Jet Propulsion Laboratory
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Incremental States for Precise On-Orbit Relative Knowledge in Formation Flight

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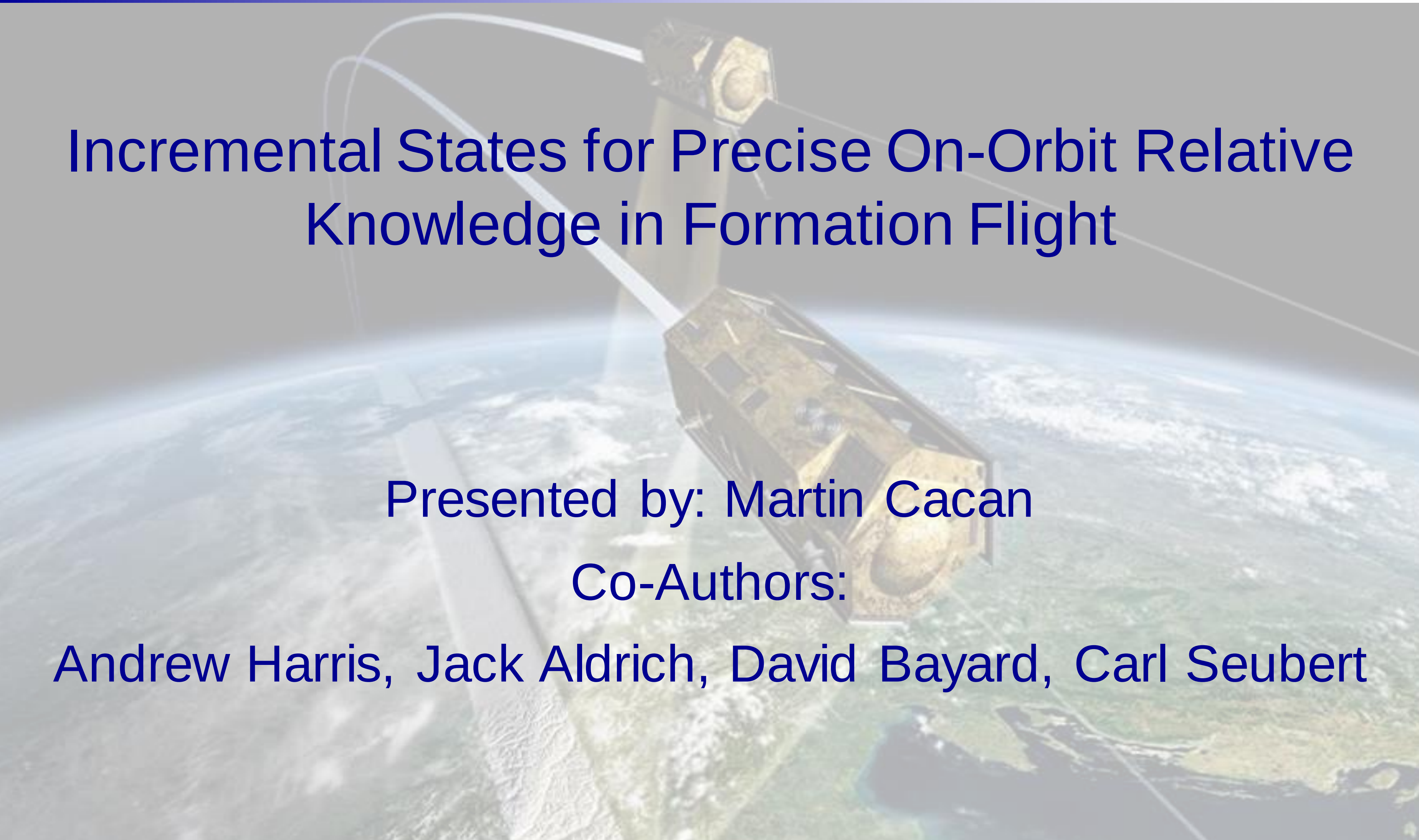
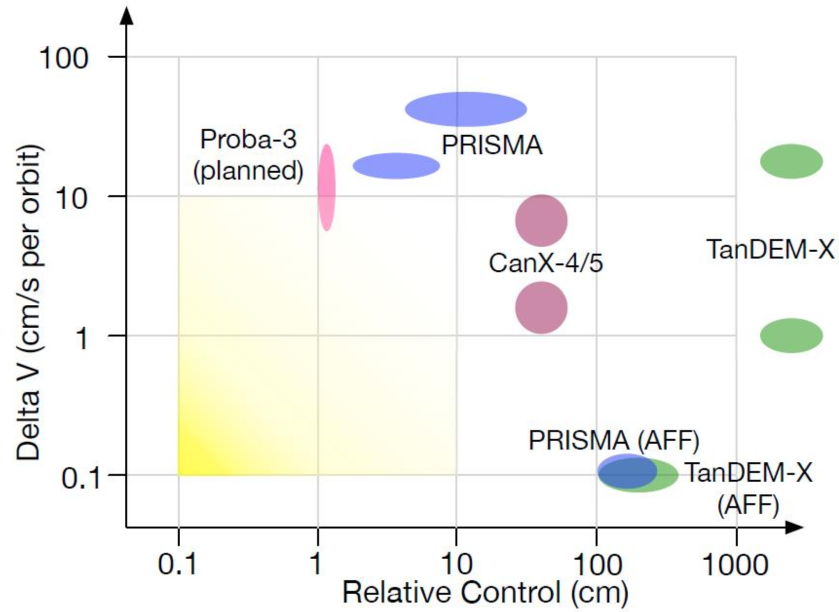
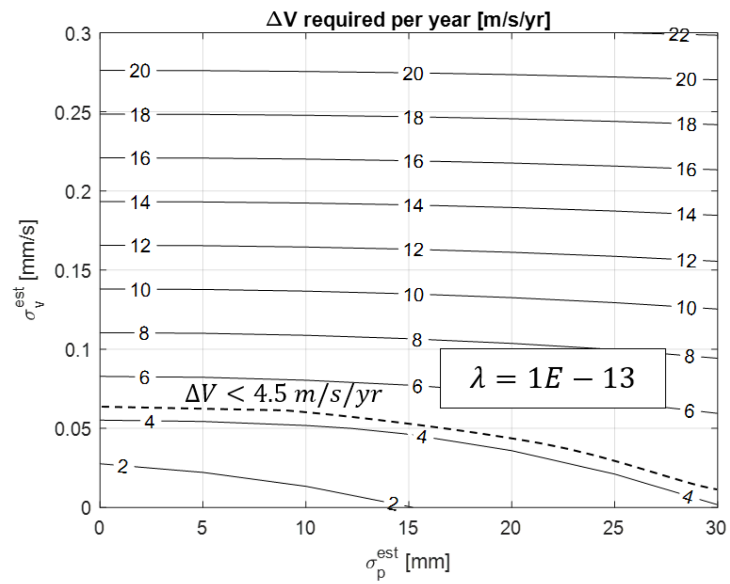


Artist Rendition of TanDEM-X and TerraSAR-X in Formation Flight. Adapted from *TanDEM-X Science Home* by Microwave and Radar Institute. Retrieved from http://www.dlr.de/hr/desktopdefault.aspx/tabid-2317/3669_read-5488/

Motivation

- Interested in formations of small satellites capable of precise on-orbit control with minimal fuel usage
- Fuel can be spent on:
  - Driving the true system to a predicted path in Guidance
  - Navigation errors which pass thru the control law to impact accuracy and fuel consumption
- Previous research found relative velocity error to be the driving factor in fuel usage and control accuracy.





Objective: Examine strategies to improve on-board navigation without raising computational costs.

Started Asking Questions:

- What are the driving factors in velocity estimation error?

- How does the choice of estimation architecture impact performance?
  - Choice of absolute or relative states?
  - When do 'relative-only' methods which assume Leader position break down?

- When do linearization errors become important? How do they compare to measurement errors?
  - How do leader-linearized formations assuming HCW break down even at small separation distances?

→ Lead down two main paths…

Overview

1. Linear KF vs. Extended KF
   – EKF provides an easy mechanism to minimize error by self-linearizing the nonlinear system about the most recent state.
   – EKF is a full state estimator. However, at the core, the algorithm finds the small change in state relative to the previous (or nominal) state.
   – Capabilities of the KF are lost when the EKF obscures the linearized system and nominal state.

Can the small – or incremental – state and associated linear system be leveraged to better incorporate measurement data?

2. Linearization Errors
   – The EKF must still linearize the system about a known operating point.
   – When do these errors dominate measurement error?
   – When do strategies that use ‘relative-only’ state representations (where the Leader’s absolute position is assumed known) break down?

Outline

1. Incremental State Architecture
   – Methodology
   – Case Study
2. Example: 2 spacecraft using GPS (absolute and carrier-difference)
3. Numerical Analysis
   – Linearization and measurement error sensitivity
   – Absolute-Relative state coupling errors and observability
4. Conclusions and Future Work

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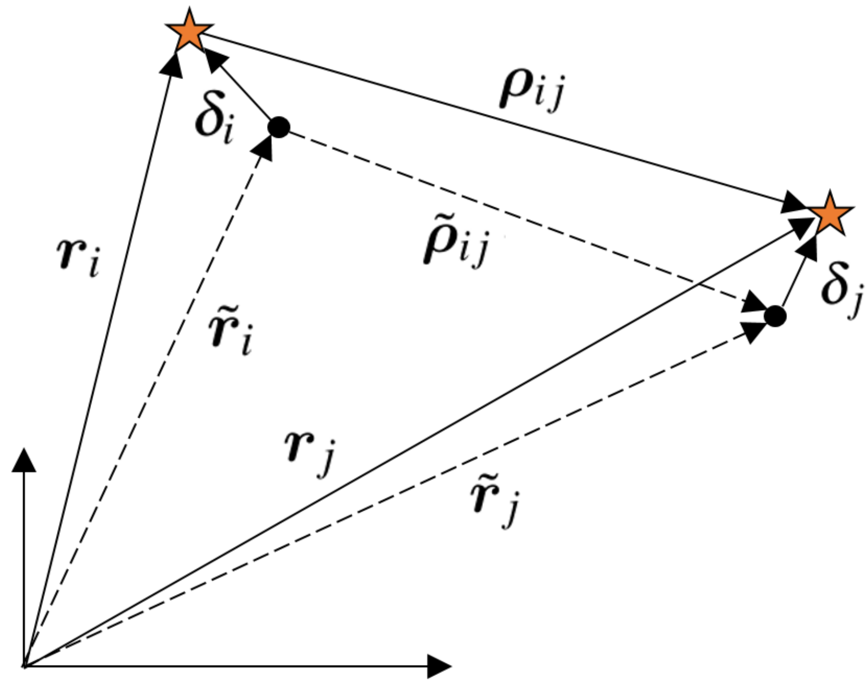
Motivating Concept

- Generalize the "re-linearization" concept used in most Extended Kalman Filters
- Consider small "incremental" states that relate a nominal to the truth
  - For EKFs, this nominal is the past estimate
  - Ideally very small, such that the state/measurement dynamics are linear
- Incremental description makes clear the points of linearization, allows the selection of "better" nominal
- Presents a standard way of selecting states, folding in measurements



Incremental States and Dynamics

- Estimate the small "incremental" states δ_i such that

$$r_i = \tilde{r}_i + \delta_i$$
$$r_j = \tilde{r}_j + \delta_j$$

- Consider the increments as representing *nearby relative orbits* obeying the same dynamics as the reference orbit ($\tilde{r}_i$):

$$x = \begin{bmatrix} \delta_i \\ \delta_j \end{bmatrix}; \quad \ddot{\delta}_i = -\frac{\mu}{\left|\left|\tilde{r}_i + \delta_i\right|\right|^3}(\tilde{r}_i + \delta_i) + \frac{\mu}{\left|\left|\tilde{r}_i\right|\right|^3}(\tilde{r}_i) = f(\delta_i, \tilde{r}_i)$$

Note: The relative dynamics between a spacecraft and its nominal trajectory ($\tilde{r}_i, \delta_i$) are the same as when the 2nd spacecraft is defined about a known leader (r_i, ρ_{ij})

- The partials of this expression with respect to the increment are:

$$\frac{\partial f}{\partial \delta_i} = \frac{\mu}{r^5}\left(3\tilde{r}_i\tilde{r}_i^{\,T} - \tilde{r}_i^{\,2} I\right) = \mathrm{F_{vp}}(\tilde{r}_i) \rightarrow F_i = \begin{bmatrix} 0 & I \\ F_{vp} & 0 \end{bmatrix}$$

Incremental Dynamics II

- When the dynamics of the absolute (incremental) state is used, the resulting matrix of partials for a two-spacecraft system of increments is:

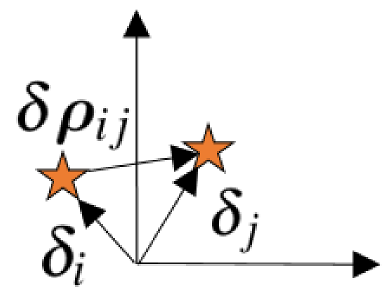
$$F = \begin{bmatrix} F_i(\tilde{r}_i) & 0 \\ 0 & F_j(\tilde{r}_j) \end{bmatrix}$$

- True dynamics are decoupled! But we care about the relative dynamics. Apply a similarity transform *T*:

$$T = \begin{bmatrix} I & 0 \\ -I & I \end{bmatrix}, T^{-1} = \begin{bmatrix} I & 0 \\ I & I \end{bmatrix}$$

$$x = \begin{bmatrix} \delta_i \\ \delta_j \end{bmatrix}; \;\; z = Tx = \begin{bmatrix} \delta_i \\ \rho_{ij} \end{bmatrix}$$

$$A = TFT^{-1} =, \begin{bmatrix} F_i(\tilde{r}_i) & 0 \\ F_j(\tilde{r}_j) - F_i(\tilde{r}_i) & F_j(\tilde{r}_j) \end{bmatrix}$$



- Results in absolute δ_i dynamics and coupled relative $\delta\rho_{ij}$ dynamics

In the incremental domain (small scale / linear) we can leverage the power of linear algebra to manipulate states and incorporate either absolute or relative measurements

Basic Case Study

- Consider a 2D system with simple dynamics and relative-only measurements:

$$x = \begin{matrix} x_1 \\ x_2 \end{matrix}, A = \begin{bmatrix} F_{11} & 0 \\ 0 & F_{12} \end{bmatrix}, H = \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}$$

- Covariance update from measurements due with an initial diagonal covariance:

$$P_k = HP_{k-1}H^T + R$$

$$P_k = \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} p & 0 \\ 0 & p \end{bmatrix} \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix} + R$$

$$P_k = \begin{bmatrix} 2p & 2p \\ 2p & 2p \end{bmatrix} + R$$

- If R is small, P_k can become singular, causing numerical issues during Kalman gain computation

Detailing this problem further to highlight the incremental state is the topic of a second paper in development

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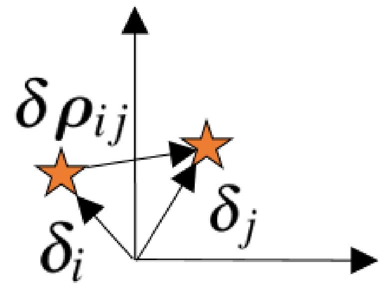
2 Spacecraft Example: CDGPS

- New state vector:

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{\delta}_i \\ \dot{\boldsymbol{\delta}}_i \\ \boldsymbol{\delta\rho}_{ij} \\ \dot{\boldsymbol{\delta\rho}}_{ij} \end{bmatrix}$$



- Restate the measurement model and its partials in terms of the new incremental state vector:

$$\phi_i^k = \left\| \boldsymbol{r}_{GPS}^k - (\tilde{\boldsymbol{r}}_i + \boldsymbol{\delta}_i) \right\| + w_\phi^k$$

$$\Delta\phi_{ij}^k = \left\| \boldsymbol{r}_{GPS}^k - (\tilde{\boldsymbol{r}}_i + \boldsymbol{\delta}_i) \right\| - \left\| \boldsymbol{r}_{GPS}^k - \left(\tilde{\boldsymbol{r}}_i + \boldsymbol{\delta}_i + \tilde{\boldsymbol{\rho}}_{ij} + \boldsymbol{\delta\rho}_{ij}\right) \right\| + \boldsymbol{w}_{\Delta\phi}^k$$

Measurement partials for relative states <u>couple into the absolute states</u>:

$$H_\phi = [LOS_i, 0_{N\times 9}]$$
$$H_{\Delta\phi} = [(LOS_i - LOS_j), 0_{N\times 3}, LOS_j, 0_{N\times 3}]$$

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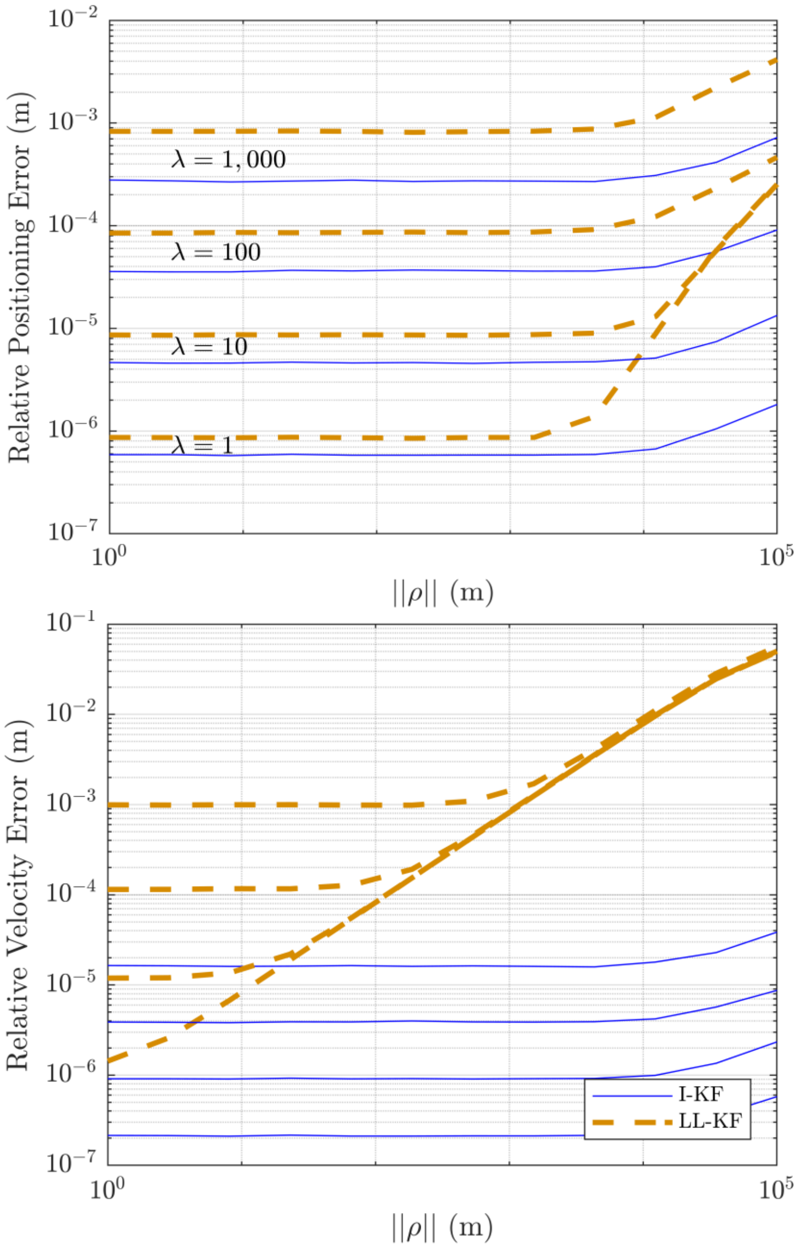
Initial Conditions + Assumptions

- LEO, eccentric orbits
- Filter dynamic, measurement models are used for truth
  - Unrealistic, but lets us compare “apples to apples”
  - Also ensures that we know which KF assumptions we’re violating
- Only examined leader-follower along-track formations
- Applicable to high precision applications with precise measurements available

Leader OE	Value
a	6,975 km
e	0.01
i	20 deg
ω	0 deg
Ω	20 deg
ν	0 deg

Covariance	Value
$R_{\Delta\phi}$	1e-6 m
R_{ϕ}	1e-3 m
Q_p	0 m
Q_v	0 m/s

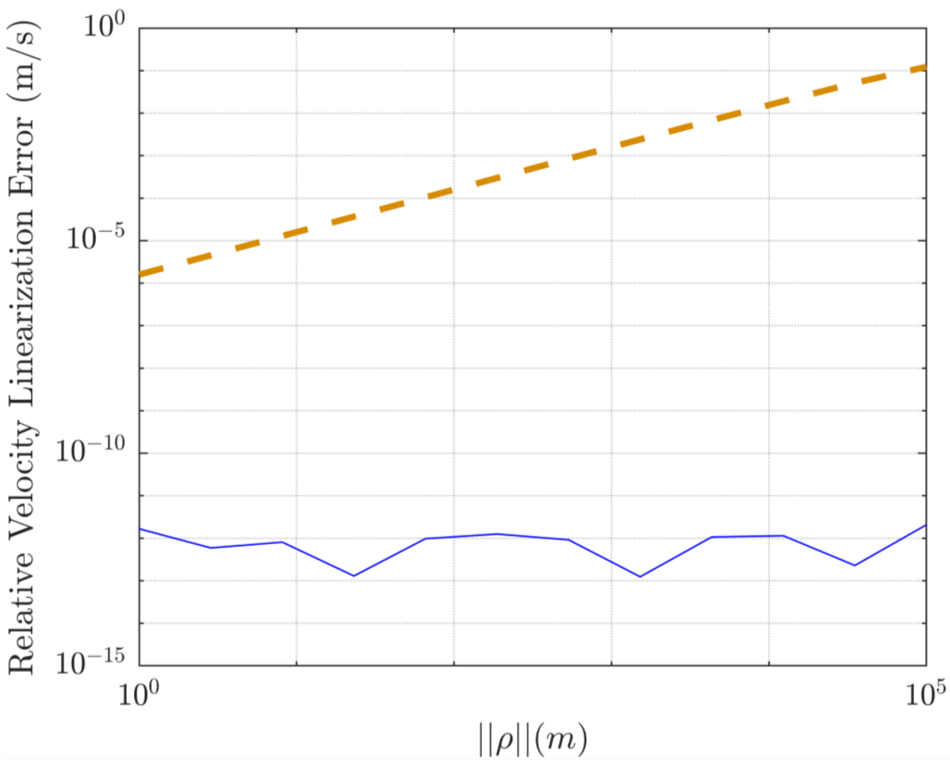
Linearization Sensitivity Analysis



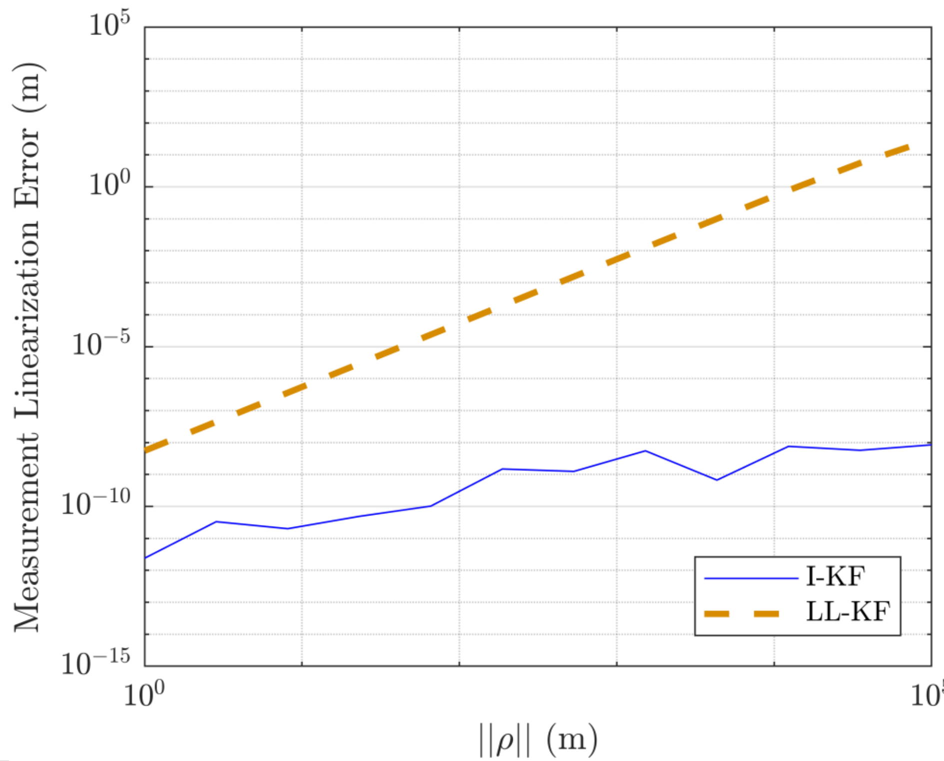
- Examine impact of linearization error on estimator accuracy versus measurement noise, formation baseline
- Scaled the measurement noise with λ:
$$R_{\phi}^{p} = \lambda R_{\phi}, R_{\Delta\phi}^{p} = \lambda R_{\Delta\phi}$$
- Linearized both filters about <u>true absolute positions</u>
- Results
  1. I-KF performance is nearly baseline-invariant
  2. I-KF is superior at inferring velocity from position measurements
  3. Results are most relevant w/ <u>precise measurements</u>

Linearization Sensitivity Analysis II

- Incremental linearization scheme is essentially invariant to the formation baseline
- Measurement model still degrades, but more slowly than LL case



Dynamic linearization error is insensitive to baseline!



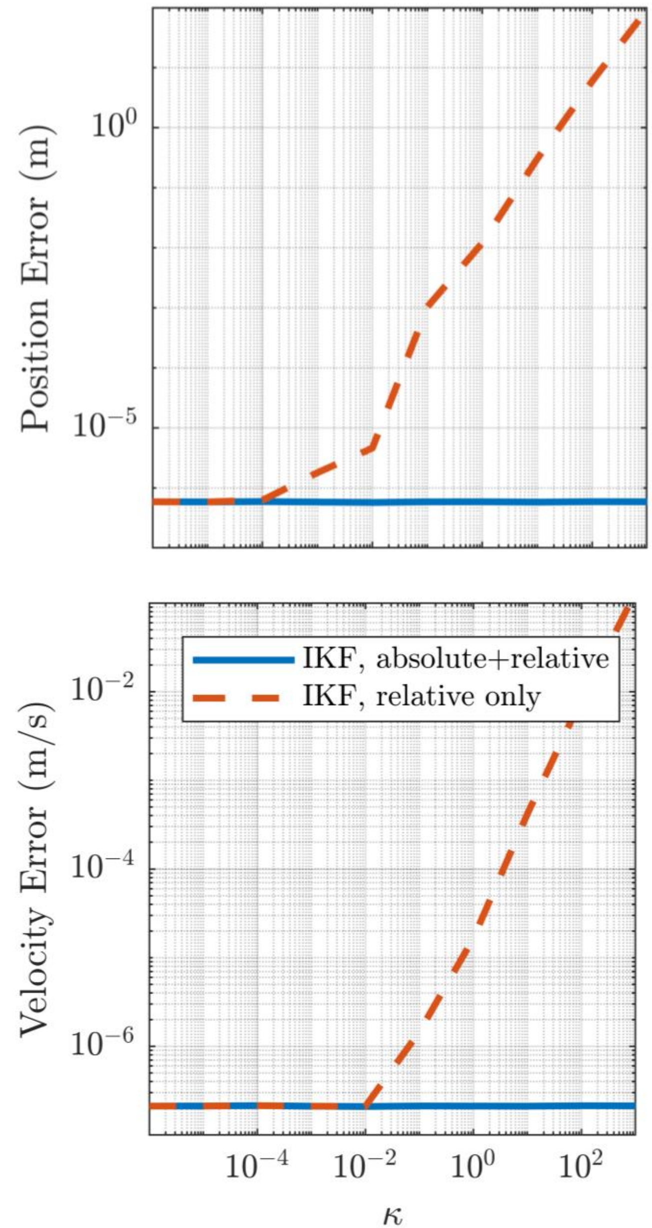
Measurement lin. Error scales more slowly w/ baseline

Nominal Error Sensitivity Analysis

- Desirable to understand the benefits of absolute+relative estimation
- Extremely high precision relative measurements ($R_{\Delta\phi} = 1 * 10^{-6}$m), coarse absolute measurements ($R_{\phi} = 1 * 10^{-3}$m)
- Add a scaled random vector to the “truth” to generate nominal position, velocity:

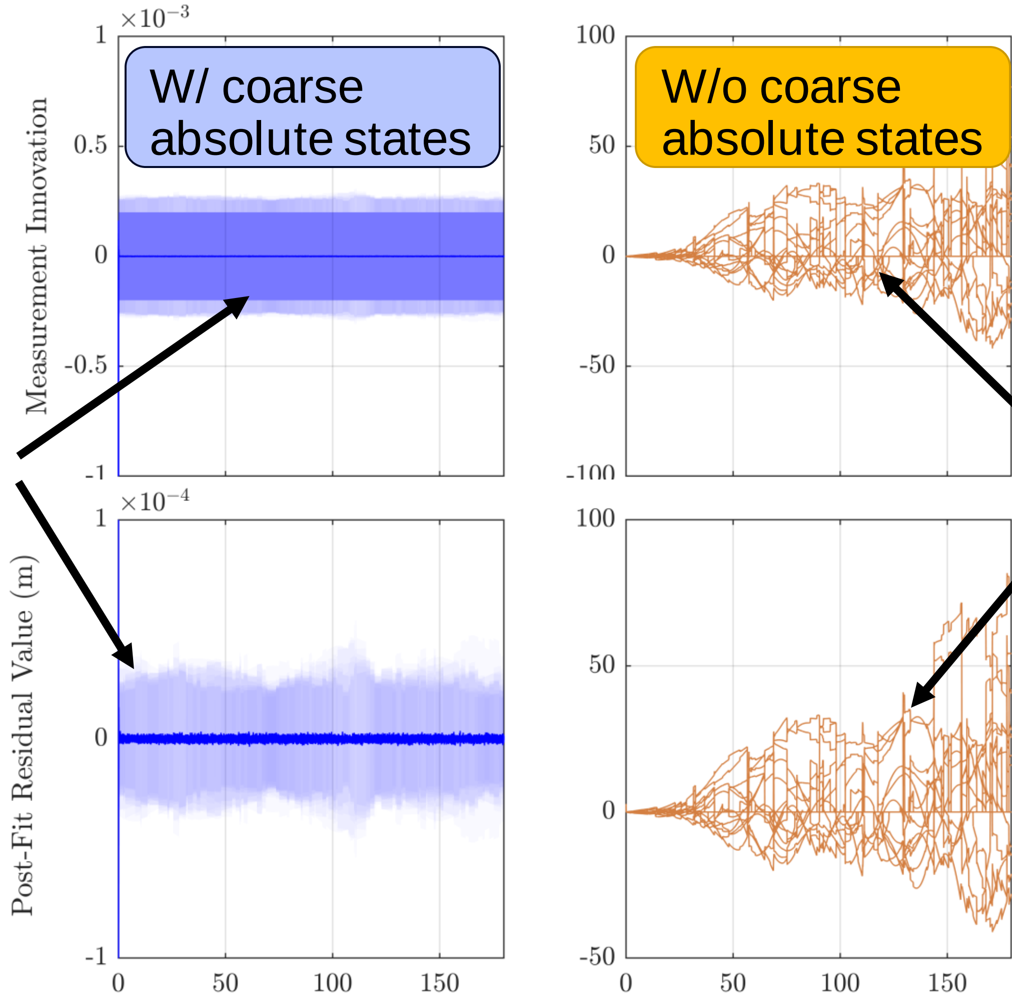
$$\tilde{\boldsymbol{r}}_0 = \boldsymbol{r}_0 + \kappa\widehat{\boldsymbol{w}}, \qquad \widetilde{\boldsymbol{v}}_0 = \boldsymbol{v}_0 + \frac{\kappa}{100}\widehat{\boldsymbol{\eta}}$$

- Comparable performance at small values of κ
- IKF w/o absolute states degrades log-linearly with disturbance magnitude



Nominal Error Sensitivity Analysis II

- Pre- and post-fit residuals show divergence due to dynamic coupling
- Could "fix" this by inflating process/measurement noise, but degraded accuracy is present regardless



Zero mean, well within 3-σ covariance bounds!

- Non-zero mean; bound are severely violated!
- Clearly shows influence of absolute/relative coupling

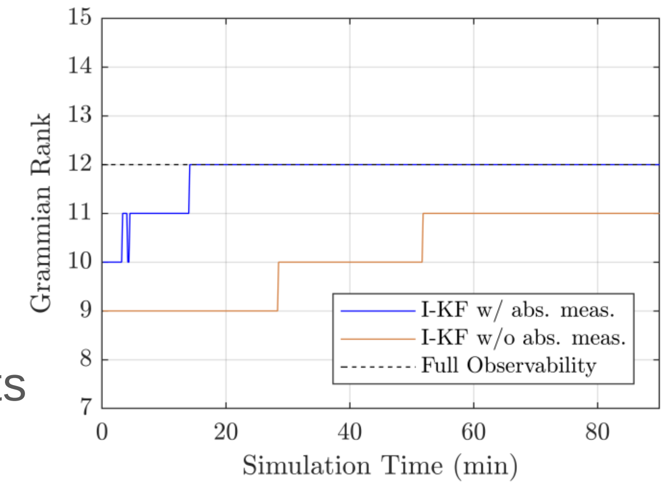
Observability Analysis

- Want to understand state observability given ONLY relative measurements. Use information matrix over the window of M points:

$$W_G = \sum_{i=M-k}^{M} \Phi(t_M, t_i)^T H_i^T R^{-1} H_i \Phi(t_M, t_i)$$

- Results:
  1. Absolute measurements are *required* for full observability
  2. Some observability in the absolute states is provided by relative-only measurements

- Partial observability from relative measurements is consistent w/ literature.
  - Full observability can be had if J2 dynamics are considered
  - Other sources of coupling in relative measurements can provide absolute information, such as inertial bearing from one spacecraft to another



Conclusions and Future Work

- Linearization error and absolute/relative coupling are important to consider for future navigation filters
- Good practice for all formation flyers, but especially missions that have very high-precision measurements available
- Need to examine filter performance with realistic disturbance and sensor models
  - Other biases might wash out performance gains
  - CDGPS errors from broadcast ephemeris can wash out other gains
  - Inter-spacecraft ranging, angles-based measurement models highly relevant
- Also need to look at performance with more than two spacecraft
  - Can play games with similarity transforms while incorporating absolute or relative measurements
  - Can these be “optimally” sequenced?

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Questions?

BACKUP SLIDES

Prior Work (LL-KF)

- Carrier-Differential GPS filter for on-board use from Alfriend et. al.
- Meter-level positioning for a two-craft formation
- Assumed state, dynamics:

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{\rho}_{ij} \\ \dot{\boldsymbol{\rho}}_{ij} \end{bmatrix}, \ddot{\boldsymbol{\rho}}_{\mathrm{ij}} = -\frac{\mu}{\left\|\tilde{\boldsymbol{r}}_i + \boldsymbol{\rho}_{ij}\right\|^3}\left(\tilde{\boldsymbol{r}}_i + \boldsymbol{\rho}_{ij}\right) + \frac{\mu}{\|\tilde{\boldsymbol{r}}_i\|^3}(\tilde{\boldsymbol{r}}_i)$$

- CDGPS measurement model from the k^{th} GPS satellite:

$$\Delta\phi_{ij}^k = \left\|\boldsymbol{r}_{GPS}^k - \tilde{\boldsymbol{r}}_i\right\| - \left\|\boldsymbol{r}_{GPS}^k - \left(\tilde{\boldsymbol{r}}_i + \boldsymbol{\rho}_{ij}\right)\right\| + \boldsymbol{w}_{\Delta\phi}^k$$

Issue #1: Dynamics and measurement model depend on *absolute position*, which is not estimated by the filter!

Issue #2: Linearizes follower motion about the leader, degrading estimate accuracy over *large separation distances*