Elliptic Edge Polygons for Observational Coverage Planning

Michael Trowbridge, Russell Knight and Elly Shao
Jet Propulsion Laboratory
California Institute of Technology
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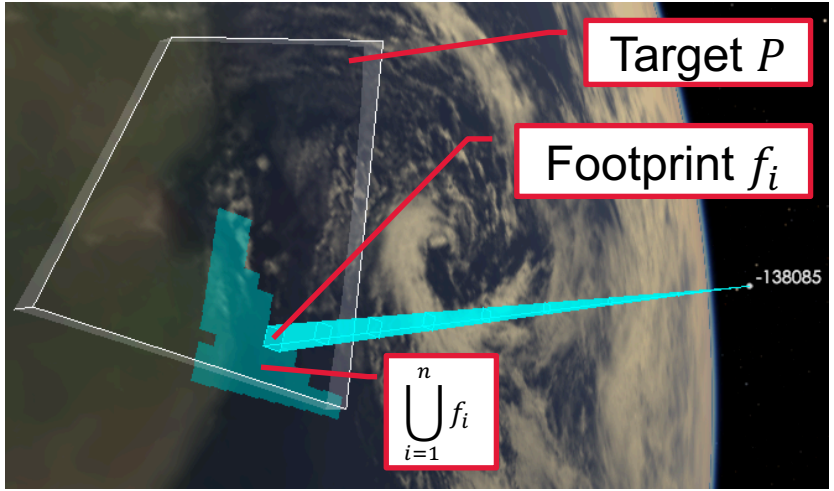
Area Coverage Planning Problem Recap

Choose observation footprints *F* to satisfy a target polygon *P*

- A (time, target, rotation) tuple defines footprint f_i
- A plan is a set of footprints $F = \{f_1, f_2, \dots, f_n\}$
- Goal state $g(n) = P \in \bigcup_{i=1}^{n} f_i$
- Heuristic for distance to the goal state
  - $h(n) = \text{area}(P) - \text{area}(P - \bigcup_{i=1}^{n} f_i)$
- Optimization formulations of the problem:
  - Smallest number of footprints (time varying footprint size)
  - Shortest duration (time varying slew cost between footprints, Shao et al. 2018)



Shao et al. 2018

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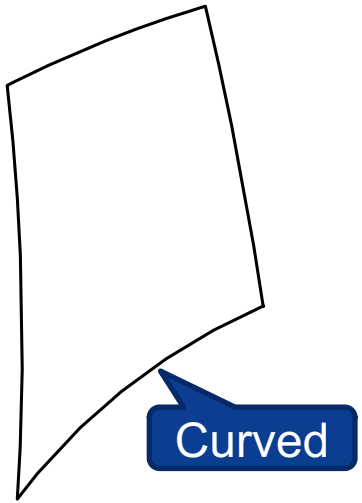
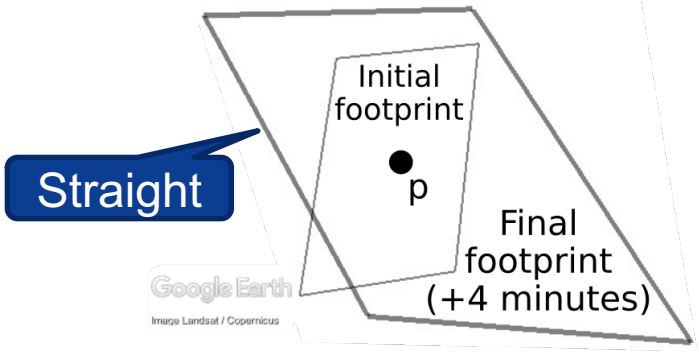
“Do the footprints really look like that?”

Chris Valicka, SPARK/ICAPS 2018, Delft, The Netherlands



Interpolated sides (22 points)
(Mitchell et al. 2018)



Uninterpolated Great Circles arc sides
(Shao et al. 2018)

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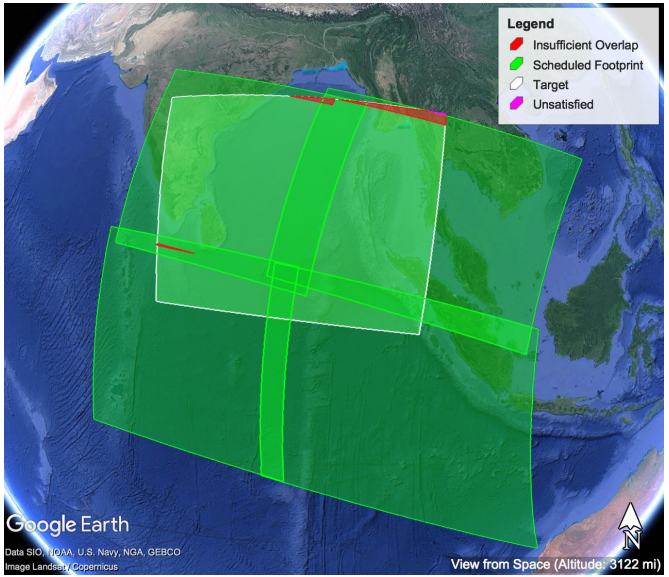
How could drawing a line affect a planner?

Neglecting curvature misinforms the distance to goal state heuristic

- Two-pass mosaic planner/scheduler
  - Fast, local search for next footprint, guided by $h(n)$
  - Slower second stage validation of overlap and coverage
- Second stage reschedules the missed fragments (suboptimal)



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Special Needs of Space Observational Coverage Planners

Things to consider when drawing footprint edges

Need to handle	Why
Geometry on an arbitrary triaxial ellipsoid	First campaign for small bodies: Churyumov-Gerasimenko 67P (Mysen 2004, Jorda et al. 2016), small moons
Polygons that contain the North or South pole	Polar ice and interesting processes on the Moon (Spudis et al. 2013) and Mars (Bibring et al. 2004)
Polygon edges that cross the antimeridian	Poles, global mapping campaigns
Edges defined by the shape of the instrument	Oblique footprints of framing instruments (Shao et al. 2018, Mitchell et al. 2018)

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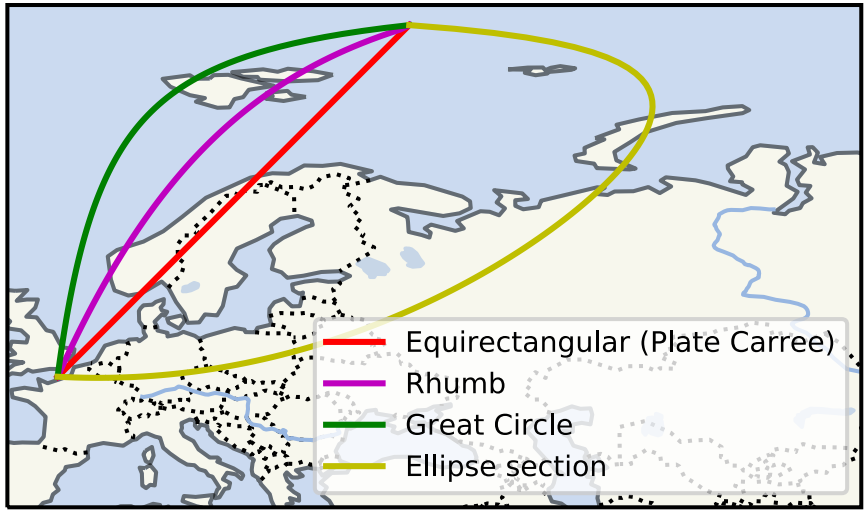
Existing Ways to Draw Polygon Edges

- **Great Circle**: intersection of plane through origin and sphere (Hachenberger and Kettner 2019)
- **Rhumb**: line (spiral) of constant compass heading (Nunes, 1537)
- **Equirectangular**: $x \leftarrow \mathrm{lat}, y \leftarrow \mathrm{lon}$ (Chamberlain and Duquette 2007)
- We propose: **Ellipse section/arc**



Quiz: which projection is this 2D map?

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Problems with Existing Line types

	Great Circles	Rhumb	Equirectangular
Pole quirks	*	Yes	Yes
Antimeridian quirks	*	Yes	Yes
Triaxial ellipsoid error	*	Yes	Yes
Polygon shape error over long distances**	Yes	Yes	Yes

*: If lat/lon is used, yes. If vectors and planes are used, no.
**: Chamberlain and Duquette published a table relating distance to approximation error (2007).

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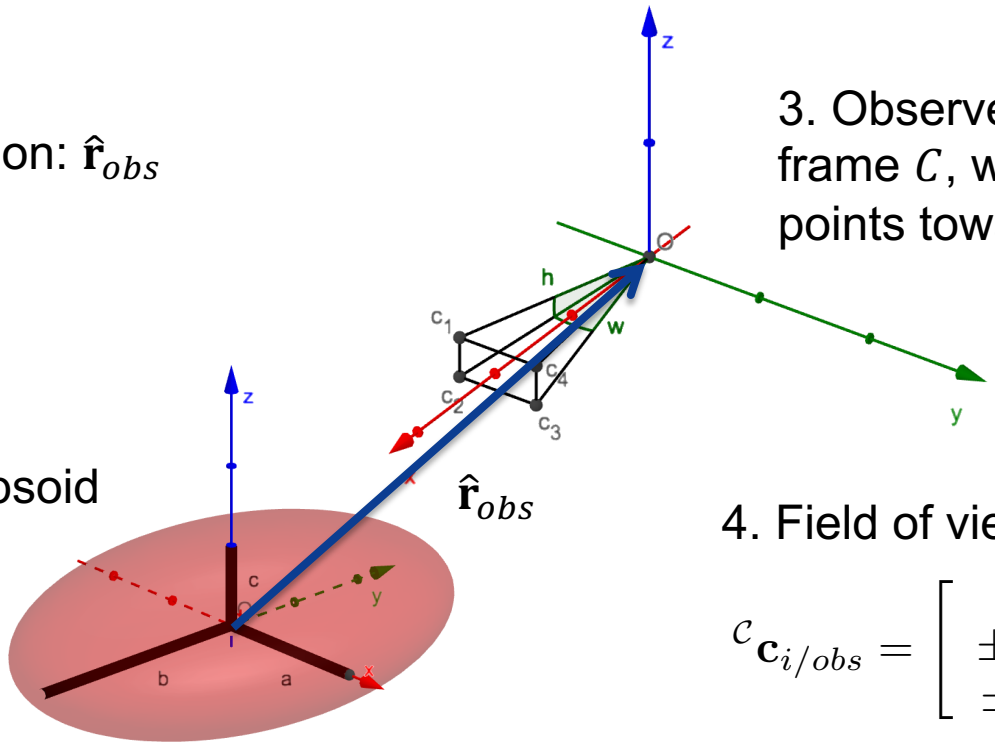
Ellipse Sections for Polygon Edges

2. Observer position: $\hat{\mathbf{r}}_{obs}$

3. Observer's camera frame C, whose x axis points toward the target



1. Target triaxial ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

4. Field of view corner vectors

$$^{\mathcal{C}}\mathbf{c}_{i/obs} = \begin{bmatrix} 1 \\ \pm\tan(w/2) \\ \pm\tan(h/2) \end{bmatrix}$$

w, h: angles in radians

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Camera frame transformation

- Recall
  - Planner chooses $\left(t, \hat{\mathbf{r}}_{tgt}, \theta\right)$
  - t determines $\hat{\mathbf{r}}_{obs}$
- Construct camera frame to body frame transform using basis vectors in body-fixed coordinates:

$$\hat{\mathbf{u}} = \frac{\mathbf{r}_{tgt} - \mathbf{r}_{obs}}{\left|\mathbf{r}_{tgt} - \mathbf{r}_{obs}\right|}$$

$\hat{\mathbf{u}}$ is a consequence of time, target

$$^{\mathcal{B}}R_{\mathcal{C}} = \left[\begin{array}{ccc} ^{\mathcal{B}}\hat{\mathbf{u}} & ^{\mathcal{B}}\hat{\mathbf{v}} & ^{\mathcal{B}}\hat{\mathbf{w}} \end{array}\right]$$

Let θ determine $\hat{\mathbf{v}}$ or $\hat{\mathbf{w}}$, use cross product to compute the last axis.

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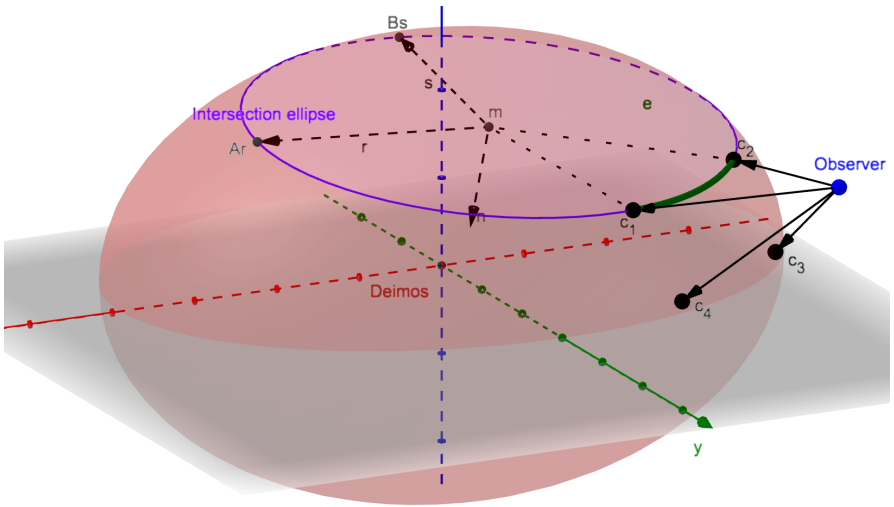
Ellipsoid-plane Intersection for Camera FOV



- $\mathbf{c}_i$ are surface intercepts of camera corners
- $(c_i, c_{i+1}, \hat{\mathbf{r}}_{obs})$ define an FOV edge plane, normal $\hat{\mathbf{n}}$
- Intersection of plane and ellipsoid has form (Klein 2012)

$$\mathbf{x} = \mathbf{m} + A\cos\theta\hat{\mathbf{r}} + B\sin\theta\hat{\mathbf{s}}$$

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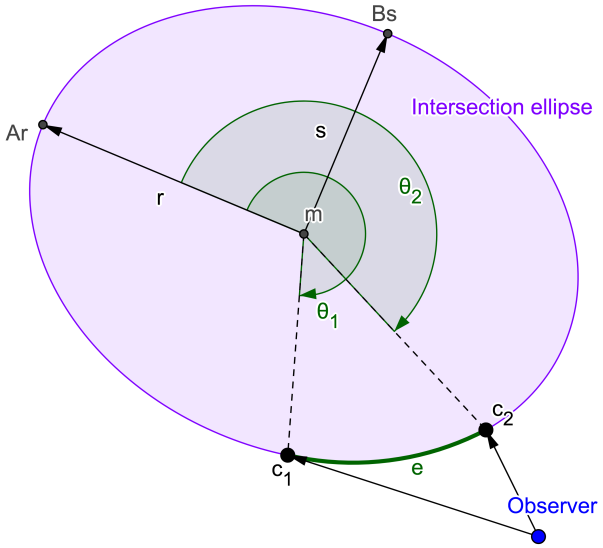
An Ellipse Arc Edge



- An FOV edge is now defined by:
  - In the ellipse plane $(\hat{\mathbf{n}}, \mathbf{m})$
  - Clock angle domain $[\theta_2, \theta_1]$

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Elliptic Edge Polygon Operations Defined in the Paper

Edge operations

- Is point on edge?
- Are arcs coplanar?
- Find intersection of arcs
- Split an arc
- Find extreme point on arc (furthest from bounding cone center)

Polygon operations

- Compute polygon bounding cone
- Point in convex polygon check

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Experiment: When Does Edge Type Matter?

Methodology

- Two observer use cases
- Compute footprints and interpolate edges with each edge type (Great Circles, Rhumb, Equirectangular, Elliptic)
- Sweep off-nadir angles until footprints don't completely intersect the target body
- Treat elliptic as truth (t) and compute a fractional polygon disagreement error for the other three (a)

$$\varepsilon = \frac{\text{area}\left(P_a \cup P_t\right) - \text{area}\left(P_a \cap P_t\right)}{\text{area}\left(P_t\right)}$$

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Experiment

Observer Cases

	Near/Narrow	**Far/Wide**
Spacecraft	SkySat-1	MRO
Body observed	Earth	Mars
Trajectory	LEO	Aerobraking
Nadir Altitude	578 km	8992 km
Field of view	$0.37° \times 0.15°$	$6° \times 6°$

- A small FOV relative to the target is
  - Near: altitude small relative to target body radii
  - Narrow: small angular field of view
- Converse for a large FOV relative to the target

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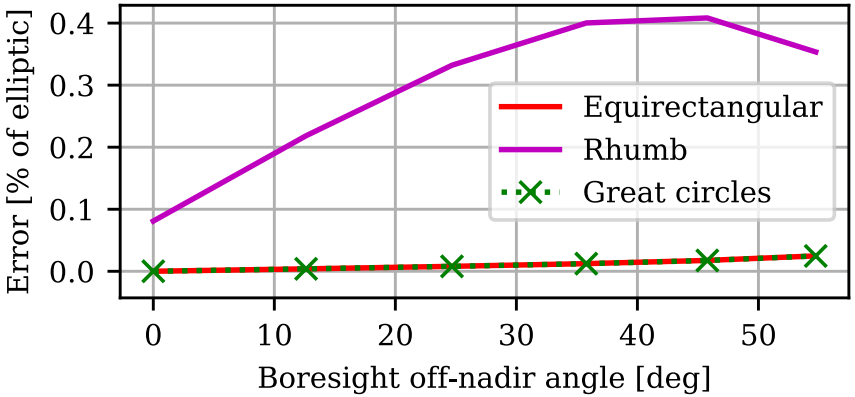
Experiment

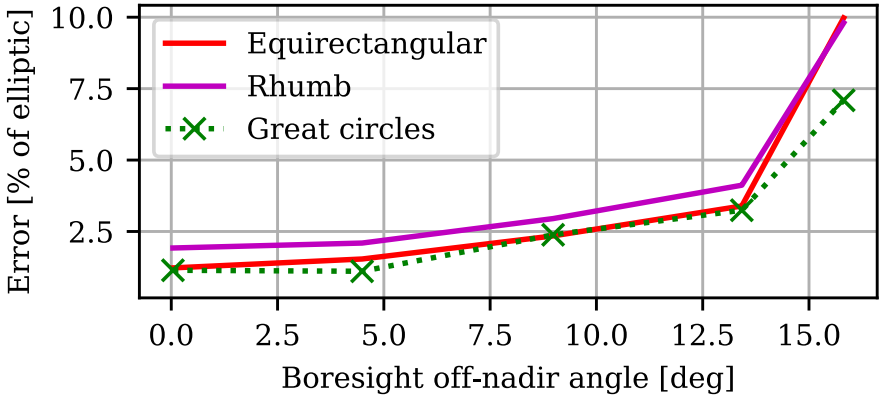
Results: Approximation Error

Near/Narrow – Earth LEO



Negligible.

Far/Wide – MRO Aerobraking



Non-negligible.

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Experiment

Results: Runtime Comparison (Near/Narrow)

- Great circles footprints
  - 24 points/footprint
  - Implementation: CLASP (Knight and Chien 2006)
  - Bounding box short-circuit
- Elliptic edge polygons are
  - ~5x faster to construct
  - ~2x faster to query
  - Less variable in runtime

	Elliptic Edge		Great Circles	
	Mean	Std. Dev.	Mean	Std. Dev.
Construct	56.3	19.7	275.8	41.8
Containment check, inside	0.5	0.5	0.9	0.5
Containment check, outside	0.2	0.4	0.4	2.4

Units: μsec

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Discussion

- Wider applicability
  - Elliptic edge operations may also be used for Great Circles arcs to avoid pole/antimeridian problems.
  - Suitable for framing instruments, but not pushbrooms
- Unexpected round trip error converting ellipse-clock angle to body fixed point (workaround in the paper)
- Future work: need a point-in-concave-polygon check before planners use elliptic edge polygons

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Conclusions

- Elliptic Edge Polygons preserve footprint curvature without adding intermediate points to the edge
- Footprint curvature is only a concern when the field of view is large relative to the target body
  - Fly-by, small body mapping, etc.

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