DIFFERENCING METHODS FOR 3D POSITIONING OF SPACECRAFT[1]

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Abstract

In this paper we describe the methods and system concepts that detect and locate in real-time the active (cooperative) and passive (non-cooperative) spacecraft, which we call targets, at the Geostationary Orbit (GEO) and at lunar distances. The methods do not assume any orbital knowledge of the target spacecraft, thus can be used for both quiescent and maneuvering spacecraft. We show by simulations that this approach can achieve meter-level three-dimension (3D) positioning accuracy with reasonable time delay measurement accuracy.

The methods leverage on existing satellites and natural features (lunar case) with accurately known positions that can act as references, and/or in the case of the GEO places a dedicated "reference" spacecraft into an eccentric geosynchronous orbit over a region of interest (e.g., sky above North America). The ground antennas track the signals transmitted or reflected by the target and the reference spacecraft to measure their signals' time difference of arrival (TDOA). There are two modes of operations:

a) "Single-differencing" – this approach assumes a transmitting radar illuminating both the reference and an uncooperative spacecraft (the target), and the ground radar antennas receiving the radar echoes to derive the relative position between the reference and the target.

b) Double-differencing – this approach applies to the scenarios when either a) the target and/or the reference spacecraft with difference clocks transmitting one-way ranging signals to the ground, or b) multiple uplink radars are used to illuminate the reference and the target.

We outline the mathematical derivations of the above schemes, and provide the accuracy performance simulations for some realistic and hypothetical scenarios. We also discuss some other applications.

I. Introduction & System Concept

In this paper we derive the theory and describe the high-level system concepts that demonstrate the feasibility of detecting active (cooperative) and passive (non-cooperative) spacecraft in real-time in the Geostationary Orbit (GEO) and in lunar orbits, where Earth's Global Navigation Satellite System (GNSS) navigation services are not available. The methods do not assume any orbital knowledge of the target spacecraft, thus can be used for both quiescent and maneuvering spacecraft. This enables tracking spacecraft under dynamic situations like during thrusting, and executing immediate correction in case of trajectory deviation. We show by simulations that this approach can achieve meter-level three-dimension (3D) positioning accuracy.

For the GEO case, this can be done by making use of existing GEO satellites as references, and/or by placing a dedicated "reference" spacecraft into an eccentric geosynchronous orbit over a region of interest (e.g. above N. America). The sky above North America is rather congested, and there is no shortage of GEO satellites with accurately known positions that are separated from their neighbors by less than one tenth degree longitude [1]. For the dedicated "reference" spacecraft approach, which is more expensive but is more controllable, one can adjust the orbit so the "reference" spacecraft can loiter around the sky back-and-forth in the vicinity of the GEO over the region of interest. In this way, the reference spacecraft can be close to any "static" GEO targets along its path.

For the lunar case, the reference can be a signal-transmitting beacon (active), and/or a prominent feature (passive) on the near-side of the lunar surface. One example of lunar prominent feature is the Tycho Crater located near the South Pole of the Moon, which has been used for the moon-bounced calibration for DSN's uplink arraying experiment [2].

On the ground, the ground antennas track the signals transmitted or reflected by the target and the reference spacecraft to measure their signals' time difference of arrival (TDOA). There are two modes of ground operations:

a) "Single-differencing" – this method assumes a transmitting radar illuminating both the reference and an uncooperative spacecraft (the target), and the ground radar antennas receiving the radar echoes measure the difference in signal arrival time to derive the relative position between the reference and the target. The term "single-differencing" is in parenthesis because it differs from the convention meaning of single-differencing, and refers to the fact that the TDOA between the radar echo reflected by the reference, and that reflected by the target is measured only once at the receiving radar. Due to the proximity between the reference and the target, this delay data type eliminates most of the systematic errors in the delay measurements when the radio waves pass through the solar plasma and the Earth's atmosphere, and also due to the instrumental delay of the receiving ground radar. The transmitted radar pulse hits the reference and the target at different time, but this time-bias is taken care of in the differencing equations. This "single-differencing" measurement technique achieves the same systematic error cancellation capability as a double-differencing approach, yet only tolerates the random error effect of a single-differencing measurement.

b) Double-differencing – this approach applies to the scenarios when either i) the target and/or the reference spacecraft with different clocks transmitting one-way ranging signals to the ground, such as formation-flying spacecraft and lunar lander-rover pair, or b) multiple uplink radars are used to illuminate the reference and the target. In both cases, in addition to the aforementioned systematic errors, the reference and the target spacecraft have different time-biases or clock-biases with respect to the ground antennas. To eliminate these biases, a double-differencing data type is created by differencing the two TDOA's from two ground antennas. A high-level concept of this approach is also discussed in Section 4 of [3].

The rest of the paper is organized as follows. Section II outline the derivations of the "single-differencing" and double-differencing algorithms. Section III provides the simulation assumptions and results. Section IV discusses the conclusion and other applications.

II. Derivation of Algorithms

The 3D positioning computation methods for the "single-differencing" approach and the double-differencing approach are outlined as follows.

A. *"Single-Differencing" (One Transmitter)*

In this problem formulation that leverages on "single-differencing" measurements, the method assumes one transmitting radar GS_0 and n receiving radar, whose positions are accurately known, where $n \geq 3$, and no three antennas lie on a straight line. Consider the i^{th} receiving antenna GS_i tracking the echoes from the target and the reference spacecraft as shown in Figure 1.

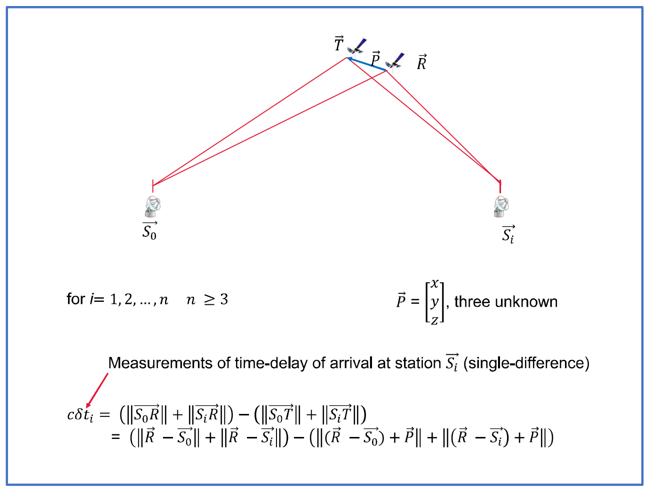

Fig 1: 3D Positioning by "Single-Differencing"

We construct the cost function

$$f_i(\vec{P}) = c\delta t_i - (\|\vec{R} - \vec{S_0}\| + \|\vec{R} - \vec{S_\iota}\|) + (\|(\vec{R} - \vec{S_0}) + \vec{P}\| + \|(\vec{R} - \vec{S_\iota}) + \vec{P}\|)$$

$$\text{for } i = 1, 2, \ldots, n \qquad (1)$$

$\|.\|$ denotes the magnitude of a vector. For example, for the relative position vector $\vec{P} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $\|\vec{P}\| = \sqrt{x^2 + y^2 + z^2}$. Also δt_i is the measured TDOA between the target and the reference spacecraft at station GS_i, and c is the speed of light.

B. Double-Differencing (Multiple Transmitters)

In this problem formulation that leverages on double-differencing, n antennas form $n-1$ independent baselines. The method assumes n receiving antennas whose positions are accurately known, where $n \geq 4$. Consider taking a subset of m baselines for satellite position determination, where $3 \leq m \leq \binom{n}{2}$. The m baselines must include at least 3 independent baselines to achieve a 3D position fix. Consider the i^{th} baseline with two receiving antennas GS_{i_1} and GS_{i_2} shown in Figure 2.

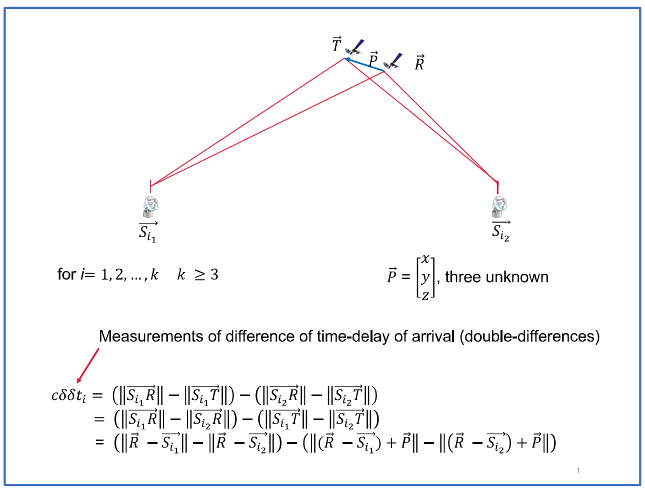

Fig 2: 3D Positioning by "Double-Differencing"

We construct the cost function

$$f_i(\vec{P}) = c\delta\delta t_i - (\|\vec{R} - \overrightarrow{S_{i_1}}\| - \|\vec{R} - \overrightarrow{S_{i_2}}\|) + (\|(\vec{R} - \overrightarrow{S_{i_1}}) + \vec{P}\| - \|(\vec{R} - \overrightarrow{S_{i_2}}) + \vec{P}\|)$$

$$\text{for } i = 1, 2, \ldots, m \qquad (2)$$

$\delta\delta t_i$ is the measured time delay of reference signal arrival between $\overrightarrow{S_{i_1}}$ and $\overrightarrow{S_{i_2}}$, minus the measured time delay of target signal arrival between $\overrightarrow{S_{i_1}}$ and $\overrightarrow{S_{i_2}}$. Note that $\overrightarrow{S_{i_1}}$, $\overrightarrow{S_{i_2}}$, and $\vec{R}$ are known quantities.

For both cases the Jacobian of the cost function can be calculated (equation 3) as follows:

$$J_{i1}(x, y, z) = \frac{\partial f_i(\vec{P})}{\partial x} \qquad (3a)$$

$$J_{i2}(x, y, z) = \frac{\partial f_i(\vec{P})}{\partial y} \qquad (3b)$$

$$J_{i3}(x, y, z) = \frac{\partial f_i(\vec{P})}{\partial z} \qquad (3c)$$

The Jacobian matrix is a $n \times 3$ matrix for "single-differencing" method, and a $m \times 3$ matrix for double-differencing method. We outline the method for double-differencing as shown below:

$$J(\vec{P}) = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial z} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} & \frac{\partial f_2}{\partial z} \\ \vdots & \vdots & \vdots \\ \frac{\partial f_m}{\partial x} & \frac{\partial f_m}{\partial y} & \frac{\partial f_m}{\partial z} \end{bmatrix} \qquad (4)$$

We then evaluate $\vec{P}$ using the Newton's Method as shown below. Using an initial guess $\overrightarrow{P_0}$,

$$\overrightarrow{P_0} = \begin{bmatrix} P_{o,x} \\ P_{o,y} \\ P_{o,z} \end{bmatrix} \quad F_0 = \begin{bmatrix} f_1(\overrightarrow{P_0}) \\ f_2(\overrightarrow{P_0}) \\ \vdots \\ f_m(\overrightarrow{P_0}) \end{bmatrix} \quad J_0 = J(\overrightarrow{P_0})$$

$$F_k = \begin{bmatrix} f_1(\overrightarrow{P_k}) \\ f_2(\overrightarrow{P_k}) \\ \vdots \\ f_m(\overrightarrow{P_k}) \end{bmatrix} \qquad J_k = J(\overrightarrow{P_k})$$

$$\Delta\overrightarrow{P_k} = (J_k{}^T J_k)^{-1} J_k{}^T F_k \qquad (5)$$

$$\overrightarrow{P_{k+1}} = \overrightarrow{P_k} - \Delta\overrightarrow{P_k} \qquad (6)$$

III. Simulations

We consider both the GEO case (36000 km) and the lunar case (360000 km). An example of 4 ground antennas at Goldstone, Alaska, Haleakala (Hawaii), and Malargue is shown in Figure 3. The reference and the target spacecraft are separated by 50 kilometers, and are located in the sky above North America. For the "single-differencing" case, we assume one transmitting radar located at Goldstone, and the receiving radars are located at Alaska, Haleakala, and Malargue. For the double-differencing case, the four ground antennas can form 6 baselines, 3 of which are independent.

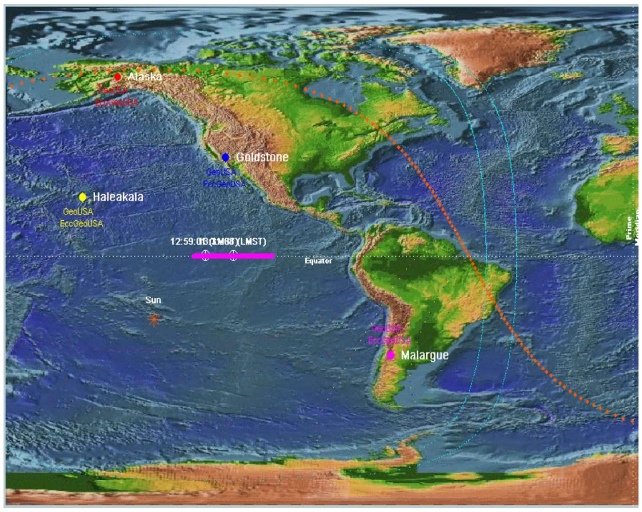

Fig 3: Example of 4 Ground Stations

To illustrate the effect of pairwise distances between ground stations on 3D localization accuracy, we use a hypothetical constellation of 4 ground stations on an idealized sphere of Earth with variable separations as shown in Figure 4.

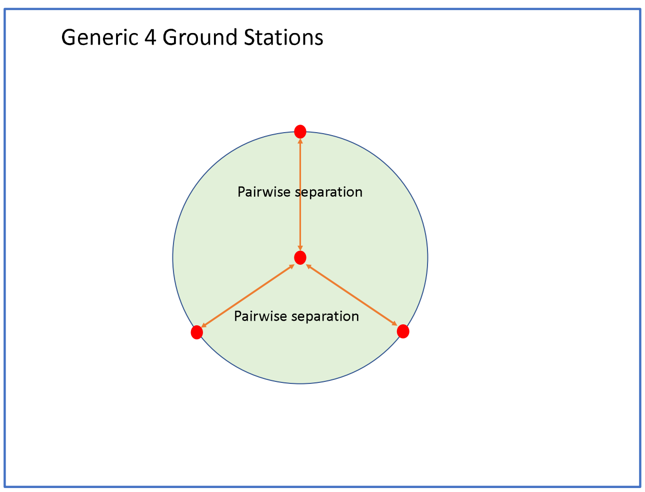

Fig 4: Hypothetical 4 Ground Stations on Sphere

We consider the ideal case that the transmitted or reflected signals from the reference and target spacecraft can be detected and identified. Also, we assume that all the systematic biases like media delays, time-offsets, and instrument delays are perfectly cancelled by the differencing processes, and we only consider the effect of ground station delay measurement errors with zero mean and Gaussian distribution. In real-life operation, the positioning accuracy would be inferior due to the residual systematic biases, but should be minimal due to the close proximity between the reference and the target, and the systematic biases would be mostly cancelled.

A. "Single-Differencing" Method:

The Root-Mean-Square error (RMSE) performance of the GEO case as a function of ground receiving station delay measurement errors at Alaska, Haleakala, and Marlargue is shown in Figure 5, and the lunar case is shown in Figure 6. For the hypothetical case (Figure 4), we assume the transmitting radar is located at the center site, and the three receiving radars are located on the circumference. The RMS error performance at the GEO distance as a function of ground station delay measurement errors and for various separation between the transmitting radar and receiving radars are shown in Figure 7, and the lunar case are shown in Figure 8.

Note that for both the GEO and lunar cases, the "single-differencing" method produces sub-meter level 3D relative localization accuracy when the delay differential measurement error is of the order of one meter[2], and better than 10-meter-level accuracy when the delay differential measurement error is of the order of 10 meters. Also, as observed in the ideal and hypothetical cases for GEO and Lunar distances (Figures 7 and 8), the RMSE performances are almost indistinguishable for transmit-receiver radar distances between 1 km and 7000 km for RMSE of 50 cm or lower. Again in real-life operation the positioning accuracy would be inferior, but not by much compared to the ideal case due to the close proximity between the reference and the target, and the systematic biases would be mostly cancelled.

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[2] The DSN's Same Beam Interferometry (SBI) system was verified to produce the MRO-ODY residuals of 1 picosecond, or 0.3 mm in range uncertainty, private communication with Jim Border.

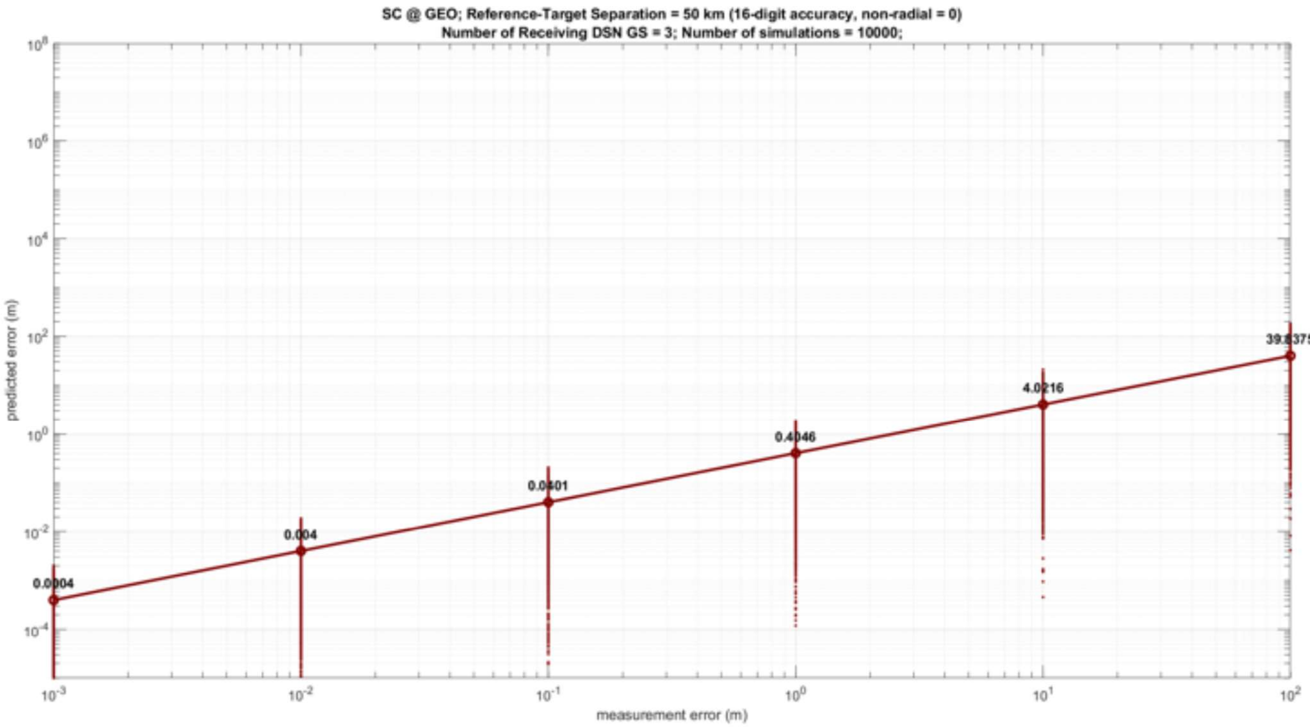

Fig 5: RMSE at GEO Distance (Realistic Case in Fig 3, Single-Differencing)

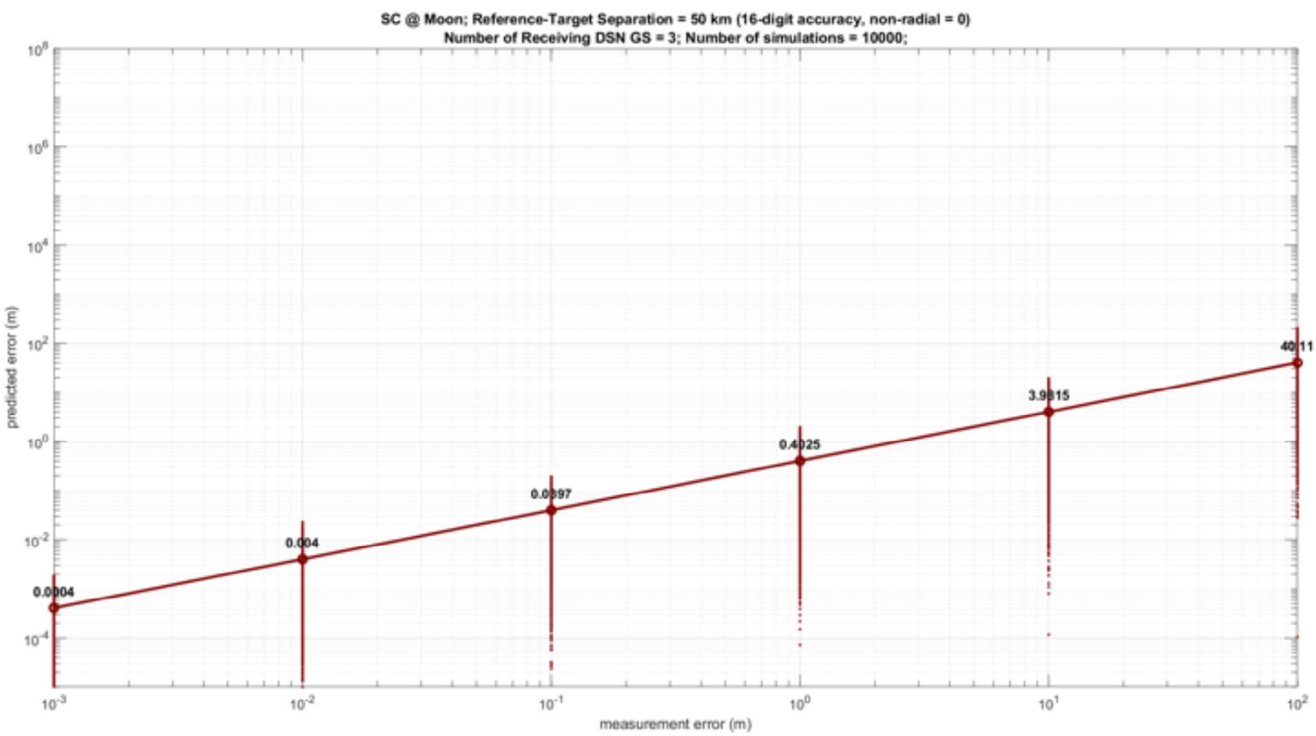

Fig 6: RMSE at Lunar Distance (Realistic Case in Fig 3, Single-Differencing)

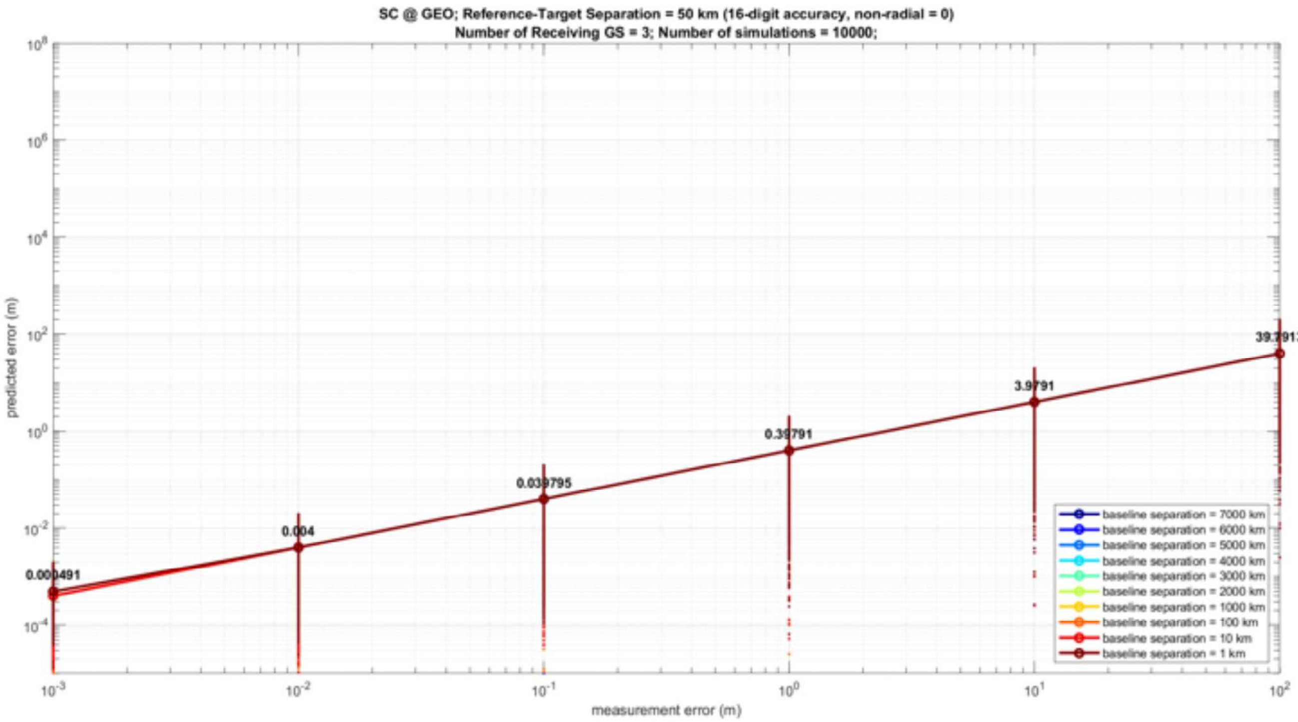

Fig 7: RMSE at GEO Distance (Hypothetical Case in Fig. 4, Single-Differencing)

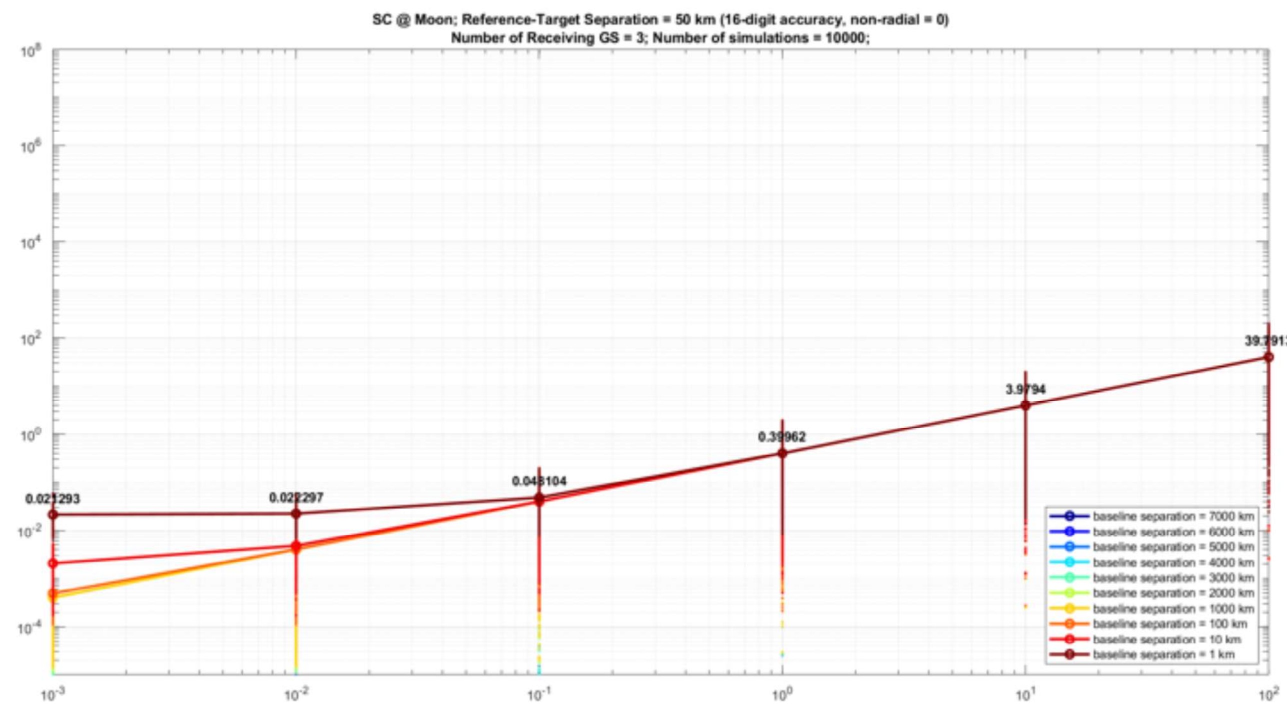

Fig 8: RMSE at Lunar Distance (Hypothetical Case in Fig. 4, Single-Differencing)

B. Double-Differencing Method:

In double-differencing, the four ground stations form 6 baselines; three of which are independent. The RMS error performance of the GEO case as a function of ground receiving station delay measurement errors at Goldstone, Alaska, Haleakala, and Marlargue is shown in Figure 9, and the lunar case is shown in Figure 10. For the hypothetical case (Figure 4), the RMS error performance at the GEO distance as a function of ground station delay measurement errors and for various separation between the center node and the circumferential nodes are shown in Figure 11, and the lunar case are shown in Figure 12.

It is quite surprising that the "single-differencing" performance is more than two order of magnitude better than the double-differencing approach. To achieve meter level accuracy in double-differencing, the delay differential measurements need an accuracy of 1 mm, which is supportable with Deep Space Network (DSN)'s high precision equipment[2]. However, it is not clear if commercial and military equipment can afford this kind of measurement accuracy. The big difference in accuracy performance between "single-differencing" and double-differencing is currently under investigation.

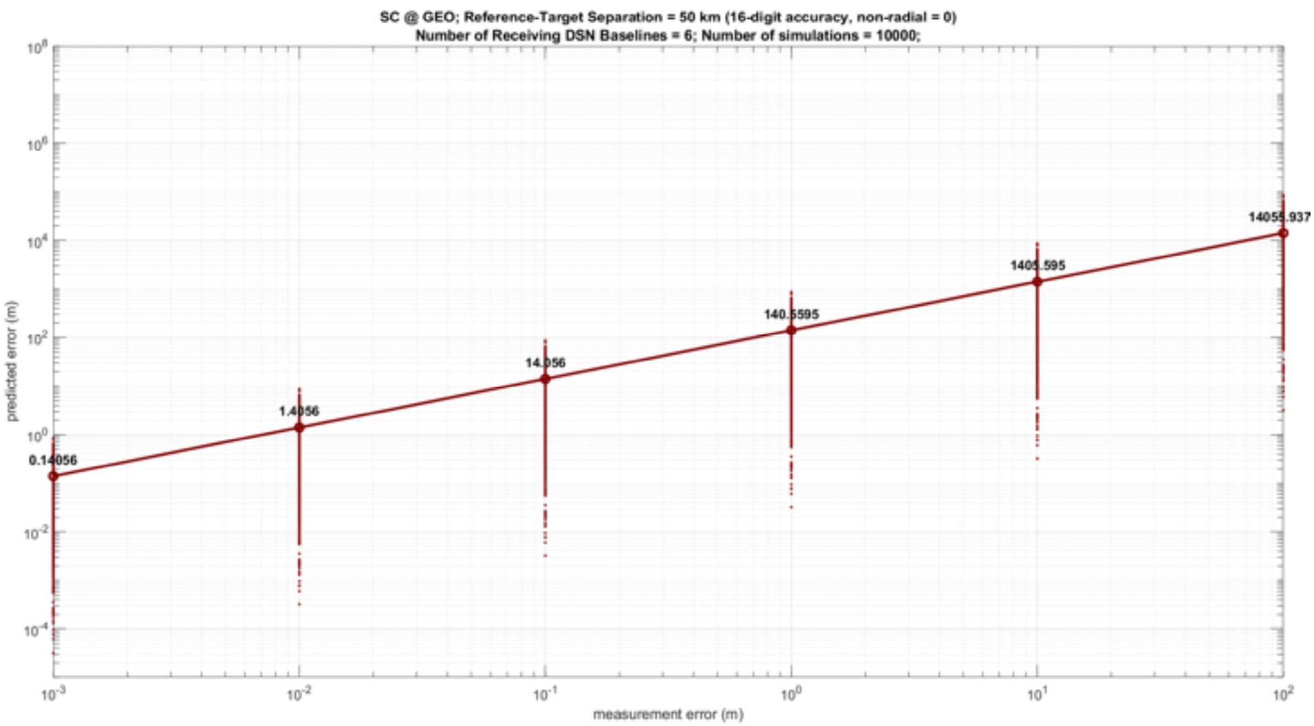

Fig 9: RMSE at GEO Distance (Realistic Case in Fig 3, Double-Differencing)

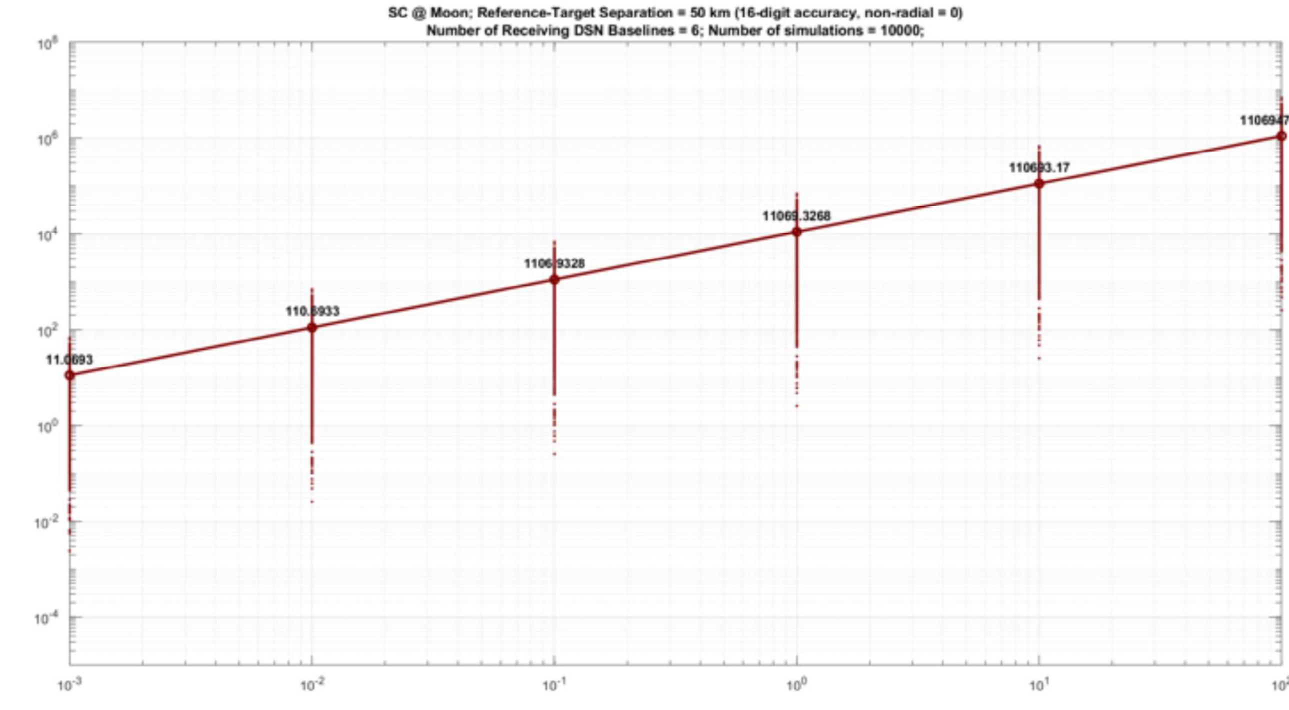

Fig 10: RMSE at Lunar Distance (Realistic Case in Fig 3, Double-Differencing)

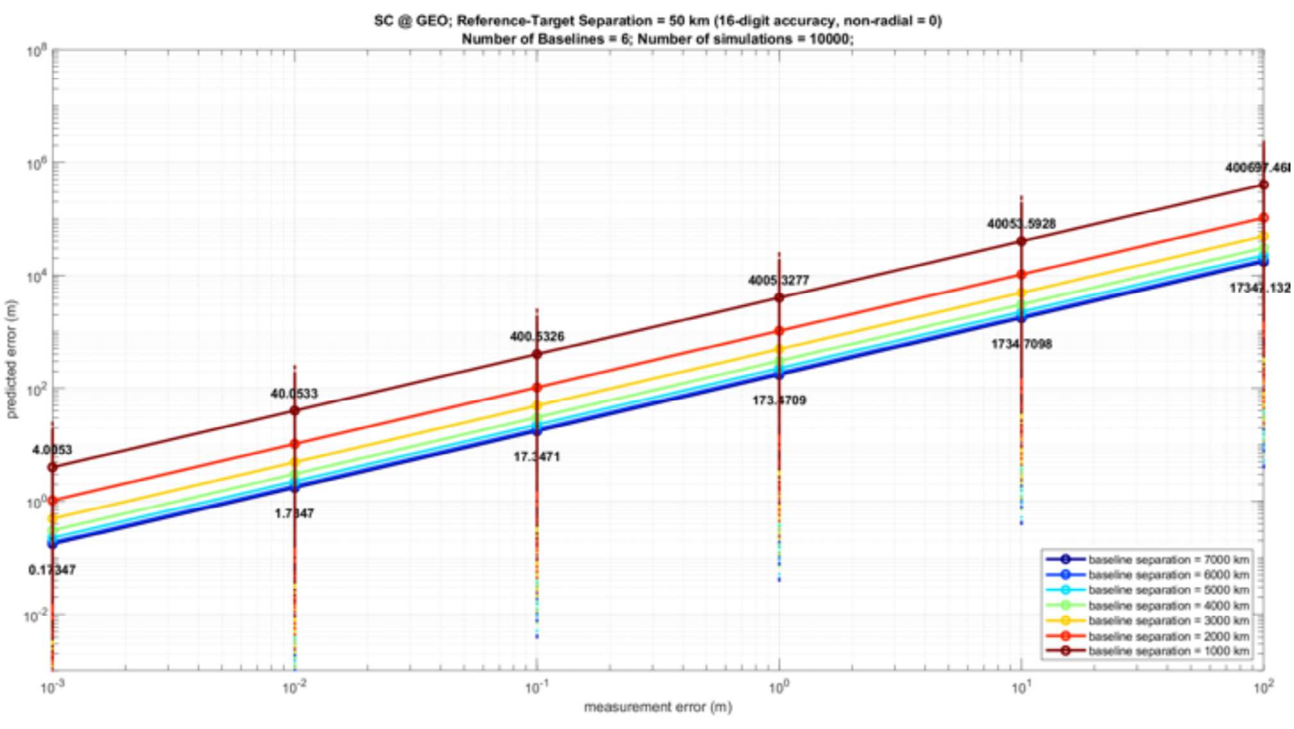

Fig 11: RMSE at GEO Distance (Hypothetical Case in Fig. 4, Double-Differencing)

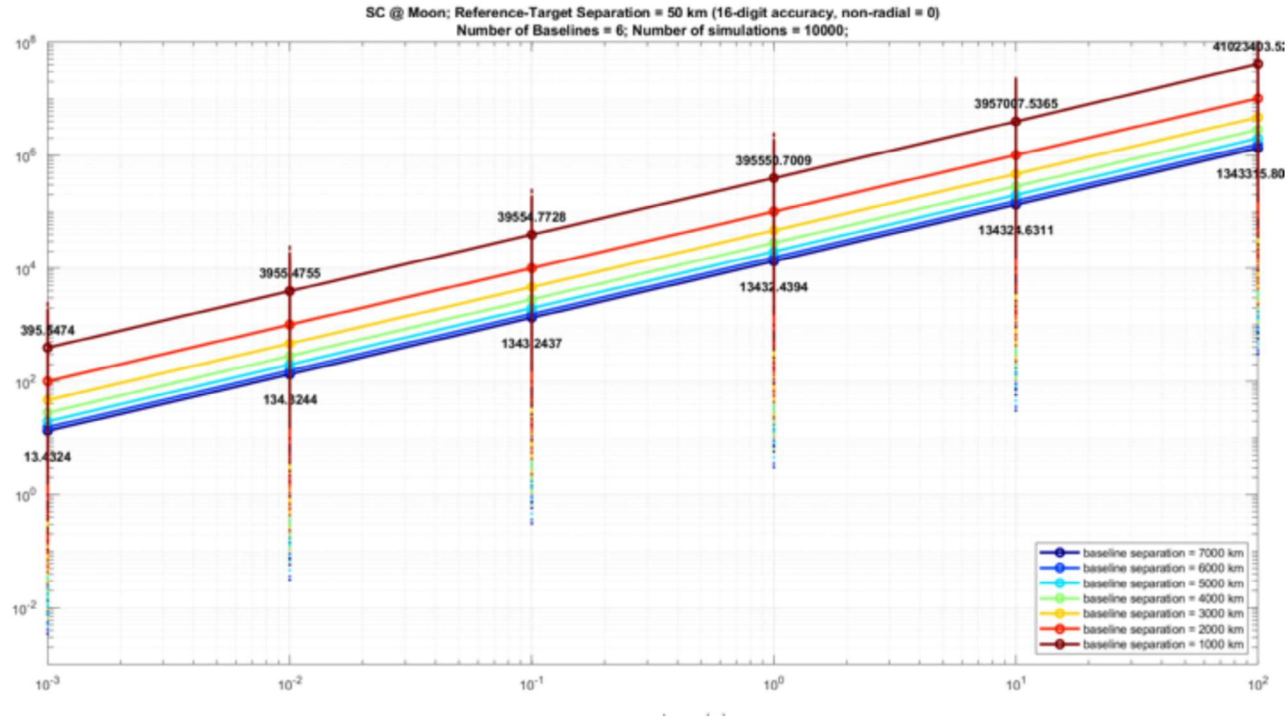

Fig 12: RMSE at Lunar Distance (Hypothetical Case in Fig. 4, Double-Differencing)

IV. Conclusion & Applications

In this paper, we describe two 3D localization methods based on "single-differencing" and double-differencing of signal arrivals between two spacecraft and multiple baselines of antennas on the ground. Simulations show that the methods can achieve meter level relative positioning accuracy for spacecraft at GEO and at lunar distance.

Given the above simulation results, as long as the delay between the target and the reference can be measured to within the accuracy levels for "single-differencing" and double-differencing as shown above, the techniques can be useful in some other civilian and military applications, where the uncooperative targets of interest are maneuvering in some fast and unpredictable manner.

For passive uncooperative spacecraft case, one example is using "single-differencing" with multiple mobile ground radars to support a friendly aircraft engaging in dogfight with an enemy aircraft. In this case, the friendly aircraft is the reference, and the enemy aircraft is the target. A transmitting ground radar illuminates both the reference and the target. The receiving radars receive their echoes and compute the relative position vector $\vec{P}$ in real-time, which provides the vital information of the direction and range of the enemy aircraft. Similarly, multiple ships and/or buoys can form multiple baselines of sonars to support a friendly submarine's cat-and-mouse pursuit of an enemy submarine. In both cases, predictive approaches like Kalman filter might not be too effective, and real-time and accurate relative positioning would be essential.

The double-differencing approach can be useful for precision and real-time detection and localization of incoming missiles. High-power and directional radars can illuminate some of the GEO satellites over the North America's sky that can serve as references, and broad-beam radars can illuminate the lower range of attitude (1000 – 2000 km) that covers the missiles' trajectories. The ground receiving radars use TDOA measurements to compute the positions of the missiles in real-time.

Other applications include orbital debris removal services [4], and precision approach radar system for airports.

References

[1] https://en.wikipedia.org/wiki/List_of_satellites_in_geosynchronous_orbit

[2] Vilnrotter, Victor et. al. “Planetary Radar Imaging with the Deep Space Network’s 34-meter Uplink Array,” IEEE Aerospace Conference, March 2011, Big Sky, Montana.

[3Cheung, Kar-Ming, Charles Lee, “Navigation Tracking with Multiple Baselines Part I: High-Level Theory and System Concepts,” submitted to IEEE Aerospace Conference 2019, Big Sky, Montana, March 2019.

[4] https://spacenews.com/orbital-debris-removal-company-astroscale-raises-50-million/

Acknowledgements

The authors would like to thank Jay Breidenthal and Victor Vilnorotter on the helpful discussion on multi-static radar.

The research described in this paper was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration. The research was supported by the NASA’s Space Communication and Navigation (SCaN) Program.

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