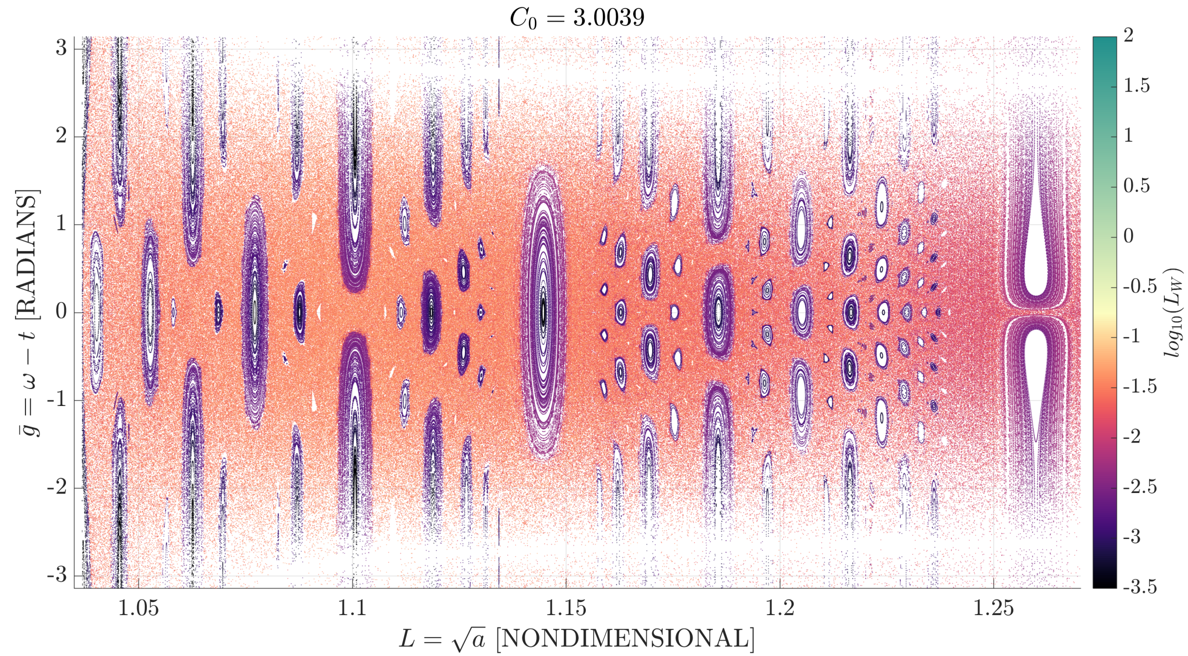

THE STABILITY OF ORBITAL RESONANCES FOR EUROPA QUARANTINE DESIGN: ESCAPE ORBIT CASE

Brian D. Anderson, Martin W. Lo, and Mar Vaquero

Jet Propulsion Laboratory, California Institute of Technology

Thursday, 1/17/2019

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Brian D. Anderson,bdanders@usc.edu

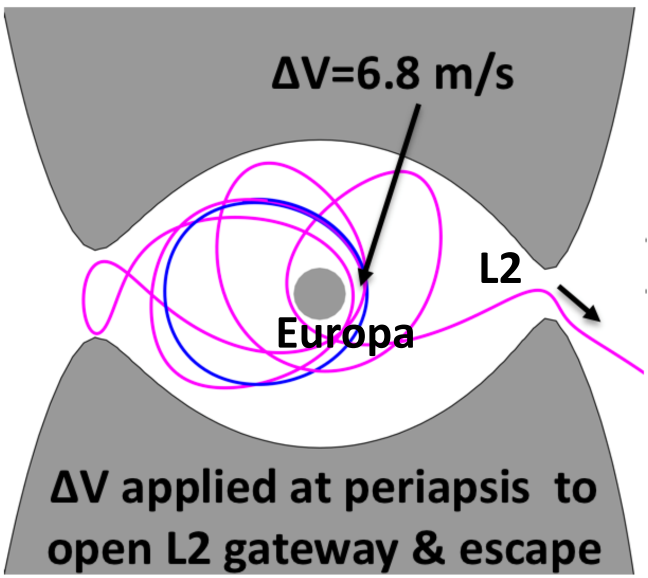
Agenda

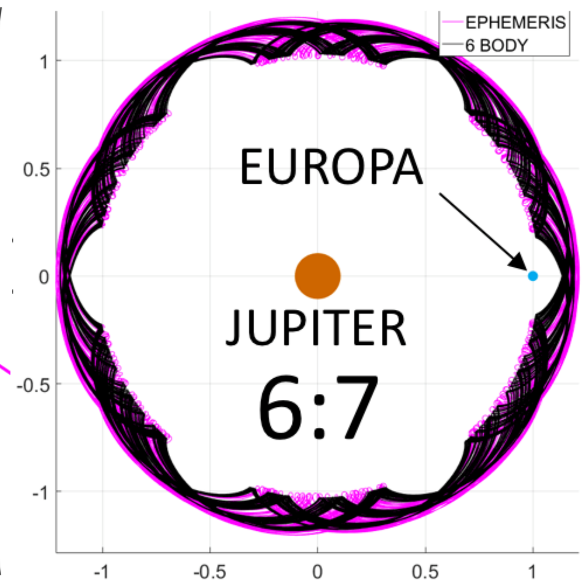
- Europa Mission Disposal Orbit
- Models: CR3BP, CR6BP, JPL Ephemeris
- Poincaré Section, Delaunay Variables
- Finite Time Stable Resonance
- Stable Resonant Orbit Abundance Estimate
- Conclusion & Future Work

The information presented about future Europa mission concepts is pre-decisional and is provided for planning and discussion purposes only.

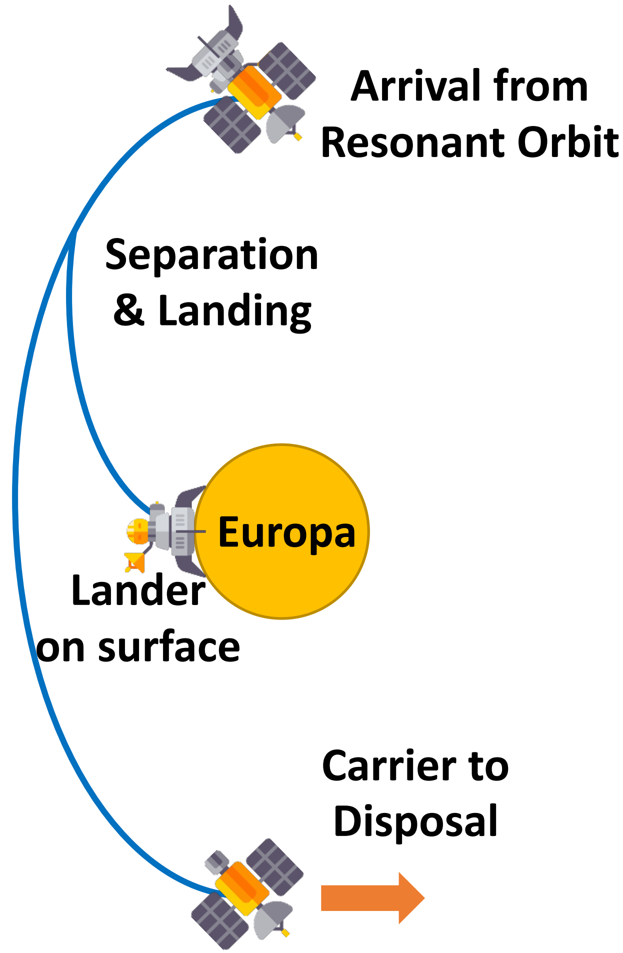
Disposal Orbit for Europa Missions

- ***PROBLEM***: Find Disposal Orbits at End of Mission, ***NO IMPACT for 12 Years***
- M. Vaquero Found: Escape L_2 to 6:7 Orbit
  - Surprisingly Easy & Cheap
- Why So Easy & Cheap? Why 6:7?
- How Many Such Orbits?





CONCEPT FOR A EUROPA LANDER



Mean Motion Resonance of Orbital Periods

- Given orbits of Europa & Orbiter with periods P_E and P_O, and integers *p, q* (relatively prime), the orbits are in exact *p:q* mean motion resonance if

$$P_E/P_O = p/q, \quad p\,P_O = q\,P_E$$

- In CR3BP, the relation is only approximate

$$P_E/P_O \sim p/q \quad pP_o \approx qP_E \implies \frac{p}{q} \approx \frac{P_E}{P_o}$$

- p/q = Best Fraction Approx. of P_E/P_O, tolerance=ϵ

$$|P_E/P_O - p/q| < \epsilon < |\, P_E/P_O - p'/q'|$$

(Best Diophantine Approx.)

Stability from Resonance of Galilean Moons

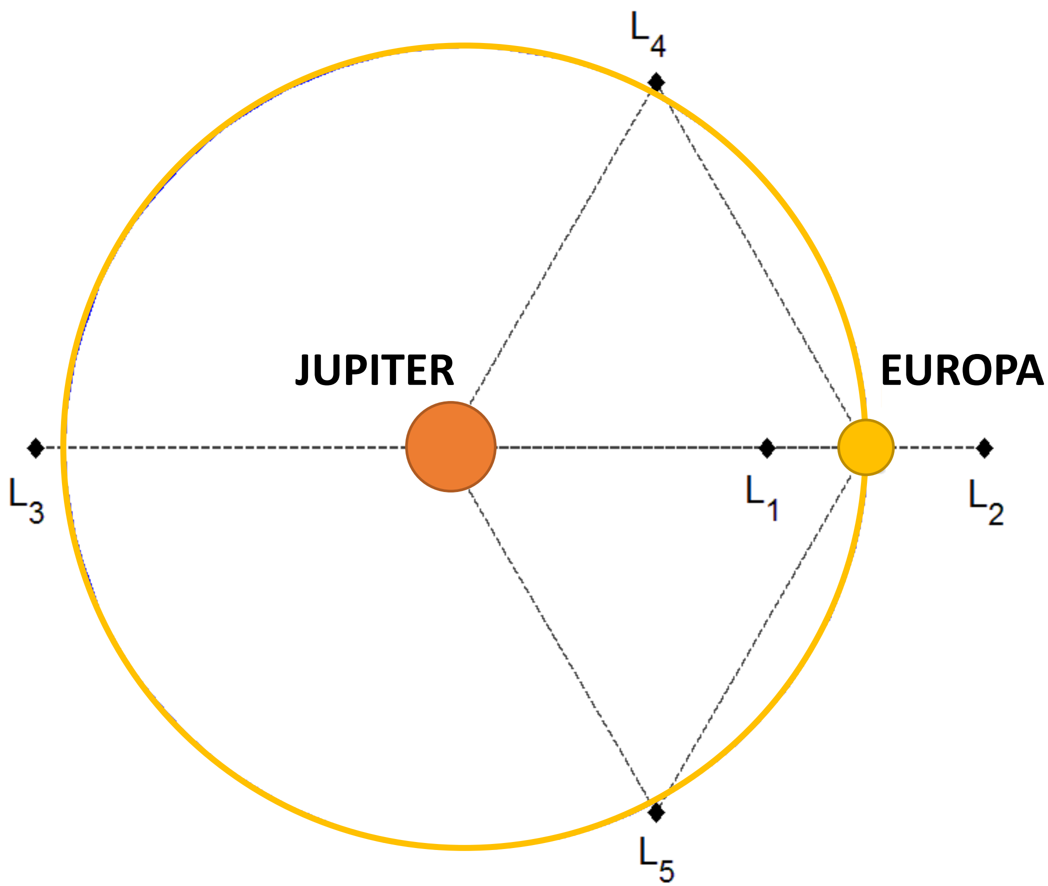
- **Conjecture (Anderson):**

  Stability from RESONANCE with ALL 4 Galilean moons
  - 1:2:4 – Io:Europa:Ganymede
    (moons in Mean Motion Resonance)
  - 3:7 – Ganymede:Callisto → 3:14 Europa:Callisto
    (moons in Near Resonance)
  - 6:7 Orbit in resonance with all Galilean moons (P=Period)

    $12\ P_{6:7} \approx 3\ P_{Callisto} \approx 7\ P_{Ganymede} \approx 14\ P_{Europa} \approx 28\ P_{Io} \approx 49$ Days
- Test Conjecture:

  Poincaré map in Circular Restricted 6-Body Problem,
  Assume Galilean moons in exact resonance (M. Lo)

CR3BP



$$\ddot{x} - 2\dot{y} = \frac{\partial U}{\partial x}$$
$$\ddot{y} + 2\dot{x} = \frac{\partial U}{\partial y}$$
$$\ddot{z} = \frac{\partial U}{\partial z}$$

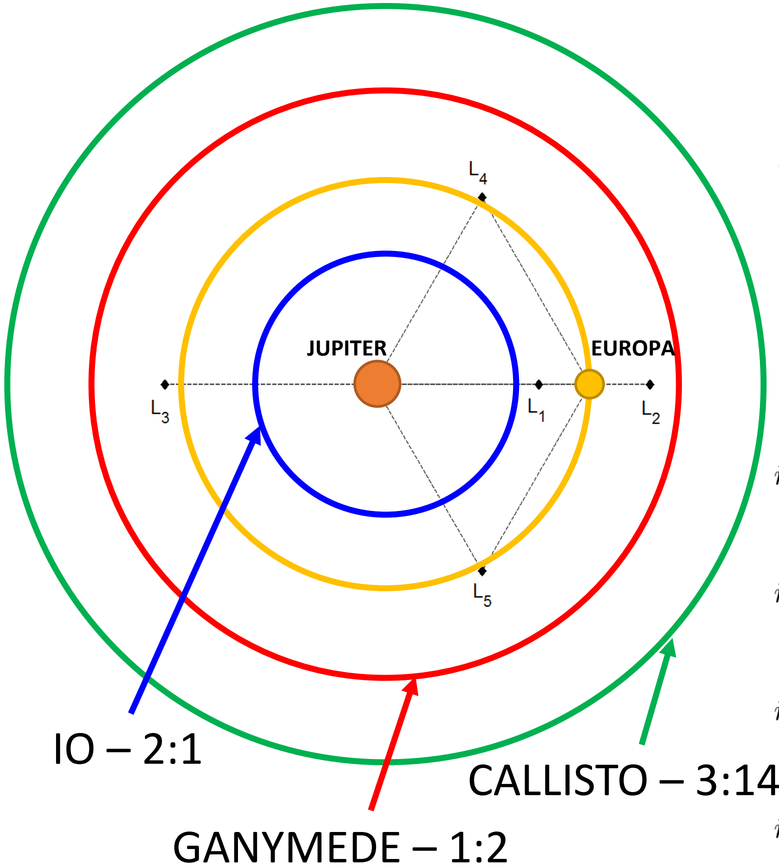
$$U = \frac{1}{2}(x^2 + y^2) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2}$$
$$r_1 = [(x+\mu)^2 + y^2 + z^2]^{\frac{1}{2}}$$
$$r_2 = [(x-1+\mu)^2 + y^2 + z^2]^{\frac{1}{2}}$$
$$\mu = m_2, 1-\mu = m_1$$

Jacobi Integral

$$C = 2U - (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$$

Galilean Moons in Exact Resonance

USC Viterbi School of Engineering



- Circular Restricted 6-Body Problem (CR6BP)

$$\ddot{x} = 2\dot{y} + x - \frac{1-\mu}{r_1^3}(x+\mu) - \frac{\mu}{r_2^3}(x-1+\mu) - \frac{\mu_4}{r_4^3}x_4 - \frac{\mu_5}{r_5^3}x_5 - \frac{\mu_6}{r_6^3}x_6$$

$$\ddot{y} = -2\dot{x} + y - \frac{1-\mu}{r_1^3}y - \frac{\mu}{r_2^3}y - \frac{\mu_4}{r_4^3}y_4 - \frac{\mu_5}{r_5^3}y_5 - \frac{\mu_6}{r_6^3}y_6$$

$$\ddot{z} = -\frac{1-\mu}{r_1^3}z - \frac{\mu}{r_2^3}z - \frac{\mu_4}{r_4^3}z_4 - \frac{\mu_5}{r_5^3}z_5 - \frac{\mu_6}{r_6^3}z_6.$$

$$\vec{r}_1 = \begin{bmatrix} x+\mu \\ y \\ z \end{bmatrix}, r_1 = |\vec{r}_1|$$

$$\vec{r}_2 = \begin{bmatrix} x-1+\mu \\ y \\ z \end{bmatrix}, r_2 = |\vec{r}_2|$$

$$\vec{r}_4 = \begin{bmatrix} x - a_4\cos(n_4 t + L_{04}) \\ y - a_4\sin(n_4 t + L_{04}) \\ z \end{bmatrix}, r_4 = |\vec{r}_4|$$

$$\vec{r}_5 = \begin{bmatrix} x - a_4\cos(n_5 t + L_{05}) \\ y - a_4\sin(n_5 t + L_{05}) \\ z \end{bmatrix}, r_5 = |\vec{r}_5|$$

$$\vec{r}_6 = \begin{bmatrix} x - a_4\cos(n_6 t + L_{06}) \\ y - a_4\sin(n_6 t + L_{06}) \\ z \end{bmatrix}, r_6 = |\vec{r}_6|$$

$$\left.\begin{aligned} \mu &= \frac{\tilde{m}_2}{\tilde{m}_1+\tilde{m}_2} \\ \mu_4 &= \frac{\tilde{m}_4}{\tilde{m}_1+\tilde{m}_2} \\ \mu_5 &= \frac{\tilde{m}_5}{\tilde{m}_1+\tilde{m}_2} \\ \mu_6 &= \frac{\tilde{m}_6}{\tilde{m}_1+\tilde{m}_2} \end{aligned}\right\} \text{MASSES}$$

$$\left.\begin{aligned} a_4 &= \frac{\tilde{a}_4}{\tilde{a}_2} \\ a_5 &= \frac{\tilde{a}_5}{\tilde{a}_2} \\ a_6 &= \frac{\tilde{a}_6}{\tilde{a}_2} \end{aligned}\right\} \text{SEMIMAJOR AXES}$$

$$\left.\begin{aligned} n_4 &= \frac{\tilde{n}_4}{\tilde{n}_2} - 1 \\ n_5 &= \frac{\tilde{n}_5}{\tilde{n}_2} - 1 \\ n_6 &= \frac{\tilde{n}_6}{\tilde{n}_2} - 1 \end{aligned}\right\} \text{MEAN MOTIONS}$$

$$\left.\begin{aligned} L_{04} &= \tilde{L}_{04} - \tilde{L}_{02} \\ L_{05} &= \tilde{L}_{05} - \tilde{L}_{02} \\ L_{06} &= \tilde{L}_{06} - \tilde{L}_{02} \end{aligned}\right\} \text{T=0 MEAN LONGITUDE}$$

EPHEMERIS MODEL

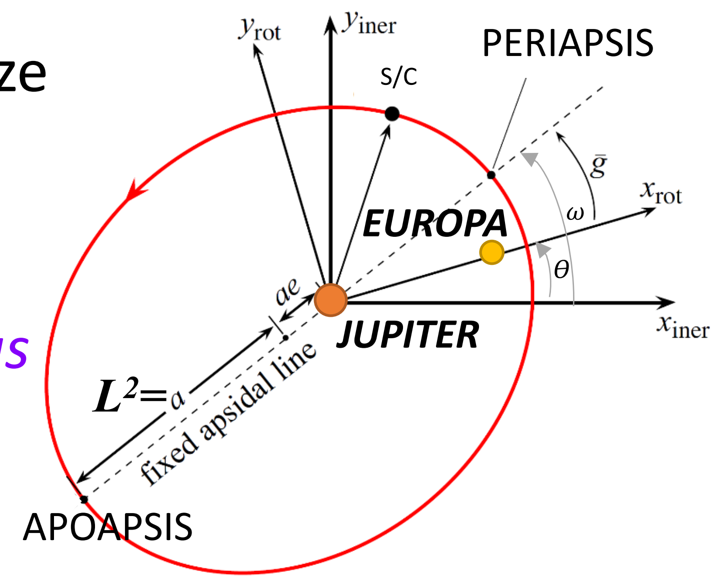
FROM JPL DE430, JUP310:

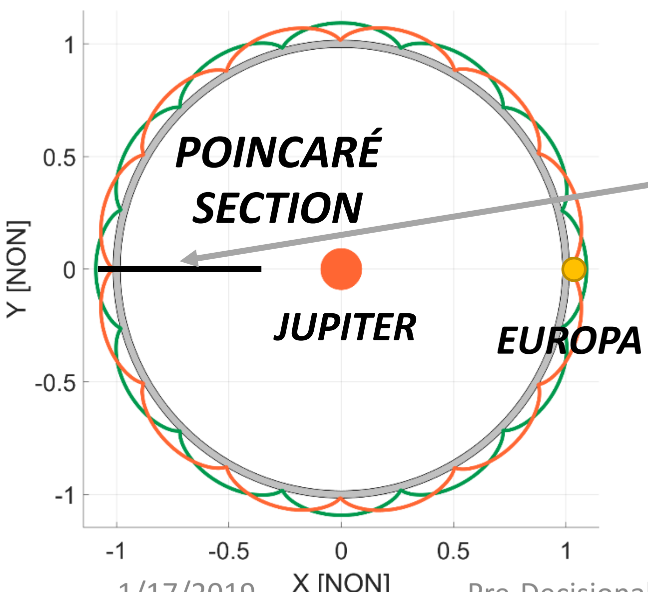
SUN+JUPITER+IO+EUROPA
+GANYMEDE+CALLISTO

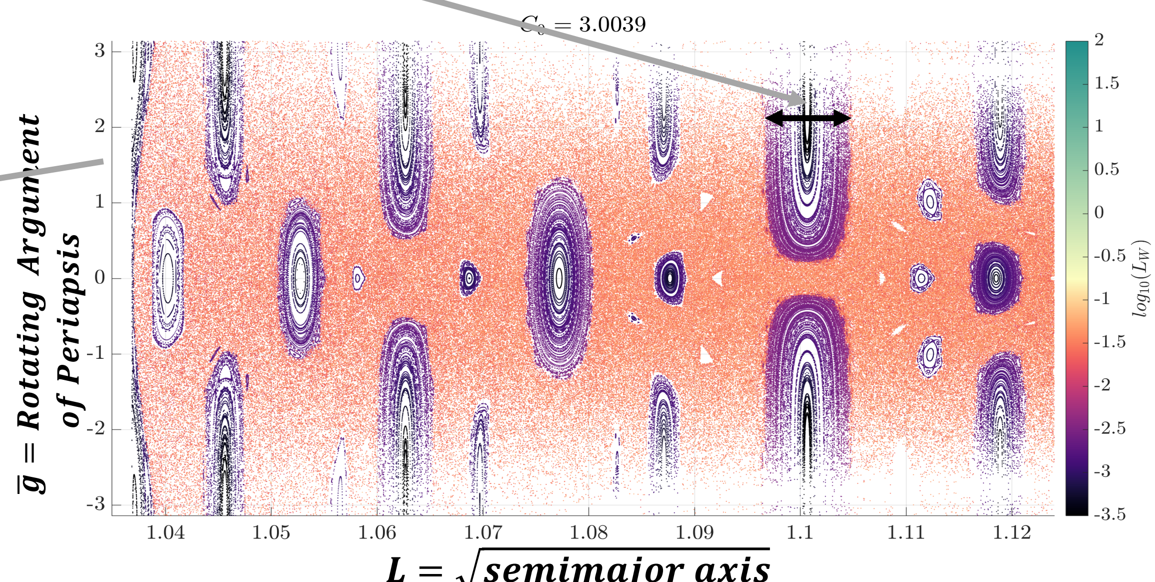
Poincaré Section, Delaunay Variables

$L = \sqrt{a}\ ,\ \bar{g} = \omega - \theta$

- $L = \sqrt{semimajor\ axis}$ measures orbit size
- Chaotic Orbits form sea of *Dots*
- Stable Resonant Orbits form *Islands*
    - *Periodic orbit at center, QPO form rings*
    - Small variation in *L* = *stable orbit*
    - *Measure stability by island L-WIDTH*







Discretize Orbit by Poincaré Map

- Problem: How to compute stability since resonant CR3BP orbits no longer periodic or quasiperiodic or stable in CR6BP?
- Solution: Replace orbit x(t) by its Poincaré Map
- Discretize Orbit x(t) by $P(x_0)$, Poincaré Map: $F: S \rightarrow S$,

  $P(x_0) = \{x_0 = x(0), F(x_0), F^2(x_0), \ldots, F^M(x_0), M \simeq 12 \text{ years}\}$
- Study resonance structure of discrete orbits $P(x_k)$:

  $\{P(x_k), x_k \in S_0 = \{ x_1, x_2, \ldots, x_N \} \subset S \}$

  S_0 = Initial Conditions for Poincaré Map F(x).

Finite Time Resonance & Stability

- $L = \sqrt{semimajor\ axis}$ represents orbit size
  - Small variation in *L* = *stable orbit*
- Define Resonance Width & Period of Orbit $P(x_k)$

$$W_k = \max_{x \in P(x_k)} |L(x) - \langle L(P(x_k)) \rangle|,$$

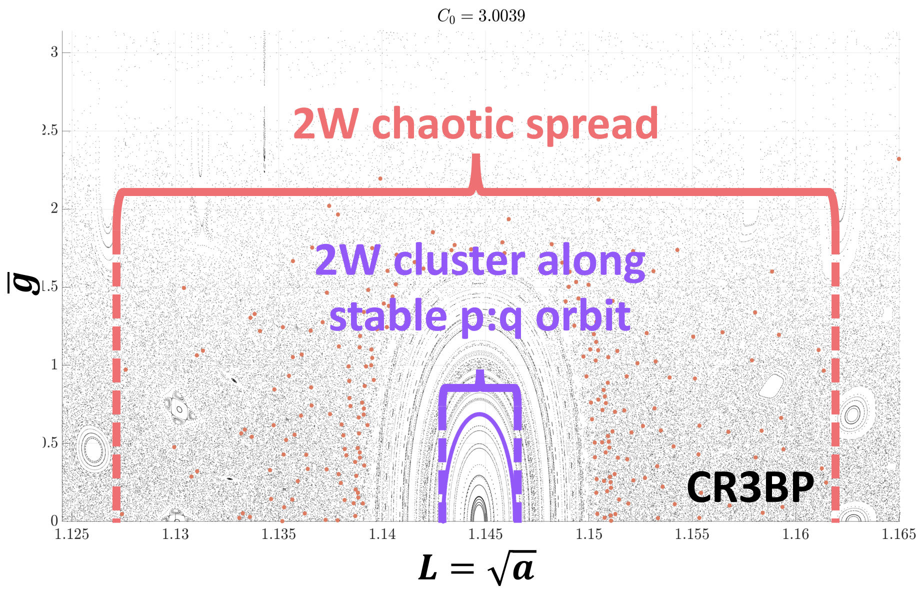
< A> = Average of A

$$FT_{12}\text{-period of } P(x_k) = 2\pi \langle L(P(x_k)) \rangle^3$$

$< L > \simeq \sqrt{a}$

- Chaotic Orbit: large W_k
- Stable Orbit: small W_k

cluster along p:q resonant orbit.



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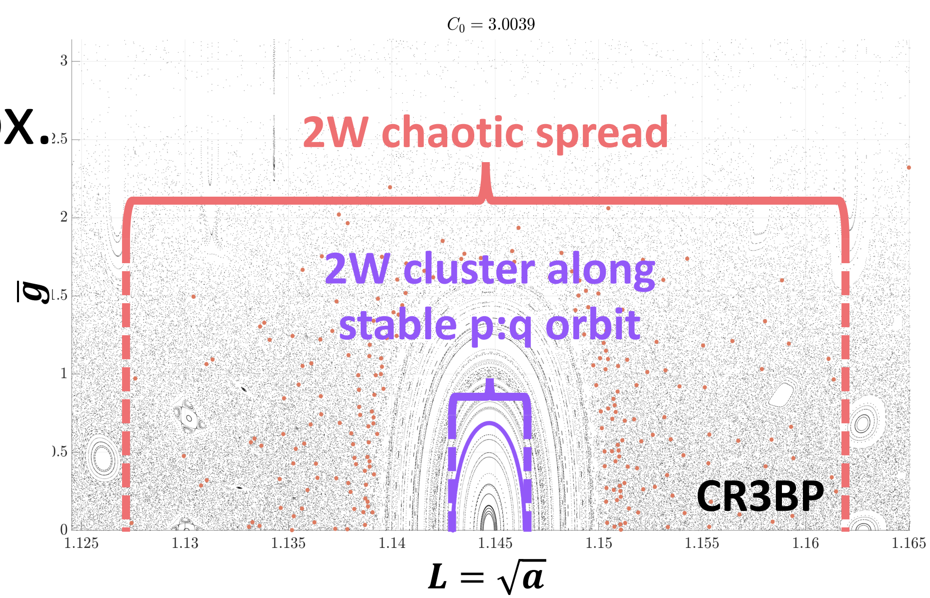
$< L > \simeq \sqrt{a}$

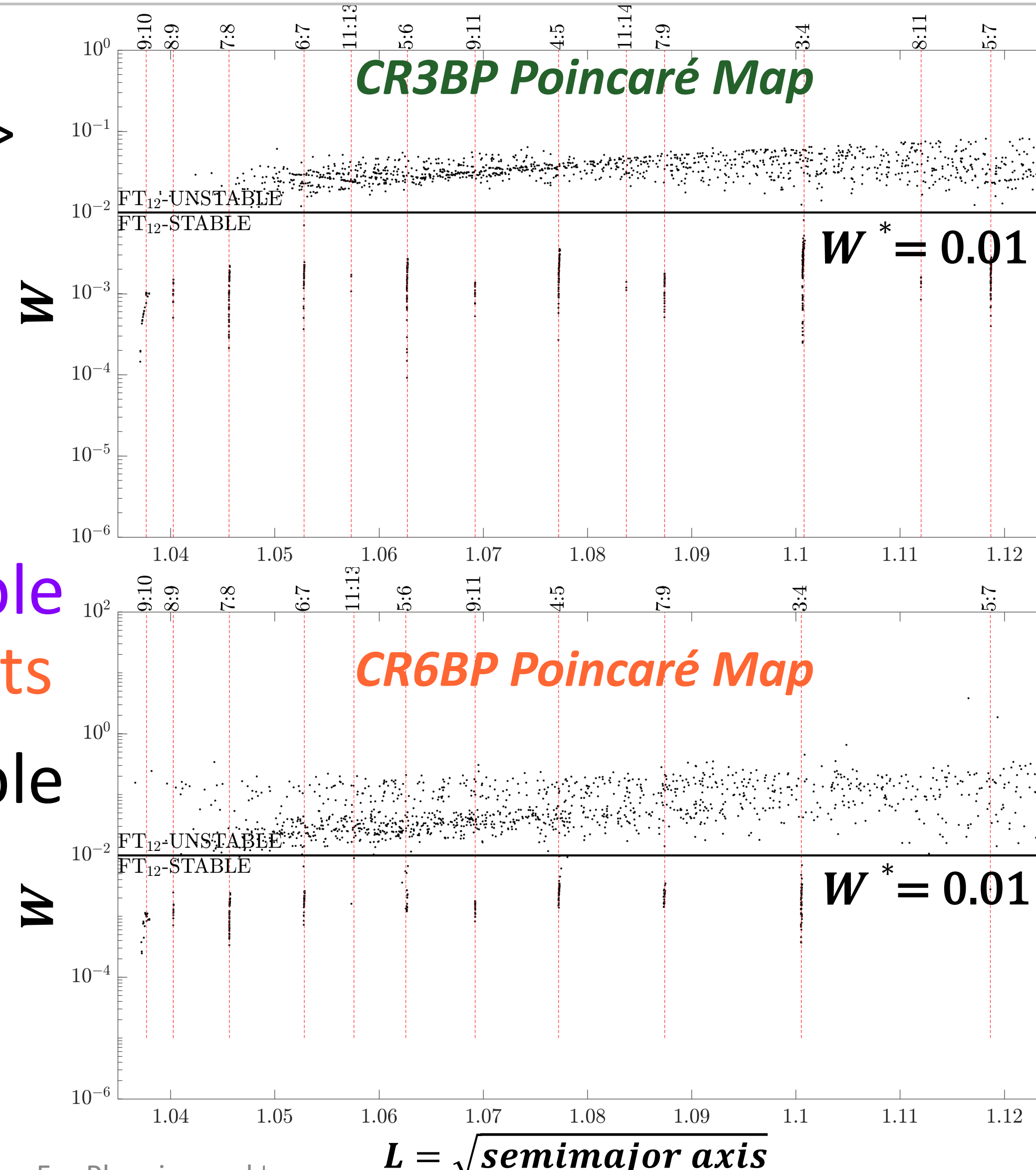
- Resonance of $P(x_k) = p_k : q_k$

  Use Best Diophantine Approx. of FT_{12}-period of $P(x_k)$.



W=Finite Time Resonance Width

- ***W*** = Max deviation from < ***L*** >
- ***W*** vs. ***L*** Poincaré map
  - 12 years
  - *CR3BP*
  - *CR6BP*
- ***W**** = 0.01 Separates stable orbits from unstable orbits
- CR6BP still has clear stable motion



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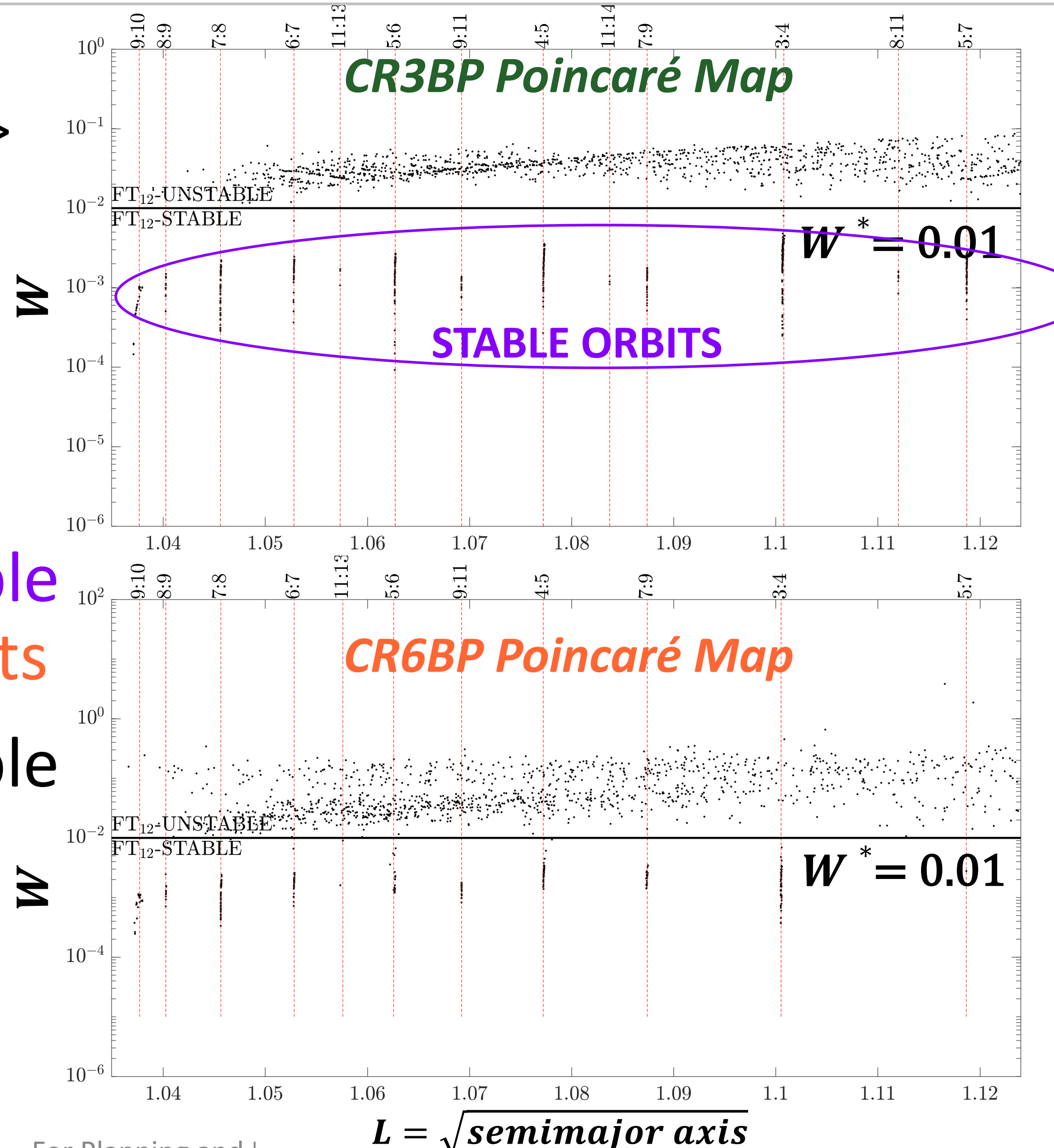

Brian D. Anderson,bdanders@usc.edu

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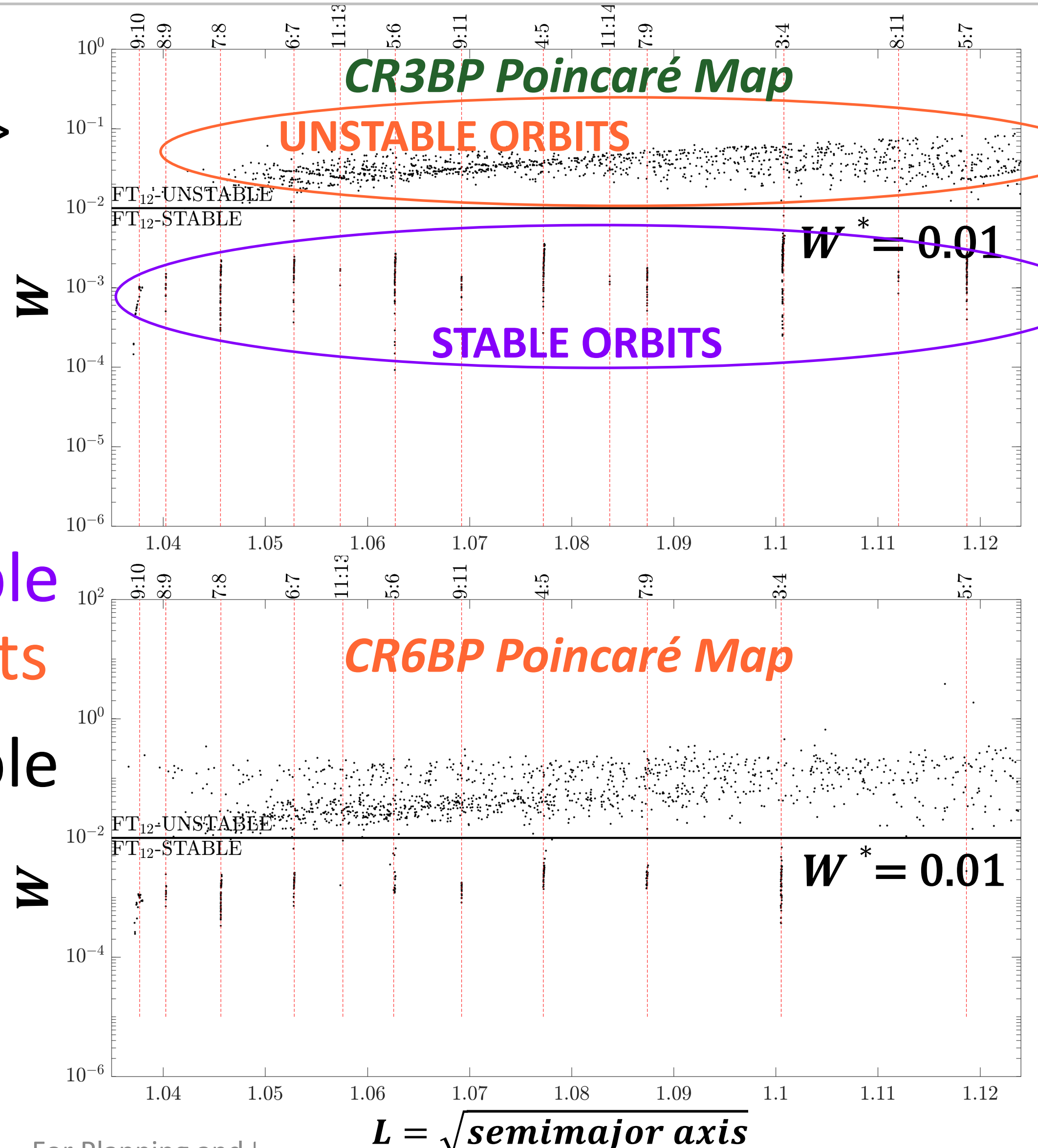

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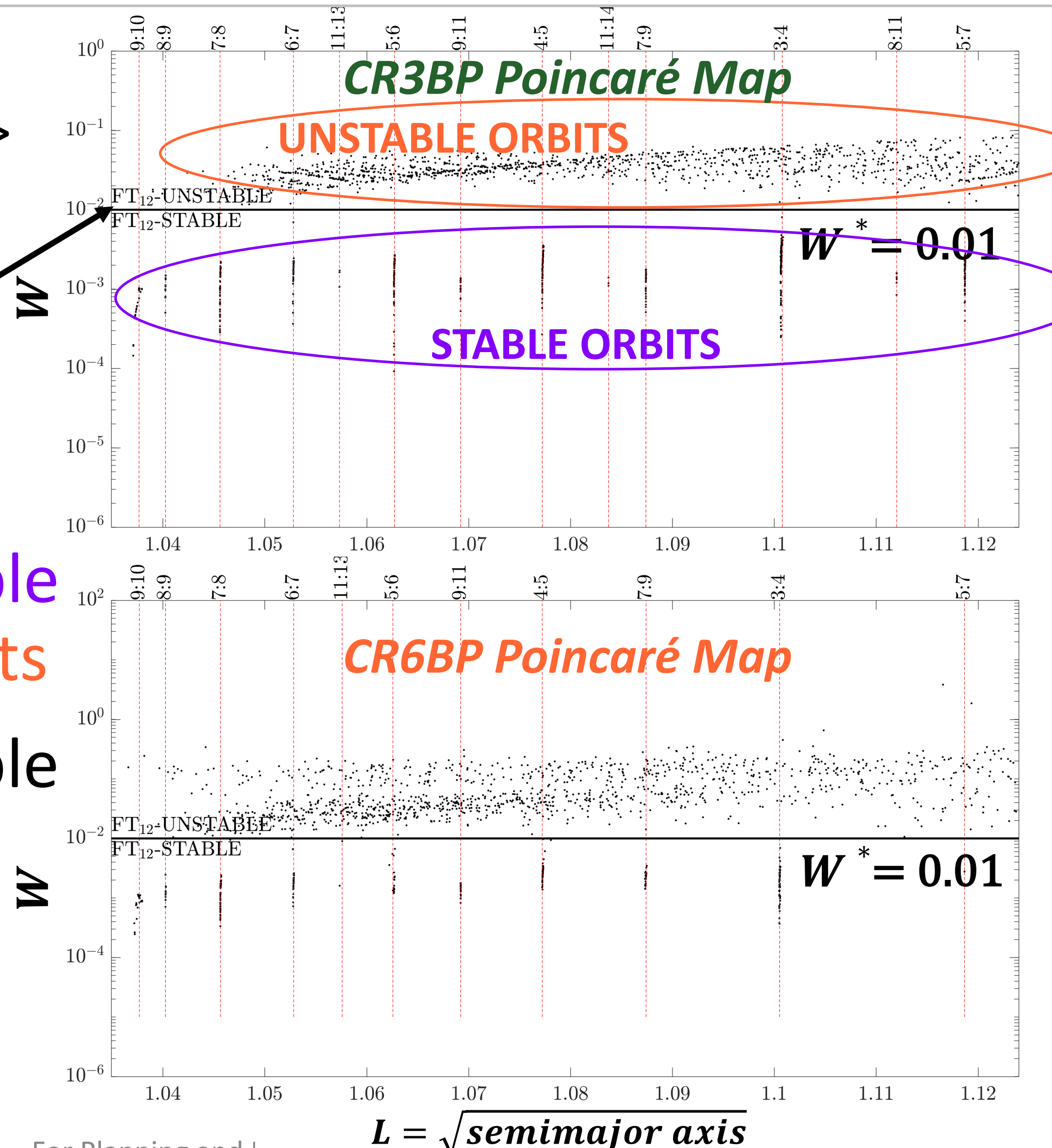

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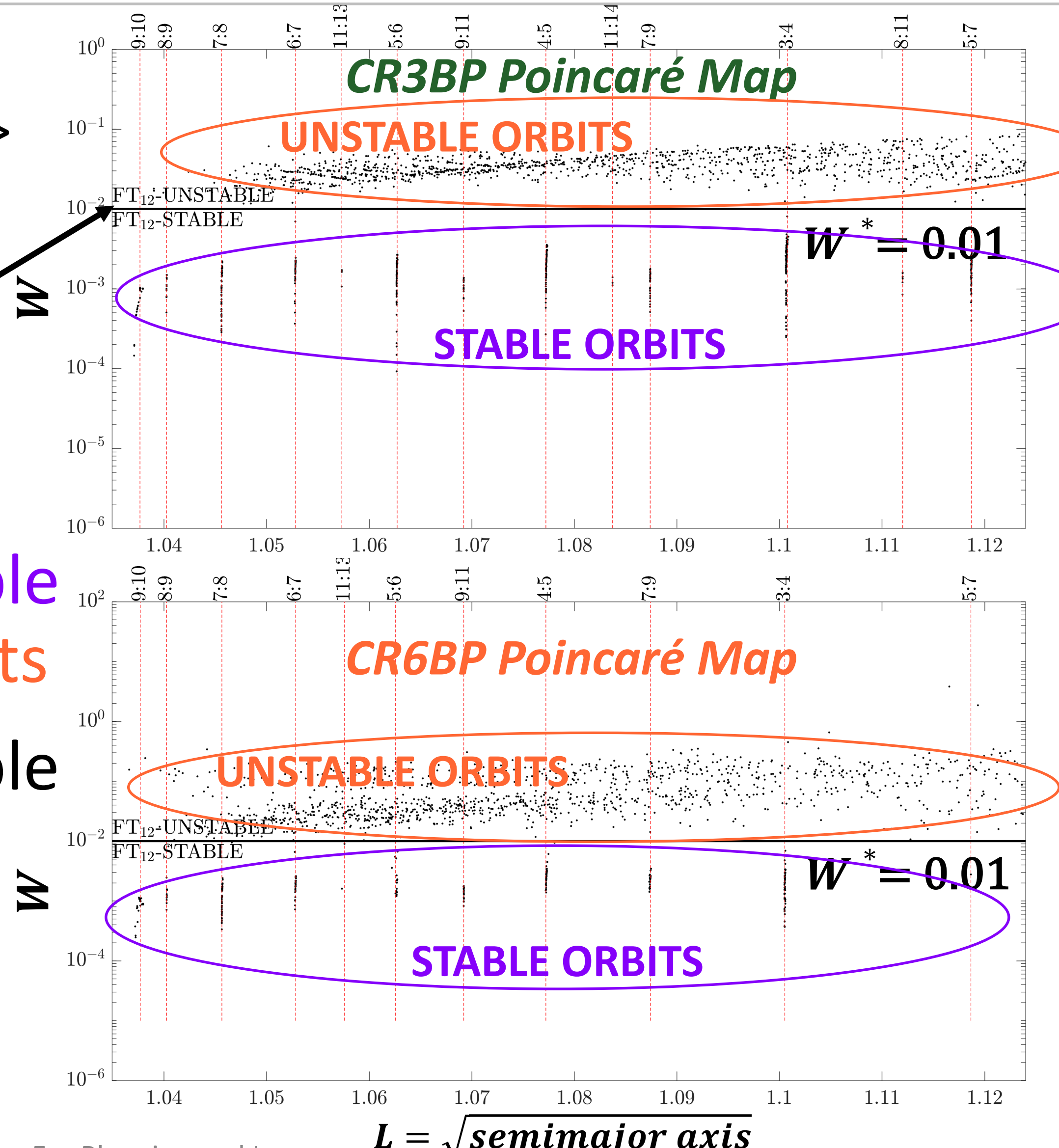
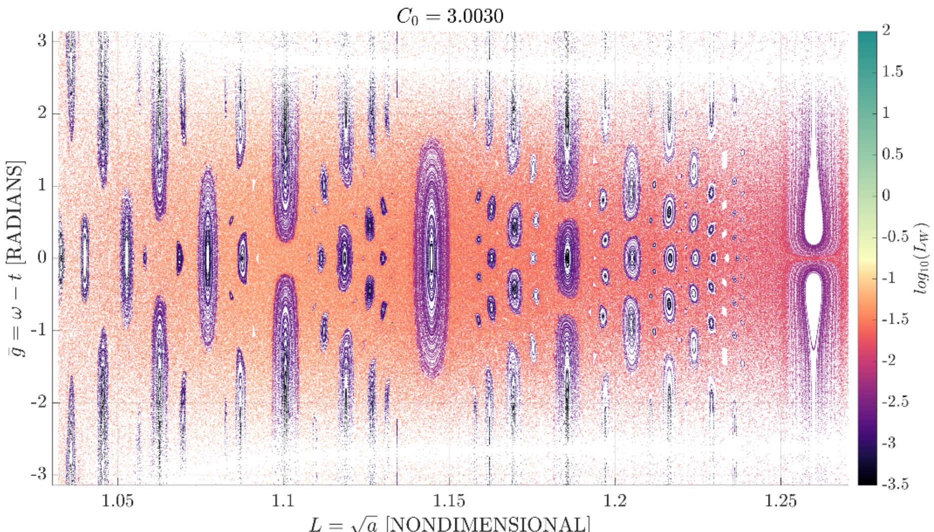


Shadow Resonances in CR6BP

- Augment Poincaré map with Resonance Width
- $color = log_{10}(Resonance\ Width)$
- Yellow-Orange = highly variable orbit size, unstable
- Purple = small orbit size variation, stable

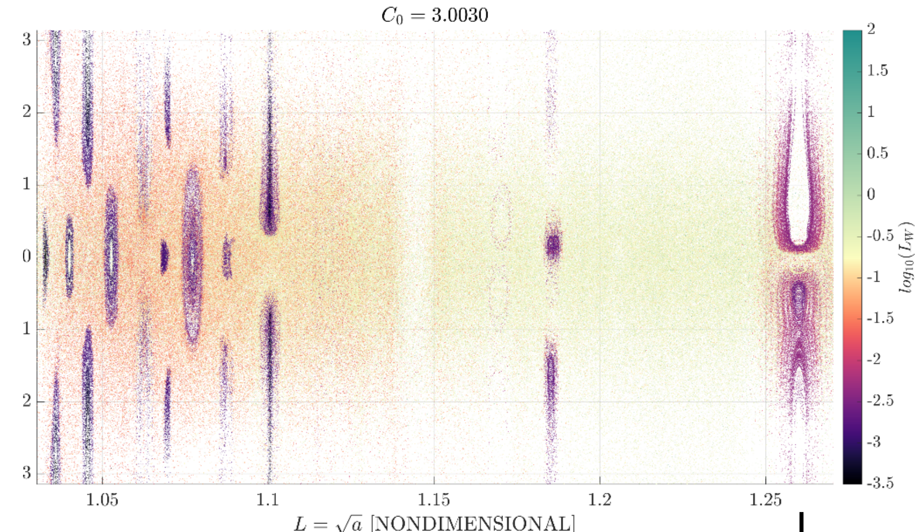
Less stable overall, but still have stable orbits

CR3BP



Ganymede Orbit Crossers

CR6BP



Ganymede Orbit Crossers

Ganymede 1:1 Resonance

$x - axis = \sqrt{semimajor\ axis}$

$y - axis = arg. of\ periapsis\ in\ rot. frame$ $\quad$ $color = log_{10}(resonance\ width)$

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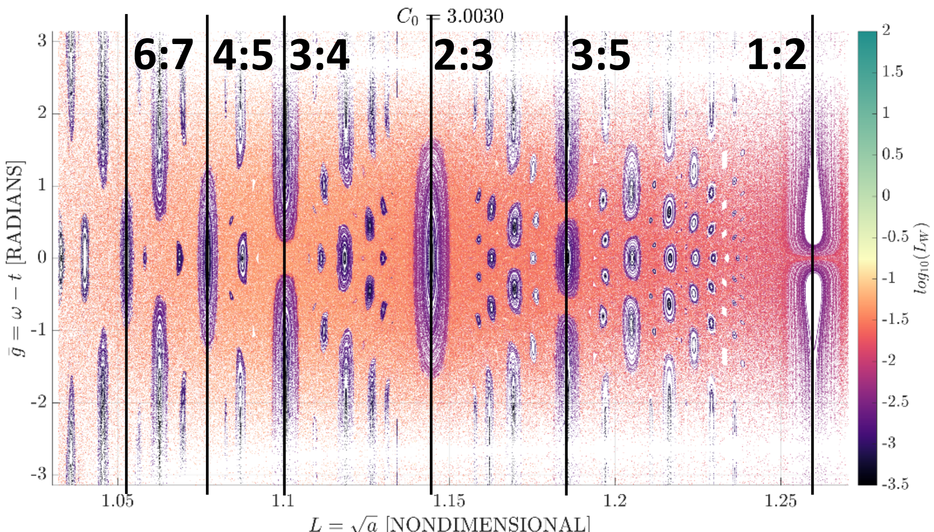

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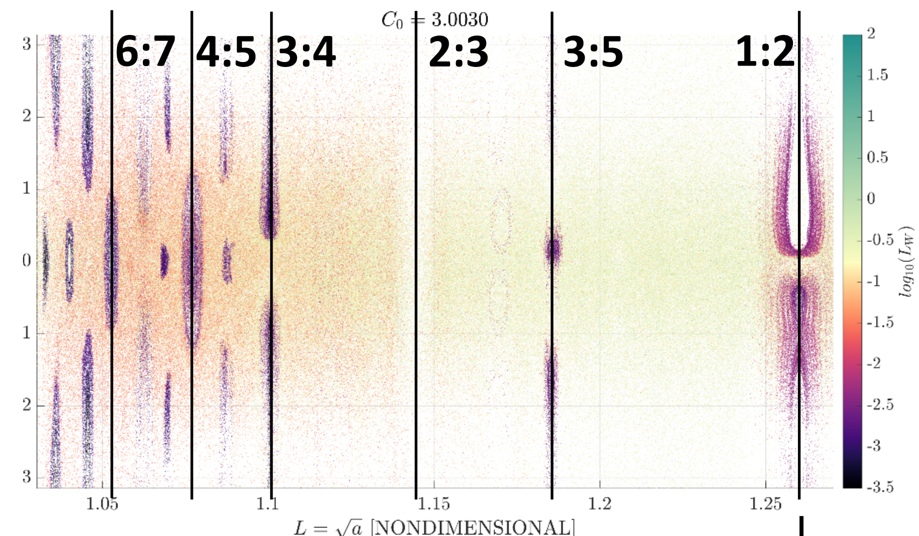

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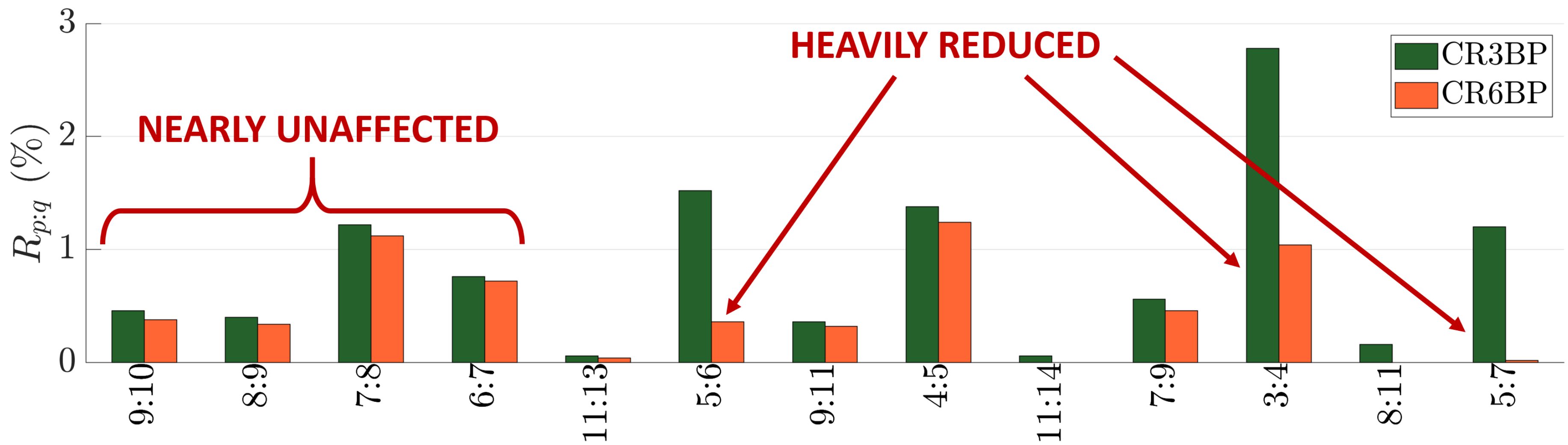
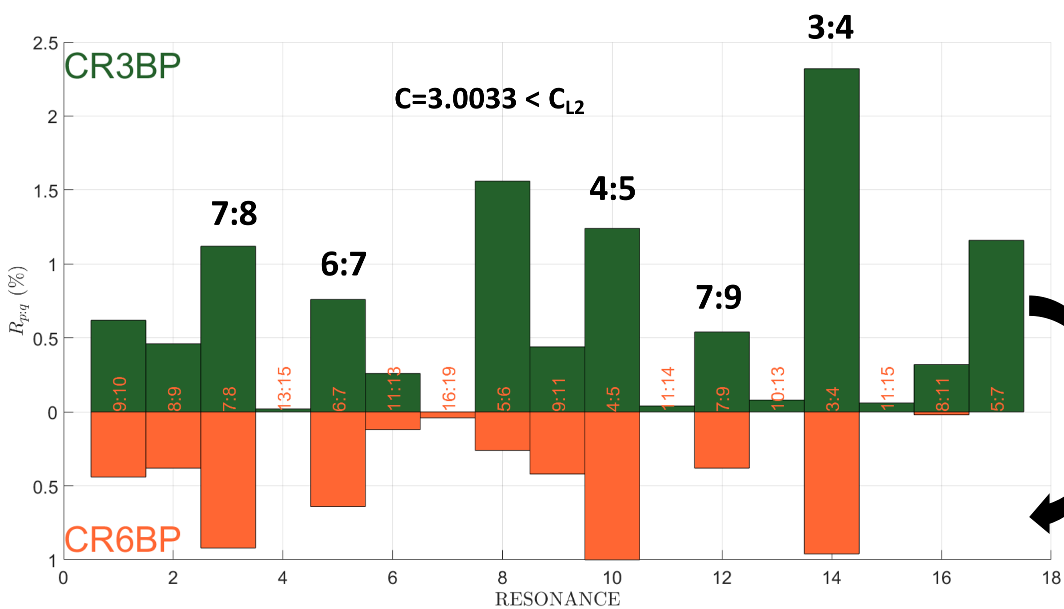
Resonance Abundance

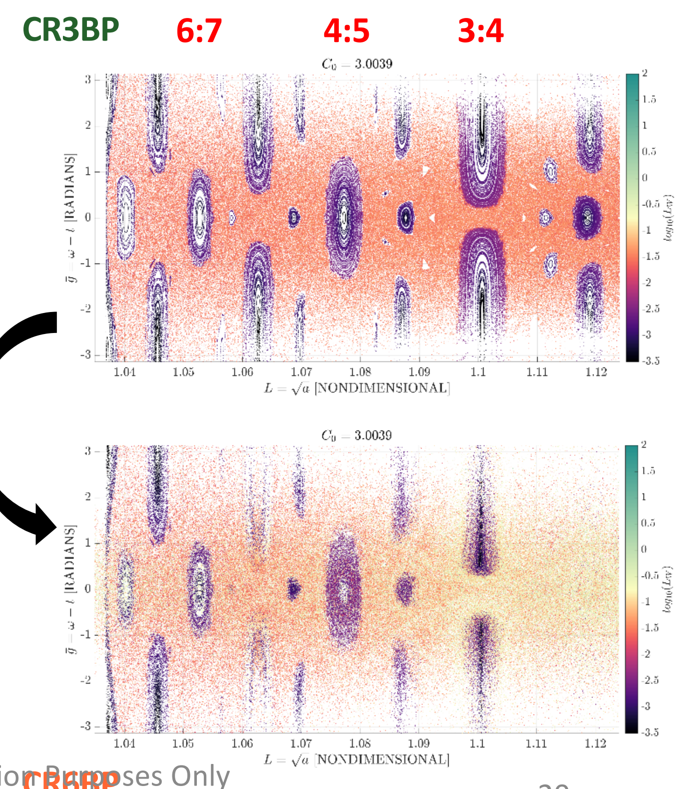
- Quantitatively compare number of stable resonant orbits in 3- and 6-body models
  - Resonance Abundance = number of stable solutions that belong to a given resonance
- Some p:q resonance abundance similar in 3- vs. 6-body
- Others significantly lower in 6-body



Testing Hypothesis, 6:7 Quarantine

- CR6BP stable resonances often have **<u>p:7k</u>** format or **<u>3k:q</u>** format
  - p:7k Europa is **2p:3k Callisto** (6:7 -> **4:1**)
  - 3k:q Europa is **14k:q Callisto** (3:4 -> **7:2**)
  - 6:7 satisfies both rules!





Brian D. Anderson,bdanders@usc.edu

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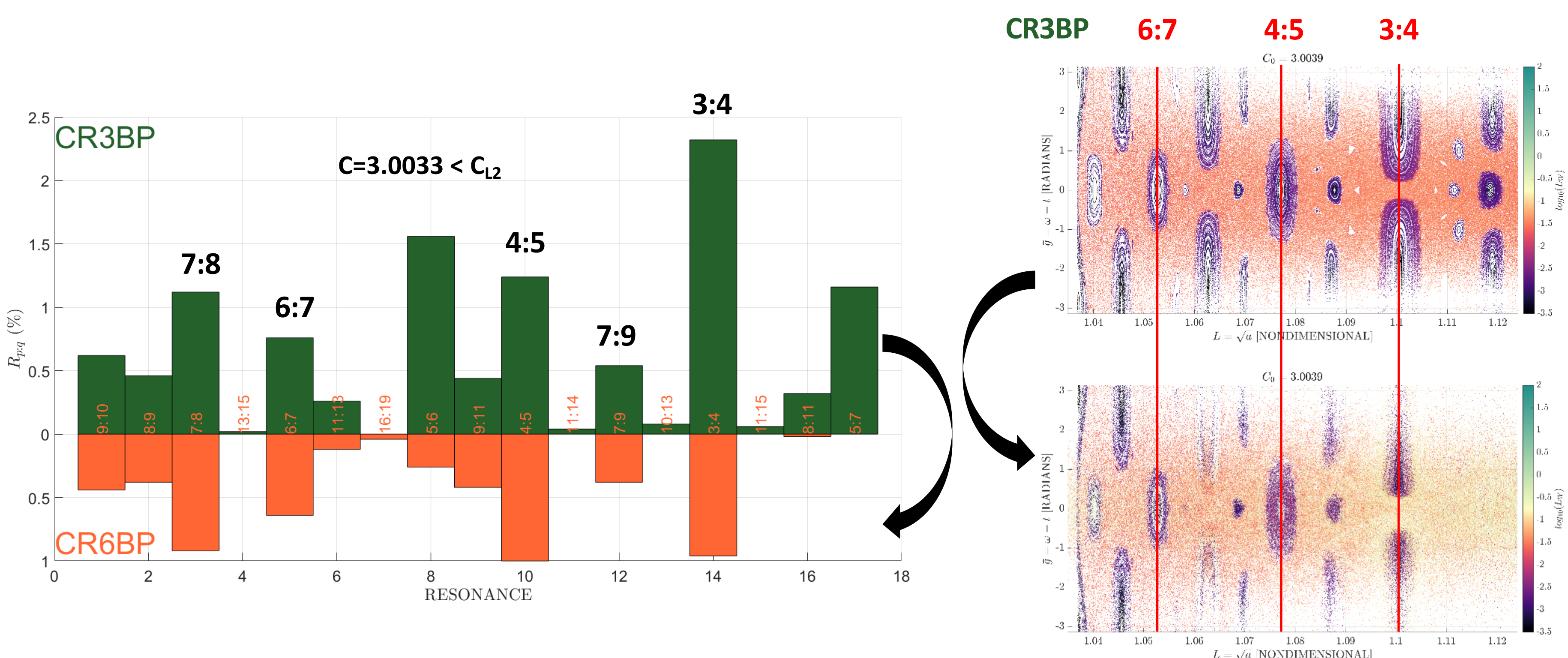
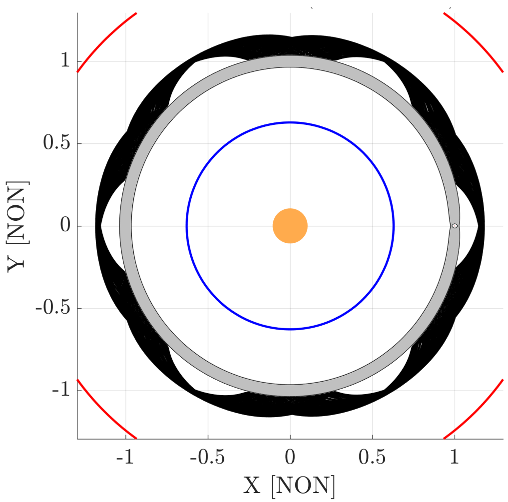
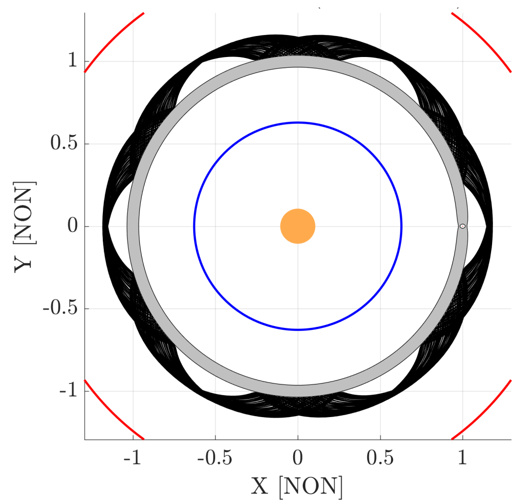


CR3BP->CR6BP->Ephemeris

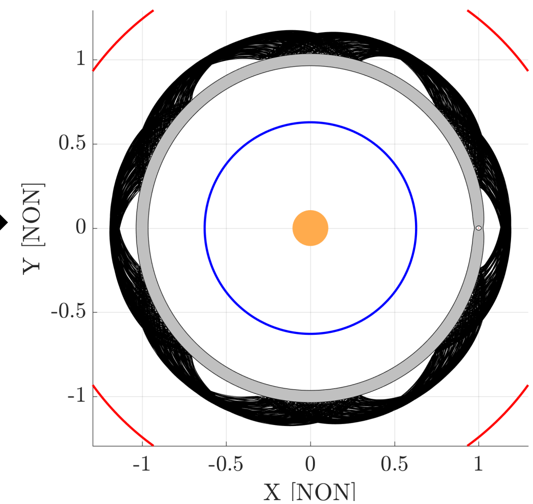
CR3BP 6:7



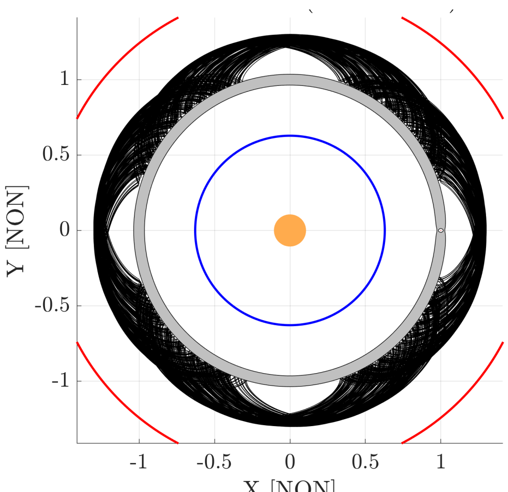
CR6BP 6:7



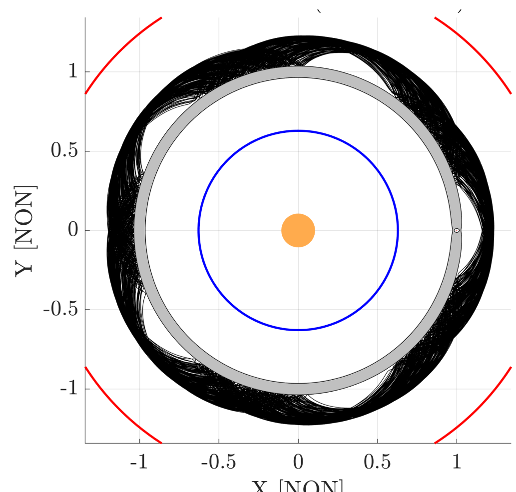
EPHEMERIS 6:7



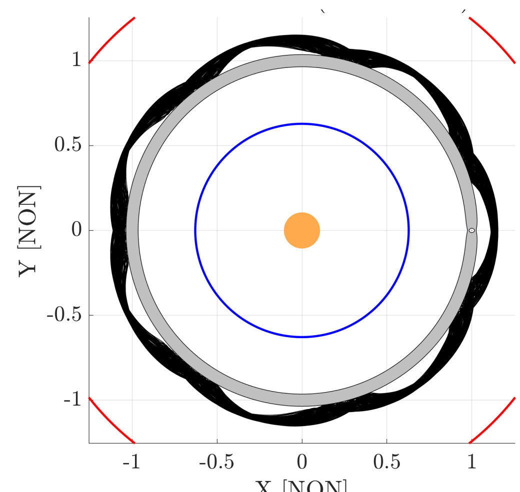
EPHEMERIS 4:5



EPHEMERIS 5:6



EPHEMERIS 7:8



Conclusion

- Answered why 6:7 disposal orbit worked so well
  - Resonance with all 4 Galilean moons needed for long term stability – Callisto can be critical
- Developed new method to estimate abundance of stable resonant orbits in Galilean Satellite System
  - Discrete Orbital Model using Poincaré Map
  - Finite Time Abundance of Resonant Orbit Algorithm
  - Restricted 6-Body Model for design of resonant Europa quarantine orbit stability carries over to ephemeris
- Future Work
  - Extend to other n-body systems (Saturn, etc.)
  - Apply to unstable orbits for tour design

QUESTIONS?

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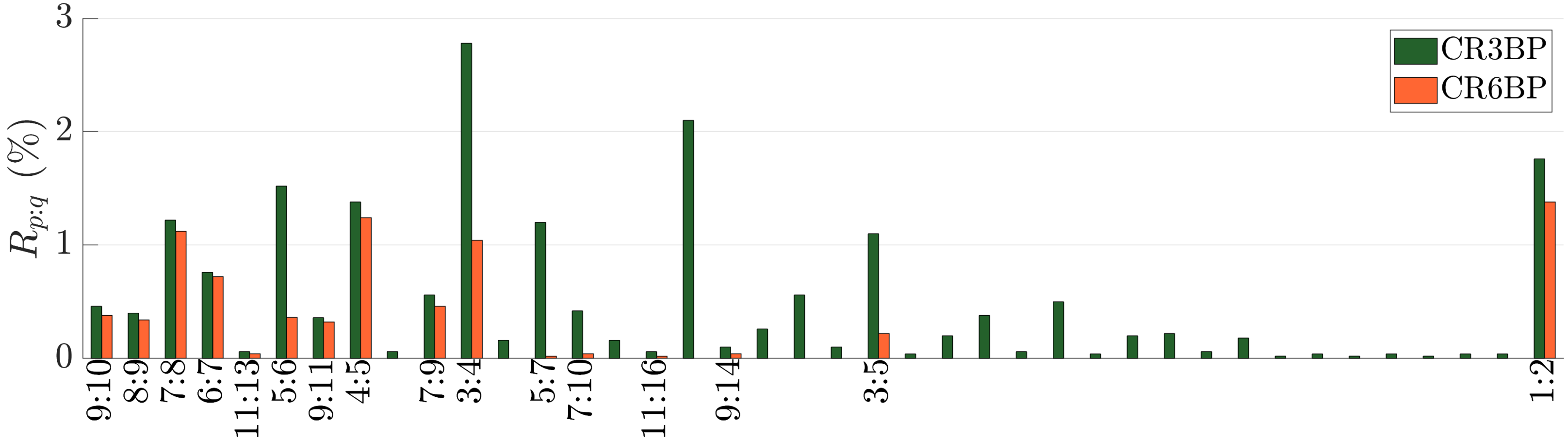
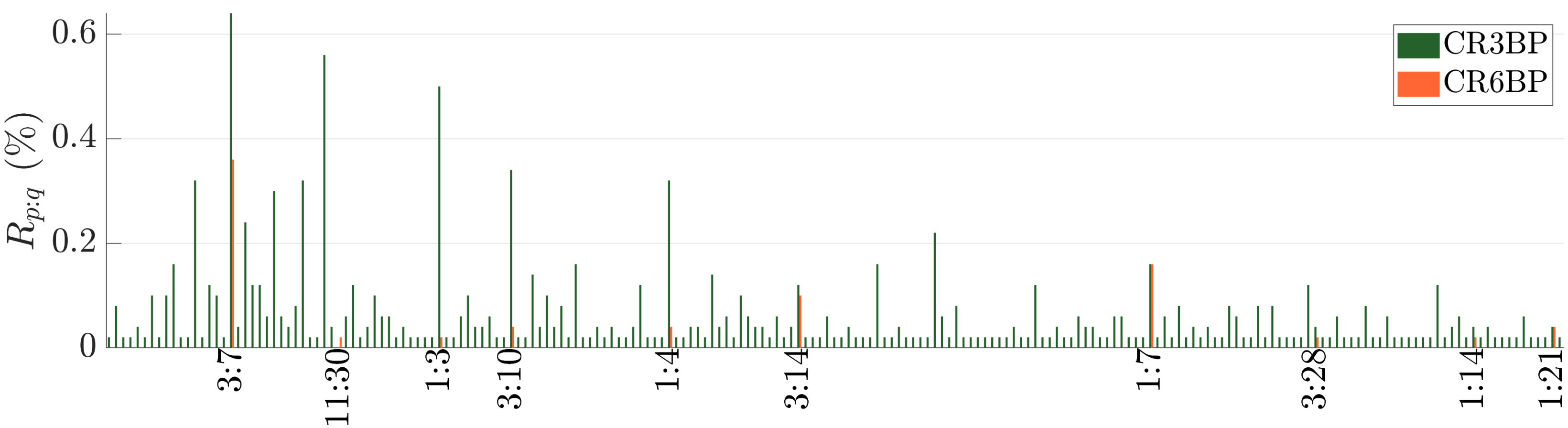
Brian D. Anderson,bdanders@usc.edu

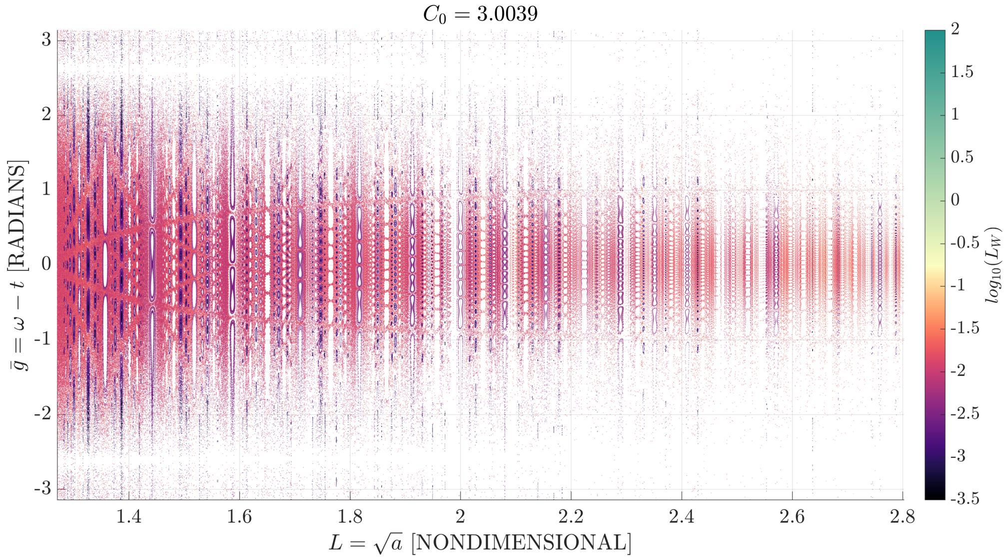
Abundance of Large Resonances

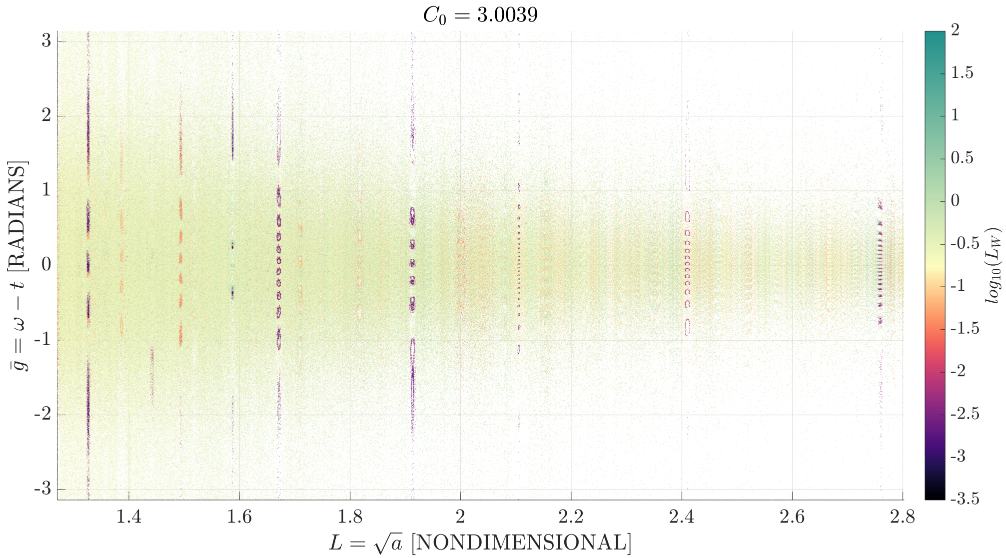




Brian D. Anderson,bdanders@usc.edu

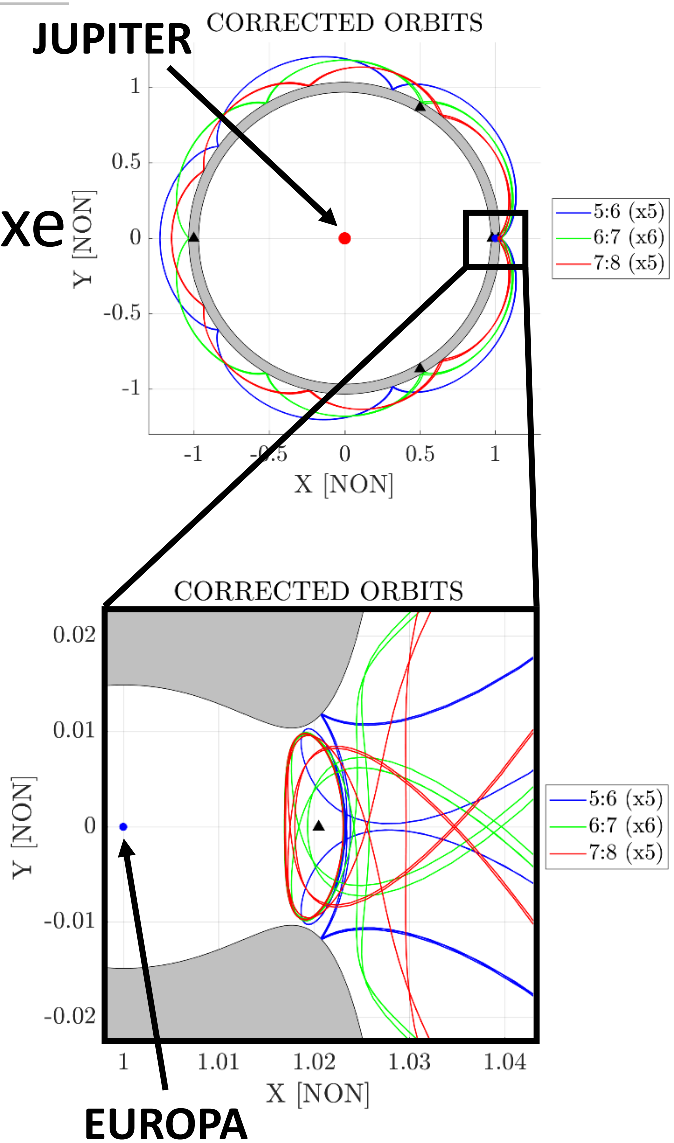
Large Resonance P. map





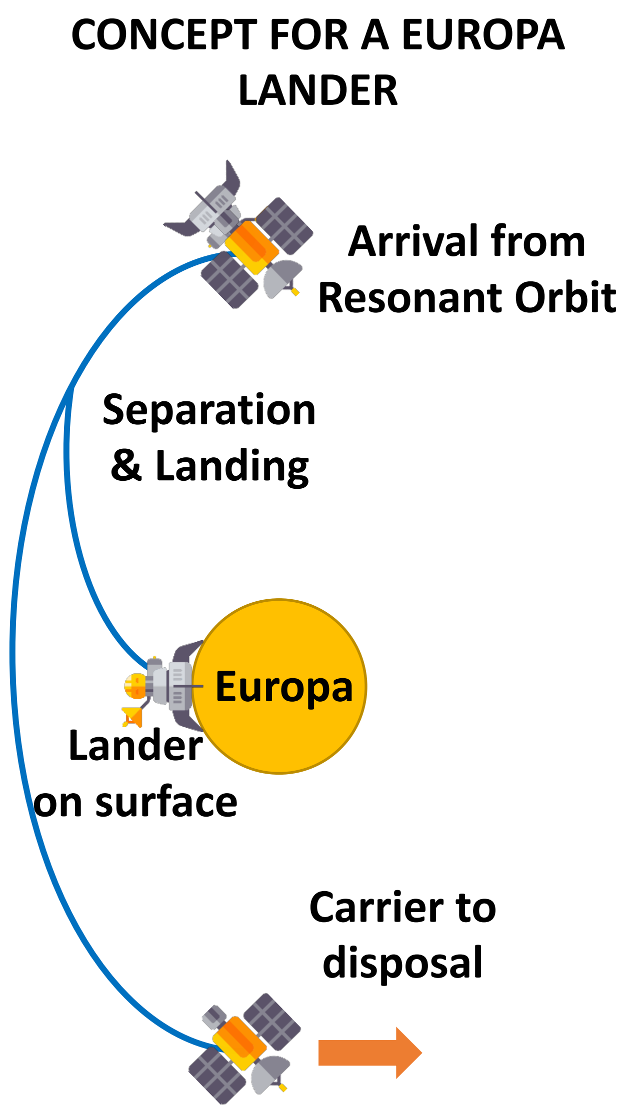
Compound Resonant Orbits

- Automated Tool Developed
- Searches for Resonant Orbits for fixed Jacobi constant
- Searches for Compound Orbits
  - Connects Lyapunov to Resonance
- Planar CR3BP
- Many orbits in each p:q resonance with varied number of loops around L-point
  - Looks very similar far from Secondary



Need Stable Disposal Orbit

- Work done for Europa Lander mission concept
- Carrier & Lander spacecraft attached
- Both enter through L_2 gateway from Resonant Orbit
- Lander separates and lands
- Carrier enters disposal orbit around Jupiter to avoid impact for 10 years for planetary protection
  - Jupiter radiation sterilizes S/C



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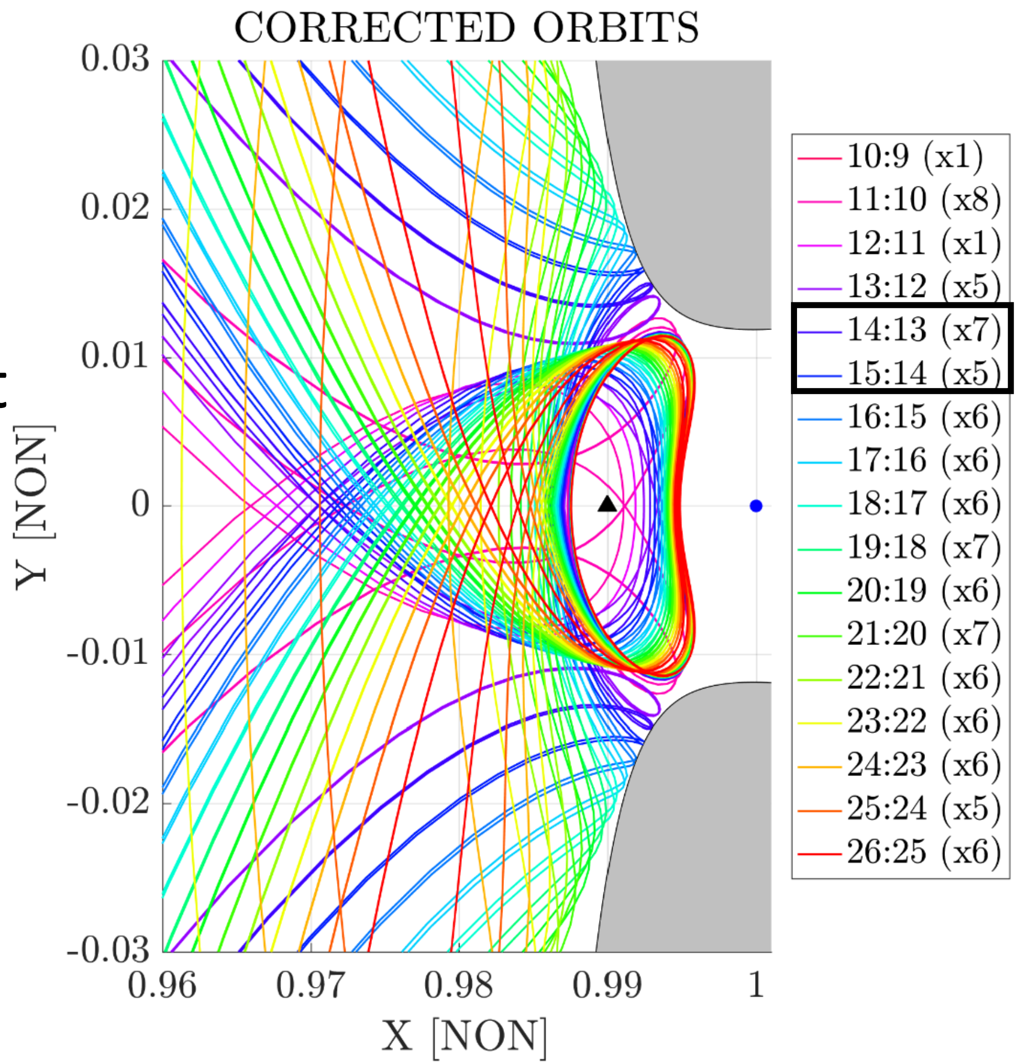
Brian D. Anderson,bdanders@usc.edu

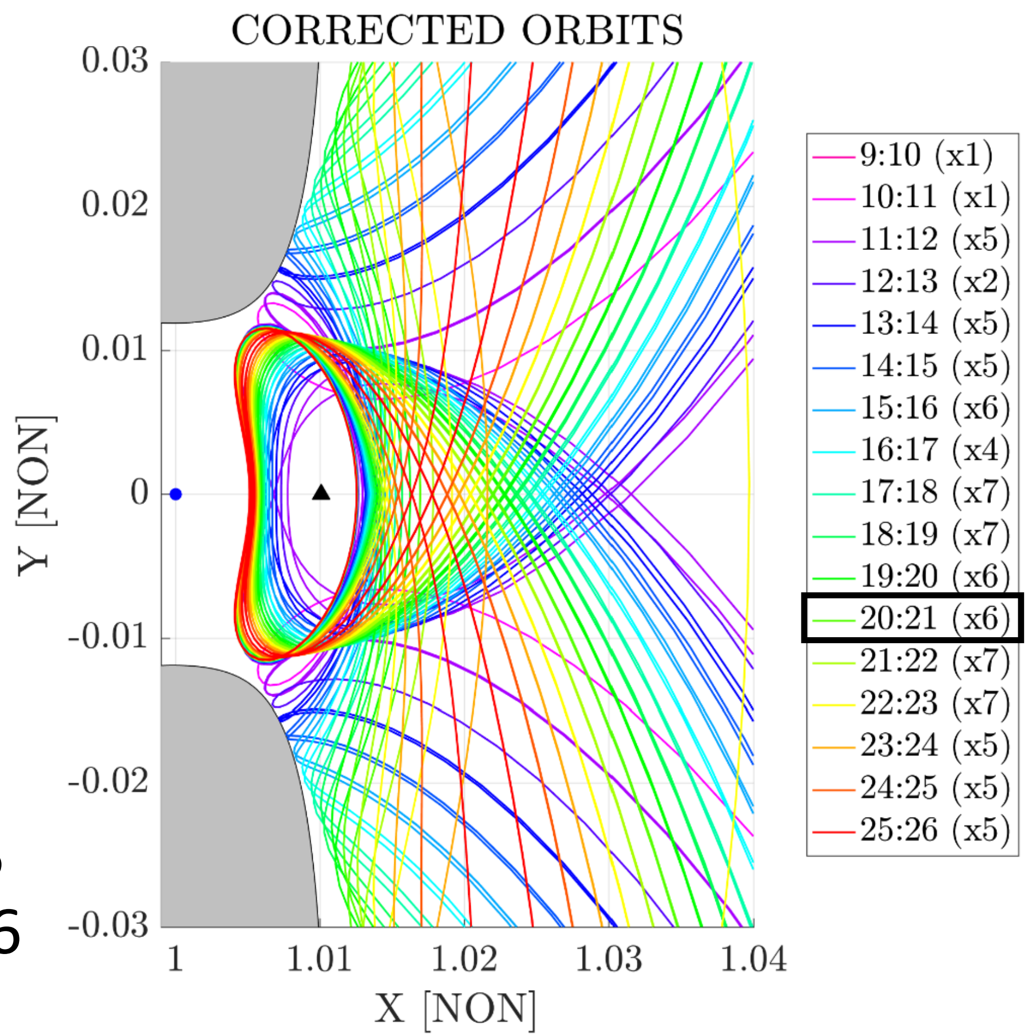
Compound Resonant Orbits

- Sun-Earth L_1
  - C = 3.0005
  - Similar to 2006 RH_{120}
- 29:27 orbit should exist since it has a period between 15:14 and 14:13
- Confirms that transit-enabling orbit exists that matches RH_{120} energy & resonance



Compound Resonant Orbits

- Sun-Earth L_2
  - C = 3.0005
  - Similar to 2006 RH_{120}
- 20:21 orbit found
- Confirms that transit-enabling orbit exists that matches RH_{120} energy & resonance
- Asteroids approaching on any resonance 10:9-26:25 can transit into 9:10-25:26 resonances



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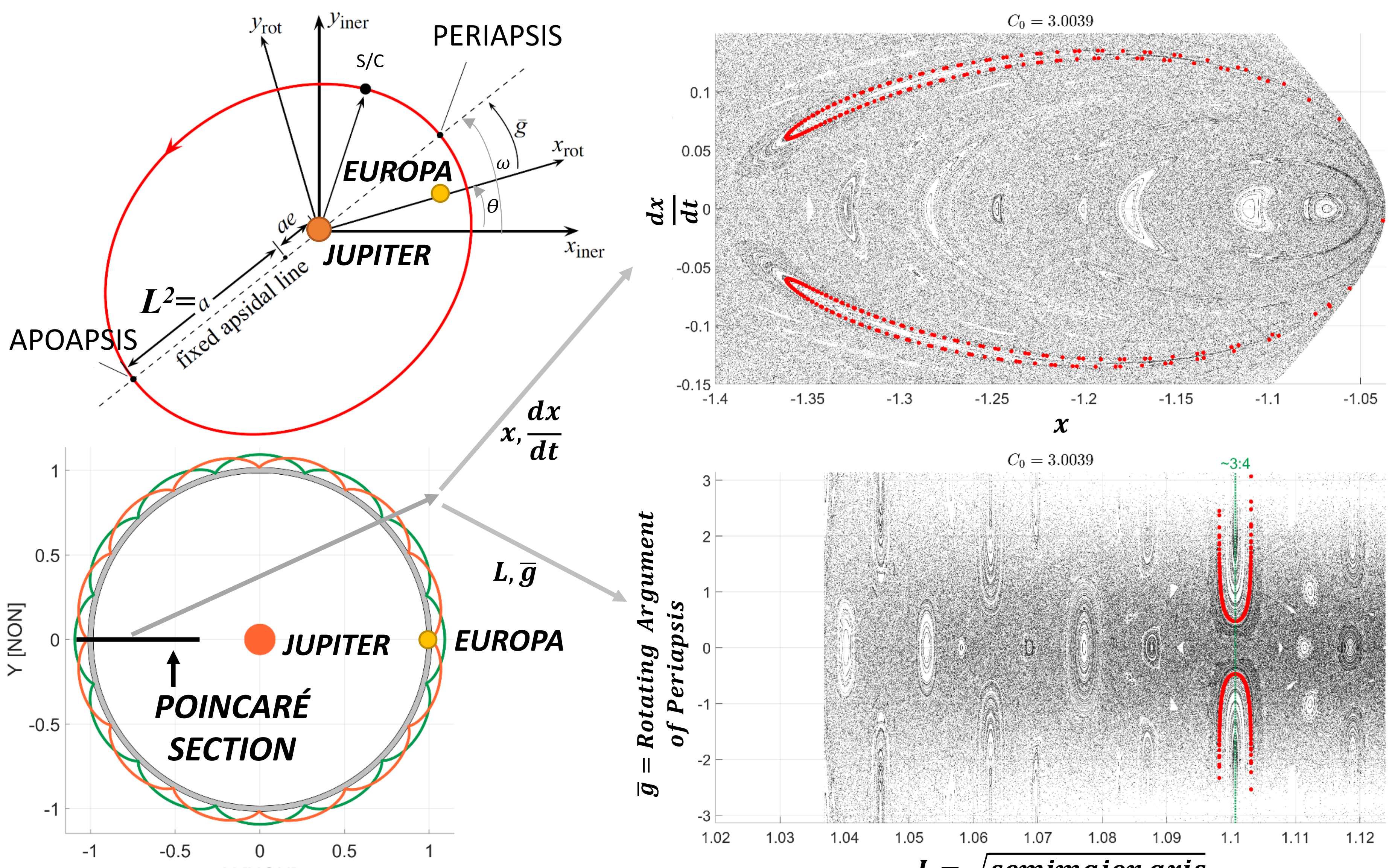
Delaunay Var. : $L = \sqrt{a}$, $\bar{g} = \omega - \theta$



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Poincaré Map