A study of the characteristics of microstrip reflectarrays as a function of the number and type of basis functions

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Introduction: Microstrip reflectarrays are commonly used in many spacecraft communication and radar systems because of their low profile and ease of deployment. In the design and analysis of large microstrip reflectarrays one often uses the infinite array model as a good approximation. An infinite array of microstrip elements is analyzed by considering two components of reflected waves. The first term due to the specular reflection is determined in the absence of conducting patches. The second term is the scattered wave due to the field radiated by the induced patch currents. The patch currents may be computed by solving the integral equation by the moment method. The required Green's functions for the multi-layer grounded periodic structure in terms of Floquet series is obtained by using the transmission line technique in the spectral domain for the TE and TM waves [1-3]. The reflection coefficient computed from this canonical problem is used in the analysis and design of large microstrip arrays, assuming that the patch size varies slowly with distance. The objective of this work is to understand the effects of the number and type of basis functions of the moment method on the reflection phase of the microstrip reflectarrays. An infinite array of microstrip patch elements on a grounded substrate layer was analyzed using the moment method.

Method: The parameters of three arrays, namely the patch dimensions, element lattice spacing, substrate dielectric constant, and thickness are shown in Table 1. The substrate thickness values in terms of the wavelength in the dielectric are 0.0659, 0.0469 and 0.0235 for the three arrays respectively. It should be noted that array 3 is extremely thin, and hence, its computed data are very sensitive to parameters such as the number of basis functions used in the moment method and the number of tetrahedra used in the finite element code, HFSS. In the HFSS computations, the phase of the reflection coefficient was obtained for a normally incident plane wave. The reference point was fixed at the location of the center of the unit cell patch at the ground plane. The analysis and the computational process employed in this work are similar to those in [1,2] except that the Green's functions are more general. By using the transmission line techniques for the TE and TM modes in spectral domain [3] the Green's functions for the multi-layer substrate-superstrate structure with an imperfect ground plane were employed. The losses in the microstrip patch conductors were accounted for by a perturbation technique. Entire domain expansion and testing functions used in this work consisted of trigonometric functions and functions exhibiting edge conditions for currents in thin conducting sheets. These functions have closed form expressions for their Fourier transforms. Different combinations of sums and products of trigonometric and edge singularity functions were employed in our study. Some representative cases discussed in Table I, are shown in Appendix. Pozar used two basis functions for each patch current [1] with an additive edge condition, as shown in case 1. In the transverse direction a uniform distribution is employed. In the second case, we enforce the edge condition along the current direction in a multiplicative sense. The third case contains all trigonometric functions enforcing edge conditions in both directions in a multiplicative sense.

Results and Discussions: Our first study was a comparison of computed values of the phase of the reflection coefficient of infinite reflectarrays for normal incidence. HFSS and the moment method with two choices of basis functions were employed. The first used four basis functions and the second two basis functions, cases 1 and 2 in the appendix. The transverse distribution in both cases is uniform. Fig.1 shows good agreement between the three results. The use of two basis functions with a multiplicative edge condition yielded results in excellent agreement with HFSS. We also investigated the use of basis functions with a multiplicative edge condition in the transverse direction, but that yielded poor results. An explanation for this behavior is as follows. The transverse distribution of the patch current near resonance is essentially a constant over most of the patch. The use of an entire domain basis function that simulates the singularities in the transverse direction would distort the near-uniform distribution over most of the patch and hence would yield poorer results.

Our subsequent study was focused on the convergence of moment method results as a function of the number of basis functions. We used up to 512 basis functions. This corresponded to 256 basis functions for each current, 16 variations in each direction, 8 of them even and 8 odd. The trigonometric functions employing multiplicative edge conditions exhibited the best convergence. A representative set of computed data is shown in Table 2. We note that 32 basis functions provide good accuracy except for the case of the 32 GHz array on 5 mil substrate. For patches away from resonance the reflection phase is less sensitive to the number of basis functions employed. Generally 2 or 4 basis functions are adequate for such purposes. We need a greater number of basis functions if the patch dimensions are closer to resonance and/or if the substrate thickness is small.

It is generally recommended that substrate thickness be greater than 0.05 wavelength in the dielectric. For thinner substrates the phase slope as a function of patch size becomes very high and hence computed results are very sensitive. A choice of 32 trigonometric basis functions, all incorporating multiplicative edge conditions generally yield very good results for the reflection coefficient. These basis functions will also yield accurate results for the cross-polar characteristics of the element pattern. Further details will be presented in the symposium.

References:
[1] D. M. Pozar, "Reflectarray Antenna and Solar Array Integration for Space-craft Applications," Final Rept. submitted to Jet Propulsion Laboratory, Mar. '97.
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[3] T. Itoh (Editor), Numerical Techniques for Microwave and Millimeter-Wave Passive Structures, John Wiley and Sons, New York, 1989.

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Parameter	Array 1 13.285 GHz	Array 2 32 GHz (10mil)	Array 3 32 GHz (5 mil)
ε_r	3.38	3.0	3.0
substrate height	0.81	0.254	0.127
Unit cell size	(11.3, 11.3)	(5.82, 5.82)	(5.82,5.82)
Res. Patch size	0.5607	0.2518	0.2593

Table 1. Physical parameters of three reflectarrays (dimensions in mm).

Number of Basis functions	Array 1 13.285 GHz	Array 2 32 GHz (10mil)	Array 3 32 GHz (5 mil)
2 (case2)	176.3	208.3	332.7
4 (case 1)	163.0	147.7	207.4
32	177.3	171.8	260.3
128	174.9	165.0	219.4
512	174.4	164.0	211.0
HFSS	180.00	180.0	240.0

Table 2. Reflection phase (degrees) for reflectarrays, normal incidence.

Appendix: Three choices of basis functions for surface currents in a patch of dimension (a, b) are:

1. Four basis functions, two for each current - additive edge condition

$J_x(x,y) = A_1 \cos(\pi\ x/a) + A_2\, [1-(2x/a)^2]^{1/2}$

$J_y(x,y) = B_1 \cos(\pi\ y/b) + B_2\, [1-(2y/b)^2]^{1/2}$

2. Two basis functions, one for each current - multiplicative edge condition

$J_x(x,y) = A_1 \cos(\pi\ x/a)\, [1-(2x/a)^2]^{-1/2}$

$J_y(x,y) = B_1 \cos(\pi\ y/b)\, [1-(2y/b)^2]^{-1/2}$

3. Trigonometric basis functions all with multiplicative edge conditions:

$$J_x(x,y) = [\sum_{m=1,3,} A_m\, g_{1e}(x,a,m) + B_m\, g_{1o}(x,a,m+1)] \bullet [\sum_{n=0,2,} C_n\, g_{2e}(y,b,n) + D_n\, g_{2o}(y,b,n+1)]$$

$$J_y(x,y) = [\sum_{m=1,3,} E_m\, g_{1e}(y,b,m) + F_m\, g_{1o}(y,b,m+1)] \bullet [\sum_{n=0,2,} G_n\, g_{2e}(x,a,n) + H_n\, g_{2o}(x,a,n+1)]$$

where $g_{1e}(x,a,m) = \cos(m\,\pi\ x/a)\,[1-(2x/a)^2]^{-1/2}$

$g_{1o}(x,a,m+1) = \sin[(m+1)\,\pi\ x/a][1-(2x/a)^2]^{-1/2}$ $\quad$ m=1,3,5... $-a/2 \le x \le a/2$

$g_{2e}(y,b,n) = \cos(n\,\pi\ y/b)[1-(2y/b)^2]^{-1/2}$

$g_{2o}(y,b,n+1) = \sin[(n+1)\,\pi\ y/b][1-(2y/b)^2]^{-1/2}$ $\quad$ n=0,2,4... $-b/2 \le y \le b/2$

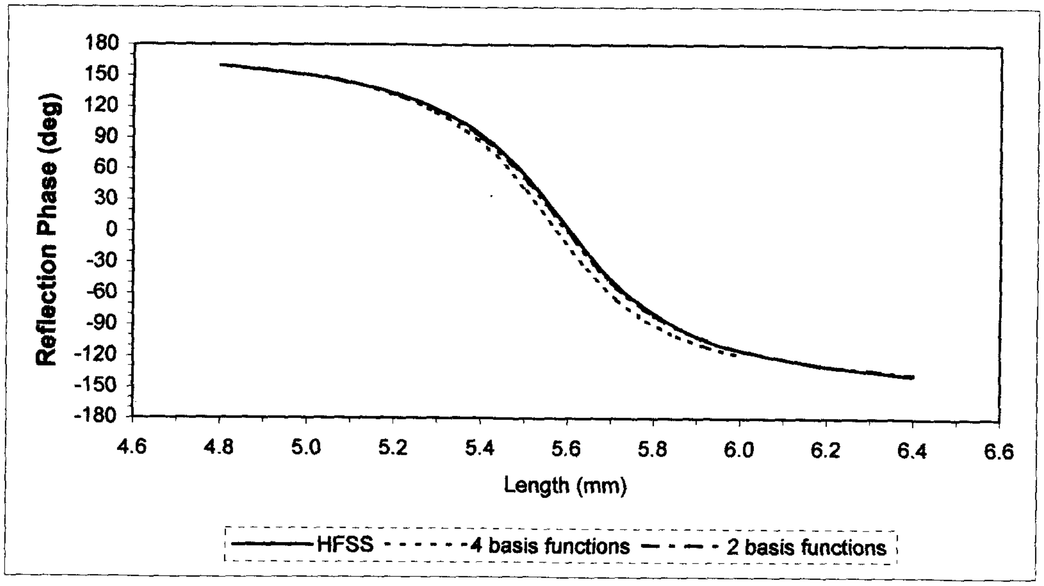

Figure 1 Reflection phase of an infinite array of microstrip patches as a function of patch width.