**Introduction:** Variations in rotation and orientation of the Moon are sensitive to solid-body tidal dissipation, dissipation due to relative rotation at the fluid-core/solid-mantle boundary, and tidal Love number $k_2$ [1,2]. There is weaker sensitivity to flattening of the core-mantle boundary (CMB) [2,3,4] and fluid core moment of inertia [1]. Accurate Lunar Laser Ranging (LLR) measurements of the distance from observatories on the Earth to four retroreflector arrays on the Moon are sensitive to lunar rotation and orientation variations and tidal displacements. Past solutions using the LLR data have given results for dissipation due to solid-body tides and fluid core [1] plus Love number [1,2,3,4]. Past detection of CMB flattening has been marginal [3,4] but is improving, while direct detection of the core moment has not yet been achieved.

**LLR Solutions:** Three decades of Lunar Laser Ranging (LLR) data are analyzed using a weighted least-squares approach. The lunar solution parameters include dissipation at the fluid-core/solid-mantle boundary, tidal dissipation, dissipation-related coefficients for rotation and orientation terms, potential Love number $k_2$, a correction to the constant term in the tilt of the equator to the ecliptic which is meant to approximate the influence of core-mantle boundary flattening, and displacement Love numbers $h_2$ and $l_2$. Several solutions, with different combinations of solution parameters and constraints, are considered.

**Love Number Determinations:** Sensitivity to the potential Love number $k_2$ comes from rotation and orientation while $h_2$ and $l_2$ are determined through the tidal displacement of the retroreflectors. Two types of LLR solutions for the Love numbers are considered. One type has the three second-degree Love numbers constrained to the ratio expected for a homogeneous incompressible sphere with $h_2:k_2:l_2$ in the ratio of $10:6:3$. The second type of solution makes $k_2$ an independent solution parameter while retaining a constraint for the two remaining Love numbers. The second type of solution, with $k_2$ a free parameter, gives $k_2 = 0.0257-0.0025$ and $h_2 = 0.029-0.013$. With the full constraint $k_2 = 0.0248-0.0024$. There is a concordant spacecraft determination of the lunar Love number $k_2 = 0.026-0.003$ which relies on variation of the gravity field [5].

Compared to the results of [4] the two LLR values for $k_2$ are somewhat smaller and have improved uncertainties. It remains true that the LLR and spacecraft determinations are larger than simple lunar elastic models predict but it is also true that the excess is comparable to the uncertainties. To explain the excess it was suggested that there may be a deep zone of low seismic speed such as a partial melt would give [4]. Further improvement in the uncertainty is needed to test this possibility.

**Dissipation from Fluid Core and Tides:** Theory and LLR solutions for lunar dissipation have been presented in [1]. The interpretation of the dissipation results invoked both strong tidal dissipation and interaction at a fluid-core/solid-mantle boundary (CMB). Several new solutions use various combinations of tide and core parameters and rotation coefficients. Of the five considered independent dissipation terms in the rotation, four are well above the noise and one is marginal. Compared to the solutions in [1], the solution parameters have changed by amounts comparable to their uncertainties.

With an analysis of the dissipation coefficients similar to that in [1], the core component is found to be somewhat stronger (core fraction $f_c=0.41$ for the principal term) and tidal Q is found to increase less with tidal period (the frequency power law exponent is -0.05). For $k_2 = 0.0257$ the power-law expression for tidal Q as a function of tidal period is $37(\text{Period}/27.212d)^{0.05}$ so the monthly Q would be 37 and the annual Q 42. Using Yoder's turbulent boundary layer theory [6] a fluid iron core would have a radius of about 345 km, but topography on the CMB would tend to decrease the inferred radius.

**Core Oblateness:** The detection of the oblateness of the fluid-core/solid-mantle boundary (CMB) would confirm the existence of a liquid core. In the first approximation CMB oblateness should influence the tilt of the lunar equator to the ecliptic plane [2] and the LLR solutions include an analytical parameter to alter this tilt. This tilt is also influenced by moment differences, gravity harmonics, and Love number $k_2$ which are also solution parameters. The solution with $k_2$ a free parameter gives a tilt correction which is 1.3 times its uncertainty while a solution with the three-Love-number constraint gives a correction 1.6 times the uncertainty. These results are stronger than past solutions [3,4] but the effect is still close to the noise. The tilt parameter anticorrelates with $k_2$ so that larger CMB oblateness corresponds to smaller $k_2$.

The tilt correction depends on the fluid core moment and the CMB flattening. The former is uncertain and there is no information about the latter apart from these LLR solutions. For a uniform iron core with a radius of about 345 km, with ratio of the fluid core to solid mantle moments $C_e/C_m = 6\times 10^{-4}$, the flattening would be of order $2\times 10^{-3}$. The oblateness scales inversely with fluid core moment. For comparison the whole Moon "dynamical flattening" based on moments is $(2C-A-B)/2C = 5.188\times 10^{-4}$ and the surface geometri-
cal flattening is $1.3 \times 10^{-3}$ [7]. The core oblateness appears to be less than either measure of the whole Moon flattening unless the fluid core moment is small. The free core nutation period would be several centuries. Considering the size of the noise, the core oblateness detection and implications are tentative.

**Core Moment of Inertia:** An analytical development in [1] presents a rotation term sensitive to the fluid core moment of inertia. This term is potentially important because it would both confirm the presence of a fluid core and it would give a direct measurement of the moment of the fluid core. It was argued that this term would be difficult to detect because it is close in frequency (81 yr beat period) to a free libration term (free precession).

The least-squares solution procedure requires partial derivatives of range with respect to core moment. The partial derivatives of the three lunar rotation components with respect to core moment have been developed using numerical integration. Solutions using these partial derivatives confirm the difficulty of detection. A solution for the ratio of fluid core to solid mantle moments gives $C_\phi/C_m = (9-19) \times 10^{-4}$. This is not a detection but it does provide a 1-σ upper limit. For a liquid iron core the value corresponds to a 360 km radius while value plus uncertainty provides a 1-σ upper limit of 460 km. These radii scale in proportion to the 1/5 power of the fluid core moment divided by density.

**Inner Core:** A solid inner core might exist inside the fluid core. Gravitational interactions between an inner core and the mantle could reveal its presence in the future. An inner core might be rotating independently or it might lock to the mantle rotation through gravitational interactions. An inner core would complicate interpretations: there would be two surfaces for solid-mantle/fluid-core/inner-core dissipation and an inner core which does not share the fluid rotation will affect core moment and flattening interpretations.

**Summary:** Adding new lunar ranges gives solutions for lunar parameters with improved uncertainties. Dissipation parameters continue to indicate a fluid core and strong tidal dissipation. The potential Love number tends to be larger than the simplest elastic models predict but the uncertainty is significant. The effect of the oblateness of the fluid core/solid-mantle boundary seems to be emerging from the noise. Direct detection of the fluid core moment remains elusive. Detection of a solid inner core is a future possibility. Additional ranges with current accuracy and future data with improved accuracy [8] should improve the determination of these lunar science effects.