Ultra-Refractive and Extended-Range One-Dimensional Photonic Crystal Superprisms

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ABSTRACT

We describe theoretical analysis and design of one-dimensional photonic crystal prisms. We found that inside the photonic crystal, for frequencies near the band edges, light propagation direction is extremely sensitive to the variations in wavelength and incident angle. Outside the photonic crystal, propagation direction of transmitted light is highly sensitive to changes in wavelength, but not incident angle. For transmitted light outside the photonic crystal, the superprism effect can be used for wavelength dispersion over entire photonic bands, rather than just near the band edges only. This extended-range superprism effect broadens the useful wavelength range, admits better optical transmission, and exhibits angular dependence on wavelength with greatly reduced non-linearity.

Keywords: photonic crystal, one-dimensional, superprism, ultra-refraction

1. INTRODUCTION

A photonic crystal[1,2,3,4] is a periodic structure consisting typically of two dielectric materials with high dielectric contrast (e.g., semiconductor and air), and with geometrical feature sizes comparable to or smaller than light wavelengths of interest. As an engineered structure or artificially engineered material, a photonic crystal can exhibit optical properties not commonly found in natural substances. Extensive research has led to the discovery of several classes of photonic crystal structures for which the propagation of electromagnetic radiation is forbidden in certain frequency ranges (photonic band gaps, or PBGs). More recently, it has also been recognized that electromagnetic radiation with frequency just outside the photonic band gaps can propagate in photonic crystals with characteristics that are quite different from those of ordinary optical materials. Lin *et al.* reported on a highly dispersive two-dimensional (2D) photonic band-gap prism in the millimeter-wave region, where a 30% change in frequency resulted in 20% change in the refractive index.[5] They attributed the large index variations to the highly nonlinear dispersions in photonic band structures near the Brillouin zone edge. While Lin *et al.* demonstrated that a 40° incident angle change resulted a 20% angle change outside the 2D photonic crystal, Kosaka and co-workers subsequently showed that in a three-dimensional (3D) photonic crystal a change of ±12° or less in incidence angle would result in a swing from -90° to 90° in the propagation angle inside the photonic crystal.[6] They also demonstrated that two light beams entering a 3D photonic crystal at the same incidence angle with slightly different wavelengths (0.99 μ m and 1 μm) diverged by 50° inside the photonic crystal[7,8]; the same pair of beams entering a conventional optical material at the same angle would diverge by less than 1° after incidence. Kosaka *et al.* attributed their observations to a different physical mechanism, explaining their results in terms of anisotropy in photonic band structures using group velocities in wave vector diagrams.[9,10,11] They called this unusually large refractive and color-dispersion capability the "superprism" phenomenon to distinguish it from previously observed photonic crystal prism effects of Lin and co-workers.[5]

The unusual propagation characteristics of electromagnetic waves in photonic crystals have been analyzed by a number of researchers. It is commonly recognized that the highly nonlinear and anisotropic dispersion of PBG materials near Brillouin zone edges can lead to strong dispersion of electromagnetic waves with frequency just outside the photonic band gaps, and can therefore be used to form highly dispersive prisms and other optical elements.[12,13,14,15,16] The photonic crystal used in the superprism demonstration is a complex three-dimensional (3D) structure fabricating using

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the "autocloning" technique.[17,18] However, Kosaka *et al.* pointed out that 2D photonic crystals should show similar phenomena.[8] This has been shown to be indeed the case by theoretical analyses[12,13,19] and experimental observations.[20] In a 1D photonic crystal, the structure is periodically modulated along the principal axis, but translationally invariant along the perpendicular directions. The multilayer geometry easily leads to highly anisotropic photonic band structure, and it should come as no surprise that the superprism phenomenon is also present in one-dimensional (1D) photonic crystals. In fact, anomalous refraction phenomena had been observed in 1D grating waveguides before photonic crystals became an active field of research.[10,11]

We were motivated to explore 1D photonic crystals because (1) their structures are exceedingly simple and therefore easier to fabricate and design, and, (2) their highly anisotropic photonic band structures can lead to ultra-refraction. In this work we analyze the superprism effect in a simple a 1D photonic crystal structure. Figure 1 shows the structure of a typical 1D photonic crystal. It consists of a periodic stack of alternating layers of two materials with different layer thickness (h_1 and h_2) and relative permittivities (ε_1 and ε_2). The period is given by $h_1 + h_2 = a$. In principle, there could be more than two layers per (repeating) period. The analysis described here also applies to structures having more than two layers per period. We will show that light propagating inside the photonic crystal with frequencies just outside the photonic band gaps can indeed exhibit ultra-refractive behavior. We will also show that strongly refractive behavior is seen for light transmitted through a 1D photonic crystal prism. Moreover, this unconventional refraction property effectively can be used for wavelength dispersion over entire photonic bands, rather than just near the band edges. This extended-range superprism effect broadens the useful wavelength range, admits better optical transmission, and exhibits angular dependence on wavelength with reduced non-linearity. A comparison between the normal and the extended-range modes of the 1D photonic crystal superprism also serves to clarify the differences and similarities between the operations of the highly dispersive photonic crystal prism of Lin *et al.*,[5] and the photonic crystal superprism of Kosaka and co-workers.[6]

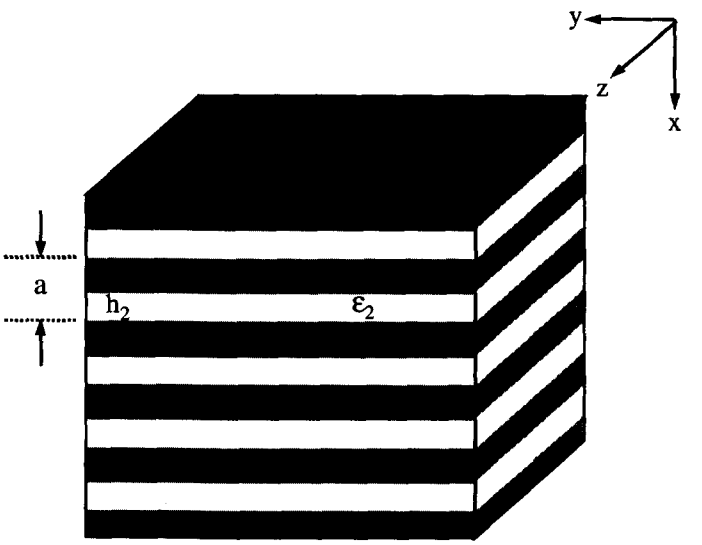

Figure 1. Illustration of a 1D photonic crystal.

2. METHOD OF ANALYSIS

To analyze superprism effects in 1D photonic crystals, we consider the schematic geometry of Fig. 2 where a collimated light ray of wavelength λ in a homogeneous medium (e.g. air) reaches the surface of the 1D photonic crystal slab at an incidence angle θ_{inc}. For simplicity we consider the case where the direction of wave propagation is restricted in the x-y plane. After entry into the photonic crystal, the light ray propagates at an angle of refraction θ_{pc}. To compute the relationship between θ_{inc} and θ_{pc} for a given wavelength, we match the frequency and tangential component of the wave vector for the incident and refracted wave across the interface using the following simple procedure. We specify the angular frequency $\omega = 2\pi f$ (f is the frequency of light) and the incidence angle θ_{inc} in the homogeneous medium. Using the relationships $\varepsilon_r \omega^2 = k_x^2 + k_y^2$ and $\tan(\theta_{inc}) = (k_y / k_x)$ we can find the wave vector in the incident medium. Here ε_r is the relative permittivity of the incident medium, and k_x and k_y are the components of the wave

vector perpendicular and parallel, respectively, to the interface between the homogeneous medium and the photonic crystal. (We assume $k_z = 0$ for simplicity.) We can compute the 1D photonic crystal dispersion relationship, or photonic band structure, $\omega(\mathbf{k})$, using the transfer matrix method[21], a standard technique commonly found in the literature. Here the angular frequency ω and the parallel component of the wave vector k_y are the same as those in the incident homogeneous medium. The transfer matrix technique allows us to find the perpendicular component of the wave vector k_x in the photonic crystal. From $\omega(\mathbf{k})$ we can then compute the photonic crystal group velocity using $\mathbf{v}_g = \nabla_k \omega(\mathbf{k})$ $= (\partial\omega(\mathbf{k})/\partial k_x,\ \partial\omega(\mathbf{k})/\partial k_z,\ \partial\omega(\mathbf{k})/\partial k_z)$. From the components of the group velocity we can determine the angle of refraction using $\theta_{pc} = \tan^{-1}(v_{g,y}/v_{g,x})$. A similar procedure, discussed in the Sec. 3.2, is use to compute the angle of propagation after the ray exits the photonic crystal.

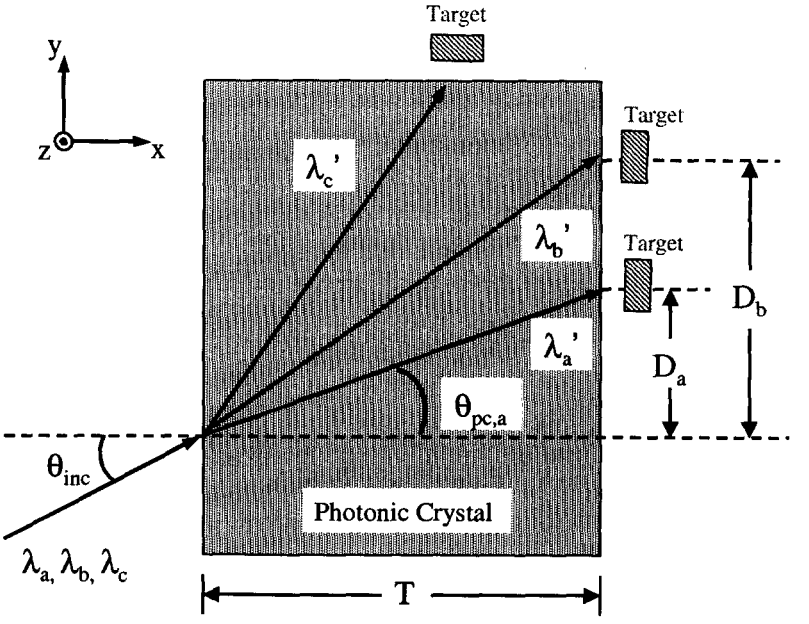

Figure 2. Illustration of how a 1D photonic crystal might be used to disperse light of different wavelengths.

3. RESULTS

In this section we will first explore the ultra-refractive properties of 1D photonic crystals, and then discuss the extended-range superprism effect. We take the specific example of a 1D photonic crystal with $\varepsilon_1 = 11.56$ and $\varepsilon_2 = 1$ (appropriate for silicon and air, respectively), and $h_1 = 0.2a$ and $h_2 = 0.8a$ (i.e., 20% of each period is occupied by the first material). Figure 3 shows the band structure for this photonic crystal, computed using a finite difference method. Angular frequency ω is plotted against the wave vector $\mathbf{k}$, in units of $(2\pi/a)$, along three different directions in the k_x-k_y plane. Throughout this paper, we describe ω in units of $(2\pi c/a)$, i.e., as normalized frequency. Since $\omega/(2\pi c/a) = a/\lambda$, the normalized frequency is sometimes written as the dimensionless quantity a/λ; we use this notation in our discussions. We note in Fig. 3 that although gaps are present in the dispersion curves along the k_x direction, they are not true omni-directional band gaps, as evident in the dispersion curves along the k_y direction. In general, 1D photonic crystals do not have a true omni-directional band gaps. However, since the superprism effect is a consequence of the non-linearity and anisotropy in the photonic band structure, the absence of photonic band gaps does not preclude anomalous refraction behavior. We examine wave propagation inside the 1D photonic crystal in Sec. 3.1, and study the behavior of transmitted waves outside the photonic crystal in Sec. 3.2.

3.1 Ultra-Refraction in 1D Photonic Crystals

Figure 2 illustrates how we could use a slab of 1D photonic crystal to separate a polychromatic beam into different wavelength components. We assume for now that the front and back surfaces of the slab are parallel, and that both surfaces are perpendicular to the principal axis of the photonic crystal. For the remaining discussion, we shall assume that the principal axis of the photonic crystal is along the x-direction. Assuming we could arrange for the wavelength components in the incident beam to be within an ultra-refractive range, then light of different wavelengths could be dispersed into a wide range of refraction angles (θ_{pc}'s) inside the photonic crystal. The figure illustrates that the wavelength components λ_c can be directed towards a different facet of the photonic crystal slab, and be separated from λ_a and λ_b. Figure 2 also illustrates that wavelength components λ_a and λ_b can be separated by having different lateral (y direction) displacements D_a and D_b along the back surface; they can be directed to different targets (e.g., detectors or ports) placed at suitable locations just outside the back surface. We note in Fig. 2 that the lateral displacement is given by $D = T\tan(\theta_{pc})$, where T is the total thickness of the photonic crystal slab.

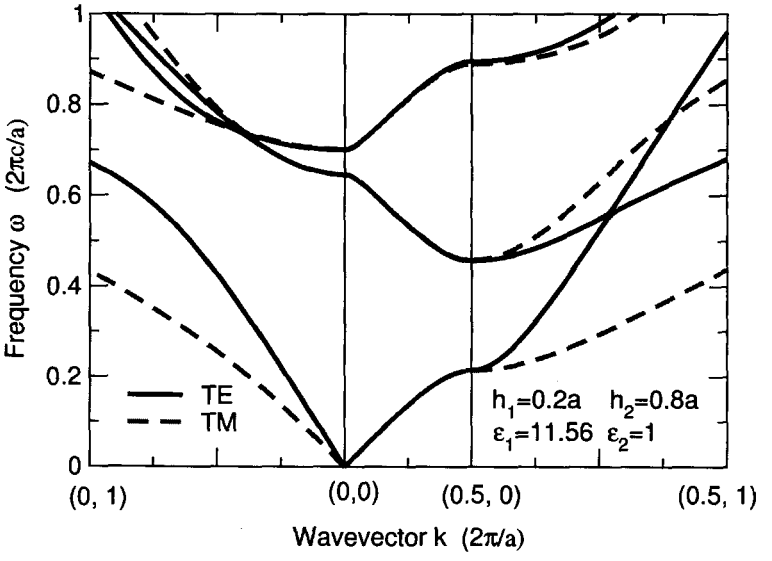

Figure 3. Band structure of a 1D photonic crystal. The lowest three frequency bands are shown.

Figure 4 shows the calculated refraction angle inside the 1D photonic crystal as a function of incident wavelength. The incidence angle is fixed at $\theta_{inc} = 10^\circ$. All angles shown are in the x-y plane, and measured with respect to the x-axis. All calculations are performed for TE-mode radiation (electric field along the z-axis); calculations for TM-modes can also be performed in a similar fashion, with qualitatively similar results. The regions in the figure labeled as "Gap 1" and "Gap 2" correspond to the gaps in frequency bands where propagation of electromagnetic radiation is forbidden inside the photonic crystal.[22] Figure 4 illustrates that in wavelength ranges just outside the gaps, the angle of refraction θ_{pc} can vary extremely rapidly as a function of wavelength, much more so than in conventional optical material, demonstrating the superprism effect or ultra-refraction in our 1D photonic crystal. It should be noted that, except in the lowest band, θ_{pc} is not monotonic within a given band.

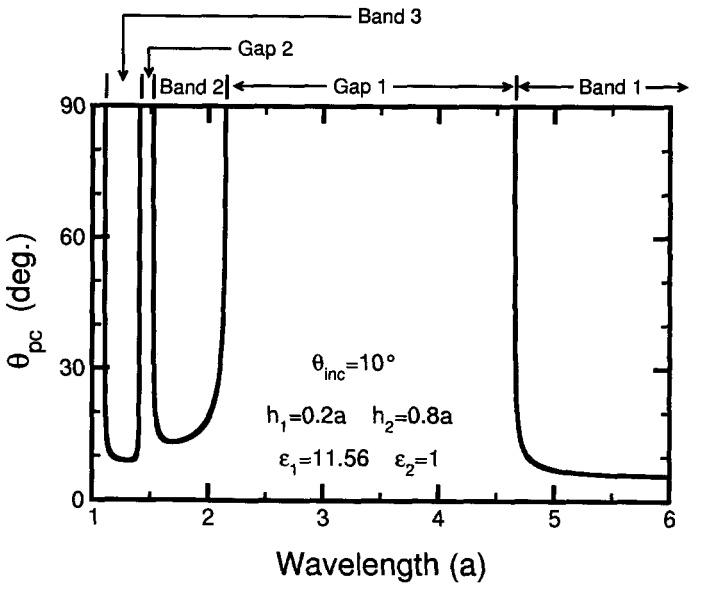

Figure 4. Propagation angle inside a 1D photonic crystal slab as a function of incident beam wavelength (in units of "*a*", the photonic crystal period).

Table 1. Ranges in which refraction angles vary rapidly (but monotonically) with wavelength. The wavelengths are given in units of the photonic crystal period "*a*," and angles in degrees. The incident angle is 10°.

	Wavelength Range		Refraction Angle Range	
	Starting λ (*a*)	Ending λ (*a*)	Starting θ_{pc} (deg.)	Ending θ_{pc} (deg.)
Range 1	4.6565	4.6798	70	20
Range 2	2.0257	2.1463	20	70
Range 3	1.5267	1.5463	70	20
Range 4	1.4048	1.4075	20	70
Range 5	1.1078	1.1156	70	20

Table 1 tabulates five distinct wavelength ranges in which the respective refraction angle varies rapidly and monotonically between 20° and 70°. Table 1 shows that, within each of the listed narrow wavelength range, two beams of light entering the photonic crystal at the same angle of incidence θ_{inc} with slightly different wavelengths can have angles of refraction inside the photonic crystal (θ_{pc}) which are substantially different, demonstrating instances of the superprism effect or ultra-refraction. Note that in the present configuration ultra-refraction occurs just outside the gaps in the spectrum shown in Fig. 4. Electromagnetic waves with wavelengths inside the gap cannot propagate inside the photonic crystal, and therefore is totally reflected back into the incident medium. These gaps correspond to the gaps in frequency bands where propagation of electromagnetic radiation is forbidden inside the photonic crystal. In Table 1, Range 1 and Range 2 respectively corresponds to electromagnetic radiation with frequency ranges just below and just above the lowest (in frequency) gap (labeled Gap 1 in Fig. 3), Range 3 and 4 surround the second gap (Gap 2), etc. Not all of the ultra-refraction ranges are shown, as Fig. 4 extends only over the wavelength from 1*a* to 6*a*. In principle, there are an infinite number of gaps with frequency ranges higher than (or wavelength ranges shorter than) the ones shown in Fig. 4. In other words, there are an infinite number of ultra-refraction ranges with wavelengths shorter than 1*a*. However, these ultra-refraction wavelength ranges are very closely spaced. For practical applications, we would probably restrict ourselves to the use of the ultra-refraction ranges surrounding the lowest few photonic band gaps.

In the model computations leading to Table 1 and Fig. 4, the wavelength and the layer widths (h_1 and h_2) were expressed in units of the one-dimensional photonic crystal period "*a*" to show that the ultra-refractive wavelength range of interest is adjusted by scaling the layer widths. By way of example, if the range of the wavelengths of interest resides between 1.527 μm and 1.546 μm, as characterized by Range 3, we set a = 1 μm (or h_1 = 0.2 μm and h_2 = 0.8 μm). If, however, we are interested in dispersion light with wavelengths between 3.053 μm and 3.093 μm, we set a = 2 μm (or h_1 = 0.4 μm and h_2 = 1.6 μm) instead. If we are interested in dispersion light in a range of wavelengths beyond 1.5 μm, we have the option of using any of the ultra-refractive wavelength ranges listed in Table 1 by selecting an appropriate value for the photonic crystal period *a*.

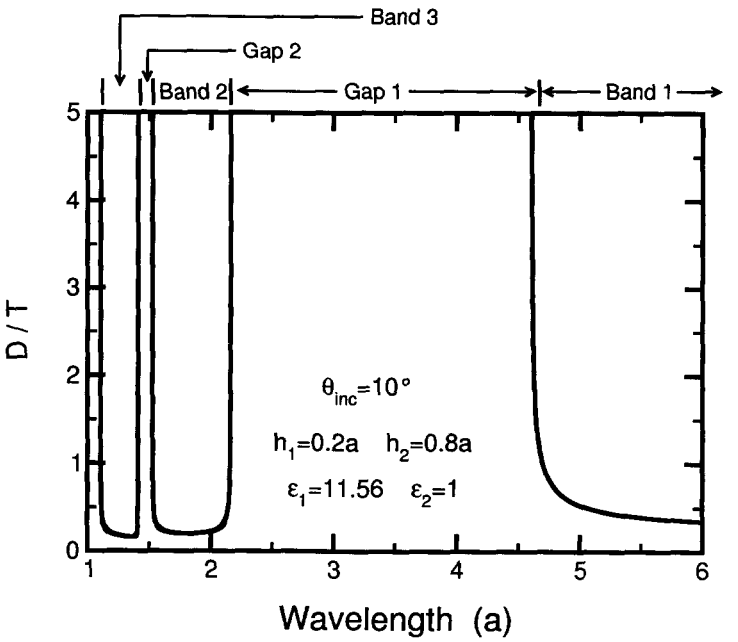

Figure 5. The ratio between lateral displacement (*D*) and crystal slab thickness (*T*) as a function of incident wavelength.

In Fig. 5 we plot the lateral displacement D in units of photonic crystal slab thickness T, i.e., D/T or $\tan(\theta_{pc})$, as a function of incident wavelength for an incident angle of $\theta_{inc}=10°$. We find that the lateral displacement also varies rapidly over narrow wavelength ranges associated with the band edges (just outside the reflection gaps), in much the same way that θ_{pc} does in Fig. 3. The plot of displacement versus wavelength underscores another feature of using ultra-refraction in this way. As wavelength approaches a gap, the displacement increases rapidly in a highly non-linear fashion. While this hypersensitivity of displacement to wavelength is precisely the effect we wish to exploit, the use of this effect would also require a high degree of precision in the location of targets (e.g. detectors). In addition, we would expect diminished transmission into the photonic crystal as wavelength approaches the gaps where transmission is not allowed. The use anti-reflection coating might be necessary to ensure that we can couple enough light into the photonic crystal to take advantage of ultra-refraction. One way to increase the coupling efficiency is to introduce appropriate artificial interface structures, as shown by the theoretical work of Baba and Ohsaki recently.[23]

Another characteristic of the superprism effect is the rapid change in the angle of refraction as the incidence angle varies. Figure 6 shows the calculated refraction angle as a function of incidence angle for frequencies just below and above the lowest band gap.[22] Slightly away from the band edge, at normalized frequencies of $a/\lambda = 0.205$ and $a/\lambda = 0.210$, a change of ±90° in incidence angle result in only a swing of approximately ±35° in refraction angle inside the photonic crystal, which is comparable to the behavior in ordinary dielectric material. However, much stronger effects are observed at frequency closer to the band edges. In particular, for $a/\lambda = 0.46$, the refraction angle inside the photonic crystal swings from -90° to 90° as the incident angle changes from -1.74° to 1.74° .

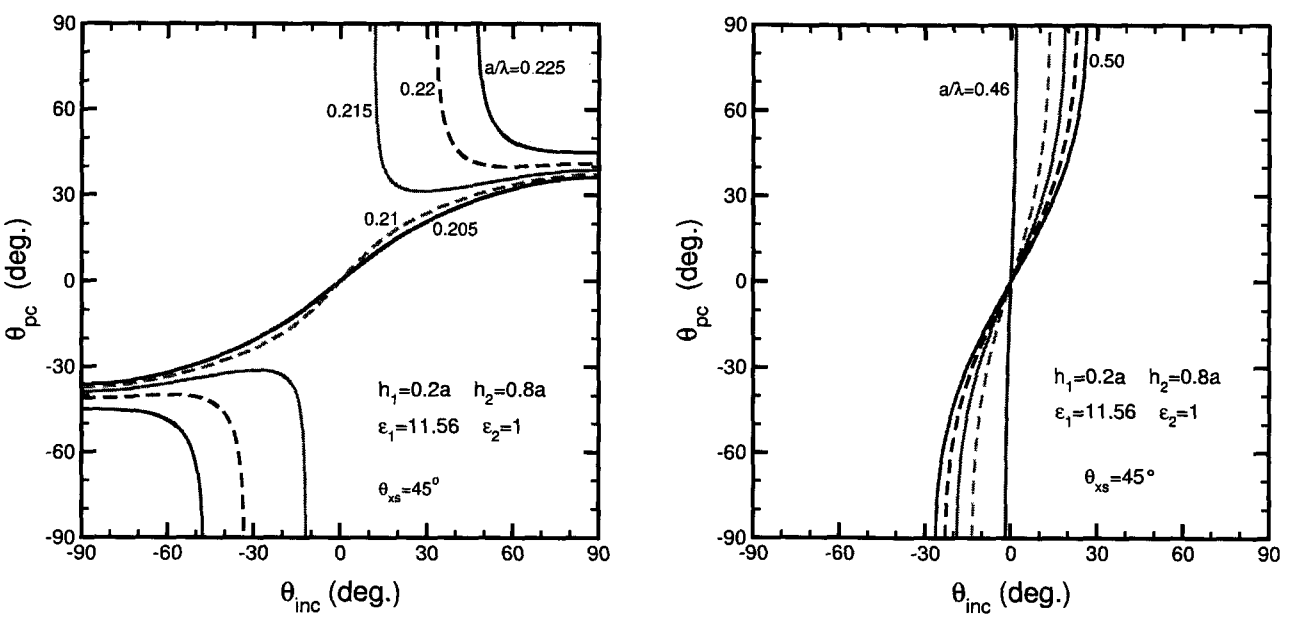

Figure 6. Angle of refraction inside the 1D photonic crystal as a function of the incidence angle. The left and right panels respectively show results for frequencies just below and above the lowest band gap. The frequencies in each panel are given as dimensionless normalized frequencies (a/λ), and are separated by equal intervals.

3.2 Extended-Range Superprism Effect in 1D Photonic Crystals

For certain applications (e.g. spectroscopy), band-edge ultra-refraction characteristics such as high non-linearity, and diminished transmission might not be desirable. Here we consider an alternative implementation that deals effectively with these issues. Figure 7 shows a prism-shaped photonic crystal. The principal axis of the 1D photonic crystal is along the x-axis. We assume that the normal vector for the incident surface is also along the x-axis, and that the normal vector of exit surface is at an angle θ_{xs} with respect to the x-axis. A beam of polychromatic electromagnetic wave entering from the incident surface at an angle θ_{inc} would be dispersed in the photonic crystal prism, and different wavelength components would exits the slanted back surface at different exit angles θ_{out}.

To compute the exit angle θ_{out}, we match the frequency and the tangential component of the wave vector $\mathbf{k}$ of the refracted and transmitted waves across the exit interface. In Sec. 2 we described a procedure for computing θ_{pc}, the refraction angle inside the photonic crystal, by calculating $\mathbf{k}_{pc}$, the refracted wave vector in the photonic crystal, from $\mathbf{k}_{inc}$, the incident wave vector. In should be noted that in general, $\mathbf{k}_{inc}$ can couple to a set of wave vector $\{\mathbf{k}_{pc}+\mathbf{G}_n\}$, where $\mathbf{G}_n = (2\pi n/a)\hat{x}$, n is an integer, are the reciprocal lattice vectors of the 1D photonic crystal. Since, in the case of the simple 1D photonic crystal under consideration, group velocities are the same for all $\mathbf{k}_{pc}+\mathbf{G}_n$'s, θ_{pc} can be determined by computing the group velocity for $\mathbf{k}_{pc}$ only.

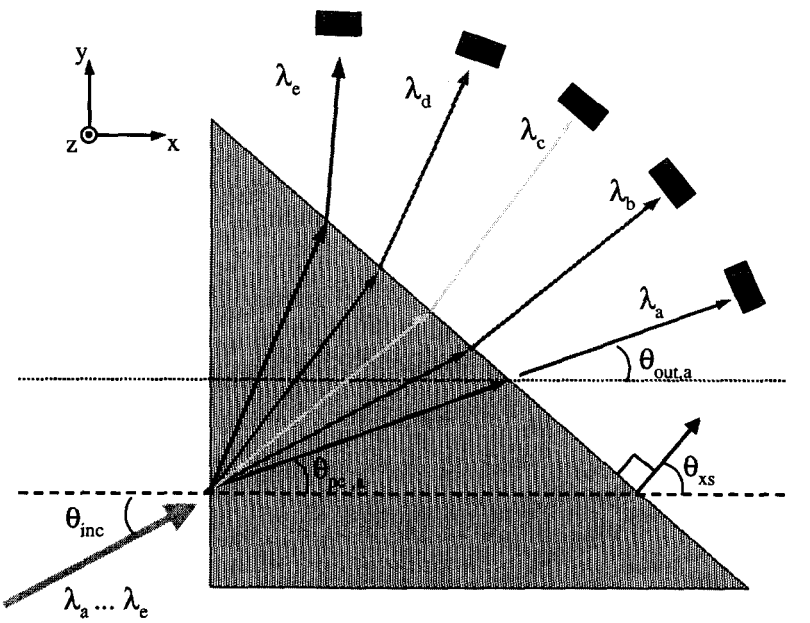

Figure 7. A 1D photonic crystal based prism, with an angled exit surface.

However, since $\{\mathbf{k}_{pc}+\mathbf{G}_n\}$ can couple to a large number of transmitted wave vectors $\{\mathbf{k}_{t,n}\}$ (analogues to 'Umklapp" terms in solid state physics), with distinct propagation directions, the determination of θ_{out} is more complicated. In general, we could use a computational electromagnetic wave method such as the rigorous coupled wave analysis (RCWA)[24] to solve the problem. However, here we use a very simple method that allows us to obtain an approximate solution with minimal computation. In phase matching across the exit surface, we simply use the extended zone scheme[25] to describe the band structure of the photonic crystal. In this approximate procedure, we select a single refracted wave vector $\mathbf{k}_{pc}+\mathbf{G}_n$ for a given frequency, and then phase match it to single $\mathbf{k}_{t,n}$, from which we compute a single propagation angle of the transmitted wave θ_{out}. This procedure yields the correct answer in the limit when $\varepsilon_1 = \varepsilon_2$, and should be a very good approximation in the case of weakly modulated structures. Our conjecture is that this procedure should describe the main contribution to the transmitted wave. Baba and Nakamura have shown in a finite difference time domain (FDTD) calculation of a 2D photonic crystal superprism that the transmitted wave consists of a main output beam as well as many diffracted waves,[16] but the diffractive waves can be largely suppressed by using additional interface structures at the exit surface. We expect similar conclusions could also be drawn for the 1D photonic crystal superprism.

Figure 8 shows the calculated exit angle as a function of incident wavelength for a set of prisms with four different exit surface orientations, characterized by θ_{xs} = 0°, 15°, 30°, and 45°. The incidence angle is θ_{inc} = 10° for all four cases. For θ_{xs} = 0°, the back and front surfaces are parallel, and, as expected, the exit angle is the same as the incident angle for all transmitted wavelengths, i.e., $\theta_{out} = \theta_{inc}$ = 10°. For the other values of θ_{xs}, the exit angle θ_{out} exhibits substantial variations as functions of transmitted wavelength, as we have seen earlier with θ_{pc} the angle of propagation in photonic crystal. For instance, as listed in Table 1, we could use the dispersion in θ_{pc} for wavelengths in Range 2, covering $\lambda = 2.1203a$ to $\lambda = 2.1592a$, or Range 3, covering $\lambda = 1.5257a$ to $\lambda = 1.5459a$. However, using the dispersion in θ_{out}, we could cover both Range 2 and Range 3, plus all the wavelengths in between, giving total coverage from $\lambda = 1.5257a$ to $\lambda = 2.1592a$, due to the fact that the variation in θ_{out} is monotonic over the entirety of each band. Although the change in θ_{out} per unit wavelength is not as dramatic as that for θ_{pc}, it is still exceedingly large compared to ordinary homogeneous material. Moreover, the use of θ_{out} broadens the useful wavelength range considerably. We also note that the dispersion of θ_{out} is much closer to being linear in wavelength, making it much more controllable than the highly non-linear variation of θ_{pc}. Since the dispersion of θ_{out} occurs within the entire transmission band, rather than at the band edges near reflection gap where transmission amplitudes diminish, we also expect better optical coupling efficiency. Note that the strong dispersion in θ_{out} is also evident in higher bands, which corresponds to longer wavelengths. Using the higher bands offers an additional advantage: it

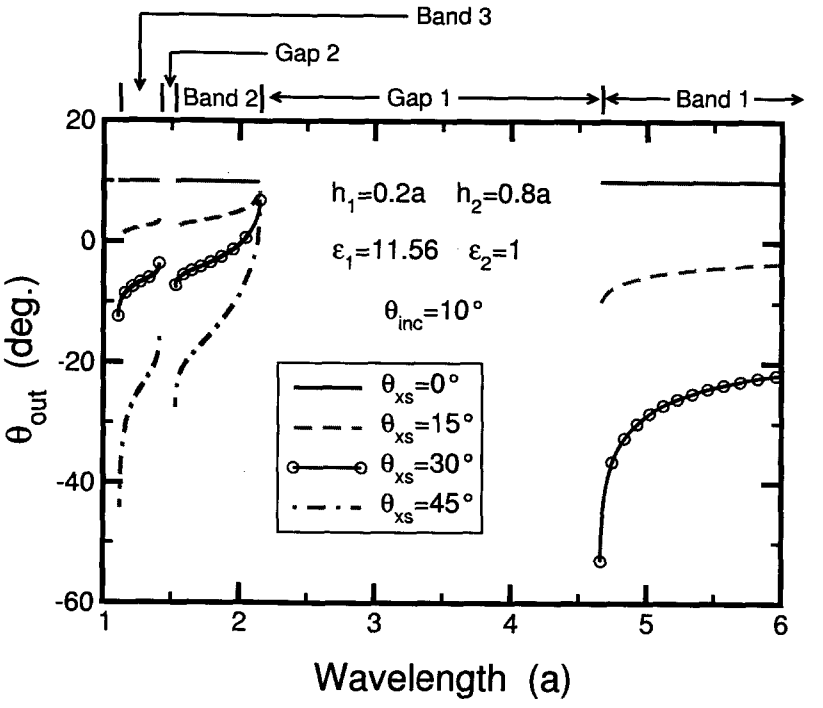

Figure 8. Exit beam angle as a function of incident wavelength, after traversing the photonic crystal prism.

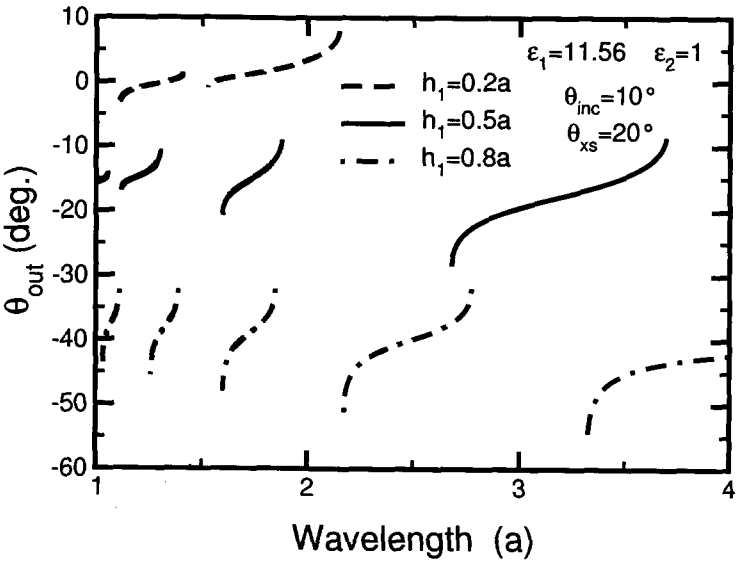

Figure 9. Exit beam propagation angle, θ_{out}, as a function of incident wavelength after traversing 1D photonic crystal prisms for three different layer-width combinations.

permits the use a larger period (a) to cover the same wavelength range. This reduced fabrication requirements since larger feature sizes are in general easier to make. However, a more rigorous investigation should used to determine whether the high bands could couple sufficiently strongly to incident radiation.

The properties of the 1D photonic crystal superprism may be tuned by changing design parameters such as layer widths and incidence beam angle. Figure 9 shows the exit angle θ_{out} as a function of wavelength for a set of three structures with different layer widths: (1) h_1= 0.2a, h_2=0.8a, in dashed lines with circles, (2) h_1= 0.5a, h_2=0.5a, in solid lines, and (3) h_1= 0.8a, h_2=0.2a, in dash-dot lines. The same angle of incidence of θ_{inc} = 10° and exit surface angle of θ_{xs} = 20° are used for all 3 cases. Figure 10 shows the exit angle θ_{out} as a function of wavelength for a structure with h_1= 0.5a, h_2=0.5a, and exit surface angle θ_{xs} = 45°. Three different incidence angle of θ_{inc} = 10°, 20°, and 30° are used. Figures 9 and 10 show that although the structure of the 1D photonic structure is simple, it still offers a rich set of design parameters, allowing opportunities for tailoring superprism characteristics.

Figure 11 shows the exit angle θ_{out} as a function of incidence angle for a 1D photonic crystal superprism with an exit surface angle θ_{xs} = 45° at selected frequencies. Note that due to the constraint imposed by θ_{xs}, the lower limit on θ_{out} is -45° . The frequencies selected for this plot are the same as those in the right panel of Fig. 6, where very large angular spreads in θ_{pc} are found for small changes in incidence angle. Here, however, we notice that the angular spread in θ_{out} is considerably smaller. For example, for normalized frequency of $a/\lambda = 0.46$, as the incident angle changes from -1.74° to 1.74° , θ_{pc} swings from -90° to 90° , while θ_{out} changes from -7.3° to -3.4° . For frequencies farther away from the band edge, the range of θ_{out}

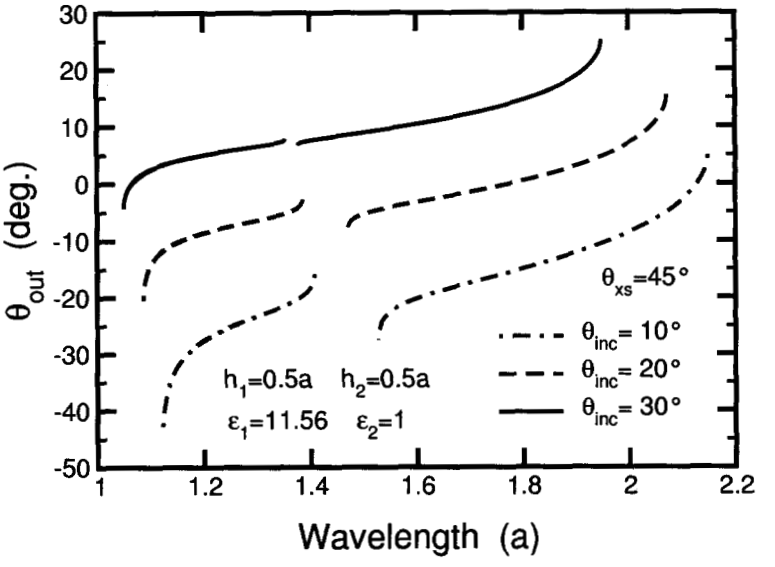

Figure 10. Exit beam propagation angle, θ_{out}, as a function of incident wavelength after traversing a 1D photonic crystal prism from three different incidence angles, θ_{inc}.

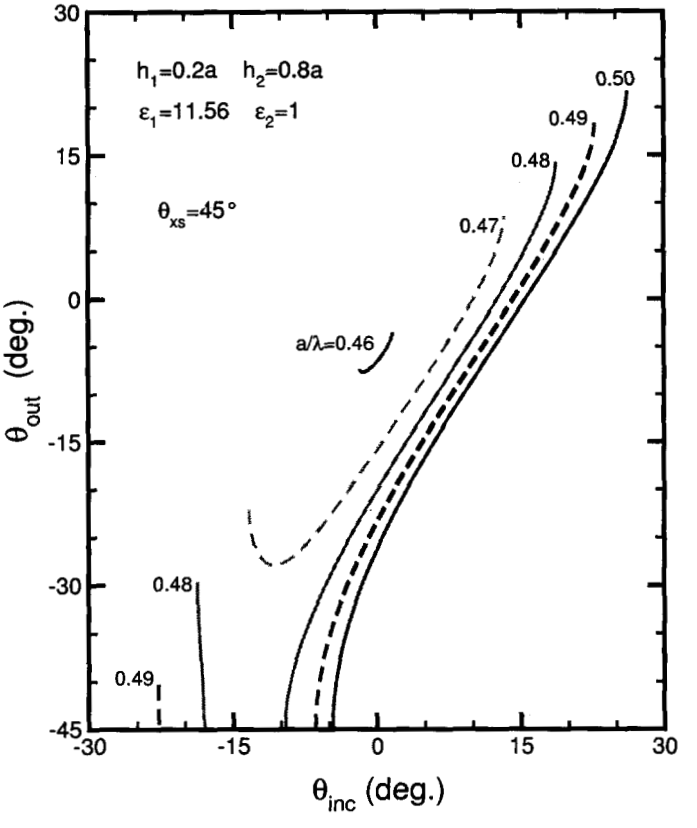

Figure 11. Exit beam propagation angle as a function of the incidence angle for frequencies just above the lowest band gap.

increases, but the corresponding range for θ_{inc} also increases. In general, we find that the degree of variations in θ_{out} as induced by changes in θ_{inc} is only comparable to that found for homogeneous material. It is also interesting to note that Fig. 11 shows that in most cases the exit beam angle is not a monotonic function of the incidence angle. This is reminiscent of the experimental observations of Lin *et al.* [5]

4. DISCUSSIONS AND SUMMARY

A strong motivation for studying 1D photonic crystal superprisms is that they are potentially easier to fabricate. The 1D photonic crystal prism might be made in a number of ways. One possibility would be to define a set of equally spaced, vertical, parallel plates using lithographical techniques. As pictured in Fig. 12, this might be implemented using a SOI (silicon on insulator) wafer,[26,27] which consists of a silicon dioxide layer sandwiched between silicon layers. The 1D photonic crystal would be defined in the top silicon layer, while the oxide layer, having a lower index than silicon, serves as a cladding layer to confine light in the photonic crystal. Since all the surfaces in the 1D photonic crystal are parallel, simple anisotropic wet etching could also be used in the fabrication process. For instance, KOH (potassium hydroxide) can be used to etch [110]-oriented silicon to produce the parallel-plate 1D photonic crystal structure inexpensively. The 1D photonic crystal structure could also be implemented in GaAs on Al_xO_y, using a procedure described in published literature.[28] The parallel plates of the 1D photonic crystal can be defined in a GaAs layer on top of $Al_{0.9}Ga_{0.1}As$ layer using electron beam lithography and reactive ion etching. The $Al_{0.9}Ga_{0.1}As$ layer can be wet oxidize to give an Al_xO_y layer, which, having a lower index than GaAs, acts as a lower cladding layer to help confine light in the GaAs photonic crystal.

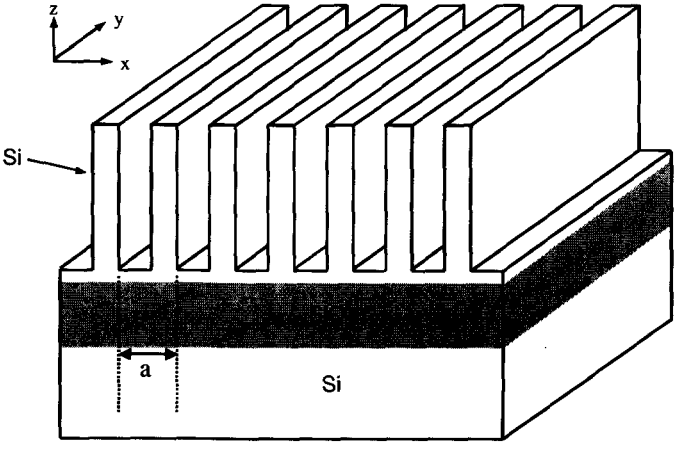

Figure 12. Illustration a 1D photonic crystal superprism fabricated on an SOI (silicon on insulator) wafer. A similar structure can also be made in GaAs.

Another reason that motivated us to explore 1D photonic crystal superprisms is that they are easy to design and analyze due to their simple structures. In fact, using the transfer matrix technique, one can easily obtain analytical expressions for the band structure of a 1D photonic crystal with two layers per period. We have described a very simple procedure for obtaining angles of refraction inside the photonic crystal, and a simple approximation for computing the direction of waves transmitted through a 1D photonic crystal prism. While these very simple methods have allowed us to study the basic properties of the 1D photonic crystal superprism, a number of other important issues still require further investigation. These issues include: coupling efficiency of incident radiation with photonic crystals, transmission efficiency, internal reflections within the prism, coupling with higher order modes and optical cross talk, effects of realistic 2D (prism) and 3D (supporting substrate) geometries, and leakage and interaction with the supporting substrate. These questions should be addressed by using rigorous electromagnetic simulations.

We expected the highly anisotropic photonic band structures of 1D photonic crystals to lead to superprism effect. Our calculations confirm this notion. We have shown that inside the photonic crystal, light propagation direction is extremely sensitive to the changes in wavelength and incident angle, for frequencies near the band edges. Outside the photonic crystal, propagation direction of transmitted light is highly sensitive to the changes in wavelength, but not particularly sensitive to changes in incident angle. It has often been pointed out in the literature [6,20] that the angular dispersion (resulting from variations in incidence angle) observed in the experiment of Lin *et al.*[5] is only comparable to that of a conventional grating, while the dispersion observed in the experiment of Kosaka *et al.*[6] is much stronger. In

fact, the term "superprism phenomenon" was coined[6] to distinguish the extra-ordinarily large angular dispersions observed in the 3D photonic crystal from the normal-sized angular dispersions observed by Lin. However, the difference is not due to the fact that a 2D photonic crystal is used in one experiment and while a 3D structure in the other. And it obviously cannot be because one experiment was interpreted in terms of band structure non-linearity while the other in terms of anisotropy. Our results, obtained using the same 1D photonic crystal structure, show that the light propagation direction *inside* the photonic crystal is far more sensitive to the incidence angle than is for transmitted light propagation direction *outside* the photonic crystal. This suggests that the key difference between the observations of Lin *et al.* and Kosaka *et al.* is simply in whether one looks at the light propagation direction *outside* or *inside* the photonic crystal.

The propagation direction of transmitted light is highly sensitive to the variations in wavelength, though not as dramatic as for the light propagation direction inside the photonic crystal. However, while the extremely large wavelength dispersion capability inside the photonic crystal occurs at the band edges, the wavelength dispersion in transmitted light extends over the entirety of each transmission band. We call this the "extended-range superprism effect." Since the extended-range superprism can operate in the middle of transmission bands, rather than at the edges, we expect it to have improved optical coupling efficiency. The extended-range superprism also exhibits significantly reduced non-linearity in angular dependence on wavelength. While our calculations are for 1D photonic crystals, we expect the extended-range superprism effect to be present in 2D and 3D photonic crystals as well, as long as it is not interfered by band crossing effects in the more complex band structures in 2D and 3D cases.

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