Feasibility of "Weak GPS" Real-Time Positioning/Timing at Lunar Distance[1]

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Abstract—**There are multiple Global Navigation Satellite Systems (GNSS's), comprising over 100 navigation satellites in the Earth's medium and high orbits. Most of these satellites have antennas that point Nadir to earth, and transmit navigation signals so vehicles on Earth's surface and in its vicinity can perform trilateration and estimate its 3-dimenional (3D) positioning.**

The sidelobes of these antennas can occasionally point to the Moon. It is postulated that a lunar vehicle carrying a large enough receiving antenna can occasionally detects and receives four or more sidelobes of these weak GNSS signals, thus enabling the vehicle to perform 3D positioning using an onboard GNSS receiver.

We propagate the orbits of the GNSS satellites from United States' Global Positioning Satellite (GPS) constellation, the Europe's Galileo constellation, and the Russia's GLONASS constellation, a total of 81 satellites. We simulate the visibility of these satellites by a lunar vehicle in a Near Rectilinear Halo Orbit (NRHO), based on the assumption that the lunar vehicle is "in-view" of a GNSS satellite as long as it falls within the 40-degree beam-width of the satellite. We also simulate the 3D positioning performance as a function of satellites' ephemeris errors and pseudo-range errors.

The preliminary results show that the lunar vehicle can "see" 5 – 13 satellites, and achieve a 3D positioning error (one-sigma) of 200 – 300 meters based on reasonable ephemeris and pseudo-range error assumptions. We also consider the case of using relative positioning to mitigate the GNSS satellites' ephemeris biases. That is, by assuming a reference receiver with accurately known positioning that is close to the lunar vehicle, and computing the relative position of the lunar vehicle relative to the reference.

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1. INTRODUCTION

There are multiple Global Navigation Satellite Systems (GNSS's), comprising over 100 navigation satellites in the Earth's medium and high orbits. The United States Global Positioning System (GPS) consists of 30+ navigation satellites in six Medium Earth Orbit (MEO) planes at 20-thousand-kilometer altitude. The European Union's Galileo System has launched 26 of the planned 30 satellites in three MEO planes at 23-thousand-kilometer orbital height. The Russia's GLONASS has 24 satellites in three MEO planes at 19-thousand-kilometer altitude.

All these navigation satellites have antennas that point Nadir to Earth, and transmit navigation signals so vehicles on Earth's surface and in its vicinity can perform trilateration, and estimate its 3-dimenisonal (3D) position. Though the GNSS's are mainly designed for Earth's terrestrial positioning applications, high-sensitivity navigation radios capable of detecting, tracking, and processing the sidelobes of GPS signals from the other side of Earth can be used for positioning of spacecraft at High Earth Orbit (HEO) including the Geostationary Earth Orbit (GEO) [1]. This enables new mission concepts that require precision positioning of spacecraft at high orbits. For example, the Sun Radio Interferometry Space Experiment (SunRISE) Mission uses an array of six CubeSats operating in GEO like one large radio telescope to study the Sun and the space environment [2][3]. Using GPS signals to position spacecraft beyond Earth's orbit has also been demonstrated by the Magnetospheric Multiscale (MMS) spacecraft [4].

It is postulated that a lunar vehicle carrying a large enough receiving antenna can occasionally track the sidelobes of four or more of the Earth's GNSS signals, thus enabling them to perform 3D positioning using an onboard GNSS receiver. It has been proposed that this "weak GPS"[2] approach can be used to perform orbit determination (OD) for the Lunar Orbiting and Transfer Gateway (LOT-Gateway) in a Near-Rectilinear Halo Orbit (NRHO). The concept is that the LOT-Gateway would carry a large antenna that receives the weak navigation signals from the Earth's direction. When the number of navigation satellites is large, there should be a

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[2] The term "weak GPS" is a misnomer. It includes navigation satellites from GPS, Galileo, and GLONASS.

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reasonable probability that the sidelobe signals from four or more satellites could be strong enough to perform positioning. Other than the opportunistic nature of this approach, another problem is bad geometry – that Earth-Moon distance (~400K km) is many times greater than the inter-satellite distances (10K – 40K km). This results in poor Position Dilution of Precision (PDOP), which is the error amplification factor in the positioning calculation. Preliminary analysis/simulation indicates that the PDOP ranges from 38 to 11854, with a mean around 100, compared to the typical values of Earth's consumer-based GPS applications of 2-3. Figure 1 shows the PDOP profile of the "weak GPS" scenario for 50 days (approximately two lunar cycles).

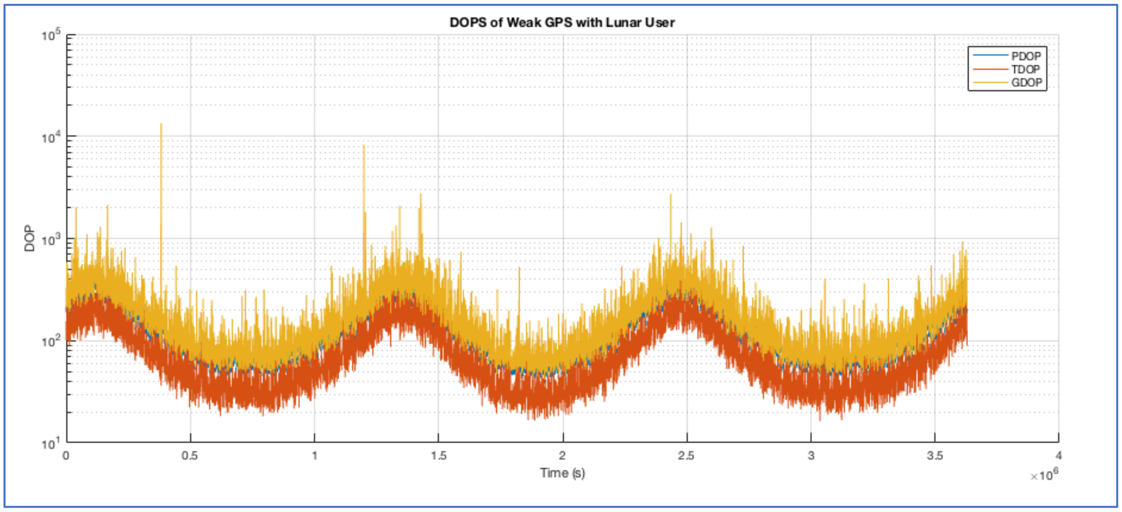

Figure 1: DOP Profile for 50 Days

One way to mitigate this problem is to put a GPS receiver (also with a large antenna) on the near-side of the lunar surface near the S. Pole, e.g. Tycho Crater. Note that the near-side of the Moon is always facing Earth, thus the GPS receiver can serve as a constant reference for the LOT-Gateway and other Moon-bound spacecraft.

The rest of the paper is organized as follows: Section II provides the detailed simulation setup, and the assumptions and caveats. Section III describes the simulation results, and Section IV discuss the conclusions.

2. SIMULATIONS, ASSUMPTIONS, CAVEATS

In this paper, we consider the case of using the "weak GPS" approach to track the LOT-Gateway when it is near the apolune of the NRHO orbit. We assume that the LOT-Gateway would carry a large Earth-pointing antenna, and one or more multi-constellation GNSS receivers. When the side-loops of the Earth's navigation satellites opportunistically point to the LOT-Gateway, the LOT-Gateway's GNSS receivers would be able to demodulate the weak GNSS signals and estimate the pseudo-ranges from Earth's GNSS satellites.

Simulating the GNSS satellites' side-loop pointing would require accurate antenna patterns. The specifications of GPS's transmitting antenna patterns are only available up to 23.5 degree off boresight [5]. Also, there is little information on antenna patterns of the navigation satellites of Galileo and GLONASS. As such, we impose an overly simplified assumption that the link between LOT-Gateway and a GNSS satellite can be closed when LOT-Gateway is within 40 degree off-boresight beam-width of the antenna of the GNSS satellite. That is, LOT-Gateway is "in-view" of the GNSS satellite.

We propagate the orbits of the navigation satellites of the United States' GPS, the European Union's Galileo System, and the Russia's GLONASS. This consists of a total of 81 satellites, see Figure 2.

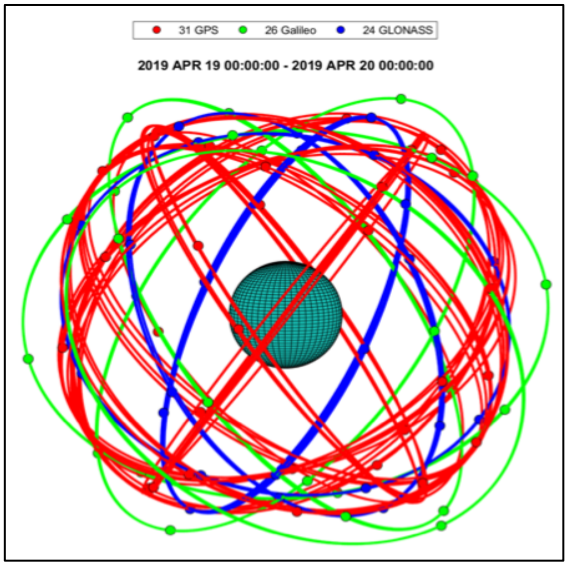

Figure 2: Orbits of GPS, Galileo, and GLONASS

For error analysis, we consider two main types of errors: receiver range estimation errors and navigation node errors. These errors are the most common error types in modern satellite navigation systems. For example, navigation node errors model imperfect knowledge in the transmitting satellite locations and clock offsets. Receiver range estimation errors model uncorrected environmental effects such as transmission medium delays, multipath, and receiver noise.

Each navigation node has a true distance d_i which is known within an error given by $d_i' = d_i + v_i$ where each v_i is an independent normally distributed random variable with mean μ and standard deviation σ_v, i.e., $v_i \sim N(\mu, \sigma_v^2)$. In actuality v_i is the norm of a random vector perturbation in the coordinates of transmission node *(x$_i$, y$_i$, z$_i$)*:

$$d_i' = \sqrt{(x_i + v_{xi})^2 + \left(y_i + v_{yi}\right)^2 + (z_i + v_{zi})^2} \quad (1)$$

$$i = 1, \cdots, n$$

With

$$v_i = \sqrt{v_{xi}^2 + v_{yi}^2 + v_{zi}^2} \qquad i = 1, \cdots, n \qquad (2)$$

And each $v_{xi}, v_{yi}, v_{zi} \sim N(0, \sigma_v^2/3)$.

Each receiver pseudo-range measurement is assumed to have a statistically independent random measurement error due to receiver noise, with a normal standard deviation σ_r that is simulated at a specified value. In other words, each pseudo-range estimation is given by:

$$r_i' = d_i' + \varepsilon_i \qquad i = 1, \cdots, n \qquad (3)$$

With $\varepsilon_i \sim N(0, \sigma_r^2)$.

We consider both absolute positioning and relative positioning. For relative positioning, we assume a reference station at Tycho Crater on the near-side of the lunar surface that would help to reduce the biases of satellites' ephemeris errors and solar plasma delay. We consider both the traditional Newton-Raphson (NR) trilateration scheme [6] and the recently proposed Geometric Trilateration (GT) scheme [7]. A position/timing solution was obtained averaging over 10,000 simulations of the Earth's GNSS tracking of the LOT-Gateway at the apolune of the NRHO orbit with the statistical receiver noise errors for pseudo-range and each navigation node location.

A range of different error conditions is shown for pseudo-range measurement error σ_r from 0 to 5.0 cm and navigation node position error σ_v from 0 m to 10 m. A receiver clock offset of *Δt* = 1/3 microseconds (100 m) was included in every simulation. We also assume a small amount of solar plasma delay (25 cm) in the pseudo-range measurements.

3. SIMULATION RESULTS

The GNSS coverage of the LOT-Gateway in terms of the number of GNSS signals that LOT-Gateway receives within 24 hours, is shown in Figure 3. With the relax and optimistic assumption of closing the link when LOT-Gateway falls within 40 degree off-boresight angle of the antenna of a GNSS satellite, the LOT-Gateway can see between 5 to 13 satellites at any time.

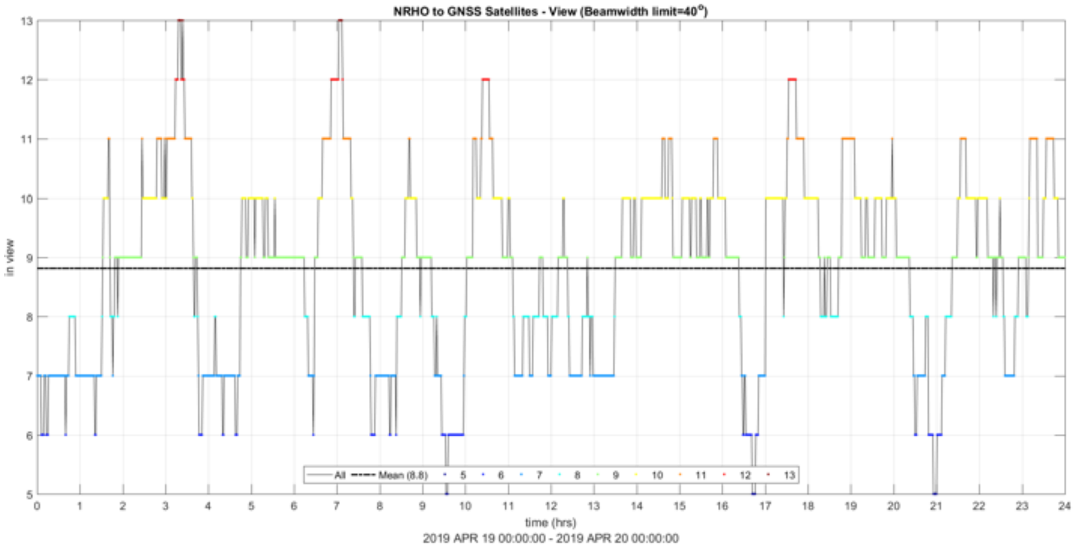

Figure 3: GNSS Coverage of LOT-Gateway

Next, we compute the Root Mean Square Error (RMSE) statistics on positioning and timing as a function of number of GNSS satellites "in-view". For the case of absolute positioning by the NR-method with GNSS ephemeris error of 30 cm, pseudo-range measurement error of 1 cm, and solar plasma delay of 25 cm, the positioning and timing error statistics are shown in Figure 4 and Figure 5 respectively.

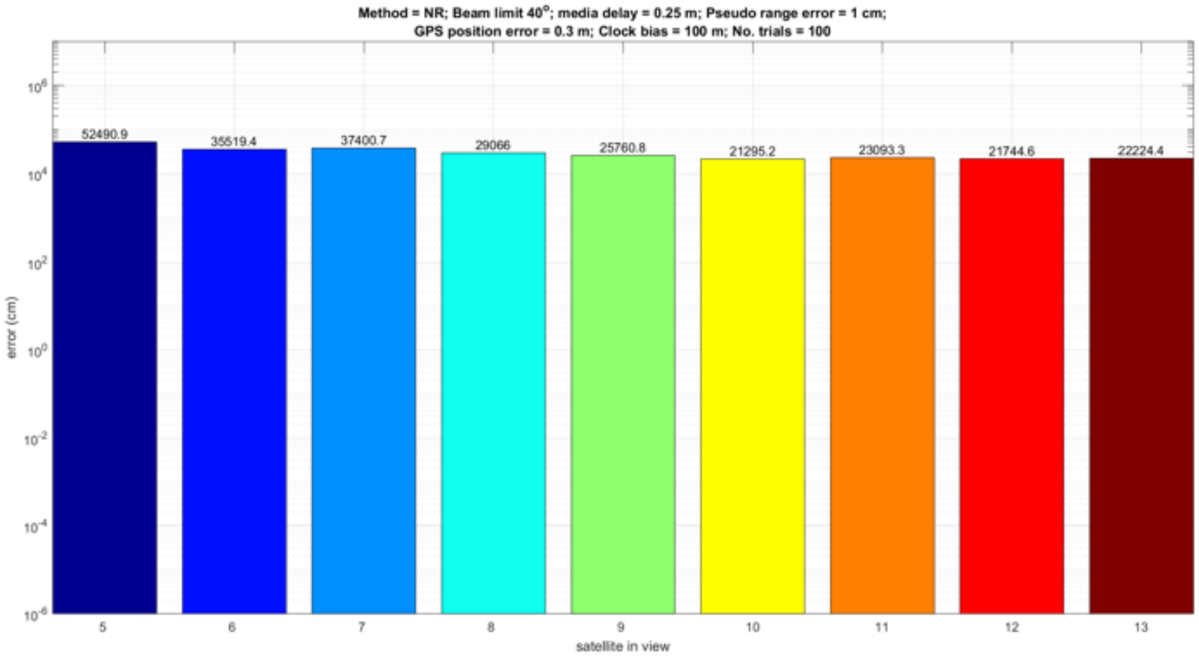

Figure 4: Positioning Error (RMSE) vs. Number of Satellites

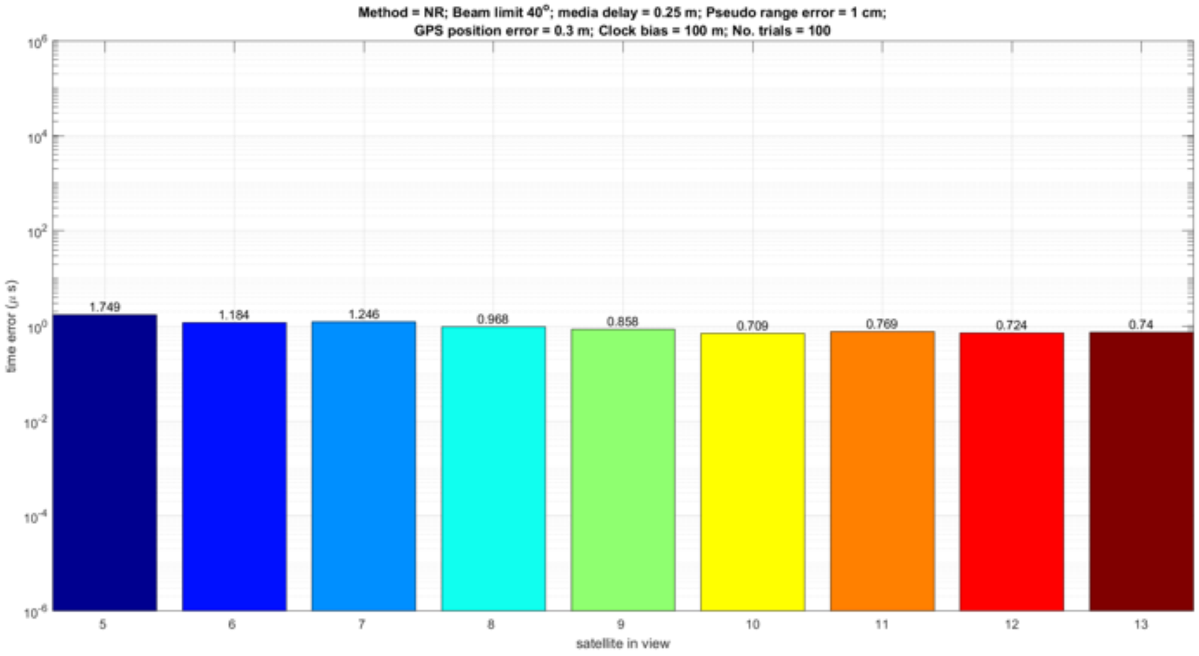

Figure 5: Timing Error (RMSE) vs. Number of Satellites

One interesting observation is that the positioning and timing errors differ by less than a factor of 3 between 5 satellites "in-view" and 13 satellites "in-view".

Next, we tabulate the positioning and timing error statistics as a function of pseudo-range measurement error and GNSS ephemeris error for the case of solar plasma delay of 25 cm, and the results are given in Table 1 and Table 2 respectively.

Recent literature [8] indicates that pseudo-range can be estimated using modern GNSS signals with an accuracy of less than 10 cm. The IGS/MGEX Report (2017) [9] shows that non real-time OD accuracy of the GPS, Galileo, and GLONASS satellites ranges from 3 cm to 30 cm. For the purpose of evaluating the real-time positioning and timing performances of the "weak GPS" approach at lunar distance, we believe a reasonable error assumption is GNSS pseudo-range measurement error of 5 cm, and GNSS satellite ephemeris error of 30 cm.

NR - Absolute Positioning

	0	0.1	0.5	1	5
0	17709	17703	17692	17701	18671
0.3	27842	27840	27835	27845	28544
0.5	40340	40339	40339	40351	40870
1	75346	75347	75352	75364	75688
5	369383	369386	369397	369412	369574
10	738714	738717	738730	738745	738892

GT - Absolute Positioning

	0	0.1	0.5	1	5
0	3282	3552	4306	5195	12238
0.3	25663	25649	25732	25731	27338
0.5	40444	40456	40478	40457	41469
1	77438	77376	77432	77418	78090
5	373314	373902	373245	373132	373196
10	742906	742975	743047	743203	743119

GT - Relative Positioning

Top Row: Pseudo Range Error (cm)
First Column: GPS Error (m)

	0	0.1	0.5	1	5
0	70	461	1812	2234	9114
0.3	22439	22390	22409	22511	23969
0.5	37203	37246	37234	37218	38147
1	74213	74160	74268	74296	74738
5	390322	370467	370379	370436	370616
10	740632	740671	740607	740648	740893

Table 1: Positioning Error (in cm)

NR - Absolute Positioning

	0	0.1	0.5	1	5
0	0.59	0.59	0.59	0.59	0.62
0.3	0.93	0.93	0.93	0.93	0.95
0.5	1.34	1.34	1.34	1.34	1.36
1	2.51	2.51	2.51	2.51	2.52
5	12.31	12.31	12.31	12.31	12.31
10	24.61	24.61	24.61	24.61	24.62

GT - Absolute Positioning

	0	0.1	0.5	1	5
0	0.11	0.12	0.15	0.18	0.41
0.3	0.86	0.86	0.86	0.86	0.91
0.5	1.35	1.35	1.35	1.35	1.39
1	2.58	2.58	2.58	2.58	2.61
5	12.44	12.46	12.44	12.44	12.44
10	24.75	24.76	24.76	24.76	24.76

GT - Relative Positioning

Top Row: Pseudo Range Error (cm)
First Column: GPS Error (m)

	0	0.1	0.5	1	5
0	0.00	0.01	0.06	0.07	0.30
0.3	0.75	0.75	0.75	0.75	0.80
0.5	1.24	1.24	1.24	1.24	1.27
1	2.47	2.47	2.47	2.48	2.49
5	13.03	12.34	12.34	12.34	12.35
10	24.68	24.68	24.68	24.68	24.69

Table 2: Timing Error (in cm)

4. CONCLUSIONS

In this paper, we propagate the orbits of the navigation satellites in GPS, Galileo, and GLONASS constellations, and simulate the "weak GPS" real-time position and timing performances at lunar distance as a function of reasonable GNSS satellite measurement error and ephemeris error. Key findings are:

1. Positioning RMSE is between 200 m and 300 m.
2. Timing RMSE is of the order of 1 μ-second.
3. Positioning and timing errors are not sensitive to the number of navigation satellites in-view, as long as it is greater than or equal to 5.
4. Use of relative positioning does not substantially improve the positioning and timing accuracy.

Note that this paper focuses on real-time positioning and timing of vehicles at lunar distance. For non-real-time applications, there might be other methods to reduce the ephemeris errors of the GNSS navigation satellites, thus leading to more accurate positioning and time solutions. We also did not consider the use of Kalman filter in this paper. We expect Kalman filters can be useful to smooth out the random errors and the outliers, but they might not be effective to estimate the slow-varying ephemeris errors of the GNSS satellites.

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BIOGRAPHY

Kar-Ming Cheung *is a Principal Engineer and Technical Group Supervisor in the Communication Architectures and Research Section (332) at JPL. His group supports design and specification of future deep-space and near-Earth communication systems and architectures. Kar-Ming Cheung received NASA's Exceptional Service Medal for his work on Galileo's onboard image compression scheme. He got his B.S.E.E. degree from the University of Michigan, Ann Arbor, in 1984, and his M.S. and Ph.D. degrees from California Institute of Technology in 1985 and 1987, respectively.*

Professor Charles H. Lee *received his Doctor of Philosophy degree in Applied Mathematics in 1996 from the University of California at Irvine. He then spent three years as a Post-Doctorate Fellow at the Center for Research in Scientific Computation, Raleigh, North Carolina, where he was the recipient of the 1997-1998 National Science Foundation Industrial Post-Doctorate Fellowship. He became an Assistant Professor of Applied Mathematics at the California State University Fullerton in 1999, Associate Professor in 2005, and since 2011 he has been a Full Professor. Dr. Lee has been collaborating with scientists and engineers at NASA Jet Propulsion Laboratory since 2000. His research has been Computational Applied Mathematics with emphases in Aerospace Engineering, Telecommunications, Acoustic, Biomedical Engineering and Bioinformatics. He has published over 65 professionally refereed articles. Dr. Lee received Outstanding Paper Awards from the International Congress on Biological and Medical Engineering in 2002 and the International Conference on Computer Graphics and Digital Image Processing in 2017. Dr. Lee also received NASA's Exceptional Public Achievement Medal in 2018 for the Development of his Innovative Tools to Assess the Communications & Architectures Performance of the Mars Relay Network.*

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