_N + a_5d_N^2] , \quad (3)$$

where

$a_3, a_4, a_5$  are the additional planar fit parameters describing quadratic and cross terms.

In our new WAAS estimation scheme (see Equation 2), we did not solve for the satellite and receiver differential biases. Instead, we used the GIM approach, outlined in Equation 1 to solve for high precision differential biases and calibrated the ionospheric range measurements before applying Equation 2. This is similar to the approach used in WAAS.

## DATA ANALYSIS STRATEGY

In our data analysis, we treated every IPP data point as if it were collocated with a WAAS IGP (so-called “pseudo IGP” approach). Subsequently, we applied the WAAS planar and quadratic fit ionospheric model algorithms to estimate the vertical ionospheric delays at each of these IPPs, treated as pseudo IGP values. Starting with the set of measurements that contributed to the planar or quadratic fits, we then computed the residual difference between the slant measurements and the estimated slant delays based on the corresponding fit, projecting the vertical TEC from the planar estimate into the line-of-site using the WAAS thin-shell obliquity factor. This residuals analysis

provides a measure of the performance of the planar and quadratic fit algorithms in reproducing slant TEC for the user.

## DATA SETS

For our data set, we chose quiet and storm days in Brazil, between July 2 and 18, 2000, respectively, using GPS receivers from the International GPS Service [IGS, 2003], and the Brazilian Network for Continuous Monitoring of GPS (RBMC). In Figure 1, we show the Kp indices for a period in 2000 indicating a major geomagnetic storm in July 13 to 16 (St. Swithin's Day).

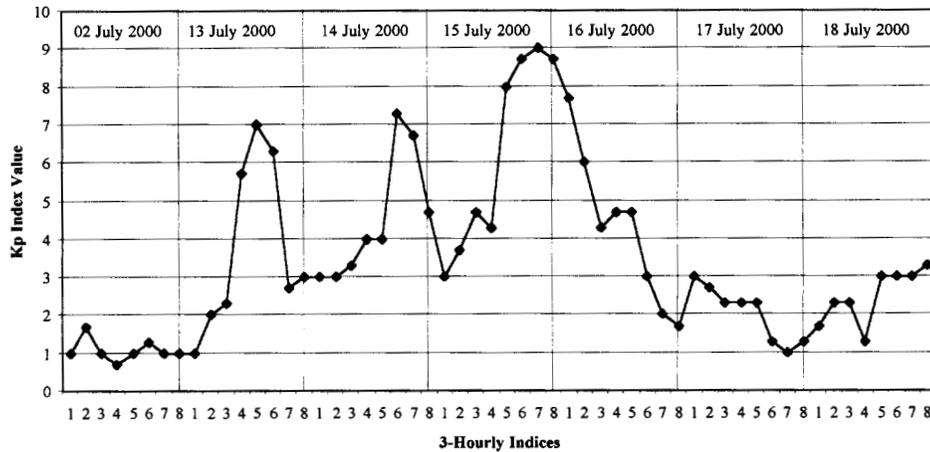


Figure 1. Behavior of Kp indices during the focus period in July 2000.

In Figure 2, we show the distribution of the GPS reference stations in Southern America for July 16, 2000. The small filled circles in red represent the sites used to provide unbiased line-of-site TEC ground-truth data.



Figure 2. Network of IGS and RBMC stations processed for July 2 to 18, 2000 time interval.

## ANALYSIS OF RESULTS

First we calibrated the satellite and receiver differential biases using the GIM method and data from the South American sector. We selected a quiet day (July 2, 2000) and a geomagnetic storm day (July 16, 2000) to illustrate the residuals using the traditional planar fit and the new proposed quadratic fit approaches. In Figure 3, we find that the slant range ionospheric delays for Brazil range between 0 and 52 meters (an elevation angle of 10 degrees was used throughout this analysis) for the storm day and it ranges between 0 and 25 meters for the quiet day. This is consistent with our earlier results in Komjathy et al. [2002b] showing quiet and storm day conditions for the March 30 and 31, 2001 period.

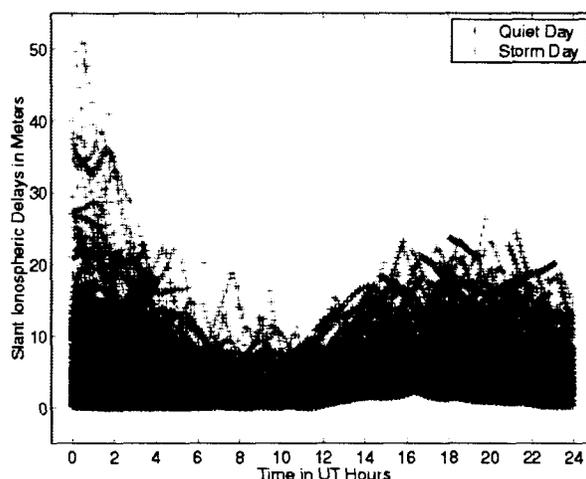


Figure 3. Slant ionospheric delay for the *quiet* (July 2, 2000) and *storm* days (July 16, 2000).

In Figures 4 and 5, we plot the planar and quadratic fit residuals in the slant domain for the quiet and storm days separately. Note that to compute residuals, the fitted vertical TEC value at an IPP location was converted to slant and differenced with the slant TEC measurement. In Figure 4, we show the performance of the planar and quadratic fits for the quiet day. We demonstrate that during post sunset hours (0 to 6 UT) the temporal variability of the ionosphere is very high. During daytime (UT 12 to 24), there is also a large variability in the residuals. When comparing the quadratic and planar fit approaches for the quiet day, we achieve a 10 percent improvement using the quadratic fit. For the quiet day, we obtained WAAS planar fit residuals less than 13 meters (1.42 meter RMS) in Brazil. When using a quadratic approach we obtained less than 9 meter residuals (1.29 meter RMS).

In Figure 5, we display a comparison for one of the worst storms during the last solar cycle. During post-subset hours (between UT 0 and 6), the planar fit residuals are as large as 5 to 15 meters during which time the storm impacted the ionosphere (see the Kp index in Figure 1). During daytime hours, we observe planar fit residuals at the 5 meter level. For the storm day, the planar fit residuals were as large as 25 meters with an RMS of 2.16 meters whereas the quadratic fit resulted in less than 11 meters and with an RMS of 1.69 meters.

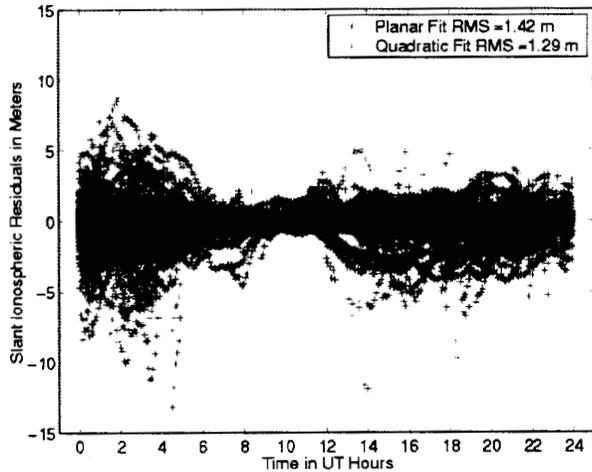


Figure 4. Planar and quadratic fit residuals in Brazil for the *quiet day*, July 2, 2000.

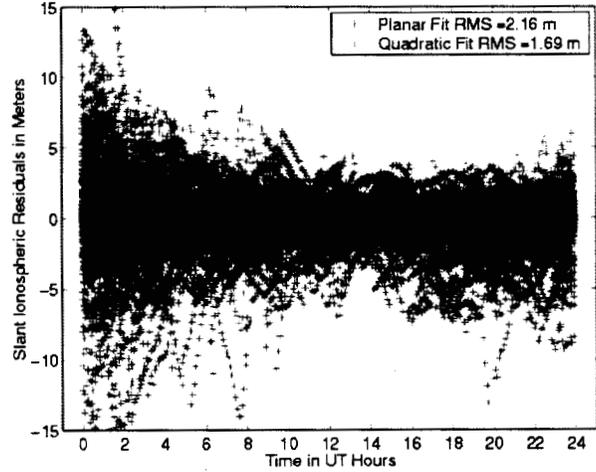


Figure 5. Planar and quadratic fit residuals in Brazil for the *storm day* July 16, 2000.

In Figure 6, we plotted the planar and quadratic fit slant residuals for the quiet day as a function of satellite elevation angle. We can clearly see that we achieved an improvement at low elevation angles over the planar fit approach when using the quadratic fit approach. Figure 7 demonstrates the case for the storm day using planar and quadratic fits. The quadratic fit appears to reduce the residuals both for low and high elevation angles.

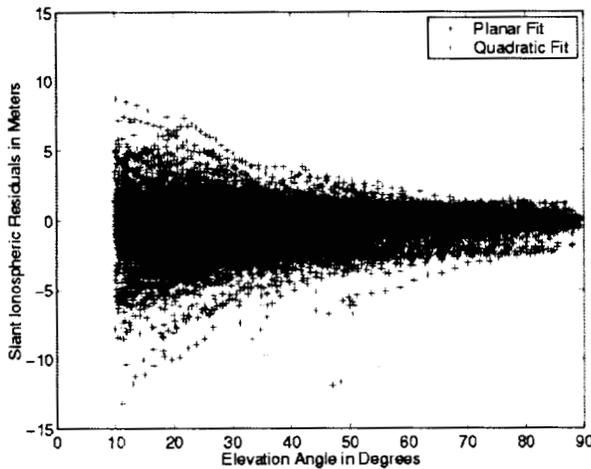


Figure 6. Slant residuals for the *quiet day* as a function of elevation angle.

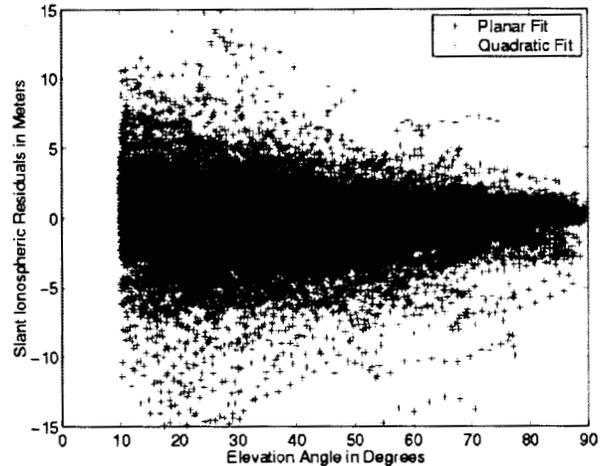


Figure 7. Slant residuals for the *storm day* as a function of elevation angle

In Figure 8 and 9, we display the standard deviation of least squares planar and quadratic fit slant residuals using all data points that went into the particular fit. We can also consider this as a measure of temporal and spatial decorrelation of the ionosphere. For the quiet day, the quadratic fit shows an improvement in sigma of slant residuals over the planar fit for all local times of the day. For the storm day, we found dramatic improvement especially during the storm time (UT 0 to 6 hours).

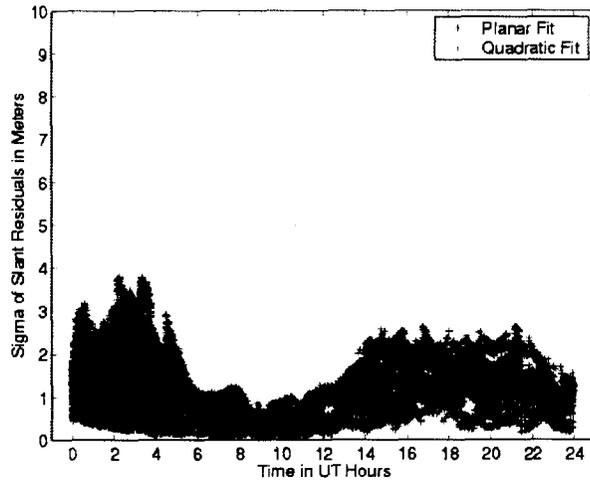


Figure 8. Sigma of slant residuals for the *quiet* day July 2, 2000

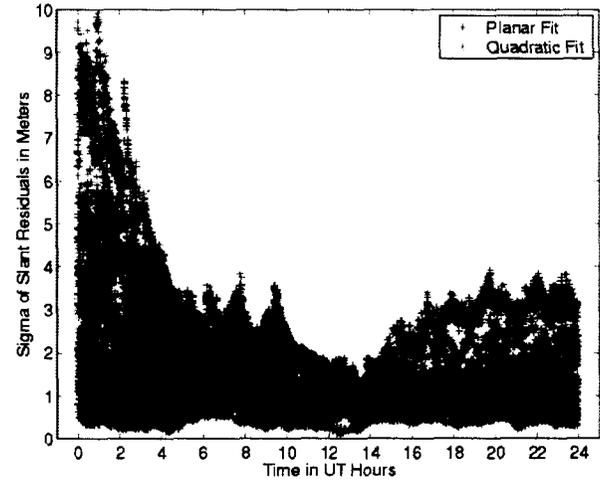


Figure 9. Sigma of slant residuals for the *storm* day July 16, 2000

In Table 1, we summarized the statistics for all the days that we processed using the planar and quadratic approaches. From the statistics, we can conclude that we achieved an improvement in the accuracy for all quiet and storm days under investigation. An overall assessment of the statistics indicate that using the quadratic approach over the planar fit, we achieved an improvement of 20 percent using the quadratic fit.

	<b>Planar Fit</b>	<b>Quadratic Fit</b>
7/2/00	1.42	1.29
7/13/00	1.65	1.21
7/14/00	1.60	1.19
7/15/00	2.03	1.57
7/16/00	2.16	1.69
7/17/00	1.40	1.08
7/18/00	1.66	1.39
Average	1.70	1.35

Table 1. RMS of daily residuals in meters.

## CONCLUSIONS

In this paper, we have compared the performance of the WAAS ionospheric planar-fit and the new quadratic-fit approach in the low latitude region where the temporal and spatial variation of the ionosphere is the highest in the world. We used data from the network of dual-frequency GPS receivers in and around Brazil. Unbiased line-of-site TEC ground-truth data were generated using JPL's Global Ionospheric Mapping (GIM) software. Using the truth data, the WAAS planar fit algorithm was evaluated by treating each observation as representing a WAAS ionospheric grid point (IGP) and computing the planar fit estimate for that IGP after excluding it from the fit.

We found slant ionospheric range delays up to 55 meters for Brazil. For the quiet day, we obtained WAAS planar fit residuals less than 13 meters (1.42 meter RMS) in Brazil. When using a quadratic approach we obtained less than 9 meter (1.29 meter RMS) residuals. For the storm day, the planar fit residuals were as large as 25 meters (2.16 meter RMS) whereas the quadratic fit resulted in less than 11 meter (1.69 meter RMS). When considering all quiet and storm days, we found that the quadratic fit approach reduced the residuals by an average of 20 percent.

## ACKNOWLEDGMENTS

This research was performed at the Jet Propulsion Laboratory/California Institute of Technology under contract to the National Aeronautics and Space Administration and the Federal Aviation Administration. We greatly appreciate the help from Dr. Eurico de Paula and Mariangel Fedrizzi (both at the Instituto Nacional de Pesquisas Espaciais, INPE) for providing us with the GPS data from Brazil.

## REFERENCES

Blanch, J., T. Walter and P. Enge. (2002). "Application of Spatial Statistics to Ionosphere Estimation for WAAS." *On the CD-ROM of the Proceedings of the National Technical Meeting of the Institute of Navigation*, San Diego, CA, January 28-30.

Dehel T. and M. Corbelli (2002). "Brazilian Test Bed: Ionospheric Analysis." Presented at the ATN/GNSS CAR/SEM Seminar, Varadero, Cuba, 6-9 May, [http://www.icao.int/nacc/meetings/atngnss2002/gnss\\_71b2\\_brazil.pdf](http://www.icao.int/nacc/meetings/atngnss2002/gnss_71b2_brazil.pdf).

Enge, P., T. Walter, S. Pullen, C. Kee, Y.C. Chao and Y.-J. Tsai (1996). "Wide Area Augmentation of the Global Positioning System." *Proceedings of the IEEE*, Vol. 84, pp. 1063-1088.

Fedrizzi, M., R.B. Langley, A Komjathy, M.C. Santos, E.R. de Paula, I.J. Kantor (2001). "The Low-Latitude Ionosphere: Monitoring Its Behavior with GPS." *On the CD-ROM of the Proceedings of ION GPS 2001, 14th International Technical Meeting of the Satellite Division of The Institute of Navigation*, Salt Lake City, UT, 11-14 September.

Hansen, A.J., T. Walter and P. Enge (1997). "Ionospheric Correction Using Tomography." *Proceedings of ION GPS-97, the 10th International Technical Meeting of the Satellite Division of The Institute of Navigation*, Kansas City, MO, U.S.A., 16-19 September, pp. 249-257.

IGS (2003). International GPS Service, <http://igsceb.jpl.nasa.gov/>, Accessed 10 March.

Iijima, B.A., I.L. Harris, C.M. Ho, U.J. Lindqwister, A.J. Mannucci, X. Pi, M.J. Reyes, L.C. Sparks, B.D. Wilson (1999). "Automated Daily Process for Global Ionospheric Total Electron Content Maps and Satellite Ocean Altimeter Ionospheric Calibration Based on Global Positioning System." *Journal of Atmospheric and Solar-Terrestrial Physics*, Vol. 61, pp. 1205-1218.

Klobuchar, J.A., P.H. Doherty, M. Bakry El-Arini, R. Lejeune, T. Dehel, E.R. de Paula and F.S. Rodrigues (2002). "Ionospheric Issues for a SBAS in the Equatorial Region." *Proceedings of the 10<sup>th</sup> International Ionospheric Effects Symposium*, 7-9 May.

Komjathy, A. (1997). *Global Ionospheric Total Electron Content Mapping Using the Global Positioning System*. Ph.D. dissertation, Department of Geodesy and Geomatics Engineering Technical Report No. 188, University of New Brunswick, Fredericton, New Brunswick, Canada, 248 pp.

Komjathy, A., B.D. Wilson, T.F. Runge, B.M. Boulat, A.J. Mannucci, L. Sparks and M.J. Reyes (2002a). "A New Ionospheric Model for Wide Area Differential GPS: The Multiple Shell Approach." *On the CD-ROM of the Proceedings of the National Technical Meeting of the Institute of Navigation*, San Diego, CA, January 28-30.

Komjathy, A., L. Sparks, A.J. Mannucci and X. Pi (2002b). "An Assessment of the Current WAAS Ionospheric Correction Algorithm in the South American Region." *On the CD-ROM of the Proceedings of the 15<sup>th</sup> International Technical Meeting of the Satellite Division of the Institute of Navigation*, Portland, OR, Sept 24-27.

Lawson, C. (1984). "A Piecewise C2 Basis for Function Representation over a Surface of a Sphere." JPL internal document.

Mannucci, A.J., B.D. Wilson, D.N. Yuan, C.H. Ho, U.J. Lindqwister and T.F. Runge (1998). "A Global Mapping Technique for GPS-derived Ionospheric Total Electron Content Measurements." *Radio Science*, Vol.33, pp.565-582.

Mannucci A.J., B.A. Iijima, L. Sparks, X. Pi, B.D. Wilson and U.J. Lindqwister (1999). "Assessment of Global TEC Mapping Using a Three-Dimensional Electron Density Model." *Journal of Atmospheric and Solar Terrestrial Physics*, Vol. 61, pp. 1227-1236.

Sparks, L., B.A. Iijima, A.J. Mannucci, X. Pi, B.D. Wilson (2000). "A New Model for Retrieving Slant TEC Corrections for Wide Area Differential GPS." *Proceedings of the ION National Technical Meeting 2000 of the Institute of Navigation*, Anaheim, CA, 26-28 January, pp. 464-474.

Sparks, L., A. Komjathy and A.J. Mannucci (2002). "Sudden Ionospheric Delay Decorrelation and Its Impact on WAAS." *Proceedings of the 10<sup>th</sup> International Ionospheric Effects Symposium*, 7-9 May.

WAAS MOPS (1999). "Minimum Operational Performance Standards for Global Positioning System/Wide Area Augmentation System Airborne Equipment." RTCA Inc. Document No. RTCA/DO-229B October 6. pp 225.

Walter, T., A. Hansen, J. Blanch, P. Enge, T.J. Mannucci, X. Pi, L. Sparks, B. Iijima, B. El-Arini, R. Lejeune, M. Hagen, E. Altschuler, R. Fries, A. Chu (2000). "Robust Detection of Ionospheric

Irregularities.” *On the CD-ROM of the Proceedings of ION GPS 2000, 13th International Technical Meeting of the Institute of Navigation, Salt Lake City, UT, 11-14 September*