Differencing Methods for 3D Positioning of Spacecraft

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Outline of Talk

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Derivations of Algorithms

- "Single-Differencing" Method
- Double-Differencing Method

Simulation Results

- "Single-Differencing" Method, GEO and Moon
- Double-Differencing Method, GEO and Moon

Conclusion, Upcoming Activities, and Other Potential Applications

Motivations and System Concept (1)

- Rationales for precision radar localization of objects in GEO and lunar orbits
  - Today's small satellites can pack very high surveillance capabilities, yet they can be difficult to detect and to locate at GEO range
  - There are malfunction satellites and orbital debris in Earth's and lunar orbits
- Current bistatic or multi-static radar approaches are based on "sum of range", and have accuracy of 100's meters or kilometers
- Deep Space Network (DSN)'s Delta Differential One-Way Ranging (ΔDOR) and GPS's geodetic survey use the concept of double-differencing to eliminate systematic biases like instrument delays, clock biases, media delays, etc., but they require a reference with accurately known location
- There is no shortage of GEO satellites above the sky of the United States
- The Moon's Tycho Crater located near the S. Pole has been used for moon-bounced calibration for DSN's uplink array experiment
- In this paper, we introduce two differential radar techniques:
  - "Single-differencing" that assumes one transmitter and multiple receivers
  - "Double-differencing" that allows multiple transmitters and multiple receivers

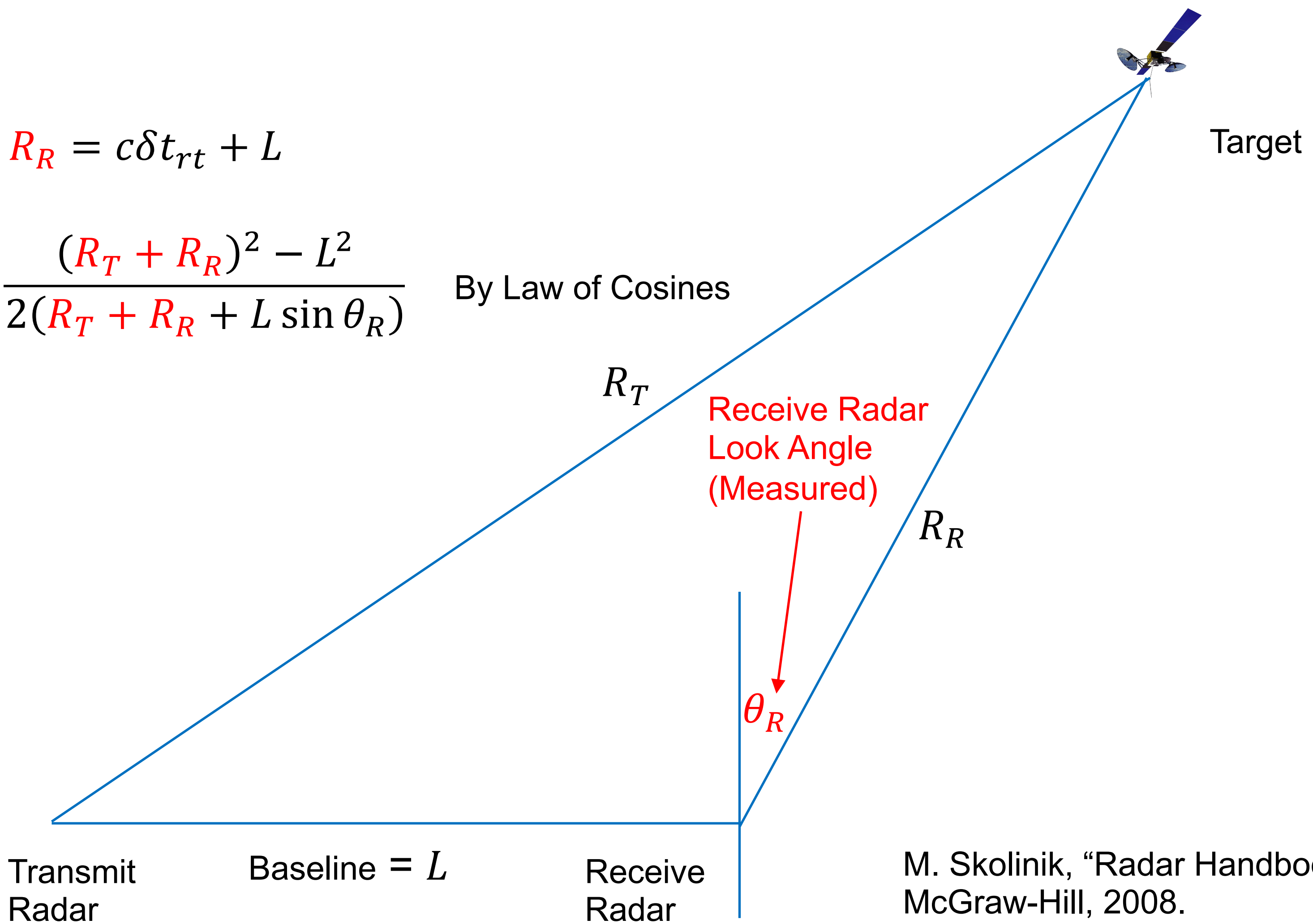
Motivations and System Concept (2)

- Current multi-static radar approach is based on sum of range

$$R_T + R_R = c\delta t_{rt} + L$$

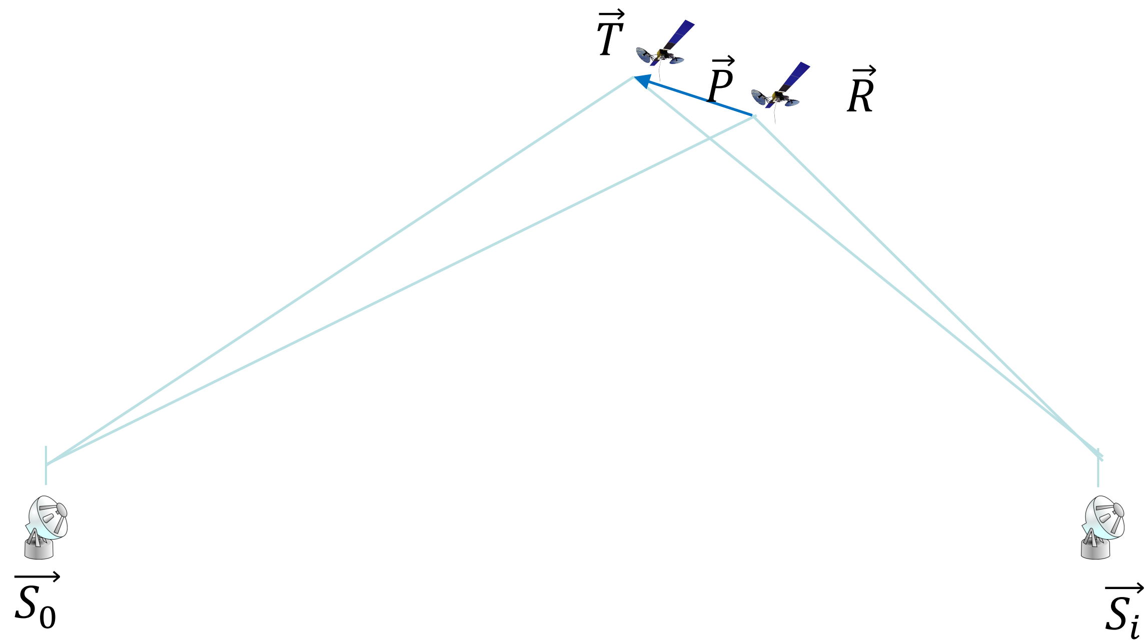
$$R_R = \frac{(R_T + R_R)^2 - L^2}{2(R_T + R_R + L\sin\theta_R)}$$

By Law of Cosines



M. Skolinik, "Radar Handbook", 3rd Edition, McGraw-Hill, 2008.

Derivation of Algorithm – "Single-Differencing"

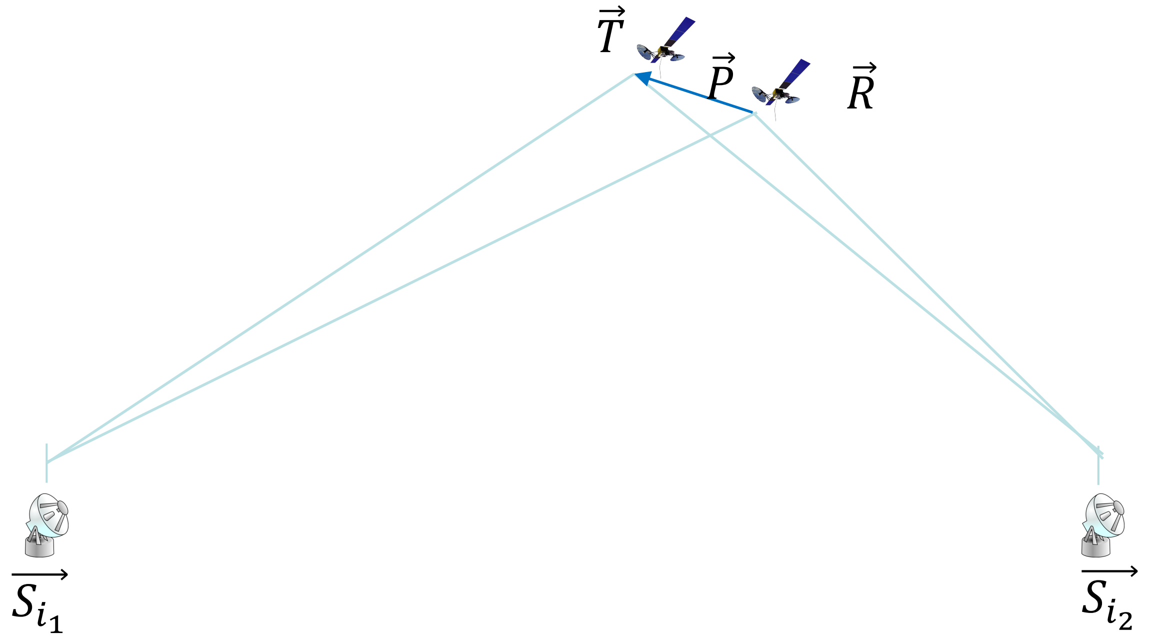


for i= $1, 2, \dots, n \quad n \geq 3$ $\quad \vec{P} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, three unknown $\quad \|\vec{P}\| = \sqrt{x^2 + y^2 + z^2}$

Measurements of time-delay of arrival at station $\overrightarrow{S_i}$ (single-difference)

$$c\delta t_i = \left(\|\overrightarrow{S_0 R}\| + \|\overrightarrow{S_i R}\|\right) - \left(\|\overrightarrow{S_0 T}\| + \|\overrightarrow{S_i T}\|\right)$$
$$= \left(\|\vec{R} - \overrightarrow{S_0}\| + \|\vec{R} - \overrightarrow{S_i}\|\right) - \left(\|(\vec{R} - \overrightarrow{S_0}) + \vec{P}\| + \|(\vec{R} - \overrightarrow{S_i}) + \vec{P}\|\right)$$

Derivation of Algorithm – "Double-Differencing"



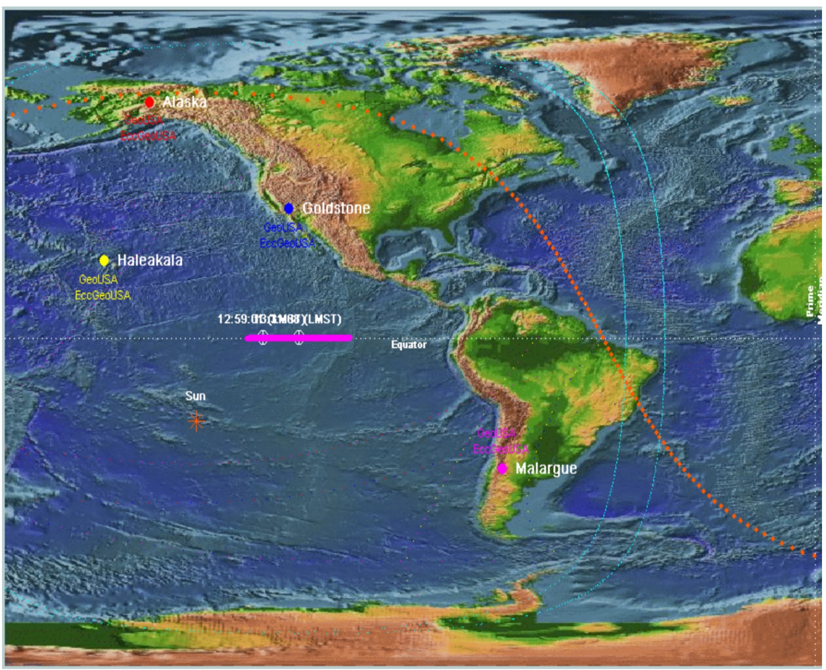
for $i = 1, 2, \ldots, k \quad k \geq 3$ $\quad \vec{P} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, three unknown $\quad \|\vec{P}\| = \sqrt{x^2 + y^2 + z^2}$

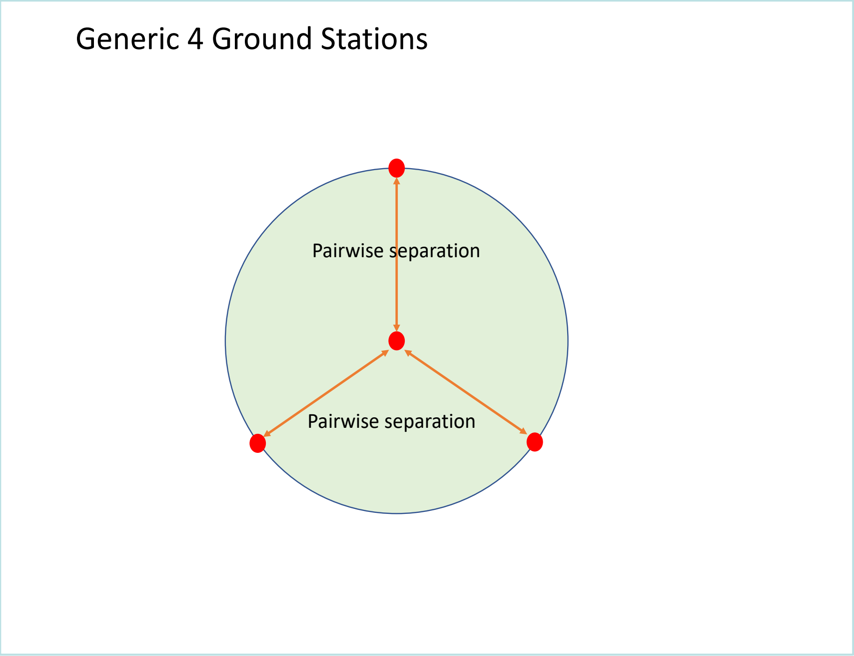
Measurements of difference of time-delay of arrival (double-differences)

$$
\begin{aligned}
c\delta\delta t_i &= \left(\|\overrightarrow{S_{i_1}R}\| - \|\overrightarrow{S_{i_1}T}\|\right) - \left(\|\overrightarrow{S_{i_2}R}\| - \|\overrightarrow{S_{i_2}T}\|\right) \\
&= \left(\|\overrightarrow{S_{i_1}R}\| - \|\overrightarrow{S_{i_2}R}\|\right) - \left(\|\overrightarrow{S_{i_1}T}\| - \|\overrightarrow{S_{i_2}T}\|\right) \\
&= \left(\|\vec{R} - \overrightarrow{S_{i_1}}\| - \|\vec{R} - \overrightarrow{S_{i_2}}\|\right) - \left(\|(\vec{R} - \overrightarrow{S_{i_1}}) + \vec{P}\| - \|(\vec{R} - \overrightarrow{S_{i_2}}) + \vec{P}\|\right)
\end{aligned}
$$

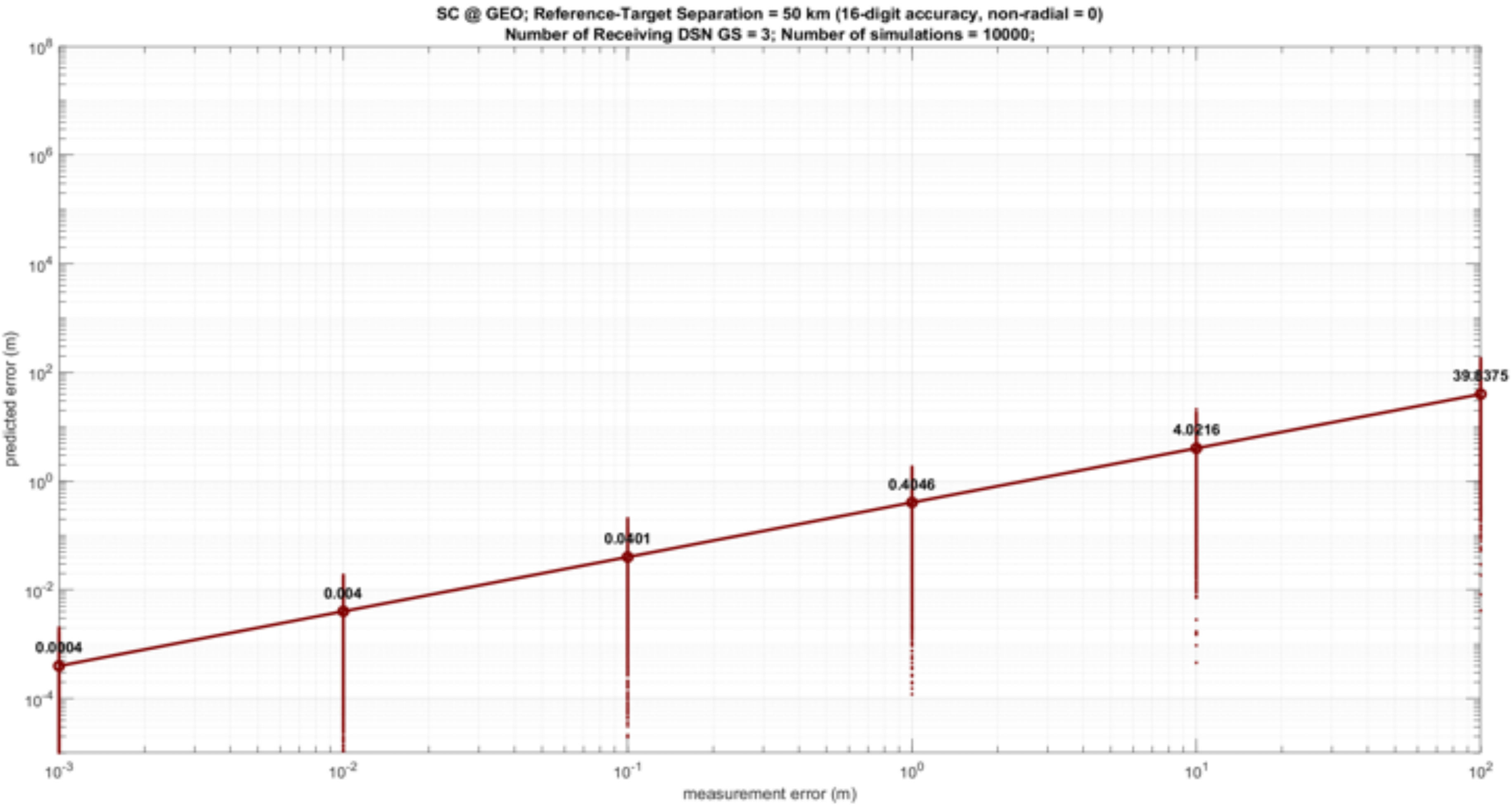
Simulations – Setup (1)

- Consider both the GEO case (36000 km) and the lunar case (360000 km)
- The Reference and target spacecraft are separated by 50 kilometers
- Consider a real case with 4 ground antennas
  - Goldstone (transmitter for single-differencing), Alaska, Hawaii, and Malargue
- Consider a hypothetical constellation of 4 ground stations on an idealized sphere of Earth with variable separation
- Caveat: assume all systematic biases cancelled by the differencing process. Simulations reflect the effect of random measurement errors only

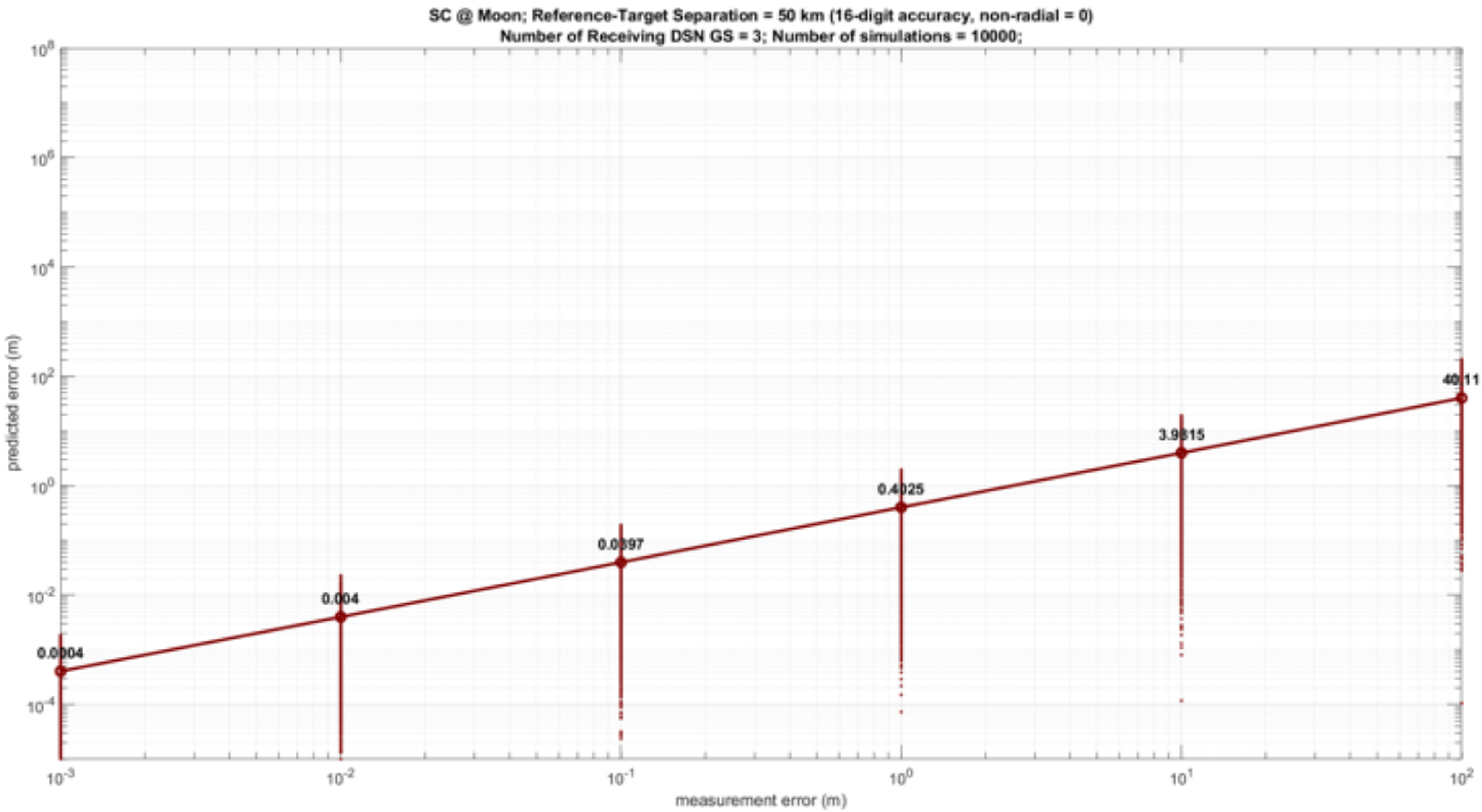




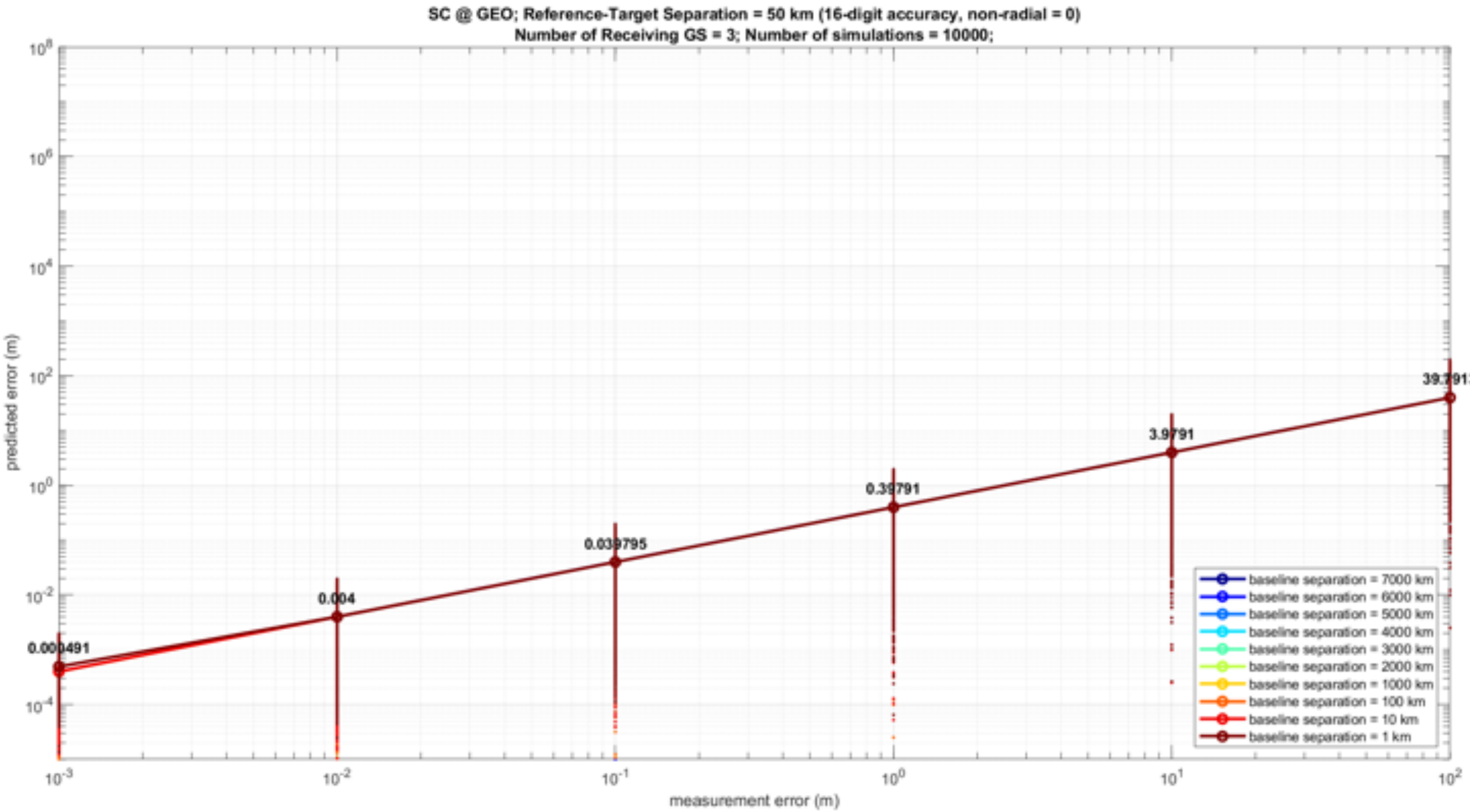
Simulations – “Single-Differencing” realistic case at GEO (2)



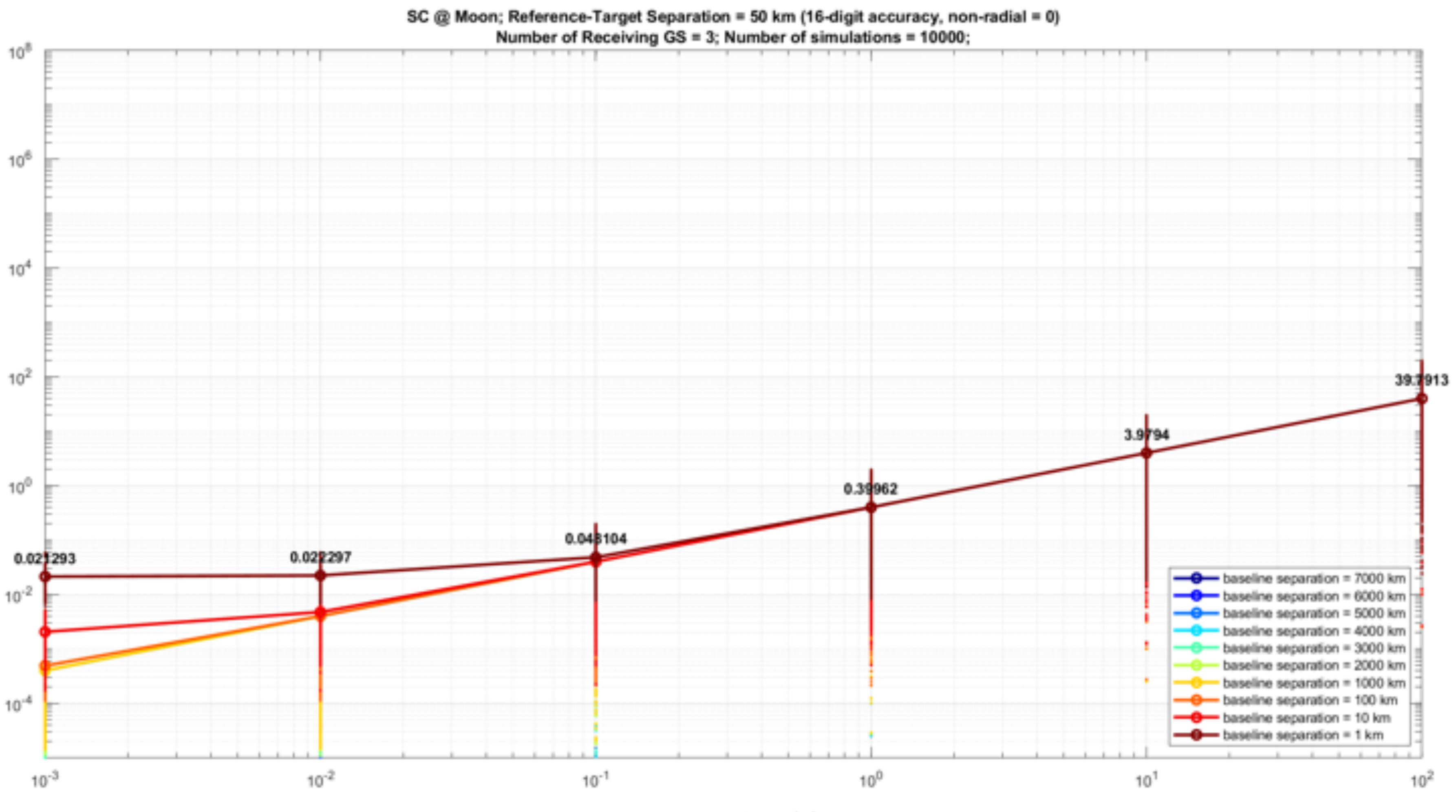
Simulations – "Single-Differencing" realistic case at Moon (3)



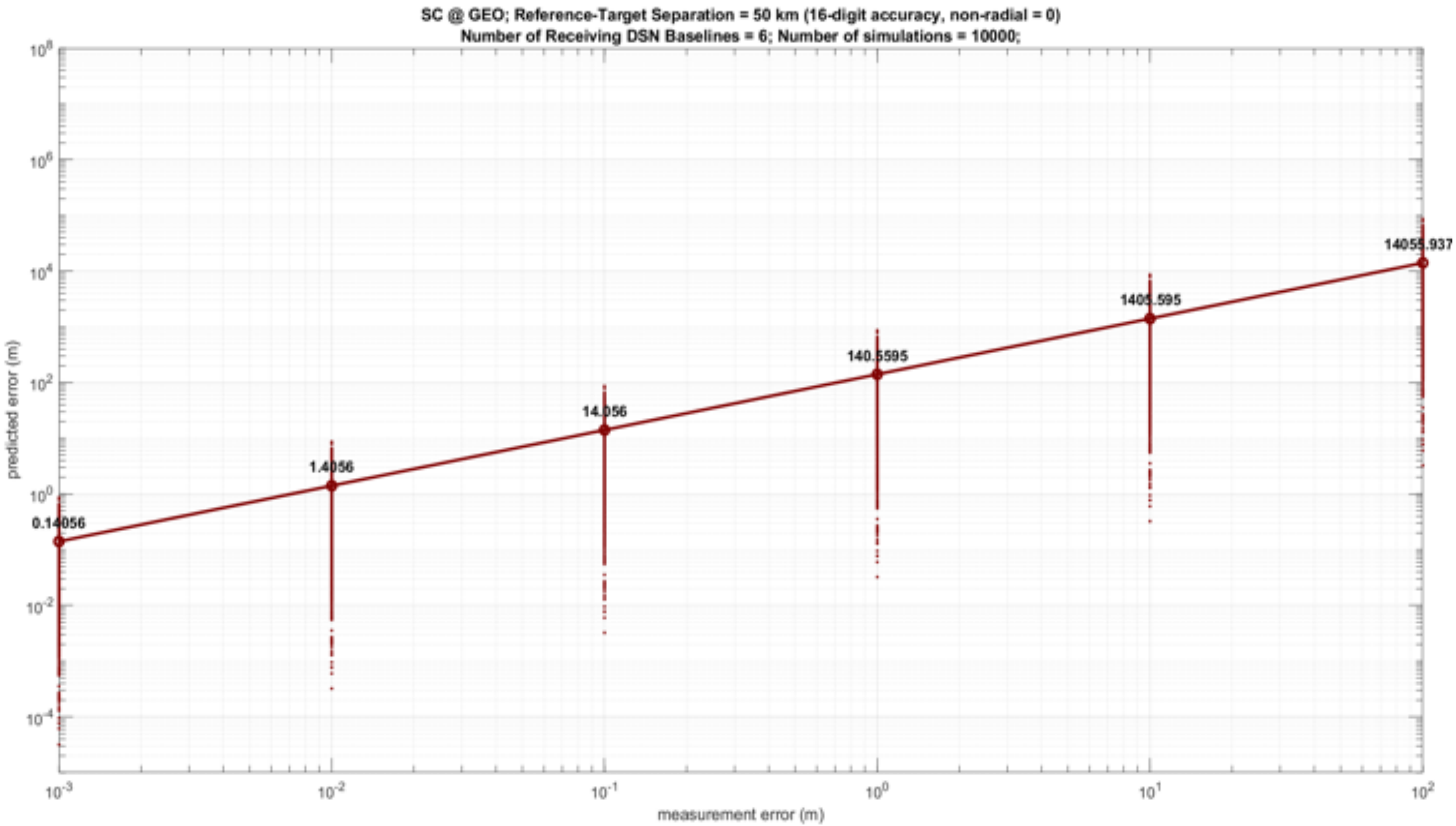
Simulations – "Single-Differencing" Hypothetical case at GEO (4)



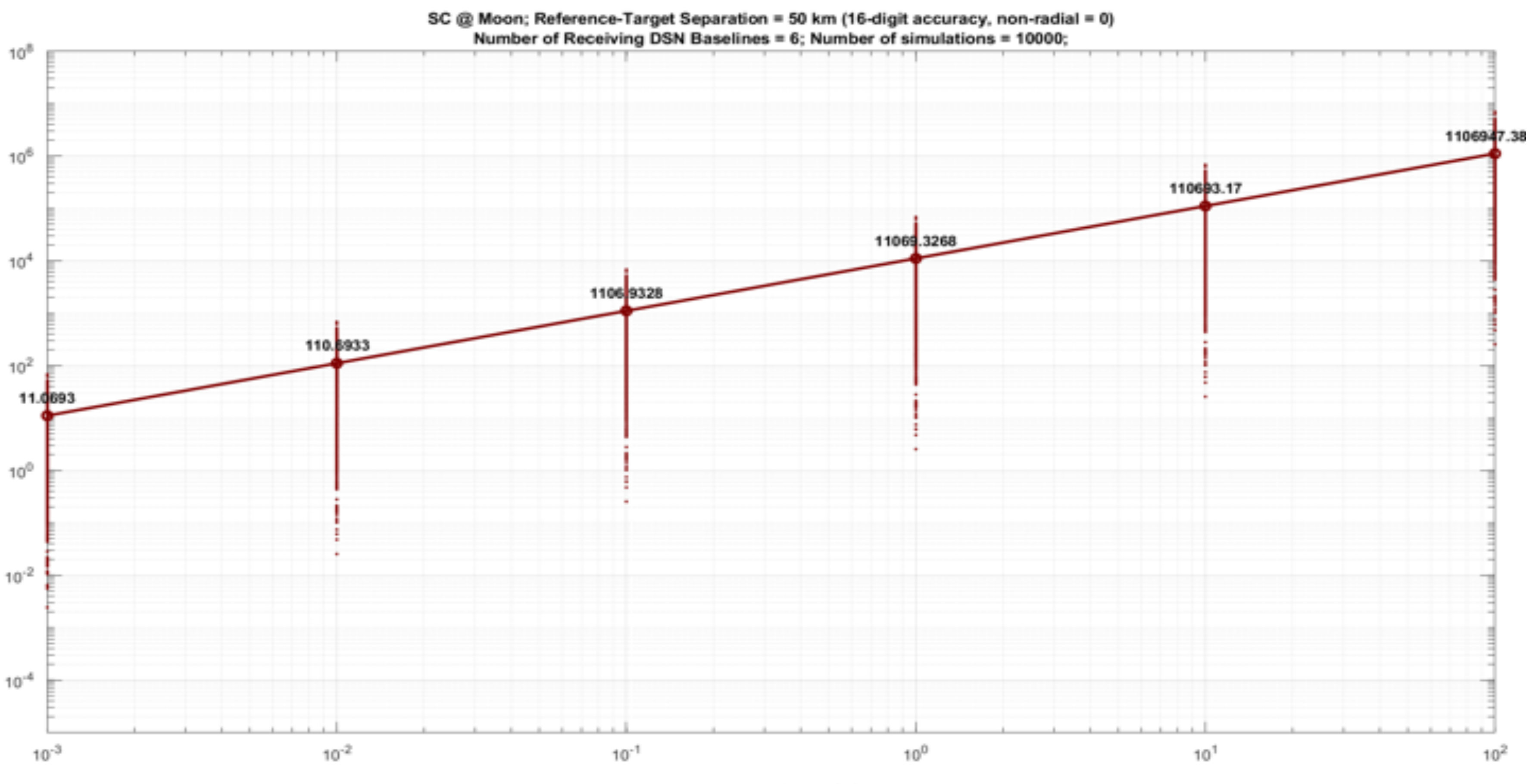
Simulations – "Single-Differencing" hypothetical case at Moon (5)



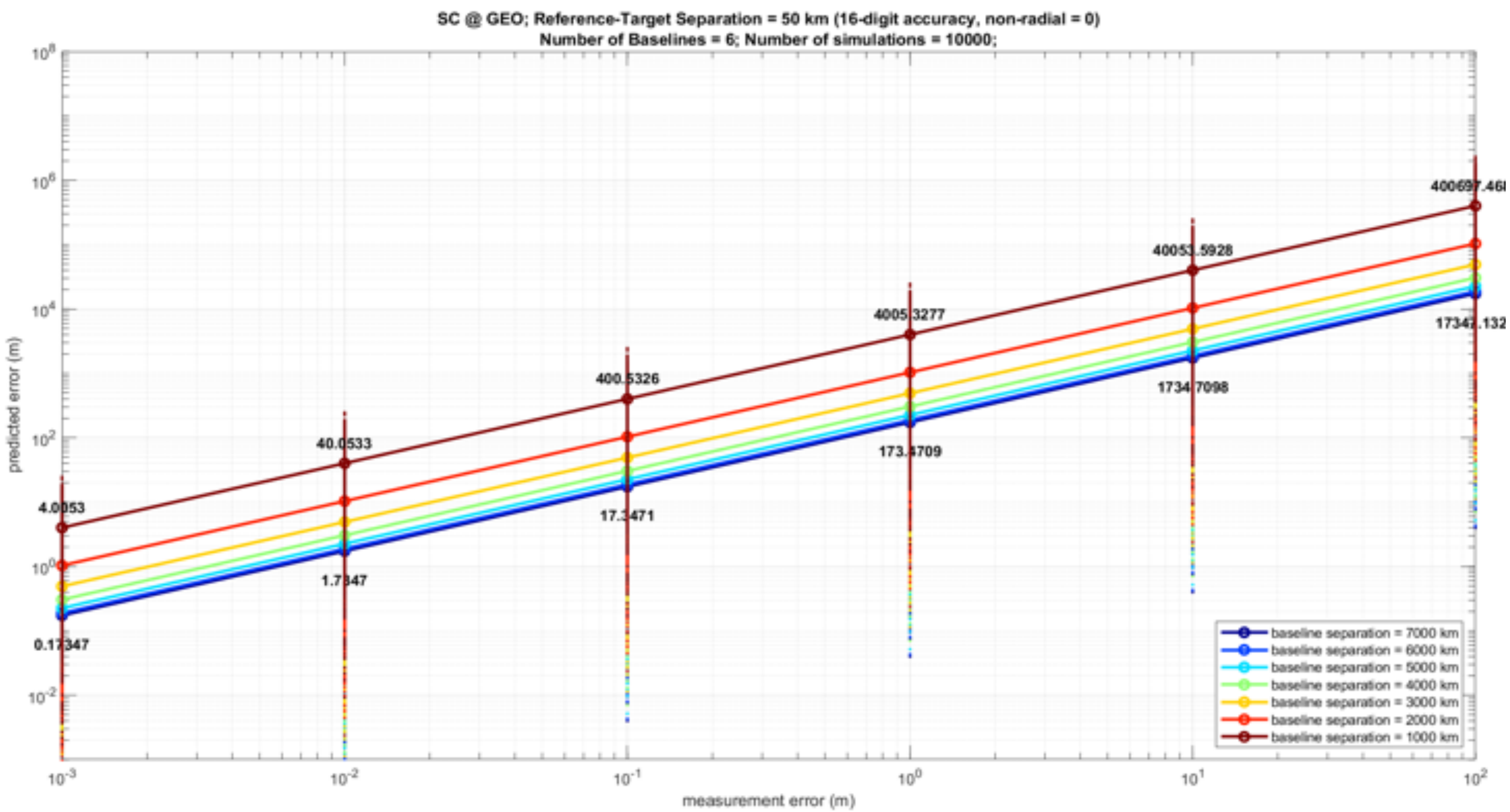
Simulations – “Double-Differencing” realistic case at GEO (6)



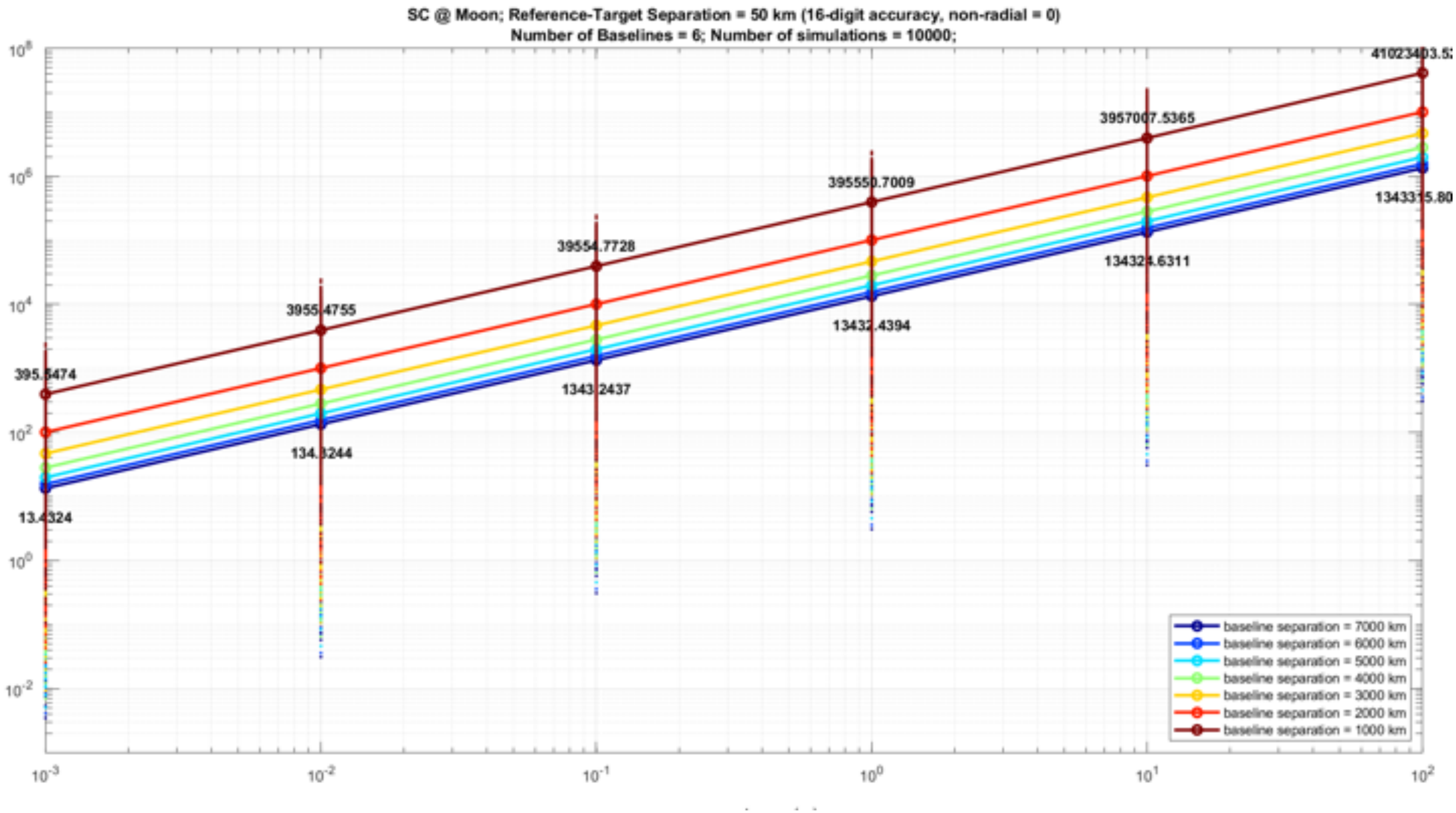
Simulations – "Double-Differencing" realistic case at Moon (7)



Simulations – "Double-Differencing" Hypothetical case at GEO (8)



Simulations – “Double-Differencing” hypothetical case at Moon (9)



Conclusion, Future Work, and Potential Applications

Conclusion

- This paper introduces two differential radar techniques to detect and to locate objects at GEO and lunar distances
- Unlike conventional multi-static radar approach that use "sum of range" for position estimation, the new techniques employ "double differencing of ranges" to eliminate most of the systematic biases to achieve high positioning accuracy

Path Forward and Other Potential Applications

- Propose to demonstrate the techniques using large antennas at DSN, VLBA, Arecibo, and Green Bank
- The "single-difference" approach can be used to support a friendly aircraft (reference) engaged in dogfight with an enemy aircraft
- Similarly, multiple ships and/or buoys can form multiple baselines of sonars to support a friendly submarine's cat-and-mouse pursuit of an enemy submarine
- The double-differencing approach can be used for precision and real-time detection and positioning of incoming missiles
  - High-power and directional radars illuminate the known GEO satellites over the N. America's sky that can serve as references
  - Broad-beam radars illuminate the lower range of altitude (1000 – 2000 km) that covers the mission's trajectory
- Other applications include orbital debris removal and precision approach radar system for airports