Coupled Receiver-Decoders for Low Rate Turbo Codes*

Jon Hamkins†, Dariush Divsalar†

October 31, 2002

Abstract

We present a rate 1/31 turbo code that achieves a bit error rate of 10^{-6} at $E_b/N_0 = -0.9$ dB. At this coding gain, insufficient energy per symbol is present for a conventional receiver to recover the carrier phase properly. We present a method to overcome this problem by coupling the receiver and decoder functions. Simulations indicate that this approach may enable the elimination or significant reduction of required residual carrier power for signaling on AWGN channels impaired by phase noise.

1 Introduction

We have recently designed a rate 1/31 turbo code that has 11.4 dB of coding gain at a Bit Error Rate (BER) of 10^{-6}. Deep space applications may require this type of high coding gain when the received power from a spacecraft is very low, such as during emergency, direct probe-to-Earth, or ultra deep space communications.

Use of the rate 1/31 turbo code use presupposes the ability to acquire a carrier signal prior to decoding. The Deep Space Network (DSN) tracks suppressed carrier signals with a conventional Costas loop with squaring loss $\frac{E_s/N_0}{1+2E_s/N_0}$, where E_s/N_0 is the symbol energy to 1-sided noise PSD ratio. The rate 1/31 turbo code operates at BER=10^{-6} at $E_s/N_0 = -15.8$ dB, which unfortunately results in a squaring loss of 13.0 dB. Without a better receiver, power would need to be diverted from the telemetry signal and put into an unmodulated residual carrier. Instead, we propose a coupled receiver-decoder that can eliminate or significantly reduce the amount of power necessary in the residual carrier.

2 Preliminaries

2.1 Signal model

This paper considers the receiving and decoding of a single channel binary signal with a residual carrier and a data subcarrier. This type of signaling is typical of a signal received from a spacecraft by the DSN [Kin96, Yue83]. The front end of the receiver performs IF down conversion, analog to digital conversion, carrier mixing, subcarrier mixing, and low pass filtering, to obtain the two discrete time signals

$$\begin{aligned} r_d[k] &= \sqrt{P_d} c_k e^{j\theta_k} + n_k \\ r_c[k] &= \sqrt{P_c} e^{j\theta_k} + n'_k \end{aligned}$$

*The work described was funded by the IPN Technology Program and performed at the Jet Propulsion Laboratory, California Institute of Technology under contract with the National Aeronautics and Space Administration.

†Communications Systems and Research Section, JPL

The first signal is the data dependent signal, being proportional to the kth transmitted code symbol, c_k; the second signal is an unmodulated carrier signal used to help establish a coherent phase reference. P_d and P_c represent the power present in the data and carrier signals, respectively, and the total power of the signal is $P_t = P_d + P_c$. The noise terms n_k and n'_k are independent and each of the form $n_I[k] + jn_Q[k]$, each component with normal distribution $N(0, N_0/(2T_{sym}))$. The carrier phase has value θ_k during the kth received symbol. For typical deep space applications, imperfect oscillators may cause the carrier phase to be time varying, often with a PSD proportional to $1/f^3$.

2.2 The Uncoupled system

In an uncoupled, non-data-aided system, the receiver tracks the carrier from the residual carrier signal, and this is used to wipe off the phase noise in the data signal. This architecture is shown in Fig. 1. The communication is unidirectional, and no information from the decoder is used by the receiver. The receiver itself is typically a phase-locked loop (PLL) that requires no delay or perhaps a single sample delay [ST95], resulting in a relatively simple signal flow through the system.

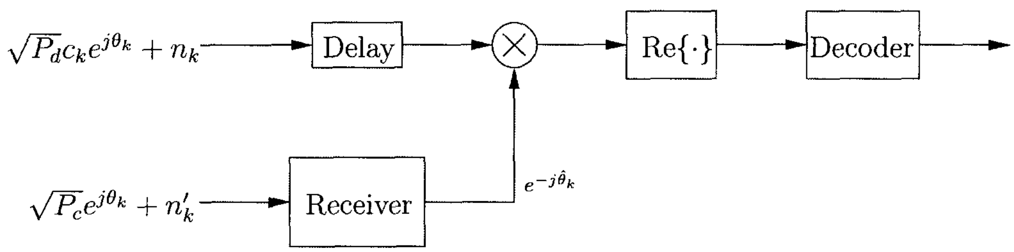

Figure 1: In an uncoupled receiver and decoder, there is a unidirectional flow of information from left to right.

3 A rate 1/31 turbo code

We have designed a rate 1/31 turbo code that achieves BER=10^{-6} at E_b/N_0 = -0.9 dB (see Fig. 2). This is competitive with the most power efficient codes known, although an additional 0.2 to 0.3 dB of gain is possible by using a lower rate, a longer interleaver, and more decoding iterations (see, e.g., turbo-Hadamard codes [PLW01]). The encoder for our code is shown in Fig. 3. The two component codes each are 32-state recursive convolutional codes with 15 outputs. Together with the systematic bit, there are 31 outputs per input bit, making a rate 1/31 turbo code. The first component code has feedback $1 + D^2 + D^5$, a primitive polynomial. The second component code is a "big numerator-little denominator" code consisting of an accumulator and taps on delayed accumulations. The performance reported here is for a code with input block size 16384, decoded with a log-MAP turbo decoder using 20 iterations. As can be seen, the performance is within about 0.6 dB of the unconstrained capacity of rate 1/31 codes, which has an asymptote at -1.494 dB.

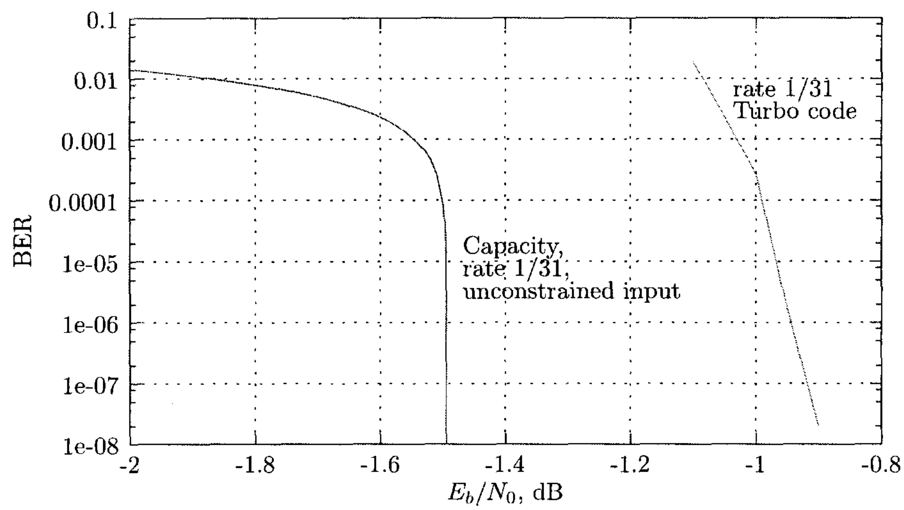

Figure 2: Performance of the rate 1/31 turbo code, assuming perfect carrier recovery.

4 Coupled systems for carrier tracking and decoding

4.1 Iterative soft data wipe-off

A coupled receiver-decoder architecture is shown in Fig. 4. The receiver begins as before: a PLL or Wiener filter tracks the residual carrier signal, wipes off the phase noise as best it can from the data signal, and sends the result to the turbo decoder. Timing recovery is accomplished with, for example, a digital transition tracking loop (DTTL), or other similar loop. The first difference arises after the first iteration of the turbo decoder, when soft data outputs are available from the turbo decoder.

For carrier phase recovery, the turbo decoder sends the soft data symbols (real numbers) back to the receiver, where they are used to softly wipe off the data signal. For example, we might have $c_k = -1$ and the turbo decoder soft output may be $\hat{c}_k = -0.9$, which results in a soft wipe-off of $-0.9r_d[k] = 0.9e^{j\theta_k} + 0.9n_k$, a close approximation of a residual carrier signal. Once the data is wiped, the signal contains a large carrier phase component that can be tracked with another Wiener filter. The refined phase estimates from the Wiener filter are used to wipe the phase noise from the original, delayed data signal. The refined data samples are then sent back to the turbo decoder for the second iteration. The soft symbols from the second iteration are sent back to the receiver in the same manner as the first, and the process is repeated. This two-way communication between the receiver and decoder results in improved performance of both the receiver and decoder.

For timing recovery, the turbo decoder outputs are used to replace the transition detector arm in the DTTL. In the usual DTTL, transitions are detected based on hard decisions of two consecutive symbols. These hard decisions are based on raw symbol-by-symbol channel input, with no coding gain. In the new scheme, the decoder estimate of the codeword has taken advantage of the underlying code. The transitions present in the hard-limited codeword estimates lead to an improved transition detector, which improves the overall performance of the tracking loop.

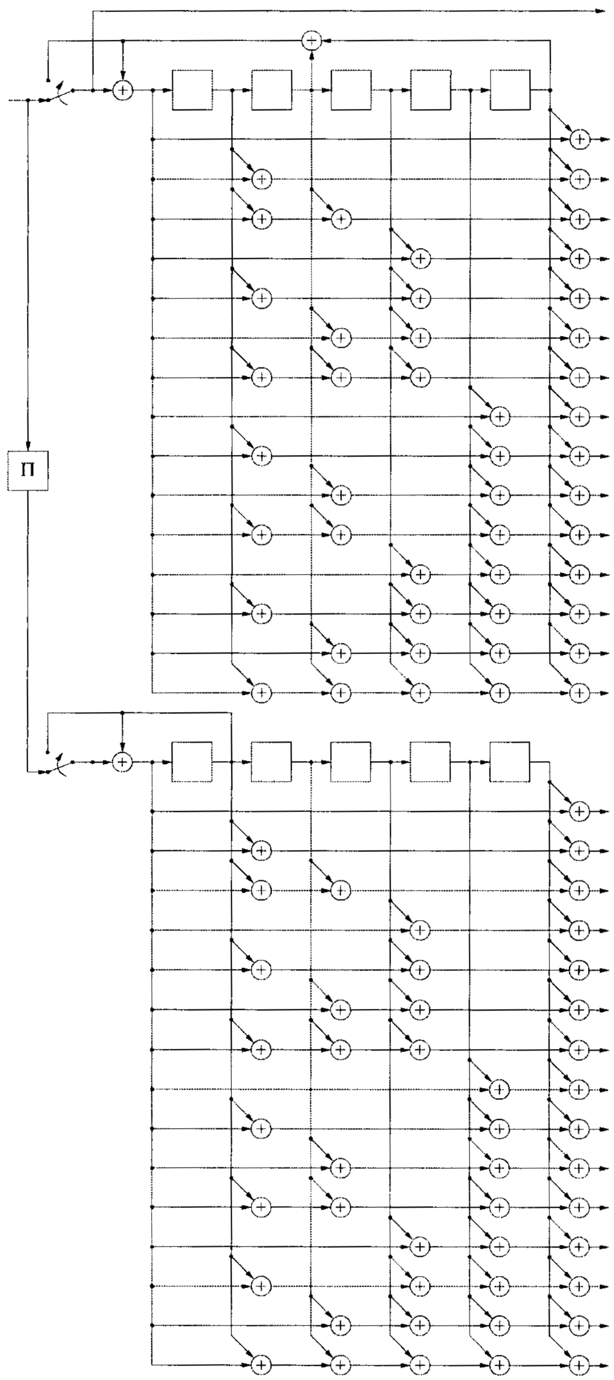

Figure 3: Encoder for a rate 1/31 turbo code. Switches are flipped during termination phase.

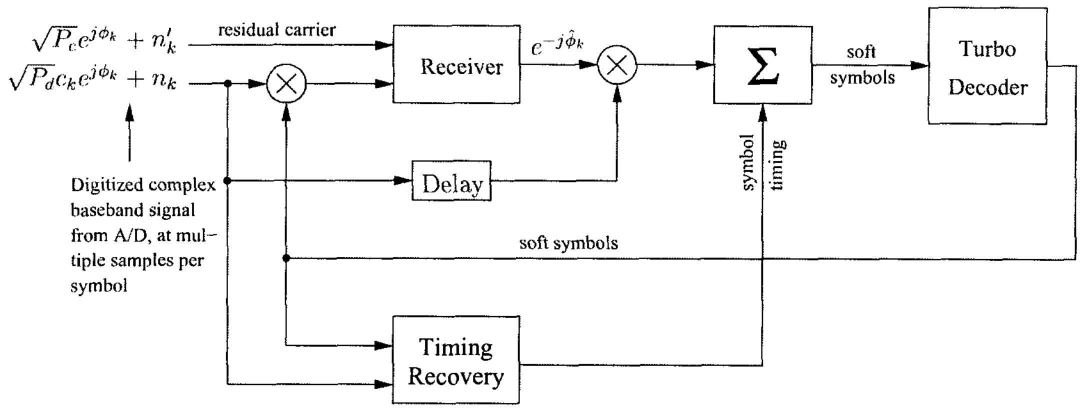

Figure 4: Architecture for coupled receiver decoder with both carrier recovery and symbol timing feedback.

4.2 Joint phase and data recovery from constituent codes

One limitation of the previous coupled receiver-decoder design is that a residual carrier is needed in order for the receiver to perform well enough for the first iteration of the turbo decoder to provide meaningful feedback. (Recall, a Costas-type loop that operates on the data modulated signal results in an unacceptable 13 dB squaring loss.) If the residual carrier power is too low, the initial soft inputs to the turbo decoder are of such low quality that it cannot recover codewords adequately.

Decoding suppressed carrier convolutionally coded signals is possible with the use of the higher complexity joint receiver-decoder [Ham99]. In that work, we determined that these schemes can boot-strap themselves into simultaneously recovering the carrier phase and decoded data, which led to a 3 to 4.25 dB power savings. However, until now, it was not clear that this work could be extended to a joint receiver-decoder scheme for suppressed carrier turbo codes. Based on the same idea, we now describe this extension.

The basic architecture is shown in Fig. 5. The idea is to perform the phase-and-data recovery boot-strapping process on each of the constituent convolutional codes of the turbo code. Although the individual codes are

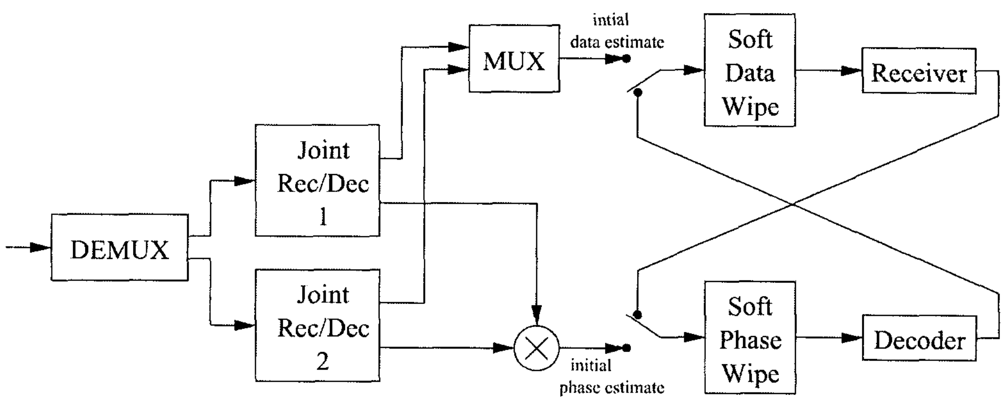

Figure 5: A coupled receiver-decoder for suppressed carrier signals.

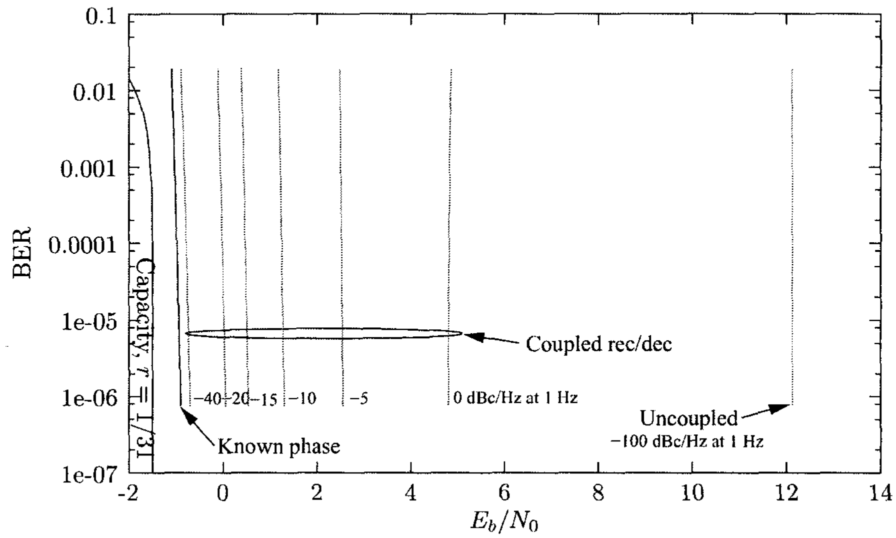

Figure 6: Performance of the coupled receiver-decoder.

weak, such low constraint-length codes are strong enough to allow adequate phase recovery from the suppressed carrier signal, even in the presence of moderately high phase noise [Ham99]. Thus, the quality of the soft input in the first iteration of the turbo code is adequate enough for the turbo decoder to provide improved feedback to the receiver. Additionally, since the complexity of the constituent codes is quite small individually — much smaller than the (7,1/2) code considered in [Ham99], for example — this joint receiver-decoder approach is a low complexity solution.

Fig. 6 shows the performance of this coupled receiver-decoder on an AWGN channel impaired by phase noise. We considered phase noise with a 2-sided PSD $S(f)$ proportional to $1/f^3$, ranging from $10\log_{10} S(1) = -100$ to 0 dBc/Hz. A moderately high phase noise of -20 dBc/Hz results in about a 1 dB loss from the ideal performance, a 12 dB improvement over the uncoupled system.

References

[Ham99] Jon Hamkins. A joint receiver-decoder for convolutionally coded BPSK. *TMO Progress Report*, 42(139):1–23, November 1999.

[Kin96] Peter W. Kinman. TLM-21 DSN telemetry system, Block-V Receiver, 810-5, rev. D. JPL document, December 1996.

[PLW01] L. Ping, W.K. Leung, and K.Y. Wu. Low rate turbo-Hadamard codes. In *IEEE Int. Symp. Inform. Theory*, page 211, 2001.

[ST95] S. A. Stephens and J. B. Thomas. Controlled-root formulation for digital phase-locked loops. *IEEE Transactions on Aerospace and Electronic Systems*, 31(1):78–95, 1995.

[Yue83] Joseph H. Yuen. *Deep Space Telecommunications Systems Engineering.* Plenum Press, New York, NY, 1983.