European Planetary Science Congress 2018
16-21 September 2018
TU Berlin, Berlin, Germany

A code for the study of gravitational aggregates with non-spherical particles

Fabio Ferrari*§, Michèle Lavagna§
* Jet Propulsion Laboratory, California Institute of Technology
§ Department of Aerospace Science and Technology, Politecnico di Milano

Jet Propulsion Laboratory
California Institute of Technology

POLITECNICO MILANO 1863

Marie Skłodowska-Curie Actions

European Commission

Presentation overview

1. Introduction
   – Research goal and motivation
2. Implementation and methods
   – Software architecture
   – Gravitational dynamics
   – Contact dynamics
   – Numerical integration
3. Applications
   – Rubble-pile asteroid
   – Granular soil interaction
4. Conclusion
   – Final highlights
   – Future work and ongoing collaborations

Introduction

Research goal and motivation

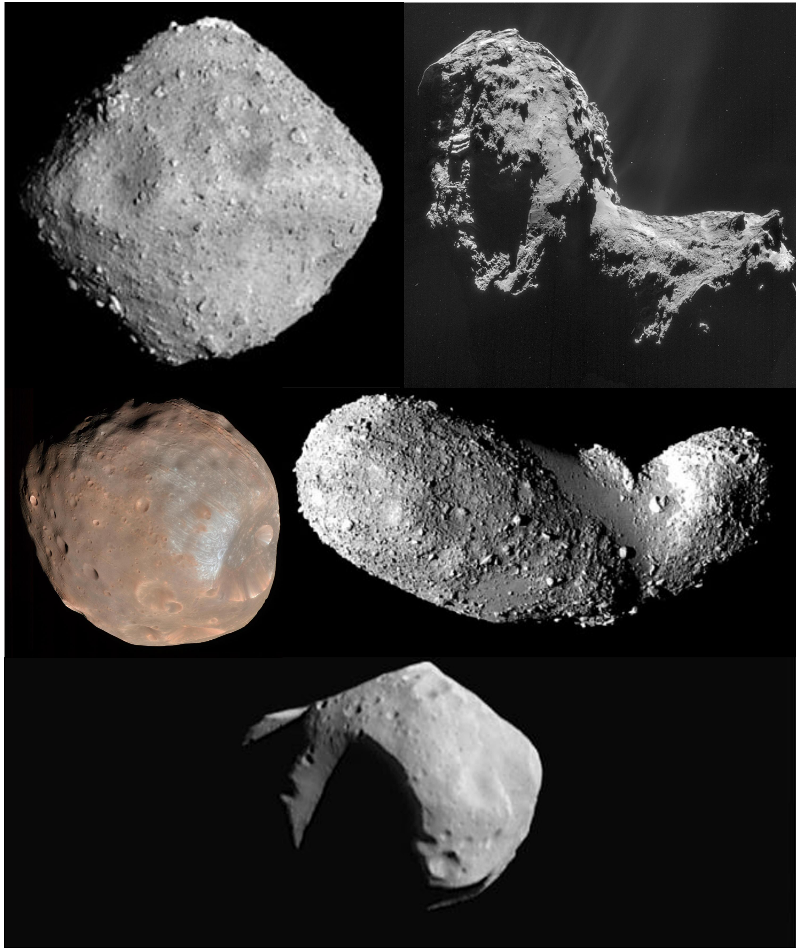
Many asteroids between 100 m and 100 km in size are likely to be gravitational aggregates “rubble pile” [Richardson et al. 2002]

Evidence from studies and in-situ observations:

- 253 Mathilde (NEAR-Shoemaker, 1997)
- 25143 Itokawa (Hayabusa, 2005)
- 162173 Ryugu (Hayabusa-2, 2018)
- 67/P Churyumov-Gerasimenko (Rosetta, 2014)
- Phobos (multiple missions)

Research goal:

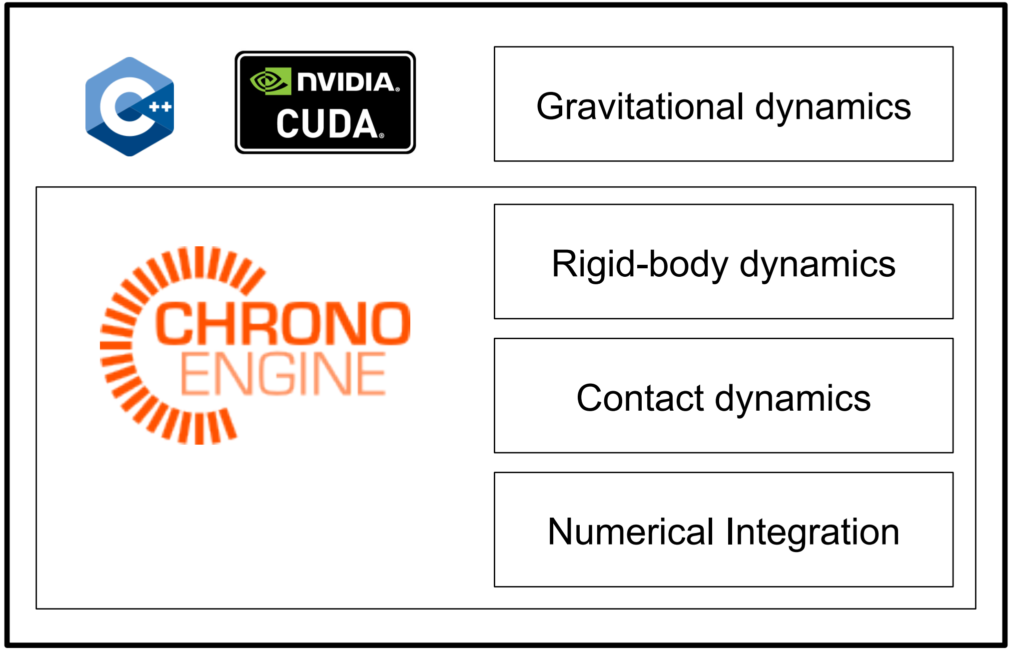
Study of rubble-pile asteroids as gravitational aggregates through numerical simulations (granular dynamics)



Implementation and methods

Implementation and methods

Software architecture



Implementation and methods

Gravitational dynamics: available methods

Gravitational dynamics

Rigid-body dynamics

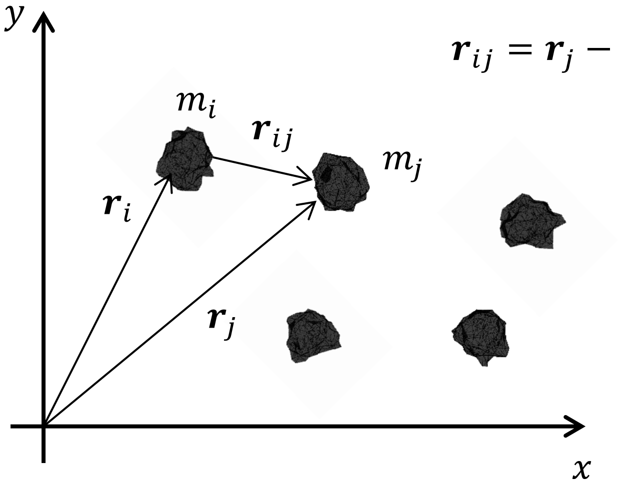
Contact dynamics

Numerical Integration

- Direct N-to-N integration
- Barnes-Hut octree (CUDA/GPU parallel)

Implementation and methods

Gravitational dynamics: direct N-to-N integration



N equations of motion

$$m_i\, \ddot{\boldsymbol{r}}_i = G \sum_{j=1, j\neq i}^{N} \frac{m_i\, m_j}{r_{ij}^3} \boldsymbol{r}_{ij}$$

<u>Features of the dynamical system</u>

- No analytical solution for the gravitational motion of N bodies
- Highly non-linear (chaotic) behavior
- Strong dependency on initial conditions
- Slow dynamics: characteristic time $T \sim \frac{1}{\sqrt{G\rho}}$

  (with $G = 6.67 \cdot 10^{-11}\, \frac{m^3}{kg\ s^2}$)

<u>Features of the numerical problem</u>

- Initial value problem
- Integration time step can be big

  $dt < \frac{T}{2} = \frac{1}{2\sqrt{G\rho}}$

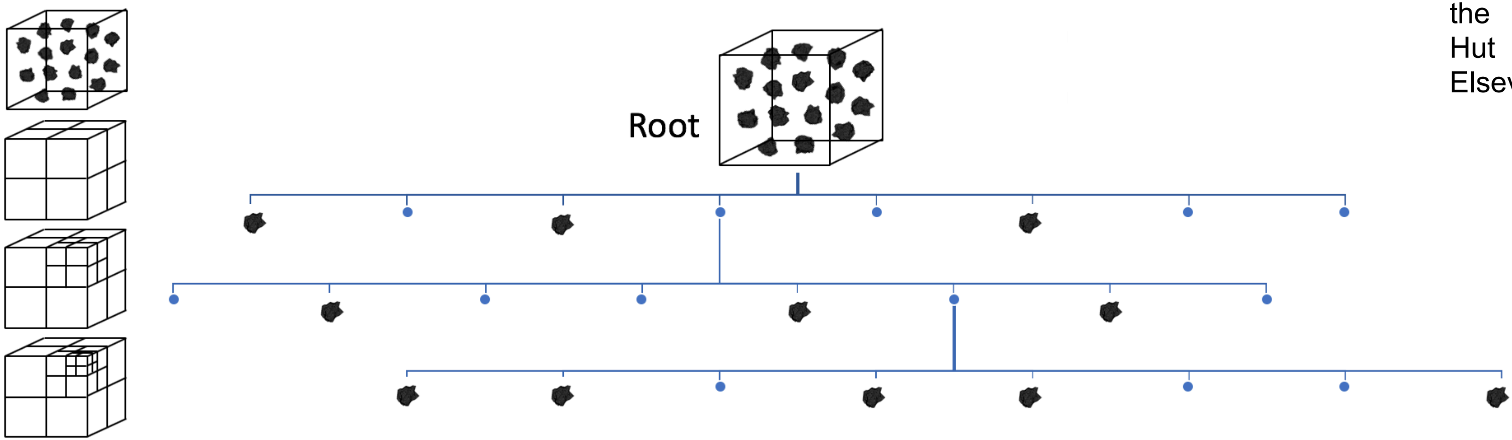
  ($dt \sim 10^3\ s$ for typical asteroids densities)

Implementation and methods

Gravitational dynamics: Barnes-Hut octree (CUDA/GPU parallel)

Kernel 1 Compute root (bounding box) → **Kernel 2** Build octree → **Kernel 3** Store body information → **Kernel 4** Sort bodies by distance → **Kernel 5** Evaluate forces

Based on the work by M. Burtscher and K. Pingali, *GPU Computing Gems*, Chapter 6: An efficient CUDA implementation of the Tree-based Barnes Hut N-body Algorithm. Elsevier Inc, 2011.



- **Nodes** correspond to **cubes** in the physical space
- Homogenous Spatial Recursive sub-division (until each extremal node has 1 or 0 particles

Implementation and methods

Gravitational dynamics: Barnes-Hut octree (CUDA/GPU parallel)

Under certain conditions, the force acting on a body, generated by a cluster of bodies can be approximated ➔ treat cluster as a single body

For each Body-Node pair $(\boldsymbol{B}, \boldsymbol{N})$: $\boxed{\theta = \dfrac{Radius\ of\ N}{\boldsymbol{R}_N - \boldsymbol{R}_B}}$

$\boldsymbol{R}_N$: position of barycenter of node ***N*** of the octree.

$\boldsymbol{R}_B$: position of body ***B***.

After choosing the accuracy ($\theta_{accuracy}$) the condition is: $\boxed{\theta < \theta_{accuracy}}$

- $\theta_{accuracy} = 0$ is the limiting case of considering all interactions between bodies
- Typical value: $\theta_{accuracy} = 0.25$ (the body-to-cluster distance is at least 4 times the radius of the cluster)

Implementation and methods

Gravitational dynamics: Barnes-Hut octree (CUDA/GPU parallel)

For each body ***B***, the tree is traversed from the root downwards.

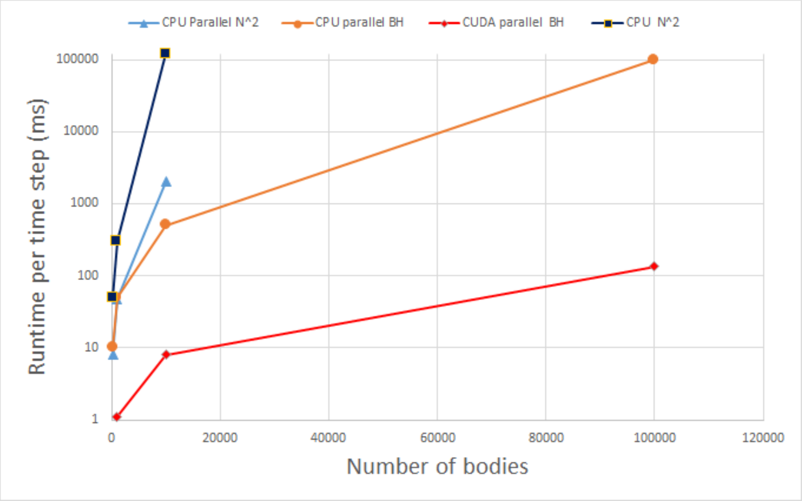
Every time a node ***N*** is encountered:

- **If *N* is a leaf:**
  body-to-body interaction
- **If *N* is internal and** $\boldsymbol{\theta < \theta_{accuracy}}$**:**
  Traversal interrupted and body-to-cluster interaction
- **If *N* is internal and** $\boldsymbol{\theta \geq \theta_{accuracy}}$**:**
  Traversal continues

Implementation and methods

Gravitational dynamics: performance

	DIRECT N-to-N	TREE CODE ALGORITHM
Computation time	$O(N^2)$	$O(N \log(N))$
Number of bodies	< 1000	> 1000
Accuracy	Accurate (integrator)	Accuracy depends on the algorithm (simulator)



CPU: Intel Core i7 6500U 3.1GHz
GPU: Nvidia GeForce 940M

Implementation and methods

Rigid-body dynamics

Gravitational dynamics

Rigid-body dynamics

Contact dynamics

Numerical Integration

- Rotational degrees of freedom
- Tensor of inertia
- Arbitrary shape

Implementation and methods

Rigid-body dynamics

N bodies, each with

- position $\boldsymbol{r}_i$
- rotation quaternion $\boldsymbol{\rho}_i$
- velocity $\dot{\boldsymbol{r}}_i$
- angular velocity $\boldsymbol{\omega}_i$

→ Generalized coordinates

$$\boldsymbol{q} = \left\{\boldsymbol{r}_i^T, \boldsymbol{\rho}_i^T\right\}^T \in \mathbb{R}^{7N}$$

$$\boldsymbol{v} = \left\{\dot{\boldsymbol{r}}^T{}_i, \boldsymbol{\omega}_i^T\right\}^T \in \mathbb{R}^{6N}$$

- mass m_i
- tensor of inertia $\mathbf{I}_i$

→

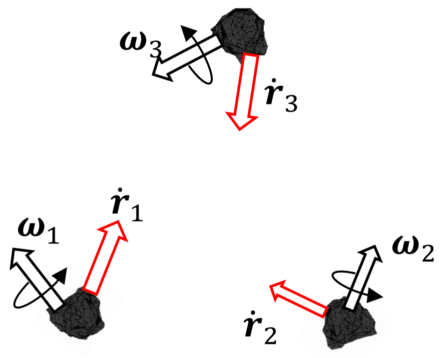
$$\mathbf{M} = [m_i] \in \mathbb{R}^{6N\times 6N}$$

$$\mathbf{J} = [\mathbf{I}_i] \in \mathbb{R}^{6N\times 6N}$$

- collision surface Ω_i

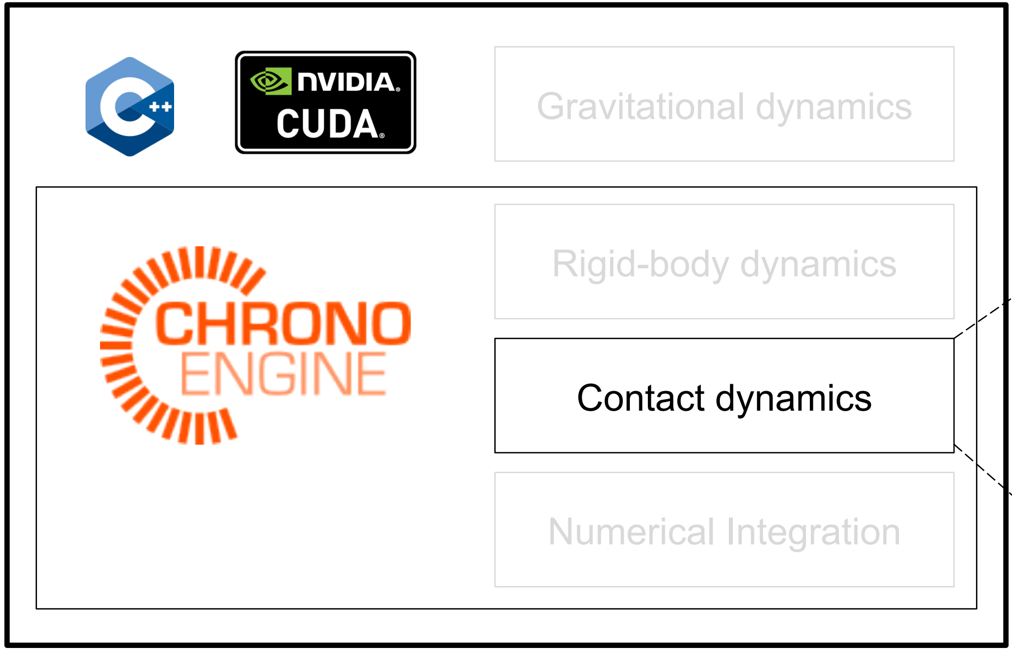
→ Shape:

- Triangulated mesh
- Convex hull
- Common geometry (sphere, box, cone,…)



Implementation and methods

Contact dynamics: available methods



- NSC: non-smooth contact (hard-body, constraint-based)
- SMC: smooth contact DEM (soft-body, penalty-based)
- Hybrid: constraint-based with compliance and damping

Implementation and methods

Contact dynamics: non-smooth dynamics (NSC)

- Equations of motion are formulated as Differential Variational Inequalities (DVI)
- Hard-body model
- Complementarity-based
- Impulse-momentum formulation
- Suitable for problems with discontinuities (rigid contacts)

$$\begin{cases} \gamma \text{ (contact) as solution of CCP} \\ \boldsymbol{v}_{\boldsymbol{n+1}} = f(\boldsymbol{q}, \boldsymbol{v}, t, \gamma) \\ \boldsymbol{q}_{\boldsymbol{n+1}} = g(\boldsymbol{q}, \boldsymbol{v}) \end{cases}$$

Parameters of the model:

- Friction (static, dynamic, spinning)
- Cohesion (value and constitutive model)
- Restitution coefficient

Credits: Tasora et al 2013

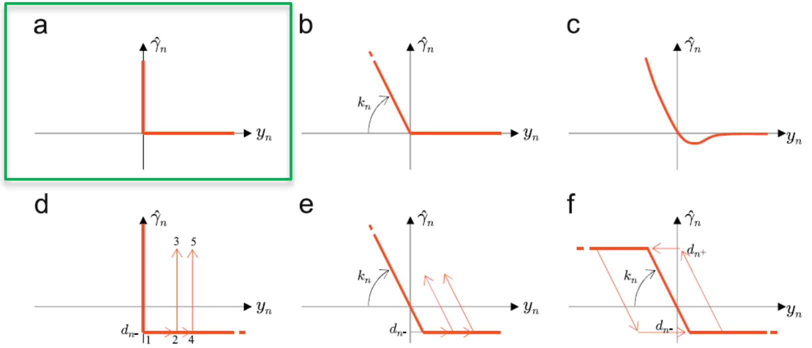


Fig. 1. Basic constitutive relations for normal reaction.

Implementation and methods

Contact dynamics: smooth dynamics (SMC)

- Equations of motion are formulated as Differential Algebraic equations (DAE)
- Soft-body model (DEM)
- Penalty-based
- Force-acceleration formulation
- Suitable for problems with no discontinuities (no rigid contacts)

$$\begin{cases} \dot{\boldsymbol{x}} = f(\boldsymbol{x}, t) \\ \boldsymbol{g}(\boldsymbol{x}, t) = 0 \end{cases}$$

ODE + AE (kinematic constraint)

Parameters of the model:

- Friction (static, dynamic, spinning)
- Cohesion (value and constitutive model)
- {Young modulus, Poisson ratio, restitution coefficient} or {stiffness and damping (normal and tangential)} and constitutive model (Hooke, Hertz)

In this case stiffness and damping are estimated based on constitutive law of material

Credits: Tasora et al 2013

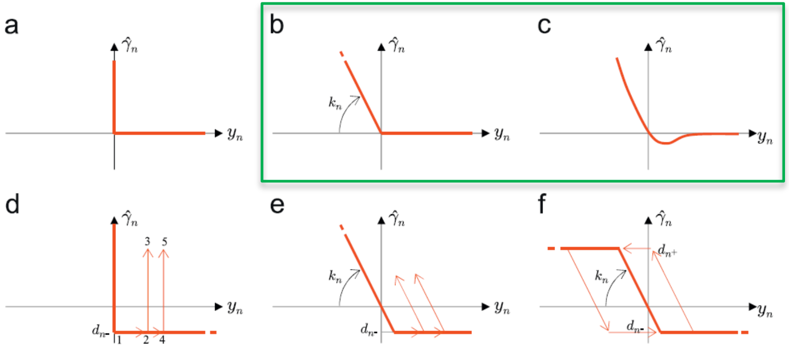


Fig. 1. Basic constitutive relations for normal reaction.

Implementation and methods

Contact dynamics: hybrid model

$$\begin{cases} \gamma \text{ (contact) as solution of CCP} \\ \boldsymbol{v_{n+1}} = f(\boldsymbol{q}, \boldsymbol{v}, t, \gamma) \\ \boldsymbol{q_{n+1}} = g(\boldsymbol{q}, \boldsymbol{v}) \end{cases}$$

- Equations of motion are formulated as Differential Variational Inequalities (DVI)
- Soft-body model (compliance and damping)
- Complementarity-based
- Impulse-momentum formulation
- Suitable for problems with discontinuities

Credits: Tasora et al 2013

Parameters of the model:

- Friction (static, dynamic, spinning)
- Cohesion (value and constitutive model)
- Restitution coefficient
- Stiffness and damping (normal, tangential, rolling, spinning), rolling friction and constitutive model

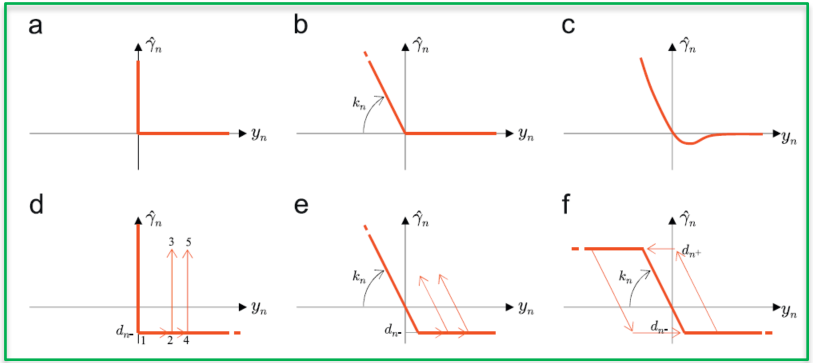


Fig. 1. Basic constitutive relations for normal reaction.

Implementation and methods

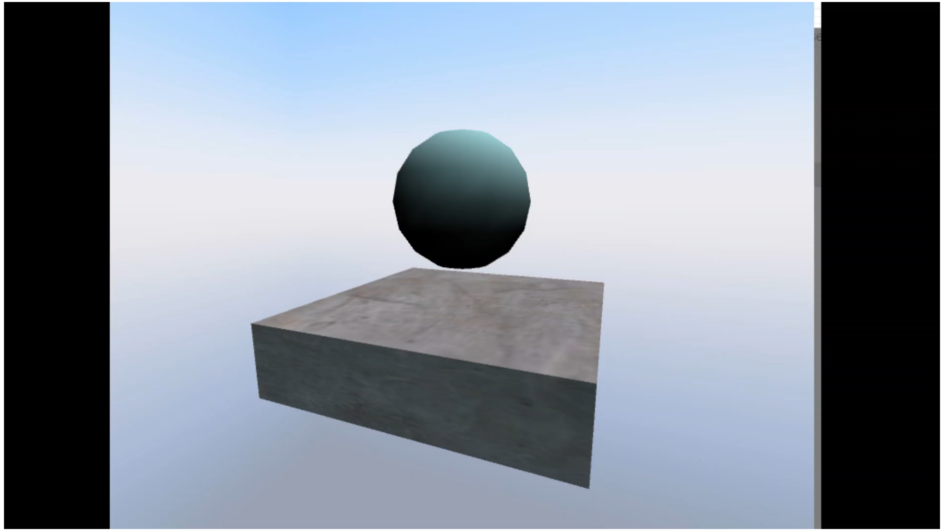
Contact dynamics: summary

		NSC	SMC	Hybrid
Formulation	Equations of motion	DVI	DAE	DVI
	Contact model	hard	soft	soft
Performance	Computational time (single time step)			
	Size of time step			
	Reproducing non-rigid contact dynamics			
	Handling complex shapes			

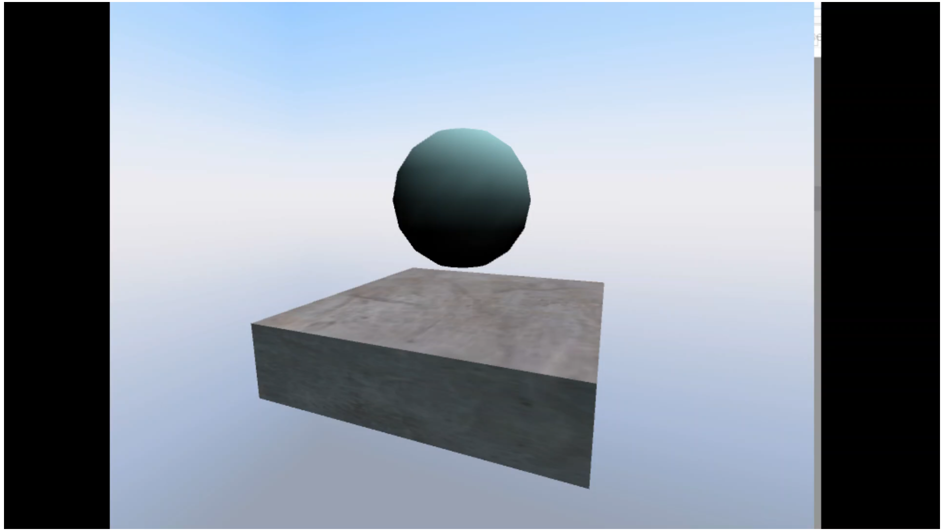
Implementation and methods

Contact dynamics: tuning the parameters

Hybrid model

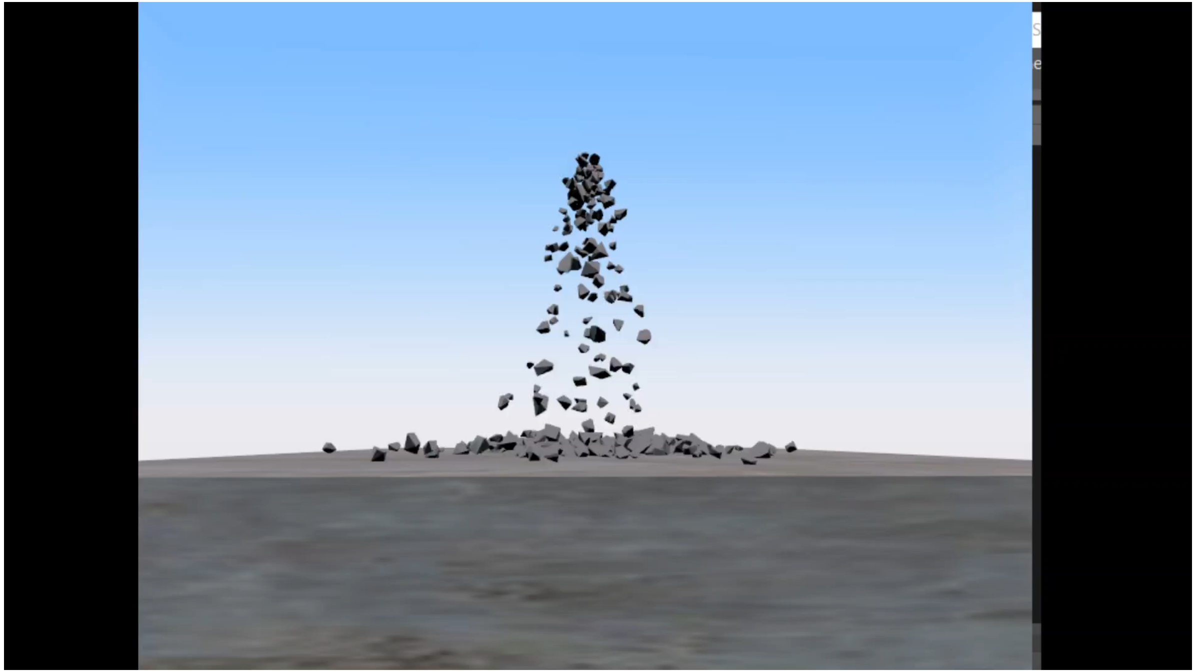


SMC (DEM)



Implementation and methods

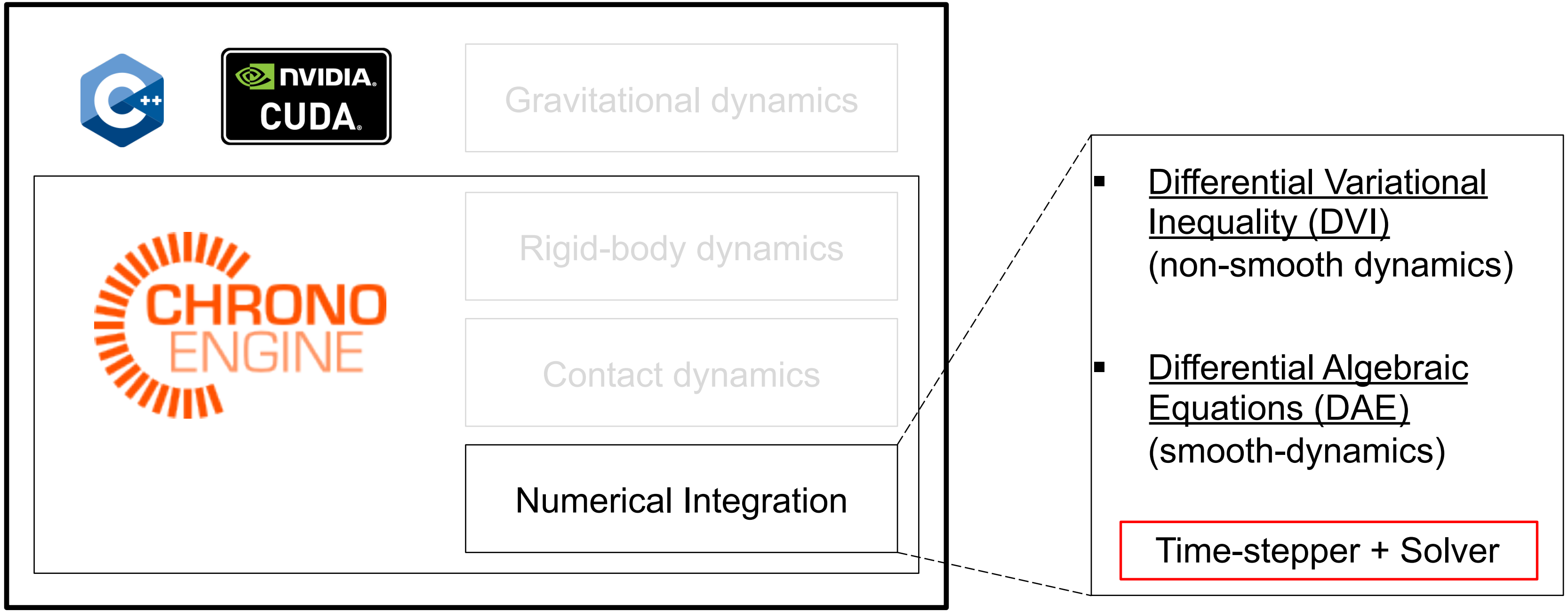
Contact dynamics: tuning the parameters



Angle of repose

Implementation and methods

Numerical integration: available methods



Implementation and methods

Numerical integration: available methods

Non-smooth dynamics (NSC)

Equations of motion are formulated as Differential Variational Inequalities (DVI)

Smooth dynamics (SMC)

Equations of motion are formulated as a Differential Algebraic Equations (DAE)

- Time-steppers:
  - Symplectic methods (semi-implicit Euler, leapfrog) → Suited for gravitational problem
  - Runge Kutta methods (RK45, explicit Euler, implicit Euler, trapezoidal, Heun) → Higher order
  - Newmark, Hilber-Hughes-Taylor → Suited for FEA problems

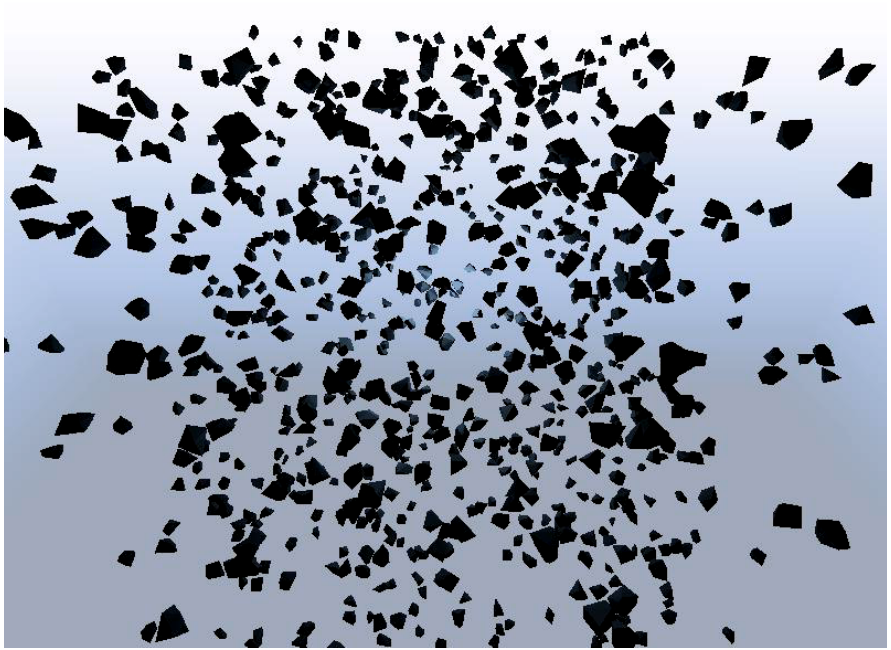
- Solvers:
  - Iterative solvers
  - Direct solvers

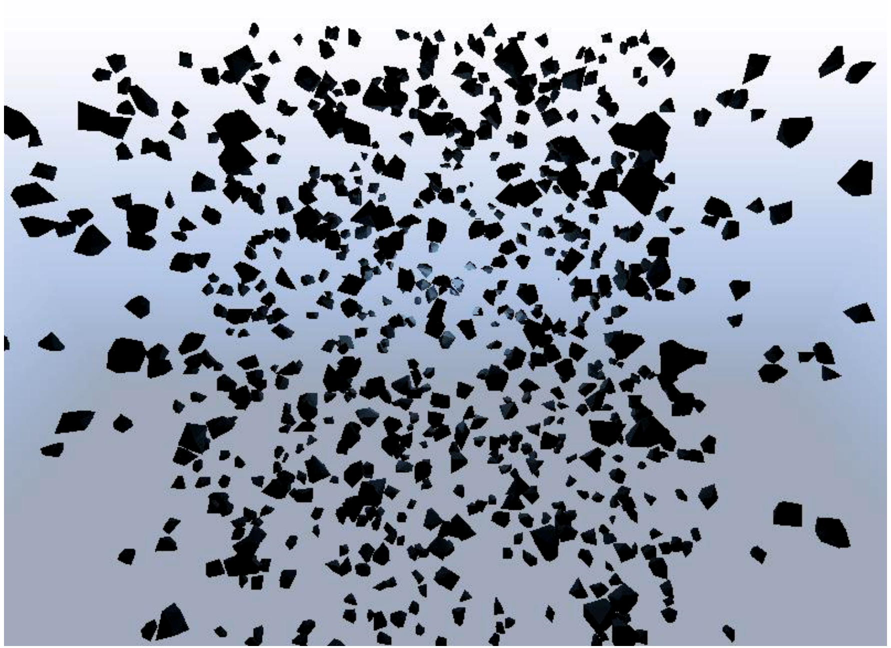
Most commonly used: good for both DVI and DAE problems

Applications

Applications

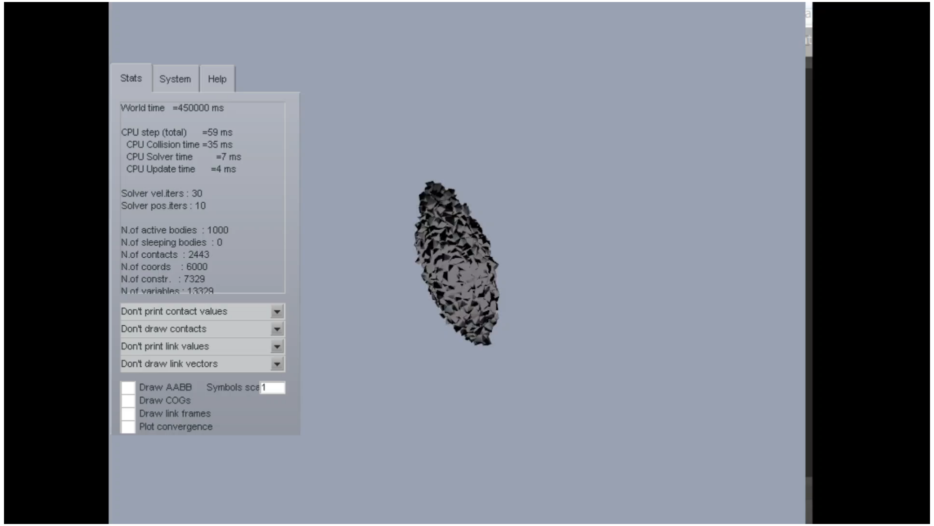
Rubble-pile asteroid

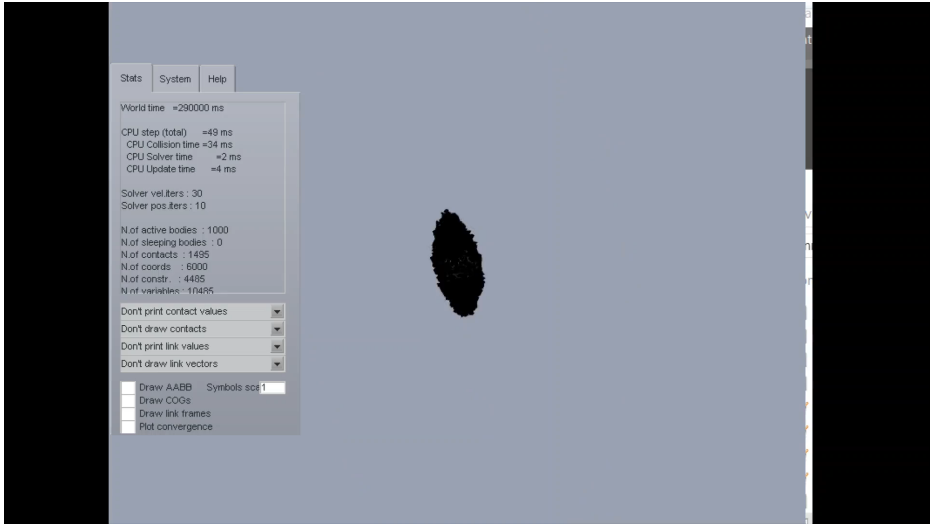




Applications

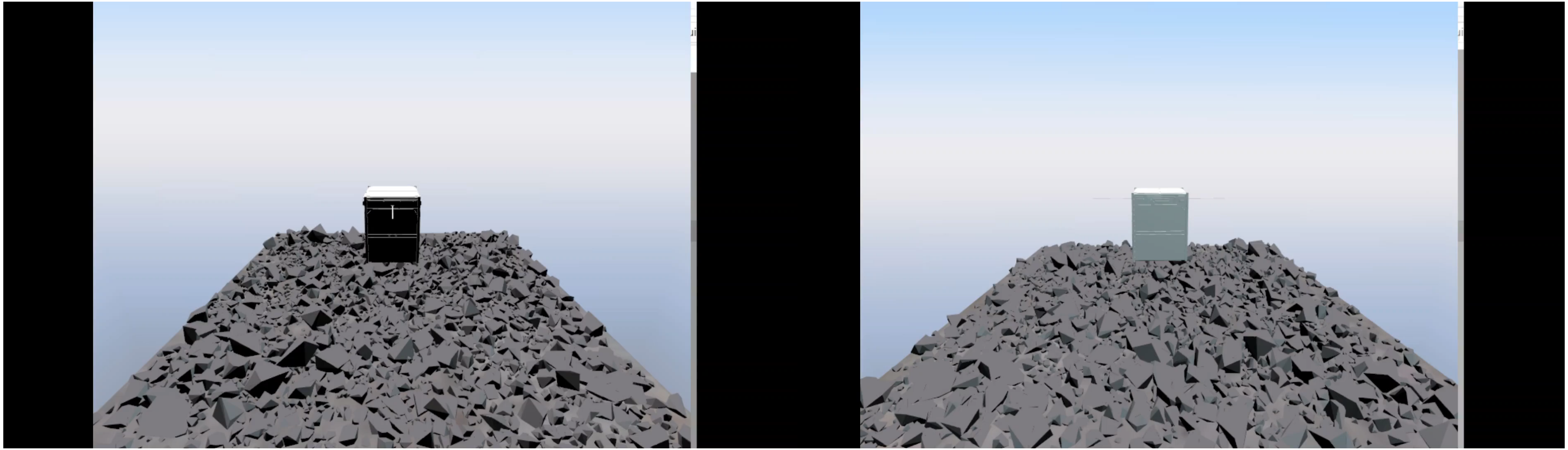
Rubble-pile asteroid





Applications

Granular soil interaction



Conclusion

FINAL HIGHLIGHTS

- Handles complex-shaped bodies
- State-of-the-art methods for gravitational dynamics: Barnes-Hut parallel GPU
- State-of-the-art methods for contact dynamics: both hard- and soft-contact models
- Great flexibility of models/methods and implementation

FUTURE WORK AND ONGOING COLLABORATIONS

- Go on with validation/benchmarking and developing effort (with Chrono::Engine team, Univ. Parma)
- Rubble pile aggregation / reconfiguration (with OCA)
- Lander/soil interaction and lander/rover mobility (JPL)
- Planetary rings dynamics (JPL)
- Rubble pile gravity field (JPL, PoliMi)

Acknowledgements

This project has received funding from the European Union’s Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 800060.

The work was initiated at Politecnico di Milano, Department of Aerospace Science and Technology.

Part of the research work was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration.

Jet Propulsion Laboratory
California Institute of Technology

POLITECNICO
MILANO 1863

Marie
Skłodowska-
Curie Actions

European
Commission