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Numerical modeling of electroactive polymer mirrors for space applications

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ABSTRACT

Thin-film mirrors are attractive for large aperture, lightweight optical system and microwave antennas operating in micro-gravity space. The surface shape of these deployable thin film structures requires control to a precision range that depends on the specific applications. For optical systems, such surfaces need to be deployed and refined in the range of submicrons. Electroactive polymers (EAP) are potential candidates for making such thin film materials. Generally, EAPs are produced in thin film form with electrodes on their major surfaces. Depending on the reflectivity of the electrodes and surface roughness of the polymer they can also be produced with mirror finishes. A controllable mirror made of single-layer EAP mirror is proposed in this paper. An analytical solution of required voltage distribution for forming a parabolic mirror from a planar film is presented. Calculations show a single layer film made of currently available EAP has the capability to control the focus distance of a 2-m mirror from infinity to 1.25 m. The results are verified by FEM model.

Keyword: Electroactive polymer, thin film mirror, thin film antenna, reflector, controllable membranes.

1. INTRODUCTION

Thin-film mirrors are attractive for large aperture, lightweight optical system and microwave antennas operating in micro-gravity space. The surface shape of these deployable thin film structures requires control to a precision range that depends on the specific applications. For optical systems, such surfaces need to be deployed and refined in the range of sub-microns. Numerous mechanisms to control mirrors were proposed [1-4]. Electroactive polymers (EAP) are one of potential candidates of the actuation materials [5].

The EAP materials can be categorized to two main classes. One is the ionic EAP [6-9]. The ionic EAPs contain electrolyte in polymer frame. They show large bending deformation at low voltage excitation. However, the actuation mechanism of these materials involves transport of ions and molecules from one side to the other of the film. The component and properties of the electrolyte have to be maintained well to keep the performance. This type EAP may not be a good candidate for space application. Another class is electric field EAP [10-12]. It includes piezoelectric, electrostrictive, ferroelectric or dielectric polymers. These materials behave like capacitors to the excitation voltage and the strain is usually proportional to the square of the applied voltage. Although they require relative high voltage (10 ~ 100V/μm) to create maximum strain, these polymers are dry and have relative stable performances. The study in this paper is focused to these type EAPs. The strain of these type EAPs can reach several even ten percents. An example is a P(VDF-TrFE)-based Co/Ter-polymer having a maximum transverse strain of ~4% [11].

One advantage of using EAP is the EAP film could be both the structure material and the actuation material of the mirrors. Generally, EAPs are produced in thin film form with electrodes on their major surfaces. Depending on the reflectivity of the electrodes and surface roughness of the polymer they can also be produced with mirror finishes.

The major advantage of the EAPs is the capability to realize distributed actuation to whole mirror surface. PVDF bimorph was suggested for distributed shape control [2]. The required electric charge for the shape control can be deposited wirelessly by using electron gun. In order to control the curvature in two directions on the mirror surface, the authors suggested exciting both sides of the bimorph. It together with the anisotropy of the PVDF material property results tremendous complexity of the control. The uniformity of the glue layer in the bimorph also is a difficult factor in practice.

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In this paper, we propose a controllable mirror made of single-layer EAP. The EAP can be an isotropic, electric field activated material. An analytical solution of required voltage distribution for forming a parabolic mirror from an initially planar film is derived. Calculations show a single layer film made of currently available EAP has the capability to adjust the focus distance of 2-m mirror from infinity to 1.25 m. The results are verified by FEM model.

2. SINGLE-LAYER CONTROLABLE EAP MEMBRANE MIRROR

The EAP bimorph actuators show impressive bending deformation. Obviously, the bimorph actuators are able to change the curvature and the shape of thin film mirror. For a non-flat mirror made of single-layer uniform EAP film and a voltage is applied, we also predict some shape change because of the strain introduced by the voltage. Although a planar EAP film with no constrains will only change its size but curvature under a uniform distributed exciting voltage, a non-uniformly distributed voltage may result into a curved shape. An example is the way to make woks in early years by hummer. Ones hit more times in the area near the center of an initially flat plate of metal than the surrounding area. It makes the center part extended more than the outside and the flat metal plate become a curved wok. The same principle can be applied to the EAP film mirror. To apply a voltage decreased from center to the edge may deform the mirror from flat to desired optical shape.

The desired optical mirror is paraboloid . We express the curve as

$$z = ar^2, \quad (0 \le r \le r_1) \tag{1}$$

Suppose an original EAP film of the same diameter deformed to the paraboloid by area extension due to the applied electric field across the thickness as sown in Figure 1.

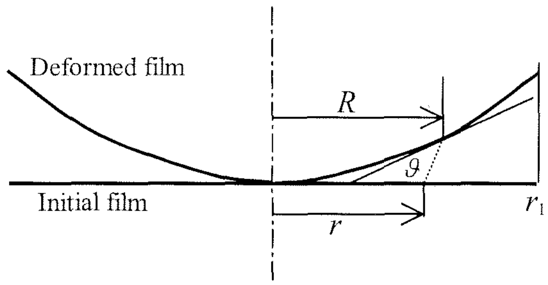

Figure 1. Paraboloid surface deformed from a planar disk.

We also assume the film is thin enough, so the bending stiffness can be neglected, i.e. we can treat it as a membrane. To avoid buckling of thin film, any negative in-plan stress (compress) should be avoided in the deformed film. Here we set the in-plan stress to be zero. Therefore, the strain in any area of the deformed film is the same as that with free boundary condition. For isotropic film materials, we have

$$S_c = S_r . \tag{2}$$

where the S denotes the strain, subscript c denotes the circumferential direction and r the tangent direction of the curve.

We define R(r) as the deformed radius of the original circle of radius of r. We have

$$S_c = \frac{R(r)}{r} - 1, \tag{3}$$

and

$$S_r = \frac{R'(r)}{\cos(\vartheta)} - 1, \tag{4}$$

where θ is the angle of tangent of the curve at radius of R. And we have

$$\frac{1}{\cos(\vartheta)} = \sqrt{1+\tan^2(\vartheta)} = \sqrt{1+(z')^2} = \sqrt{1+4a^2R^2}\ . \tag{5}$$

The differential equation is established as

$$R'(r) = \frac{R}{r\sqrt{1+4a^2R^2}}\,. \tag{6}$$

The differential equation has a solution as

$$\sqrt{1+4a^2R^2} - a\tanh\left(\frac{1}{\sqrt{1+4a^2R^2}}\right) = \ln(r) + c\,, \tag{7}$$

where the c is a constant that can be determined by boundary condition. If we set the diameter of the deformed mirror equal to the initial, the boundary condition is as

$$R(r_1) = r_1\,, \tag{8}$$

and the constant is

$$c = \sqrt{1+4a^2r_1^2} - a\tanh\left(\frac{1}{\sqrt{1+4a^2r_1^2}}\right) - \ln(r_1)\,. \tag{9}$$

A computed example is a mirror of 2 m in diameter. The target parabolic curve is $z = 0.2r^2$. The required strain to from such a mirror from a planar film of the same diameter is calculated and shown in Figure 2. As predicted, the required extension of the film has a maximum at the center and decreased with the radius. The maximum value is 4% that is reachable by the current available EAP material. The focus distance is

$$l = \frac{1}{4a}\,, \tag{10}$$

and equals to 1.25 m for this example. The f-number of the mirror, which is the ratio of the focus distance over the diameter, is

$$f = \frac{1}{8ar_1}\,. \tag{11}$$

This example mirror is operating at *f*/0.625.

A further investigation of the Eq.(7) and Eq.(9) could find that the required maximum strain is a function of ar_1 i.e. a function of f-number. The function is computed numerically and illustrated in Figure 3.

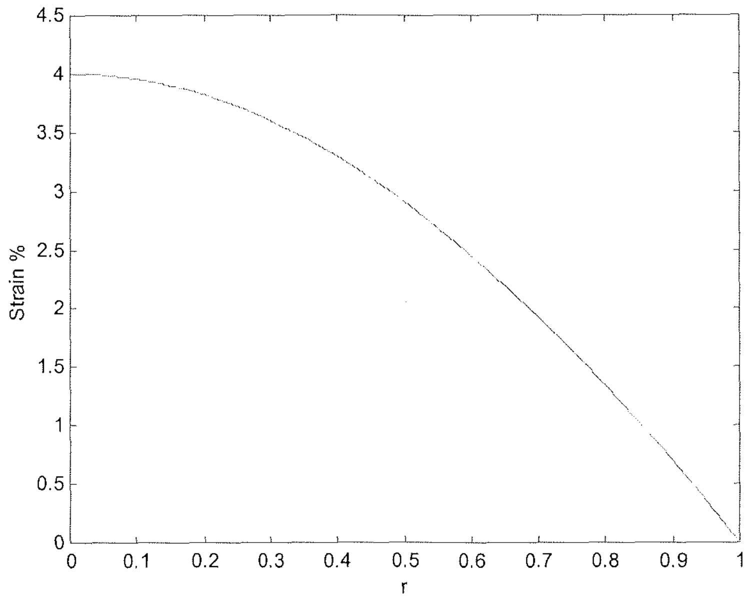

Figure 2. Required extension strain to form the parabolic mirror with $z = 0.2\ r^2$ from a planar sheet.

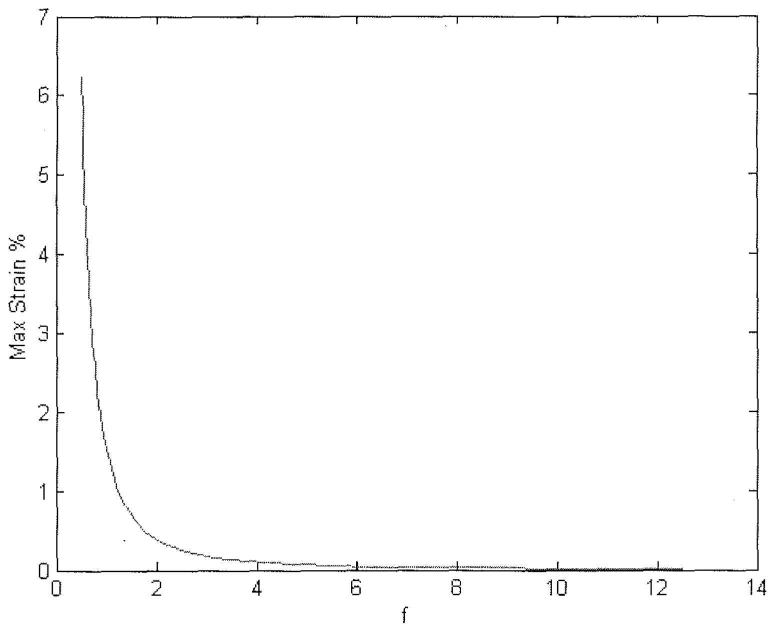

Figure 3. Maximum required extension strain as function of f-number of the parabolic mirror formed from a planar sheet.

3. FINITE ELEMENT SIMULATION

Finite element model was developed to verify the analytical results. Two EAP films with different thickness were simulated by FEM model. The geometry and material properties used in the simulation are listed in Table 1. The planar films were expressed by 100 shell elements (Shel-151, ANSYS). The extension strain created by the electric field in the EAP films was simulated by thermal expansion by set proper material thermal coefficient and corresponding temperature change. "Large Deflection" function of the ANSYS was activated to take the geometry nonlinear effect in count. The shell element does not support 'buckling' solution. Therefore, the direct solution of this model will be a planar film with in plan strain and stress. Physically, it usually is an unstable state for thin film or membrane because of compress stress in certain area. Any out-plan perturbation will let the film bends to a stable state, either one side or the other. To get the stable state solution, an artificial pressure had added first and was taken out after the first step solution. The pressure mimicked an initial side push on the film.

Table 1. Parameters used in FEM simulation

Film NO.	Diameter (m)	Thickness (μm)	E (Gpa)	Poisson's ratio σ
1	2	100	1	0.3
2	2	10	1	0.3

The results calculated by the FEM model are shown in Figure 4. The curve is fitted by parabolic formula as

$$z = ar^2 + b \tag{12}$$

No difference between the FEM results and the fitted curve can be found in the scale of the figure 4 for both films No.1 and No.2. We concluded that the analytical formula is correct. However, a close view of the data shows aberration. The analytical solution is derived under the membrane assumption. Therefore, the thickness, which introduced non-zero bending stiffness to the shell, may result into aberration from the parabolic curve. The parabolic aberration defined as the difference of the surface from the paraboloid is presented in Figure 5 and 6 for film thickness of 100 μm and 10 μm respectively. The both aberration are in the level of micrometer and only have small difference. Because the coordinate shifts 0.2 m in the FEM simulation, the parameters of the target curve is a = 0.2 and b = -0.2. The departures of parameters of the fitted curve from the target parabolic curve, which are in the level of 10^{-6}, are listed in Table2. The difference between the two films is also small. The comparison of the results for the two films implies that the effect of thickness, when it is less than 100 μm for 2 m diameter mirror, is rather small and the aberrations shown in both figures and the errors of the parameters of the fitted curves are mainly due to the accuracy of the numerical model.

Table 2. The departure of the parameters of the fitted curves from the target curve (a=0.2; b=-0.2)

Film No.	Thickness (μm)	Δa	Δb
1	100	4.097×10^{-6}	-1.472×10^{-6}
2	10	4.166×10^{-6}	-1.303×10^{-6}

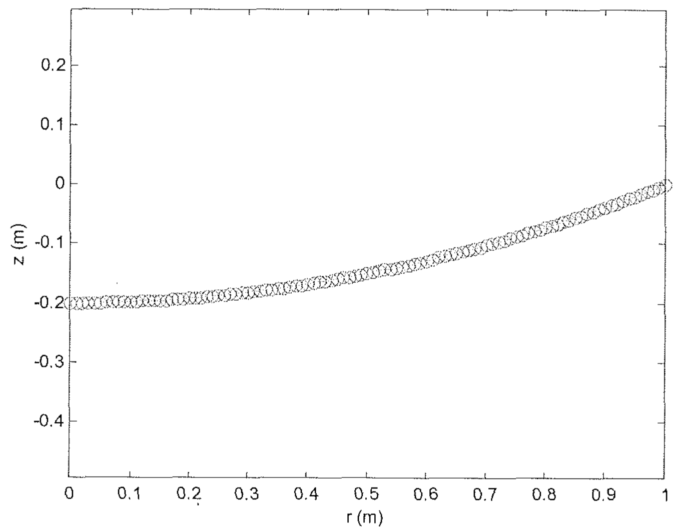

Figure 4. Deformed shape computed by FEM model (circle) and fitted curve of $z = ar^2 + b$ (line).

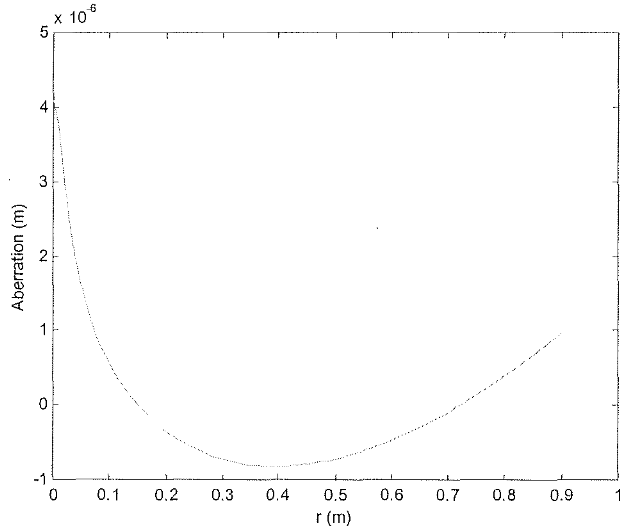

Figure 5. Aberration of the FEM results from the paraboloid, film thickness = 100 μm.

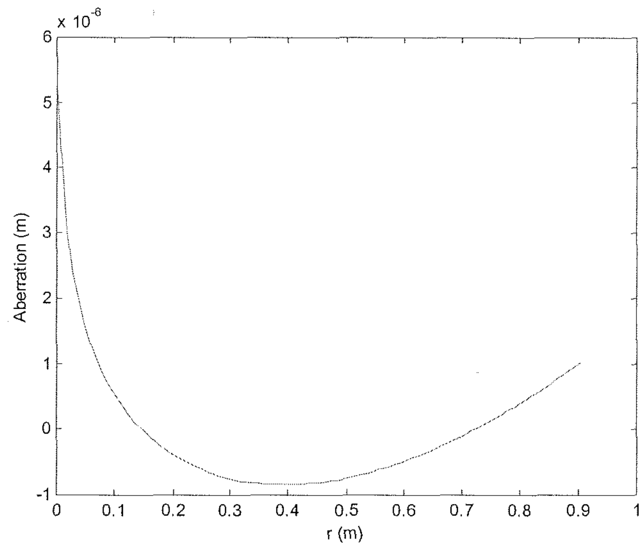

Figure 6. Aberration of the FEM results from the paraboloid, film thickness = 10 μm.

4. CONCLUSIONS

A controllable mirror made of single-layer EAP film is proposed in this paper. Applying distributed voltage to the backside of the EAP mirror can control the shape of the mirror. The voltage may be applied wirelessly by using electron gun as suggested in literature [2]. This control mechanism could be utilized along or combined with other control methods such as pressure in inflatable structure.

An analytical solution of required voltage/strain distribution for forming a parabolic mirror from a planar membrane is found explicitly. Computed results show a mirror made of single layer EAP film with maximum strain of 4% is able to control the focus distance of a 2-m mirror from infinity to 1.25 m.

The analytical solutions were verified by FEM model numerically. The FEM simulated results confirmed that the solution was correct. The numerical results also implied that the effect of the thickness of the film was not significant for a 2-m mirror if the thickness of the film is less than 100 μm. An improvement of the accuracy of the FEM model is required to determine the thickness effect in the level of submicron.

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REFERENCES

1. J.W. Wangner, G.S. Agnes, E. Magee, "Optical metrology of adaptive membrane mirrors," *J. Intell. Mater. Sys. Struc.* v 11, pp 837-847, 2000.
2. J. W. Martin, et al, "Distributed sensing and shape control of piezoelectric bimorph mirrors," *J. Intell. Mater. Sys. Struc.* v 11, p 744-757, 2000.
3. C.H. Jenkins, D.K. Marker, "Surface precision of inflatable membrane reflectors," *J. Solar Energy Eng. ASME,* v 120, p 298-305, 1998.
4. S.A. Dimkov, B.V. Kislitsyn, "Mathematical model of a variable-curvature thin-film mirror," *J. Opt. Technol.* v 67, p 229-234, 2000.
5. Y. Bar-Cohen (Ed.), *Electroactive Polymer (EAP) Actuators as Artificial Muscles - Reality, Potential and Challenges*, pp. 1-671, SPIE Press, Vol. PM98, 2001.
6. S. Nemat-Nasser, C. Thomas "Ionic Polymer-Metal Composite (IPMC)," Topic 3.2, Chapter 12, "Electroactive Polymer (EAP) Actuators as Artificial Muscles - Reality, Potential and Challenges," ibid, pp. 139-191, 2001.
7. K. Onishi, S. Sewa, K. Asaka, N. Fujiwara, K. Oguro, "Morphology of electrodes and bending response of the polymer electrolyte actuator," *Electrochimica Act* v 46, p 737-743, 2000.
8. X. Bao, Y. Bar-cohen, S. Lih, "Measurements and macro models of ionomeric polymer-metal composites (IPMC)," *Proc. of SPIE*, v 4695, p 220-227, 2002.
9. Y. Bar-Cohen, X. Bao, S. Sherri, S. Lih, "Characterization of the electromechanical properties of Ionomeric Polymer-Metal Composite (IPMC)," *Proc. of SPIE*, v 4695, p 286-293, 2002.
10. R.C. Costen, J. Su, J.S. Harrison, "Model for bending actuators that use electrostrictive graft elastomers," *Proc. of SPIE*, v 4329, p 437-444, 2001.
11. Z.-Y. Cheng, H.S. Xu, T. Mail, M. Chung, Q.M. Zhang, R.Y. Ting, "P(VDF-TrFE)-based electrostrictive Co/Ter-polymers and its device performance," *Proc. of SPIE*, v 4329, p 106-116, 2001.
12. Q.M. Zhang, H. Li, M. Poh, F. Xia, Z. -Y. Cheng, H. Xu, C. Huang, "An all-organic composite actuator material with high dielectric constant," *Nature*, v 419, p 284-287, 2002.