QUANTUM DETECTION OF COHERENT-STATE SIGNALS IN THE PRESENCE OF NOISE

Victor A. Vilnrotter and Chi-Wung Lau

Abstract

A general method for solving an important class of quantum detection problems will be presented and evaluated. The quantum theory for detecting "pure states" for communications purposes has been developed over two decades ago, however the "mixed state" problem representing "signal plus noise" states has received little attention due to its great complexity. Here we develop a practical model for solving the mixed-state problem using a discrete approximation to the coherent-state representation of signal plus noise density operators. The problem is formulated in terms of quantum "measurement states" whose detection performance is optimized via generalized rotations in Hilbert space. An efficient algorithm for carrying out the required numerical optimization will be described. The algorithm will first be applied to the binary signal plus noise problem for which exact performance results are known. Next, this newly developed technique will be applied to a previously unsolved quantum "mixed state" detection problem, namely the detection of ternary signals in the presence of thermal noise fields.

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1. INTRODUCTION

The classical communications model assumes that deterministic signals are observed in the presence of additive Gaussian noise. This model is adequate for describing communications at radio frequencies, where quantum effects are not readily detectable. However, at optical frequencies quantum effects become the dominant source of error, and must be taken into account. An approach consistent with the principles of quantum mechanics starts by quantizing the received electromagnetic field, and seeks to determine those measurements on the received field that achieve best results, such as minimizing the average probability of detection error. Exact solutions for the case of pure coherent states for both binary and higher-dimensional signals can be found in the literature [1, 2, 3, 4]. The quantum mechanical solution for pure-state signals can be formulated in terms of Kennedy's orthogonal "measurement states", involving optimization of the detection operator over the signal subspace [2, 3]. Recently, efficient algorithms have been developed to carry out the required optimization, and applied to high-dimensional signals of interest in optical communications, including the well-known "pulse position modulation" (PPM,) and "dense signal sets" such as QPSK and BPSK-PPM, as described in [5, 6].

The addition of thermal noise fields to the received coherent-state field complicates the problem significantly, and hence exact solutions to the quantum "signal-plus-noise" problems are generally not known. One class of problems that can be solved numerically is the class of binary coherent state signals in the presence of thermal noise fields, where the solution involves finding the positive eigenvalues of the difference of density operators under the two hypotheses in matrix form [1, 3]. However, for higher dimensional signals, solutions are not readily available.

Here we present a technique that can be used to evaluate the performance of higher dimensional signal sets observed in the presence of noise. Our solution involves an approximation to the density operators, followed by detection in terms of optimized measurement states defined over the signal plus noise subspace. The detection operators are expressed in terms of orthogonal measurement states, and first applied to binary signals whose performance in the presence of noise is known, for purposes of evaluation and comparison. The detection operators are then applied to a three-signal "ternary" detection problem, and performance determined in the presence of noise. Although currently applied only to ternary signals due to computational complexity, it is clear that this technique can be extended to higher dimensional signal sets, providing an ability to determine optimum performance in the presence of noise for a large class of signals.

2. QUANTUM REPRESENTATION OF SIGNAL PLUS NOISE

2.1 Coherent-State Representation of Signals

Coherent states, representing electromagnetic radiation produced by physical devices such as lasers, are an important class of states for optical communications [7]. It is well known that the coherent states of a single mode of radiation $|\alpha\rangle$ can be expressed in the form of a superposition of orthonormal states $|n\rangle$, known as the number states:

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$$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2} \sum_{n=0}^{\infty} \frac{\alpha^n}{(n!)^{1/2}} |n\rangle \tag{1}$$

The coefficients of the number states are governed by the complex parameter α. Each number eigenstate $|n\rangle$ contains n photons, and hence the probability of obtaining exactly n photons as the outcome of an experiment can be computed as

$$|\langle \rho | n\rangle|^2 = \langle n | \rho | n\rangle = e^{-|\alpha|^2} \frac{|\alpha|^{2n}}{n!} \tag{2}$$

For any n, this expression specifies Poisson probabilities for the number of photons, with average number of photons equal to $|\alpha|^2 \equiv N_s$. Coherent states are not orthogonal, as can be shown by considering the overlap between two arbitrary coherent states, $|\alpha_1\rangle$ and $|\alpha_2\rangle$. Orthogonality requires the overlap to vanish, however for coherent states the squared magnitude of the overlap is not zero, but given by the following expression:

$$|\langle \alpha_1 | \alpha_2 \rangle|^2 = \left| e^{-(|\alpha_1|^2+|\alpha_2|^2)/2} \sum_n \sum_m \frac{\alpha_1^n}{\sqrt{n!}} \frac{(\alpha_2^*)^m}{\sqrt{m!}} \langle n | m \rangle \right|^2 = \left| e^{-(|\alpha_1|^2+|\alpha_2|^2-2\alpha_1\alpha_2^*)/2} \right|^2 = e^{-|\alpha_1-\alpha_2|^2} \tag{3}$$

Equation (3) demonstrates that there is always some overlap between coherent states, regardless of how great the average photon count in each state may be.

2.2 Coherent-State Representation of Signal plus Noise

The density operator for optical signal plus background radiation in a single mode of the received field can be represented in terms of a continuum of coherent states as [1]

$$\rho = (\pi N)^{-1} \int \exp(-|\alpha - \mu|^2 / N) |\alpha\rangle\langle\alpha| d^2\alpha \tag{4}$$

where μ is the complex envelope of the coherent-state vector representing the signal in the absence of thermal noise. In the plane of complex numbers, α, the Gaussian weight function is centered over the origin when there is no signal, and shifted to the complex number μ when a signal is present, where $|\mu|^2$ is the photon energy of the signal. An approximate discrete representation of equation (4) can be obtained by replacing the integral with an infinite sum, yielding:

$$\begin{aligned} \rho^* &\cong (\pi N)^{-1} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \int_{A_{jk}} \exp(-|\alpha - \mu|^2 / N) |\alpha_{jk}\rangle\langle\alpha_{jk}| d^2\alpha \\ &= \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} P_{jk} |\alpha_{jk}\rangle\langle\alpha_{jk}| \end{aligned} \tag{5}$$

where $P_{jk} = (\pi N)^{-1} \int_{A_{jk}} \exp(-|\alpha - \mu|^2 / N) d^2\alpha$, $|\alpha_{jk}\rangle$ is the approximating discrete coherent state with complex parameter α_{jk}, obeying the constraint $\sum_{j,k} |\alpha_{jk}|^2 = N$, where N is the average number of photons in a single mode of the noise field. We can interpret the sum in equation (5) as defining a countable set of coherent states, each

occurring with probability defined by the integral of the Gaussian weight function over the area associated with the approximating coherent state. The probability P_{jk} of receiving a coherent state within a specified region of the α-plane, A_{jk}, is given by the integral of the weight function over that region. This model will be referred to as the "discrete coherent-state", or DCS, approximation.

For coherent states separated by a distance of one unit in the α plane, equation (3) evaluates to 0.368, and decreases rapidly for greater distances. This suggest the possibility of approximating the continuous α plane with discrete samples on a regular grid of points separated by one unit, for applications where relatively large regions of the α plane are involved. In effect, this approximation collapses small neighborhoods of unit diameter onto a single coherent state located at specific grid point. A graphical representation of the DCS approximation is shown in Fig. 1, illustrating the approximate density operator for thermal noise near the origin of the complex α-plane. Since coherent states separated by one unit or more have relatively small overlap, samples of the α-plane on a grid separated by one unit may be adequate under some conditions, thereby reducing the complexity of the representation. This model is expected to yield accurate results for cases where the Gaussian weight function is broad enough to have significant probability at a radius of one unit or more from the center. At the same time, the sampled model is not expected to yield good results for cases where the noise energy is so small that only the central coherent state has a significant probability of occurrence: in that case, the model effectively reduces to the noiseless case, and denser sampling is required to adequately represent the α plane. The notation for labeling the coherent-state centers anticipates the application of this model to detection, where the absence of signal energy is generally referred to as the "null hypothesis" or H_0: a cluster of coherent-states around the origin represents the density operator for noise only, or H_0, hence the common zero in the subscripts signifying the null hypothesis.

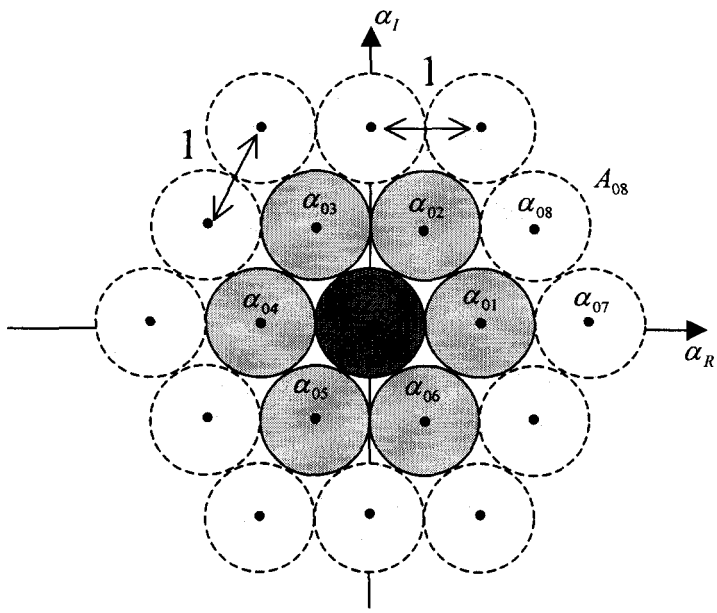

Fig. 1. Discrete coherent-state (DCS) representation of zero-mean noise

Further noting that coherent states with α - parameter far from the center of the weight distribution are not very likely to occur, it is often reasonable to ignore these low-probability states. With Gaussian weight distribution as in equation (5) and using the numbering scheme defined in Figure 1, this implies that there exists a K for either density operator such that $\forall k \geq K, \Pr[|\alpha_{ik}\rangle] \leq \varepsilon$. By selecting a suitably small ε and adopting the numbering scheme of Fig. 1, the density operators of equation (5) can be approximated by finite sums of discrete coherent states.

2.3 Quantum Detection in the Presence of Noise

The DCS noise model is first applied to binary "on-off keying" (OOK). Under "hypothesis zero", denoted by H_0, only thermal noise is received, whereas under "hypothesis one", H_1, a coherent-state signal plus thermal noise enters the receiver. Using the approximation in (5), and renaming the subscripts to reflect the hypotheses as shown in Fig. 1, the following approximate density operators are obtained :

$$\begin{aligned} H_0: &\quad \rho_0^* = \sum_{k=0}^{K} P_{0k} \,|\, \alpha_{0k} \rangle \langle \alpha_{0k} \,| \\ H_1: &\quad \rho_1^* = \sum_{k=0}^{K} P_{1k} \,|\, \alpha_{1k} \rangle \langle \alpha_{1k} \,| \end{aligned} \tag{6}$$

These density operators define the probability of occurrence of coherent states approximating the noise distribution, with coherent state $|\, \alpha_{0k} \rangle$ occurring with probability P_{0k} under H_0, and $|\, \alpha_{1k} \rangle$ with probability P_{1k} under H_1. This formulation leads to an approximate model according to which any one of a large number of coherent states could occur under either hypothesis due to the presence of noise.

Optimal detection of quantum states can be formulated in terms of projections onto orthonormal "measurement states" as described in [2, 3, 4, 5, 6]. In our model, the detection operator for the binary signal plus noise problem spans the entire signal plus noise subspace of Hilbert space. It is denoted by W_2, with components the projection operators Π_0 and Π_1, corresponding to the two hypotheses H_0 and H_1:

$$W_2 = \Pi_0 + \Pi_1 \tag{7a}$$

$$\Pi_0 = \sum_{j=0}^{K-1} |\, w_j \rangle \langle w_j \,|; \qquad \Pi_1 = \sum_{j=K}^{2K-1} |\, w_j \rangle \langle w_j \,| \tag{7b}$$

The measurement state components of these operators are orthonormal quantum states of the form $|\, w_0 \rangle, \cdots, |\, w_{(K-1)} \rangle$, and $|\, w_K \rangle, \cdots, |\, w_{(2K-1)} \rangle$, respectively. These detection operators are projectors, hence their eigenvalues are either zero or one, as shown by the following example:

$$\Pi_0 \,|\, w_m \rangle = \sum_{j=0}^{K-1} |\, w_j \rangle \langle w_j \,|\, w_m \rangle = \delta_{jm} \,|\, w_m \rangle \tag{8}$$

Application of the detection operator to the received coherent state results in one of the eigenvalues of the detection operator as the outcome: if the outcome of the measurement is a non-zero eigenvalue of Π_0 then H_0 is selected, otherwise the receiver decides that H_1 was received. Detection operators for the general M-hypothesis problem can be formulated in a similar manner. Now there are M densities, denoted by $\rho_0, \rho_1, \cdots, \rho_{M-1}$, and the detection operator for the M hypothesis problem, W_M, is of the form:

$$W_M = \sum_{m=0}^{M-1} \Pi_m \tag{9a}$$

$$\Pi_m = \sum_{j=mK}^{(m+1)K-1} | w_j \rangle \langle w_j | \tag{9b}$$

These detection operators correspond to a receiver structure whose performance can be determined using numerical techniques. The detection operator W_M is simultaneously applied to the entire signal plus noise subspace, encompassing all hypotheses. The probability of correct detection is found by first determining the expected values of the component projection operators, then averaging over the hypotheses. For example, given H_0, the probability of obtaining a non-zero eigenvalue of Π_0 is equal to the expected value of Π_0:

$$\begin{aligned} P[C \mid H_0] = E(\Pi_0) &= \sum_{j=0}^{K-1} \langle w_j \mid \rho_0 \Pi_0 \mid w_j \rangle \\ &= \sum_{j=0}^{K-1} \langle w_j \mid \sum_{l=0}^{K-1} P_{0l} \mid \alpha_{0l} \rangle \langle \alpha_{0l} \mid \sum_{m=0}^{K-1} \mid w_m \rangle \langle w_m \mid w_j \rangle \\ &= \sum_{j=0}^{K-1} \sum_{l=0}^{K-1} P_{0l} \langle w_j \mid \alpha_{0l} \rangle \langle \alpha_{0l} \mid w_j \rangle \\ &= \sum_{j=0}^{K-1} \sum_{l=0}^{K-1} P_{0l} \mid \langle w_j \mid \alpha_{0l} \rangle \mid^2 \end{aligned} \tag{10}$$

The probability of correct detection is obtained by averaging the conditional probabilities for each hypothesis: $P(C) = \sum_m P(H_m) P[C \mid H_m]$ where $P(H_m)$ are the a-priori probabilities. The average probability of error follows as $P(E) = 1 - P(C)$.

It is interesting to compare the receiver for the DCS model developed here and the application of the measurement states optimum for the pure-state (noiseless) case. In the pure-state formulation there is only one measurement-state associated with each hypothesis, hence correct detection occurs if the noise-corrupted signal, represented as one of K coherent states in a given cluster, projects onto the single measurement state associated with that cluster. However, in the DCS formulation, projection of the received coherent state onto any of the measurement states associated with that hypothesis results in correct detection. For the pure-state case, the conditional probability of correct detection contains only K terms when the DCS model is used, denoting the projection of the received noise-corrupted coherent-states onto a single measurement state for the transmitted hypothesis. For example, when H_0 is transmitted, the conditional probability for "pure projection" is given by the expression

$$P(C \mid H_0) = \sum_{l=0}^{K-1} | \langle w_0 | \alpha_{0l} \rangle |^2 \tag{11}$$

By contrast, the conditional probability of correct detection in the DCS formulation contains K^2 positive terms, since now there are K measurement states, and all possible distinct projections onto every measurement state are used, as shown in equation (10), resulting in greater conditional probability of correct detection.

In general, the DCS detection operators described above do not necessarily minimize the average probability of error. Best performance is obtained when the measurement states are optimized so as to maximize the average probability of correct detection. Since the detection operators consist of "rigid" orthogonal measurement states, the required

optimization can be formulated in terms of rotations of the detection operators in Hilbert space. The details of the numerical optimization via rigid rotations of the detection operators is the subject of the following section.

3. OPTIMIZATION OF THE DETECTION OPERATORS

Numerical optimization of the detection operators consists of finding the orientation of the orthogonal measurement states which yield the maximum probability of correct detection, or equivalently, minimum probability of error. As shown in previous articles [5, 6], the solution to the general K hypotheses problem can be found by rotating the measurement states in a K dimensional Hilbert space, and calculating the probability of correct detection for each rotation until the global maximum is found. To summarize the original approach, the optimization problem can be solved iteratively starting with two dimensions, since it is possible to rotate the projections of a vector in any lower dimension without affecting the projections of that vector onto the higher dimensions. This principle is illustrated in Figure 2, where the projection of an arbitrary vector V_1 in the X-Y plane is given by $V_1 \cos(\psi)$. The vector V_1 can be rotated such that the projection onto the Z axis is unchanged while the X and Y projection components vary. Generalizing this result to any K dimensional vector, the projections onto the (K-1) coordinate axes can be varied by rotating each of the (K-1) measurement coordinates without affecting the projection onto the next higher dimension.

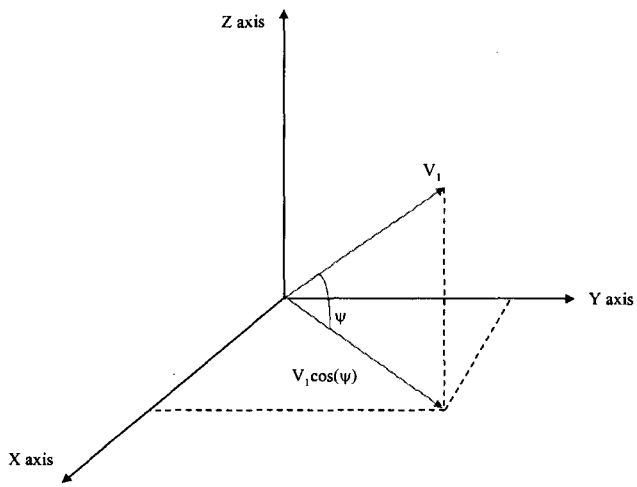

Fig. 2. Vector Rotation

Using this dimensional independence, the measurement states are incrementally rotated and the error probability calculated until the angle yielding the minimum probability of error is found for each dimension. To further simplify the search, each dimension only requires a rotation in a single plane. This plane is defined by the K-th dimensional axis and the K-1 bisector. The optimum measurement-state orientation for each dimension can be determined from the lower-dimensional solution, and this procedure continued until the complete K-dimensional solution is obtained. However, when the signal states are not symmetrically placed, such as the case for OOK or BPSK in the presence of noise, the diagonal rotation algorithm described does not provide the true optimal orientation of the measurement states. In order to find the optimal orientation, the diagonal rotational algorithm described above must be extended.

The original rotation algorithm described above, and documented in [6], must be modified to accomodate the optimization for the non-symmetrical case. The measurement state orientation and probability of error solved for the symmetric case is used as an initial condition. The measurement axes are then incrementally rotated about each axis pair in the lower-dimensional space from 0 to 360 degrees. With each rotational increment, the angle of rotation and probability of error for the new orientation is measured. The new measurement state orientation is accepted only if the probability of error is lower than that before. This operation is carried out for both directions, clockwise and counterclockwise.

To illustrate, the example for a set of binary signals in ten dimensional Hilbert space is shown. Each signal state will be augmented with 4 additional coherent states to account for the effects of noise, so that $K = 5$. Each cluster representing a noise-corrupted signal in the DCS model will contain 5 signals as depicted in Figure 3a.

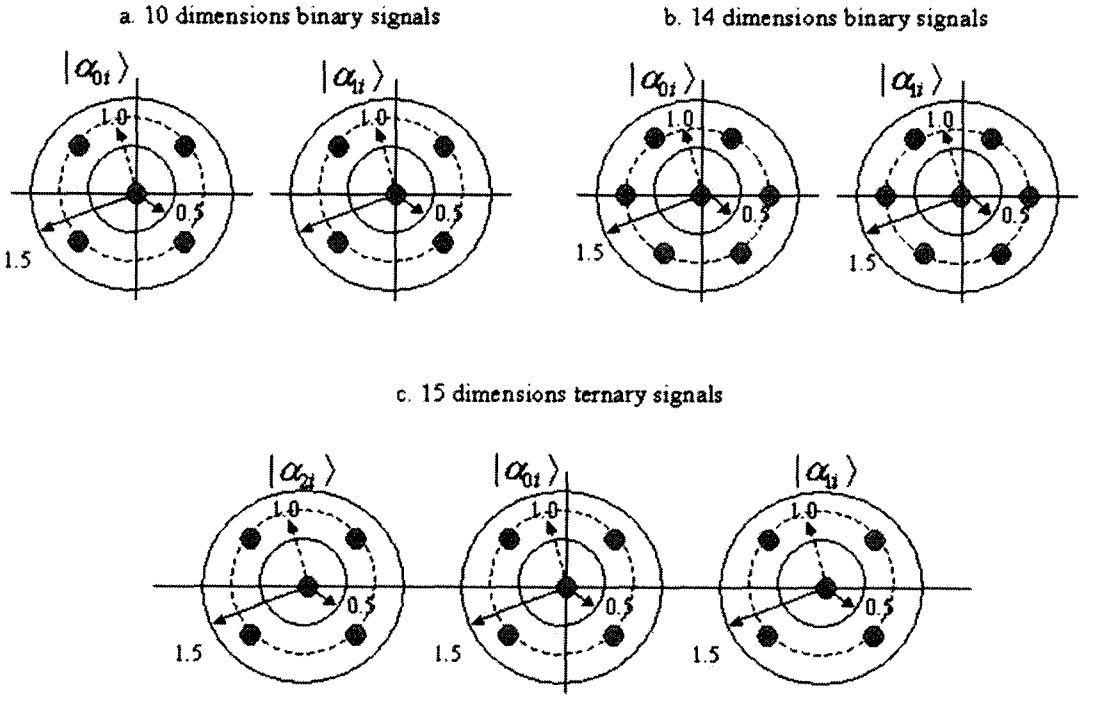

Figure 3. Placement of approximating coherent states for the DCS model with $K = 5$, and the sectors used to calculate probabilities of occurrence

For the binary DCS problem, this yields a total of 10 signal states, $|\alpha_{0i}\rangle$, $i = 0, 1, 2, 3, 4$, and $|\alpha_{1i}\rangle$, $i = 5, 6, 7, 8, 9$. Correspondingly, there will be a total of 10 measurement states $|w_i\rangle$, i = 0, 1, ..., 9. In order to provide an approximation to the sampling scheme described earlier, a two circle scheme was adopted for the probability distribution. The probability for the primary coherent state representing the signal, in the smallest circle in Figure 3a, was set to be 0.5. The four coherent states surrounding the primary coherent state were located at a radius of 1.0 from the center. The final circle was set at a radius of 1.5. The probability distribution for each state was then computed as follows. The probability of occurrence of the states under H_0 is governed by the equation

$$\hat{\rho}_0 \cong (\pi N)^{-1} \sum_{k=0}^{K} \int_{A_{0k}} e^{-|\alpha_{0k}|^2/N} |\alpha_{0k}\rangle\langle\alpha_{0k}| \, d^2\alpha \tag{12}$$

Integrating this equation over a circle of radius r yields the equation

$$P(r) = 1 - e^{-\frac{r^2}{N}} \tag{13}$$

which can be used to determine the probabilities associated which each coherent state in the cluster by evaluating $P_{jk} = (\pi N)^{-1} \int_{A_{jk}} \exp(-|\alpha - \mu|^2 / N) d^2\alpha$ over the appropriate regions. This serves as the initial starting point for the second optimization algorithm. The measurement states are incrementally rotated about each higher dimensional axis pair in the signal set from 0 to 360 degrees. For each increment rotation, the probability of error is calculated using equation (10). Using the 10-dimensional setup as an example, the algorithm begins by rotating the measurement states in the *X-Y* plane from 0 to 360 degrees. The angle yielding the minimum probability of error is determined and the orientation of the measurement states is set at the minimum angle. The new measurement state orientation is then rotated in the *X-Z* plane, the minimum probability of error is determined and the measurement state orientation is set correspondingly. This rotation process is repeated until all axis pairs have been rotated about. The process is then repeated with the rotation in negative direction from 0 to –360 degrees. After all the rotations have been completed, a minimum probability of error will be found which is the optimal orientation of the measurement states. This approach can be generalized up to *K* dimensions by systematically rotating about each axis pair and finding the minimum probability of error.

4. NUMERICAL RESULTS

Performance of the optimized detection operators for the DCS noise model have been evaluated, using the state-space optimization technique described above. The results for OOK and BPSK signals using both 5-dimensional and 7-dimensional clusters in the DCS model ($K = 5$ and $K = 7$) are shown in Figures 4 and 5. In both figures, dashed curves represent the results for K = 5, and solid curves for K = 7. The exact error probabilities obtained previously via the eigenvalue method are labeled "Q-Opt" in both figures. Comparisons of optimum quantum detection with classical detection techniques such as photon counting and coherent detection are treated elsewhere [1, 4, 5] where it is shown that classical detection techniques are typically 3 -5 dB worse than optimum quantum detection.

With OOK signaling, the received signal field is in a coherent state $|\alpha\rangle$ under hypothesis one, and state $|0\rangle$ under hypothesis zero, as shown in Fig. 3a. Thermal noise entering the receiver along with the signal generates the approximate density operators defined in equation (6). The performance of the optimum quantum detector for OOK signals in the absence of noise have been evaluated previously using measurement-state techniques, and documented in [4]. The approximate error probabilities obtained with the DCS technique are compared with the known performance of the optimum quantum detector, obtained by evaluating the eigenvalues of the difference of the density operators under the two hypotheses [1, 3]. As expected, even small amounts of thermal radiation degrades optimum receiver performance significantly: an average noise count of 0.1 photons per symbol degrades performance by more than 2 dB, whereas higher levels of noise corresponding to 0.3 photons per symbol lead to over 4 dB performance degradation.

The DCS technique yields accurate error probabilities for values that exceed the small probability mass remaining outside the largest approximating ring of the five-state DCS approximation in each cluster. These probabilities can be found using equation (13): for the higher noise case of $N = 0.3$ the probability outside the first ring is 6×10^{-4}. It is clear from Fig. 4 that these values are consistent with the floor of the DCS error probabilities. For the lower noise case, $N = 0.1$ and $N = 0.03$, the floor is due primarily to numerical errors resulting from the rather large (one degree) rotation increments used in the algorithms. This was verified by recomputing a few points with 0.1 degree increments, resulting in an order of magnitude reduction in the noise floor. Therefore, lower error probabilities can be reached with the optimized rotation algorithm by using smaller increments, but at the expanse of computation time.

The performance of the "pure projection" operator defined by equation (11) has also been evaluated. Note that for moderately high modal noise counts corresponding to $N = 0.1$, the probability of bit error for the pure projection algorithm reaches a floor at an error probability of 0.1 over the range considered, whereas the DCS algorithm continues to track the optimum solution until a much lower limiting value is reached, as shown in Fig. 4.

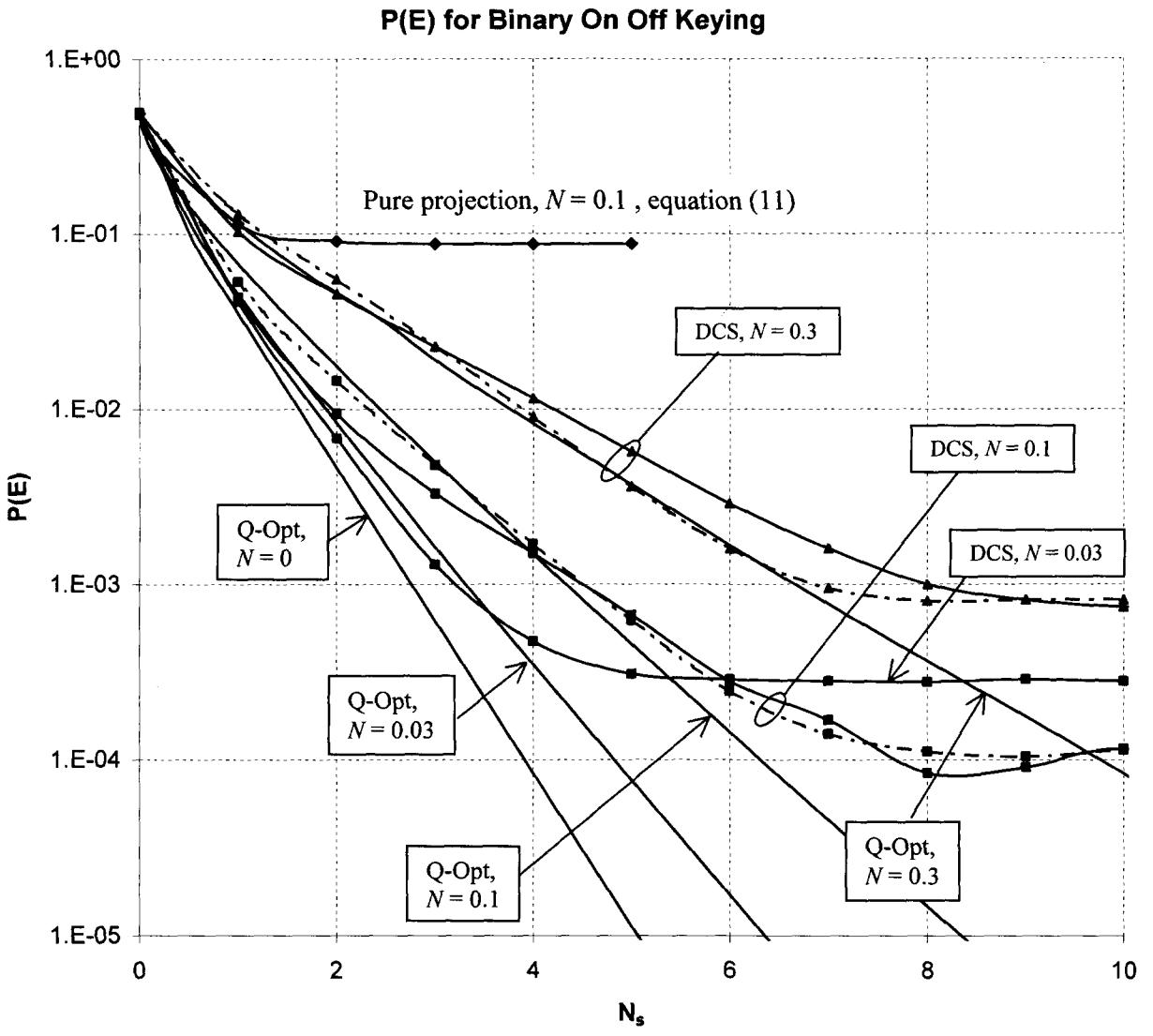

Figure 4. Performance of approximate DCS algorithm and optimum quantum detector for Binary OOK signals, in the presence of noise. $K = 5$ and 7.

Error probabilities for BPSK signals using both five and seven dimensional DCS noise models are shown in Fig. 5, and compared to the optimum quantum detector as a function of the average number of photons per symbol. For this modulation, the state of the received field in the absence of noise is denoted by $|\alpha\rangle$ under hypothesis one, and by $|-\alpha\rangle$ under hypothesis zero. Five dimensional clusters were treated as before, but in the seven-dimensional representation the areas of the sectors were equalized by increasing the radius of the central disk, in order to attain better probability weights for the approximating coherent states. It is again evident that the DCS approximation yields accurate results for error probabilities that exceed the uncounted probability outside the first DCS ring. Note that BPSK signals require only about half the average signal photons per symbol to reach a given error probability, when compared to OOK signaling. However, since BPSK signals transmits a signal pulse under both hypotheses whereas OOK transmits a signal only under the "signal hypothesis" and nothing under the alternative, BPSK modulation actually requires on the average

twice as many photons per symbol as OOK. When compared on the basis of average transmitted power, both modulation schemes exhibit similar performance.

Finally, the DCS algorithm has been applied to the problem of estimating the performance of the optimum quantum receiver for ternary signals, in the presence of noise. This signaling scheme is defined by three signal states, namely $|\alpha\rangle$, $|-\alpha\rangle$ and $|0\rangle$. The DCS noise model now consists of three clusters in the α - plane, placed as shown in Fig. 3c. The DCS algorithm places fifteen measurement states near the discrete coherent states in such a way as to maximize the probability of correct detection. Application of the DCS algorithm yields the error probabilities for weak (N = 0.03), moderate (N = 0.1), and strong noise (N = 0.3) cases shown in Fig. 6. For the strong noise case, an error floor is reached at about 10^{-3}, again due to the probability mass ignored outside the second ring in the DCS model. For the low-noise cases, the error floor is dominated by the rotation increments in the optimization algorithm. Since the current limit of the optimization algorithm is fifteen dimensions, clusters of five coherent states were used for each hypothesis (K = 5). Direct comparison with other results was not possible, however, since no previous solution to this problem could be found. The extension to higher dimensional signal sets is conceptually straightforward using the DCS technique described here, but requires extending the software to still higher dimensions.

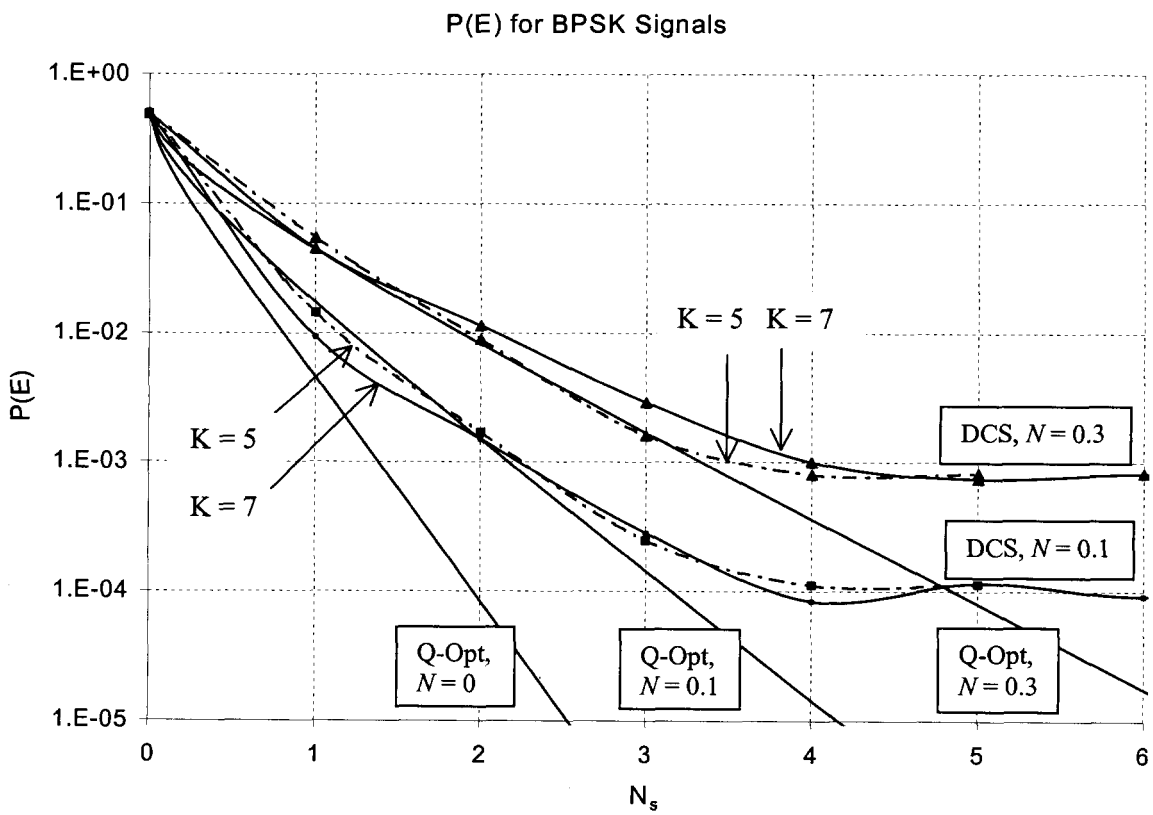

Figure 5. Performance of approximate DCS algorithm with BPSK signals, compared to known solutions for optimum quantum detection in the presence of noise. K = 5 and 7.

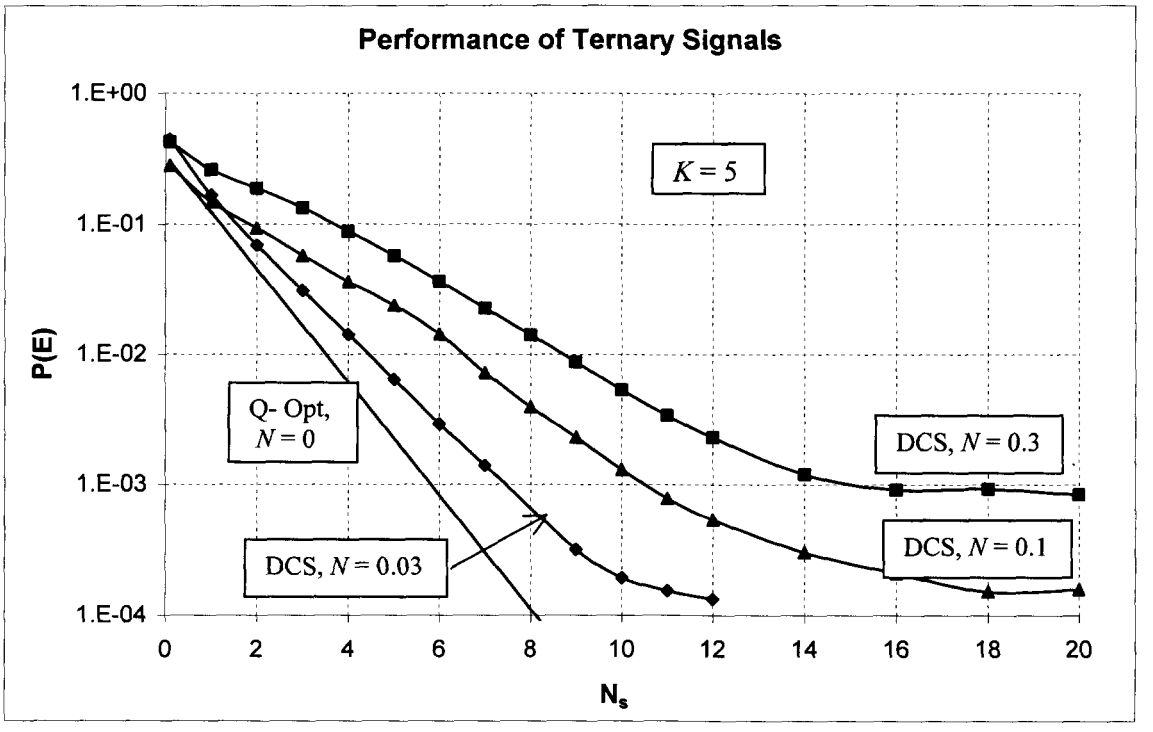

Figure 6. Performance of the optimum quantum detector for ternary signals, in the presence of noise. $K = 5$.

5. CONCLUSIONS

A new technique has been developed to evaluate the performance of the optimum quantum receiver for coherent state signals in the presence of thermal noise. The continuous density operators for signal plus noise in the coherent-state representation were approximated by discrete operators. Detection operators for the discrete coherent states were defined in terms of orthogonal measurement states, and the probability of error minimized by rotating the detection operators in Hilbert space. Optimum performance of the new DCS algorithm was verified using binary modulation formats whose performance in the presence of noise is well known [1]. The DCS algorithm was then applied to ternary modulation, whose performance in the presence of noise was not previously known, and performance curves for several noise conditions were obtained. This technique shows great promise for determining the performance of optimum quantum detectors for higher dimensional signals in the presence of noise, such as pulse-position modulated product-state signals and dense signal sets [5] contemplated for free-space communications, where no alternative methods for evaluating performance are available currently.

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