Jet Propulsion Laboratory
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Joint Estimation of Starlight and Exoplanet Signals

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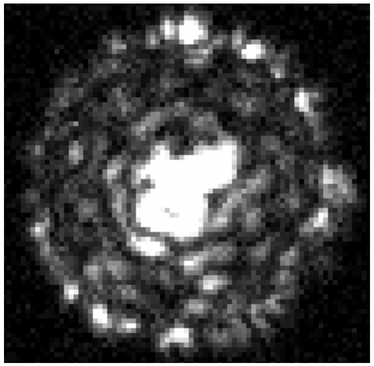
The decision to implement the WFIRST mission will not be finalized until NASA's completion of the National Environmental Policy Act (NEPA) process. This document is being made available for information purposes only.

Outline

1. Background
2. Comparison of Signal Extraction Methods
3. Wavefront Correction Differential Imaging (WCDI)
4. WCDI Lab Demo
5. WCDI Simulation for WFIRST CGI
6. Next Steps

What's in an Image?

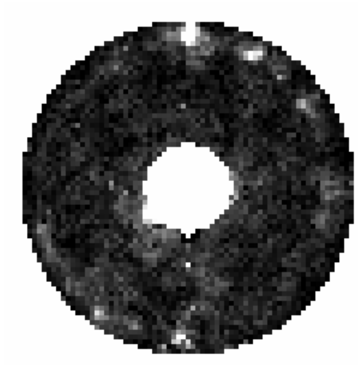
measured intensity



(JPL HCIT lab image)

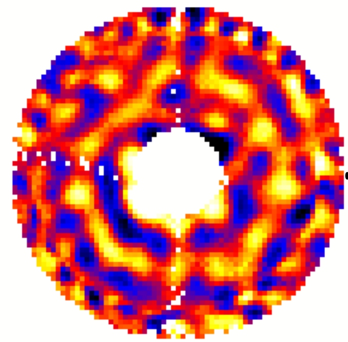
=

Incoherent Light
(exoplanets, disks, background)



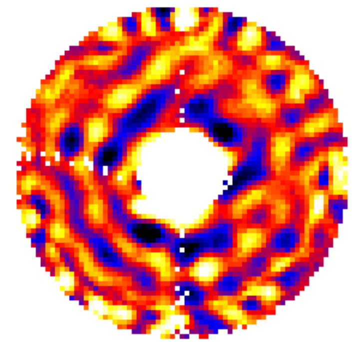
+ |

Starlight:
Real{E}



$+i$

Starlight:
Imaginary{E}



|^2

Image Credit:
Brian Kern & Eric Cady

2 Estimation Problems:

1) **Science**: How to extract exoplanets & disk signals?

2) **Engineering**: How to estimate stellar E-field (to then control it).

It can be the same question!

→ Coherent Differential Imaging (CDI)

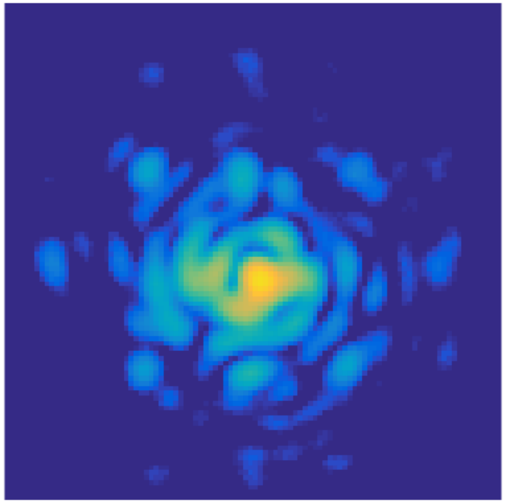
Stellar E-field Estimation

Why aren't we using CDI already?

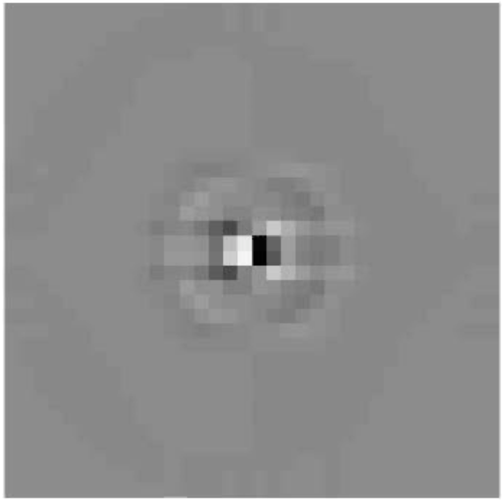
For control, to estimate stellar E-field from intensity image:

We use **phase diversity** with DMs:

Initial PSF

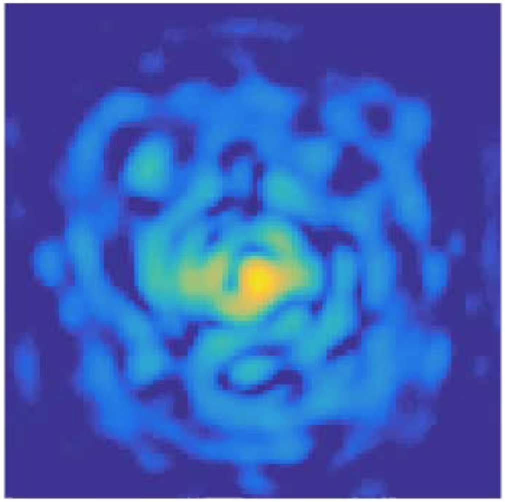


ΔDM Voltages for Probe 1



(10 nm P-V surface)

PSF for Probe 1



But more light means…

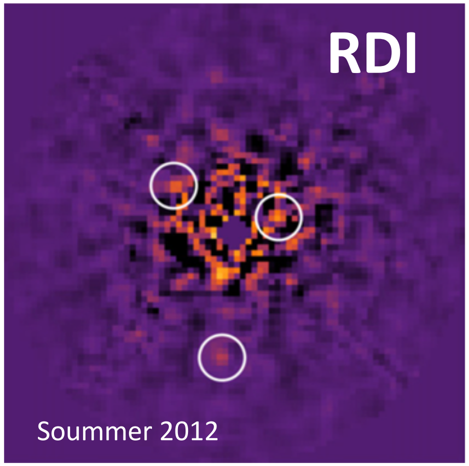
➢ More shot noise

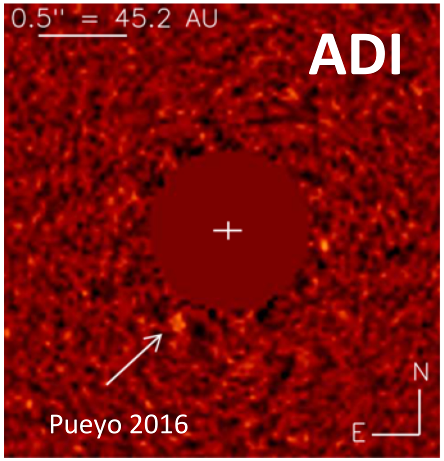
[STScI HiCAT data]

Noise Comparison for Differential Imaging

Method of Differential Imaging	How It Works
Reference (**RDI**)	Subtract off starlight template built from PSF library.
Angular (**ADI**)	Roll telescope/sky. Subtract non-rotating stellar speckles.

- RDI and ADI are more efficient if we are shot noise limited.





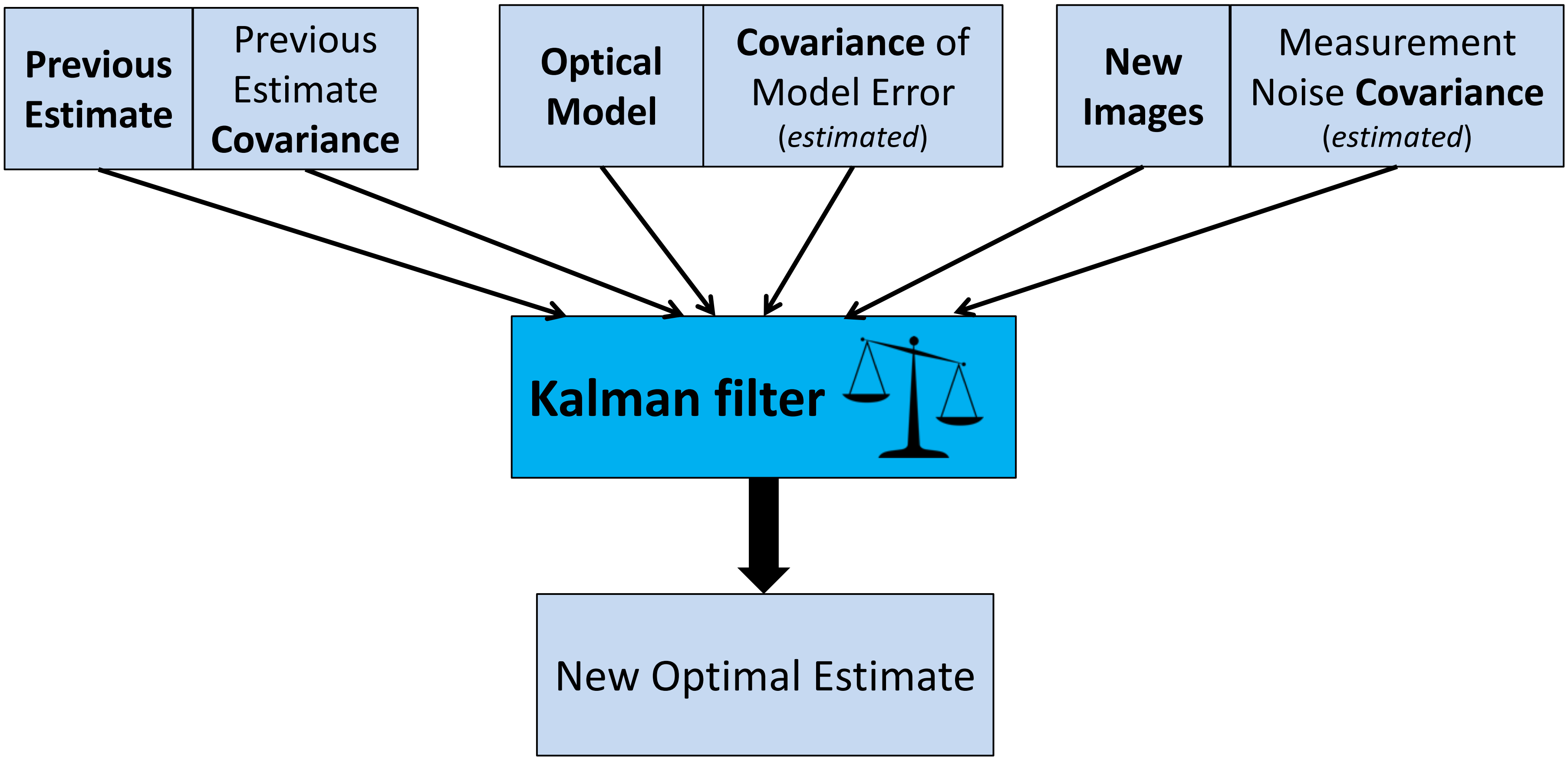
- *But we aren't!* We are speckle stability limited.

Solution: **Wavefront Correction Differential Imaging (WCDI):**

- Modulate ***and suppress*** starlight while ***estimating science targets*** and starlight.
- How?

- **Kalman filtering**

Kalman Filtering



- Provides **faster correction**
- Uses **all** prior **information**
- Optimally* **filters out noise**

Kalman JBE 1960
Groff & Kasdin JOSA-A 2013
Riggs et al. JATIS 2016

*optimal for Gaussian noise and linear processes

See also Sun et al., “Identification of the focal plane wavefront control system using E-M algorithm” *Proc. SPIE*, 2017.

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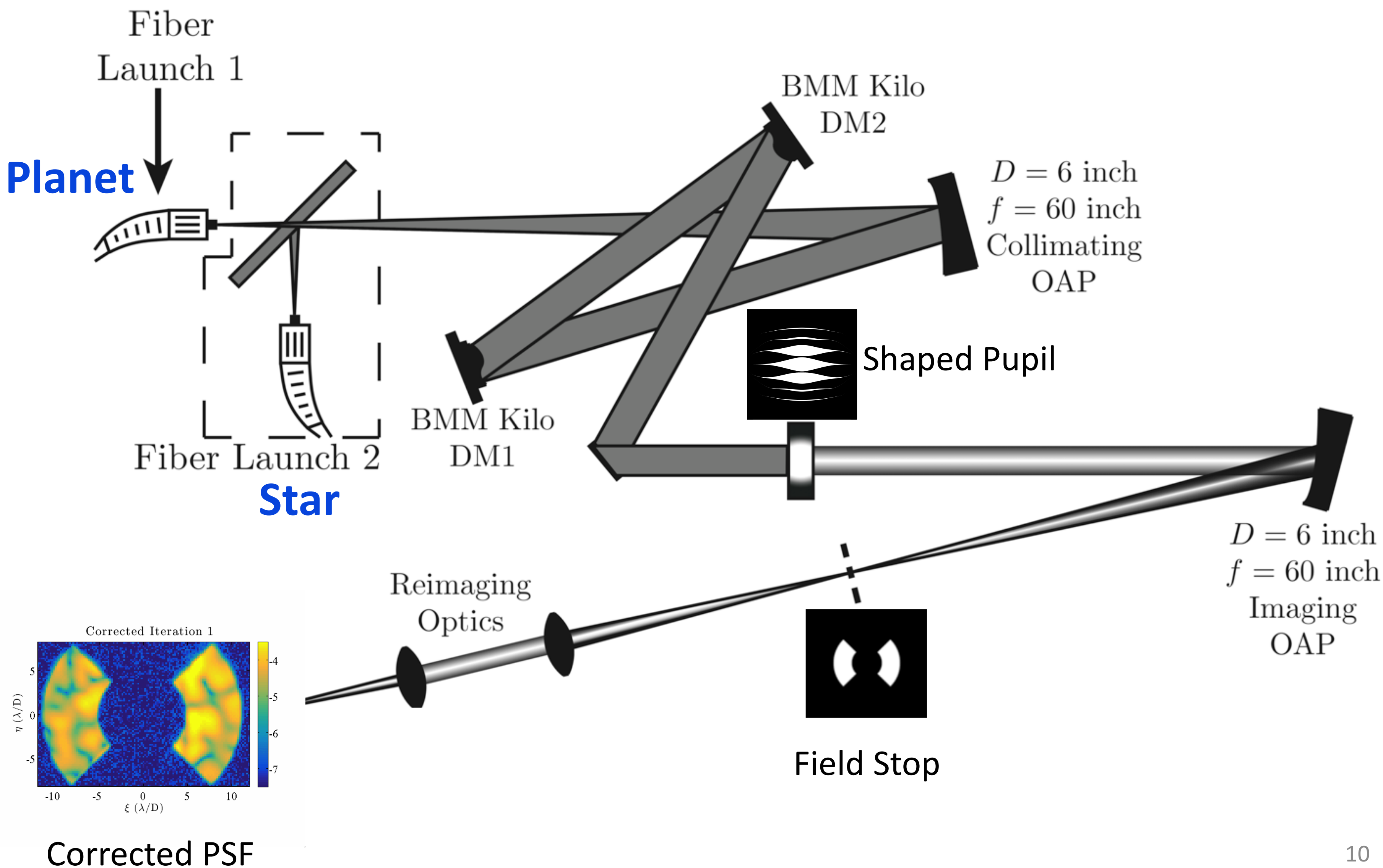
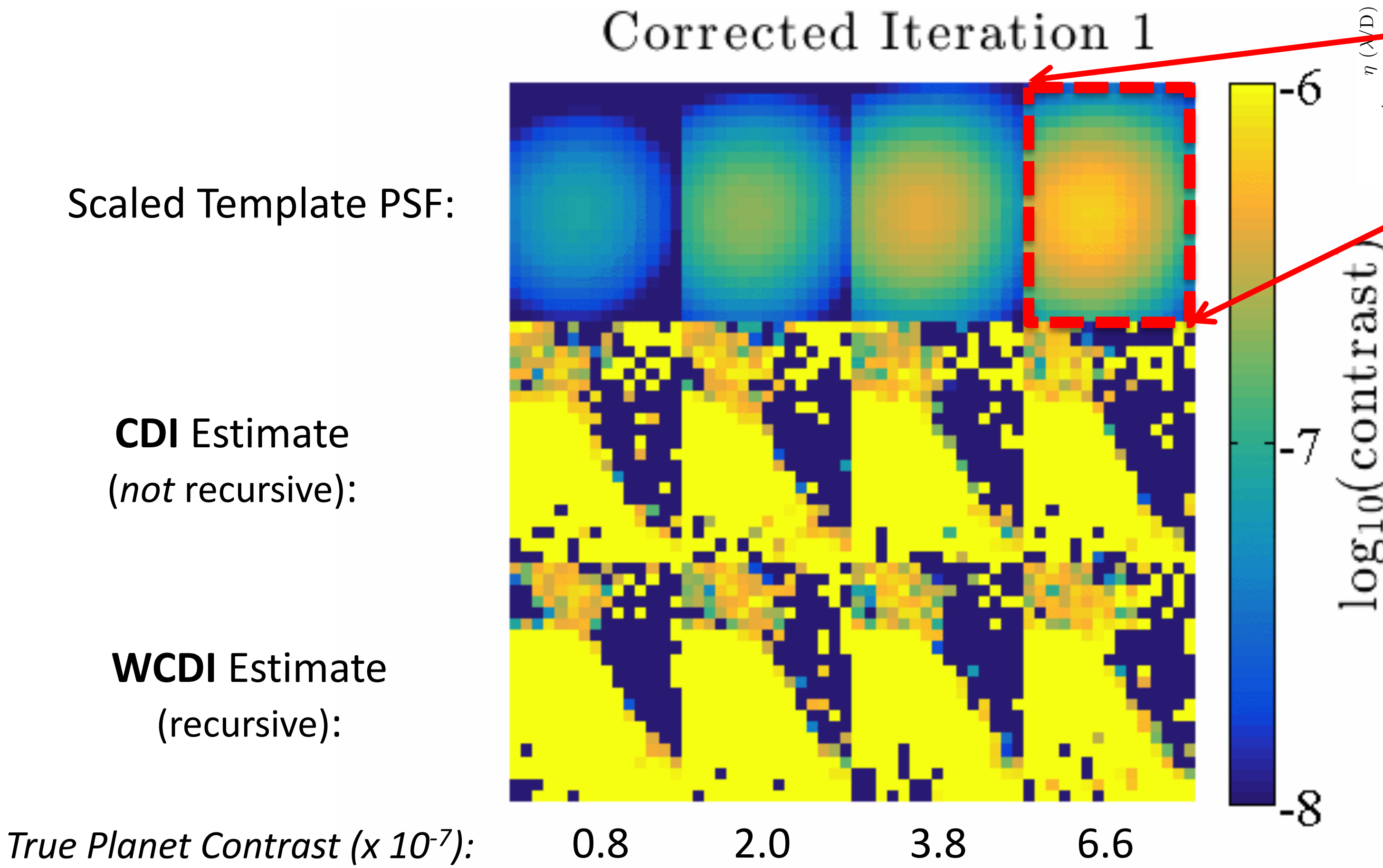
Planet Extraction in Princeton's HCIL

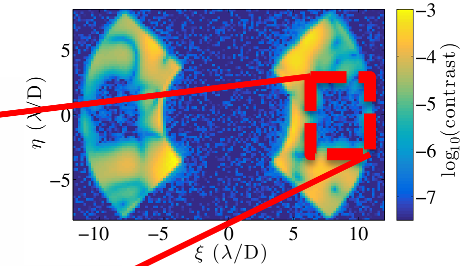
Faint pseudo-planet injected into testbed



CDI and WCDI in Princeton's HCIL

- Planet-like signal injected *into the testbed* with laser
- 4 trials at different planet contrasts





Planet is in right-side dark hole

Riggs et al., JATIS 2016

➢ **Planet is found using wavefront correction images!**

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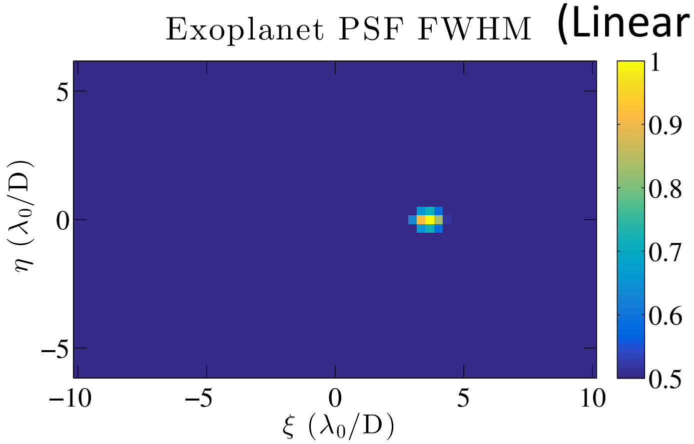
Statistical Analysis of WCDI in Simulation

- **Monte Carlo** WFSC simulations:
- Simple, static optical model of CGI's SPC
  - Photon shot noise only

- **100 trials** with & without faint planet
- **Low flux**: 1 photon/image/pixel (at planet peak)
- Compare **detection statistics**.

(Linear Scale)



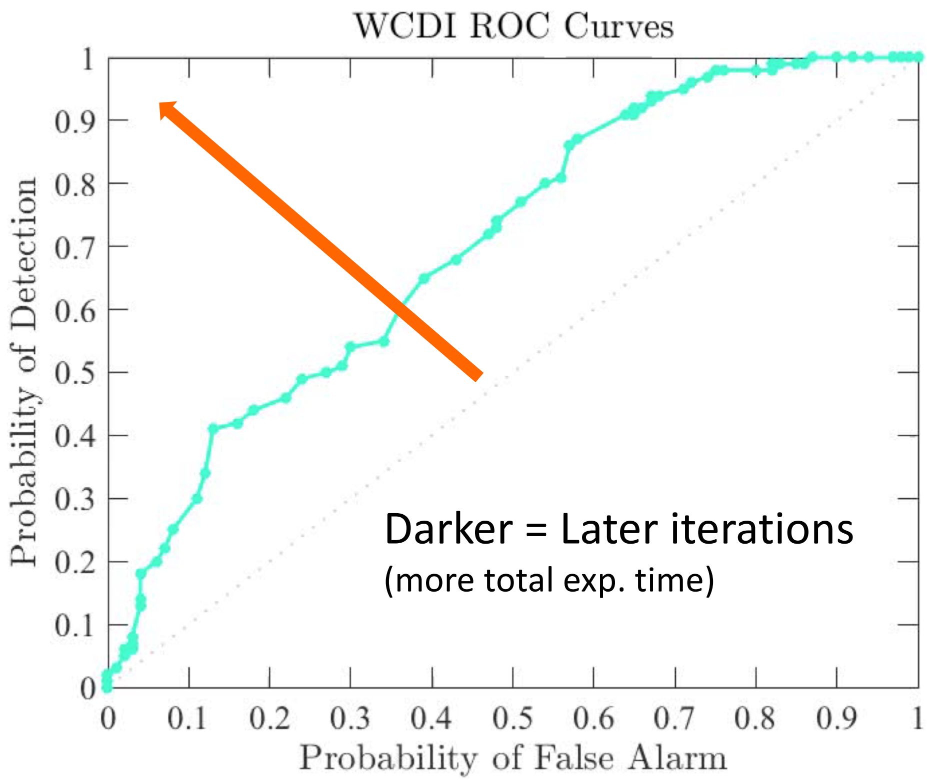
- 11 pixels within FWHM

ROC and AUC Curves

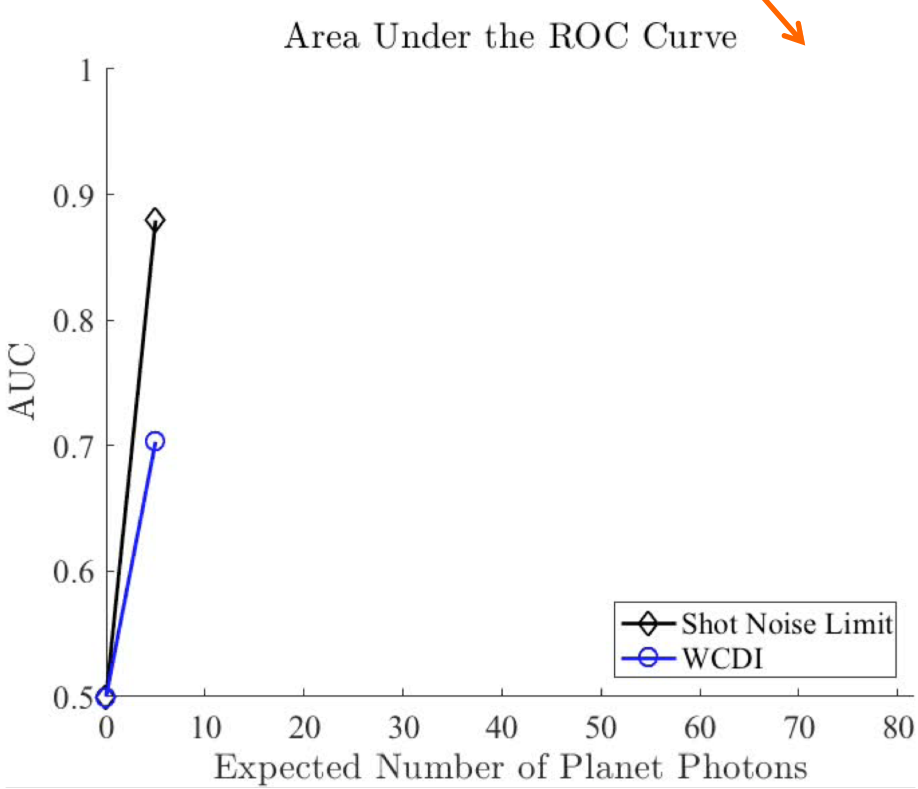
Case with 3e-10 Contrast Exoplanet

(~3x below residual starlight)

Receiver Operator Characteristic (ROC) Curve:



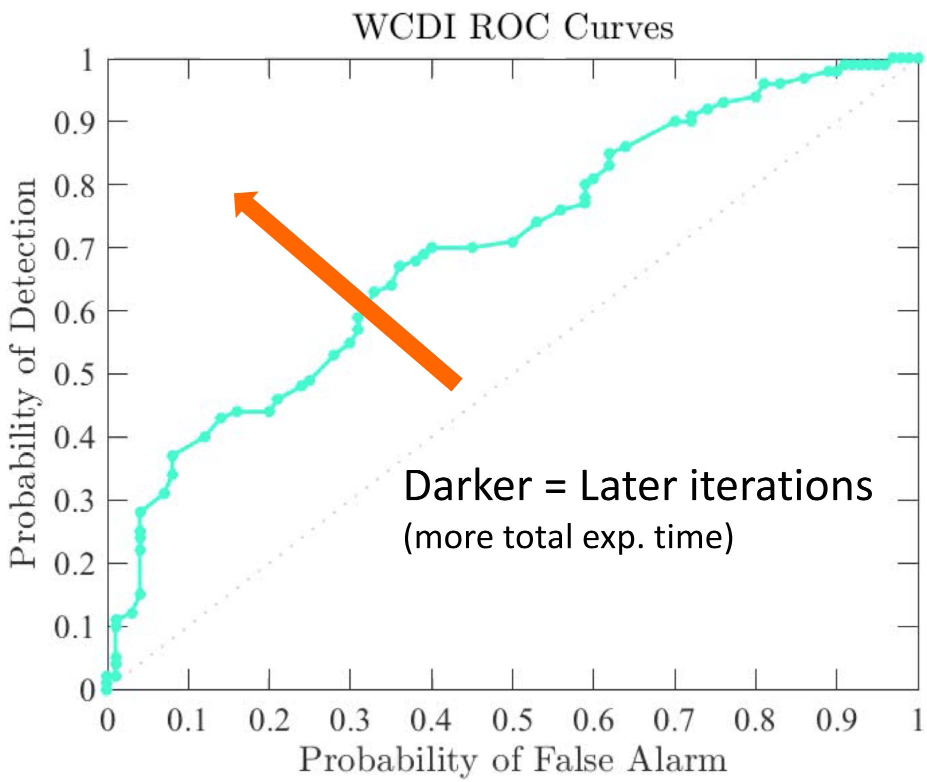
AUC=1 means perfect classification of signals

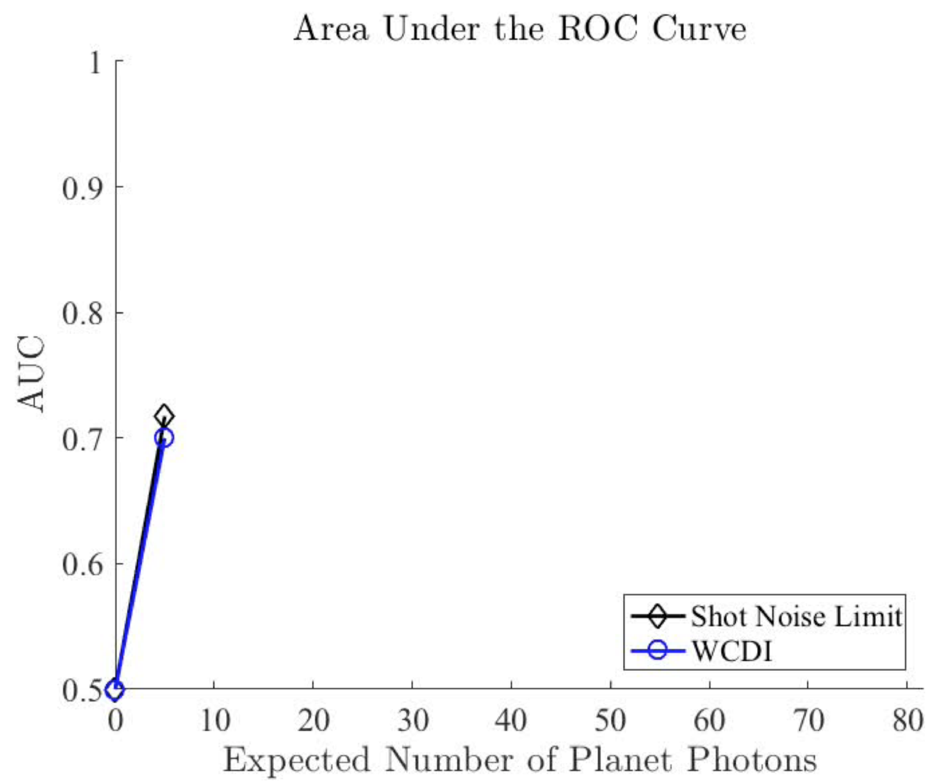


ROC and AUC Curves: Case 2

Case with 1e-10 Contrast Exoplanet

(~10x below residual starlight)





WCDI: Next Steps

Next Steps for WCDI

- For WFIRST CGI: Compare performance directly to chopping schemes with ADI and RDI.

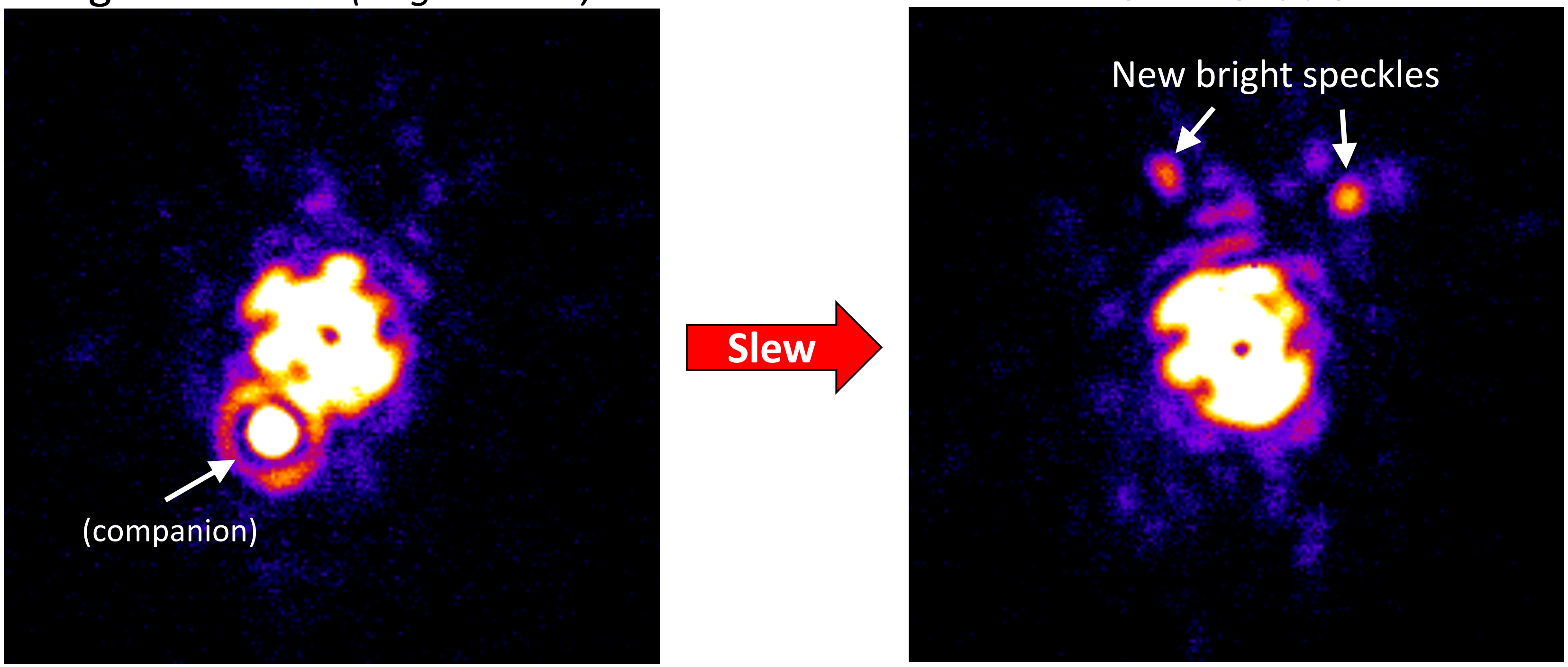
- Simulate performance of WCDI with ground and future space telescopes.

Dynamic Speckles at Keck NIRC2

Pointing angle changes the primary mirror segment alignment

→ Speckles appear!

High Elevation *(Regular PSF)* | Low Elevation

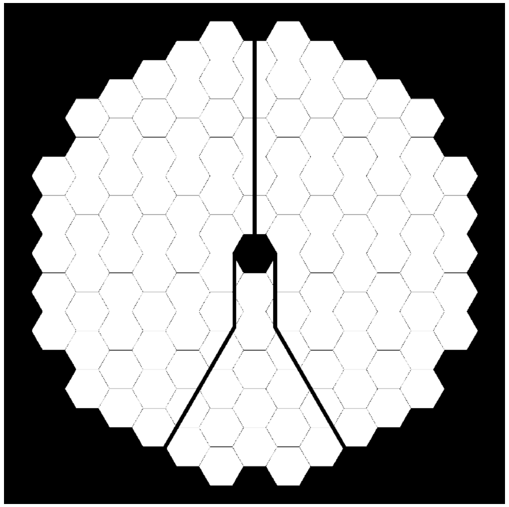


NIRC2 Images from Garreth Ruane

- Need **on-sky** WFSC to **suppress new speckles** from **slewing** or **thermal drift**
  - True for ground- and space-based segmented telescopes *(e.g., **LUVOIR**)*
  - ➢ **Use WCDI as alternative to RDI and ADI when limited by speckle stability.**

Summary

- **<u>Wavefront Correction Differential Imaging (WCDI)</u>**
  - Enables science during wavefront correction
    - Can improve WFIRST CGI science if slews/rolls affect contrast
  - Possible game-changer for ground- and space-based imaging, especially for segmented apertures



Backup Slides

Wavefront Correction: AO

Correct phase aberrations from atmospheric turbulence and imperfect optical surfaces

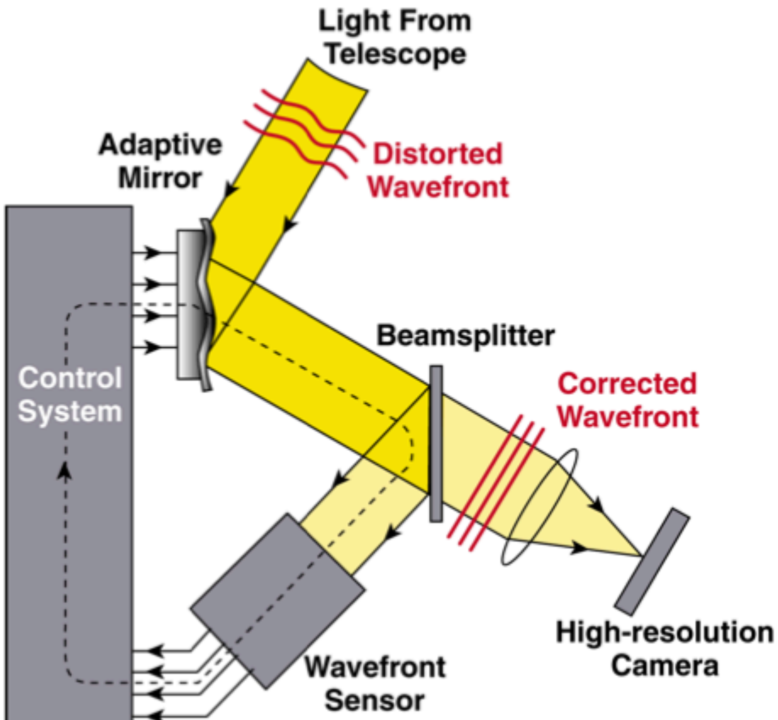


Image Credit: CfAO Summer School

Adaptive Optics (AO):

1. Measure phase errors with wavefront sensor (WFS)
2. Apply opposite shape on DM

Main issues for high-contrast imaging:

- Aberrations after WFS not sensed and corrected
- AO corrects only phase errors
  - Can reach only $\approx 10^{-5}$ contrast

Kalman Filter (KF)

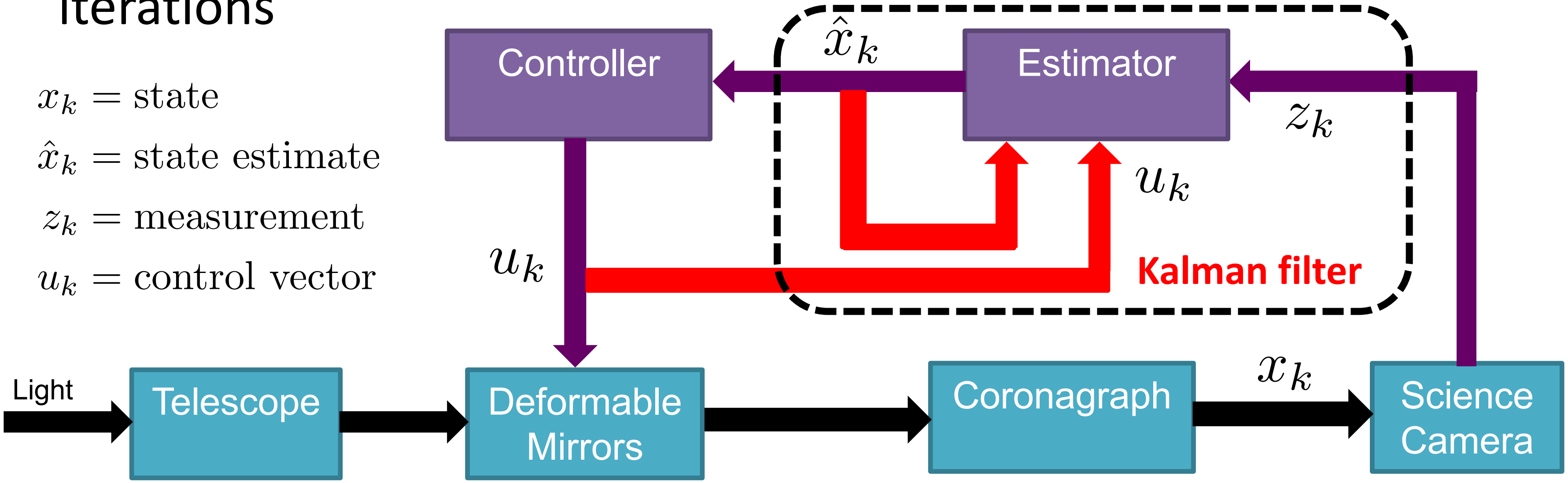
- BPE ignores previous estimates
- KF optimally combines previous data with new measurements
- KF essentially averages out noise over many correction iterations

$x_k = \text{state}$

$\hat{x}_k = \text{state estimate}$

$z_k = \text{measurement}$

$u_k = \text{control vector}$



Kalman Filter Equations

$$\hat{x}_k(-) = \hat{x}(+)_{k-1} + \Gamma u_{k-1}$$
$$P_k(-) = P_{k-1}(+) + Q_{k-1}$$

Model-based updates of state x & state covariance P

$$K_k = P_k(-)H_k^T[H_k P_k(-) H_k^T + R_k]^{-1}$$

Kalman gain: Balances model error and measurement error

$$\hat{x}_k(+) = \hat{x}_k(-) + K_k[z_k - H_k \hat{x}_k(-)]$$
$$P_k(+) = [\mathbb{I} - K_k H_k] P_k(-)$$

Measurement-based updates of x & P

Q and R are tuning values

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Pair-wise Probing

- Subtract +/- probed images to isolate cross term between probe signal and unknown stellar E-field

Give'on+ 2007

$$\underbrace{\begin{bmatrix} \Delta I_{k,1} \\ \vdots \\ \Delta I_{k,N_{pp}} \end{bmatrix}}_{= z_k} = 4 \underbrace{\begin{bmatrix} \mathcal{R}\{p_{k,1}\} & \mathcal{I}\{p_{k,1}\} \\ \vdots & \vdots \\ \mathcal{R}\{p_{k,N_{pp}}\} & \mathcal{I}\{p_{k,N_{pp}}\} \end{bmatrix}}_{= H_k} \underbrace{\begin{bmatrix} \mathcal{R}\{E_k\} \\ \mathcal{I}\{E_k\} \end{bmatrix}}_{= x_k} + \begin{bmatrix} n_{k,1} \\ \vdots \\ n_{k,N_{pp}} \end{bmatrix}$$

Measured — Model-based — Unknown

$$k = \text{Correction iteration \#}$$
$$j = \text{Probe \#}$$
$$\mathbf{p_{k,j}} = \mathbf{G_k u_j} = \text{probe field at camera}$$
$$I_{k,j} = \text{Measured intensity}$$
$$I_{inco} = \text{Incoherent intensity}$$
$$n_{k,j\pm} = \text{Measurement noise: shot, readout, dark current}$$

$$z_k = H_k x_k + n_k$$

Linear Least Squares Starlight Estimate:

Batch Process Estimator (BPE)

$$\boxed{\hat{x}_k = (H_k^T H_k)^{-1} H_k^T z_k}$$

<u>Incoherent estimate:</u>

$$\hat{I}_{inco} = I_{meas} - |\hat{E}_{star}|^2$$

- ***WFS gives us the incoherent signal for free***
  - Coherent differential imaging (**CDI**)
  - Real-time image processing
- ***Exoplanets are in the incoherent signal!***

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Pair-wise Probing

$$k = \text{Correction iteration \#}$$
$$j = \text{Probe \#}$$
$$\mathbf{p_{k,j}} = \mathbf{G_k u_j} = \text{probe field at camera}$$
$$I_{k,j} = \text{Measured intensity}$$
$$I_{inco} = \text{Incoherent intensity}$$
$$n_{k,j\pm} = \text{Measurement noise: shot, readout, dark current}$$

- Estimate light at each pixel separately
- Take images for +/- probe shapes on DM:

$$I_{k,j\pm} = |E_k \pm p_{k,j}|^2 + I_{inco,k} + n_{k,j\pm}$$
$$= |E_k|^2 + |p_{k,j}|^2 \pm 2\mathcal{R}\{E_k^* p_{k,j}\} + I_{inco,k} + n_{k,j\pm}$$

- Subtract +/- probed images to isolate cross term (heterodyne gain)

$$\Delta I_{k,j} = I_{k,j+} - I_{k,j-} = 4\mathcal{R}\{E_k^* p_{k,j}\} + n_{k,j}$$
$$= 4\begin{bmatrix}\mathcal{R}\{p_{k,j}\} & \mathcal{I}\{p_{k,j}\}\end{bmatrix}\begin{bmatrix}\mathcal{R}\{E_k\} \\ \mathcal{I}\{E_k\}\end{bmatrix} + \begin{bmatrix}n_{k,j}\end{bmatrix}$$

At least 2 probes (since 2 unknowns)

$$\underbrace{\begin{bmatrix}\Delta I_{k,1} \\ \vdots \\ \Delta I_{k,N_{pp}}\end{bmatrix}}_{= z_k} = 4\underbrace{\begin{bmatrix}\mathcal{R}\{p_{k,1}\} & \mathcal{I}\{p_{k,1}\} \\ \vdots & \vdots \\ \mathcal{R}\{p_{k,N_{pp}}\} & \mathcal{I}\{p_{k,N_{pp}}\}\end{bmatrix}}_{= H_k}\underbrace{\begin{bmatrix}\mathcal{R}\{E_k\} \\ \mathcal{I}\{E_k\}\end{bmatrix}}_{= x_k} + \begin{bmatrix}n_{k,1} \\ \vdots \\ n_{k,N_{pp}}\end{bmatrix}$$

Measured   Model-based   Unknown

Give'on+ 2007

$$= \quad z_k = H_k x_k + n_k$$

Least Squares Estimate:
Batch Process Estimator (BPE)

$$\boxed{\hat{x}_k = (H_k^T H_k)^{-1} H_k^T z_k}$$

Pair-wise Probing Error Analysis

- Pair-wise probing is efficient
  - Brighter probes → higher **homodyne gain** → approaches *fundamental shot noise limit*

Noise Equivalent Contrast (NEC) = contrast resolution level from estimation

$$\mathrm{NEC} = \underbrace{\frac{1}{F_{\mathrm{pk}} t_{\mathrm{tot}}}}_{\text{Fundamental shot noise limit}} \left(1 + \underbrace{\frac{Z + D_c + N_{exp}\sigma^2_{\mathrm{ron}}}{p^2}}_{\text{Measurement noise over probe intensity}} \right)$$

$$\begin{aligned} F_{pk} &= \text{Stellar flux} \\ t_{tot} &= \text{Total exposure time for probed images} \\ p^2 &= \text{Probe intensity} \\ N_{exp} &= \text{\# of exposures per image} \\ \sigma^2_{\mathrm{ron}} &= \text{Read noise variance} \\ Z &= \text{Background light} \\ D_C &= \text{Dark current signal} \end{aligned}$$

Example: For $p^2 >> E^2$, if expose long enough to get (on average) 1 photon at 10^{-8} contrast, you can estimate down to 10^{-8} contrast.

- Estimate accuracy set by:
  - Nonlinearities
  - Model error (of DM & optical system)

Groff, Riggs, et al. 2015

The Kalman Filter (KF)

- BPE ignores previous estimates
  - KF optimally combines previous estimate with new measurements using models of system and noise
  - Provides **faster correction** and **more robustness** to measurement noise

Kalman Filter Equations (per pixel)

$$\hat{x}_k(-) = \hat{x}(+)_{k-1} + \Gamma u_{k-1}$$
$$P_k(-) = P_{k-1}(+) + Q_{k-1}$$

Model-based updates of state x & state covariance P

$$K_k = P_k(-)H_k^T[H_k P_k(-) H_k^T + R_k]^{-1}$$

Kalman gain: Balances model and measurement error

$$\hat{x}_k(+) = \hat{x}_k(-) + K_k[z_k - H_k \hat{x}_k(-)]$$
$$P_k(+) = [\mathbb{I} - K_k H_k] P_k(-)$$

Measurement-based updates of x & P

Groff & Kasdin 2013

Incoherent estimate is still not recursive:

$$\hat{I}_{inco,k} = I_k - |\hat{E}_k|^2$$

Unprobed image

Starlight estimate

Exoplanets are in the incoherent signal!

Kalman Filter (KF)

- BPE ignores previous estimates
- KF optimally combines previous data with new measurements
- Enables faster correction and robustness to measurement noise

Kalman Filter Equations (per pixel)

$$\left.\begin{aligned}\hat{x}_k(-) &= \hat{x}(+)_{k-1} + \Gamma u_{k-1} \\ P_k(-) &= P_{k-1}(+) + Q_{k-1}\end{aligned}\right\}$$

Model-based updates of state x & state covariance P

$$\left.K_k = P_k(-)H_k^T[H_k P_k(-) H_k^T + R_k]^{-1}\right\}$$

Kalman gain: Balances model and measurement error

$$\left.\begin{aligned}\hat{x}_k(+) &= \hat{x}_k(-) + K_k[z_k - H_k\hat{x}_k(-)] \\ P_k(+) &= [\mathbb{I} - K_k H_k]P_k(-)\end{aligned}\right\}$$

Measurement-based updates of x & P

Groff & Kasdin 2013

Matrix	Representation	Dimension
Linearized State Response	$\Phi = \mathbb{I}$	2×2
Linear Observation	H_k	$N_{pp} \times 2$
Linearized Complex Response of Probing DM	G	$1 \times N_{act}$
Linearized Response of Probing DM	$\Gamma = \begin{bmatrix} \mathcal{R}\{G[1]\} \cdots \mathcal{R}\{G[N_{act}]\} \\ \mathcal{I}\{G[1]\} \cdots \mathcal{I}\{G[N_{act}]\} \end{bmatrix}$	$2 \times N_{act}$
Disturbance Response	$\Lambda = \Gamma$	$2 \times N_{act}$
State Covariance (Time Update)	$P_k(-) = E[(x_k - \hat{x}_k(-))(x_k - \hat{x}_k(-))^T]$	2×2
State Covariance (Measurement Update)	$P_k(+) = E[(x_k - \hat{x}_k(+))(x_k - \hat{x}_k(+))^T]$	2×2
Process Noise	$Q_k = \Lambda E[w_k w_k^T]\Lambda^T$	2×2
Sensor Noise	$R_k = E[n_k n_k^T]$	$N_{pp} \times N_{pp}$
Kalman Gain	K_k	$2 \times N_{pp}$

Incoherent estimate is not recursive:

$$\hat{I}_{inco,k} = \underbrace{I_k}_{\text{Unprobed image}} - \underbrace{|\hat{E}_k|^2}_{\text{Starlight estimate}}$$

Exoplanets are in the incoherent signal

EKF Equations

Measurement Vector:

$$z_k = \begin{bmatrix} I_k & I_{k,1+} & I_{k,1-} & \cdots & I_{k,N_{pp}+} & I_{k,N_{pp}-} \end{bmatrix}^T$$
$$= h(x_k) + n_k$$

Riggs et al. 2016

Quadratic Measurement Function:

$$h(x_k) = \begin{bmatrix} |E_k|^2 + I_{inco,k} \\ |E_{k,1+}|^2 + I_{inco,k} \\ |E_{k,1-}|^2 + I_{inco,k} \\ \vdots \\ |E_{k,N_{pp}+}|^2 + I_{inco,k} \\ |E_{k,N_{pp}-}|^2 + I_{inco,k} \end{bmatrix} \approx \begin{bmatrix} |E_k|^2 + I_{inco,k} \\ |E_k + Gu_1|^2 + I_{inco,k} \\ |E_k - Gu_1|^2 + I_{inco,k} \\ \vdots \\ |E_k + Gu_{N_{pp}}|^2 + I_{inco,k} \\ |E_k - Gu_{N_{pp}}|^2 + I_{inco,k} \end{bmatrix}$$

Linearized Observation Matrix:

$$H_k = \left.\frac{\partial h(\hat{x}_k)}{\partial \hat{x}_k}\right|_{\hat{x}_k = \hat{x}_k(-)}$$

Extended Kalman Filter Equations

$$\hat{x}_k(-) = \hat{x}(+)_{k-1} + \Gamma u_{k-1}$$
$$P_k(-) = P_{k-1}(+) + Q_{k-1}$$
$$K_k = P_k(-) H_k^T [H_k P_k(-) H_k^T + R_k]^{-1}$$
$$\hat{x}_k(+) = \hat{x}_k(-) + K_k[z_k - \boxed{h(\hat{x}_k(-))}]$$
$$P_k(+) = [\mathbb{I} - K_k H_k] P_k(-)$$

- Nearly same form as KF's
- Different matrix definitions because of different x & z

Iterated Extended Kalman Filter (IEKF)

- **Problem**: EKF estimates known to be biased
- **Solution**: Iterating the EKF can reduce the bias error
  1. Run EKF
  2. Relinearize about new estimate
  3. Re-compute H & K.
  4. Re-compute x & P.
  5. Repeat steps 2-4 until estimates converge.

Iterated Extended Kalman Filter (IEKF) Equations

$$H_{k,i} = \left.\frac{\partial h(x)}{\partial x}\right|_{x=\hat{x}_{k,i}(+)}$$

$$K_{k,i} = P_k(-)H_{k,i}^T[H_{k,i}P_k(-)H_{k,i}^T + R_k]^{-1}$$

$$\hat{x}_{k,i+1}(+) = \hat{x}_k(-) + K_{k,i}\big(z_k - h(\hat{x}_{k,i}(+)) - H_{k,i}[\hat{x}_k(-) - \hat{x}_{k,i}(+)]\big)$$

$$P_{k,i+1}(+) = [\mathbb{I} - K_{k,i}H_{k,i}]P_k(-)$$

Princeton HCIL

High Contrast Imaging Laboratory (HCIL)

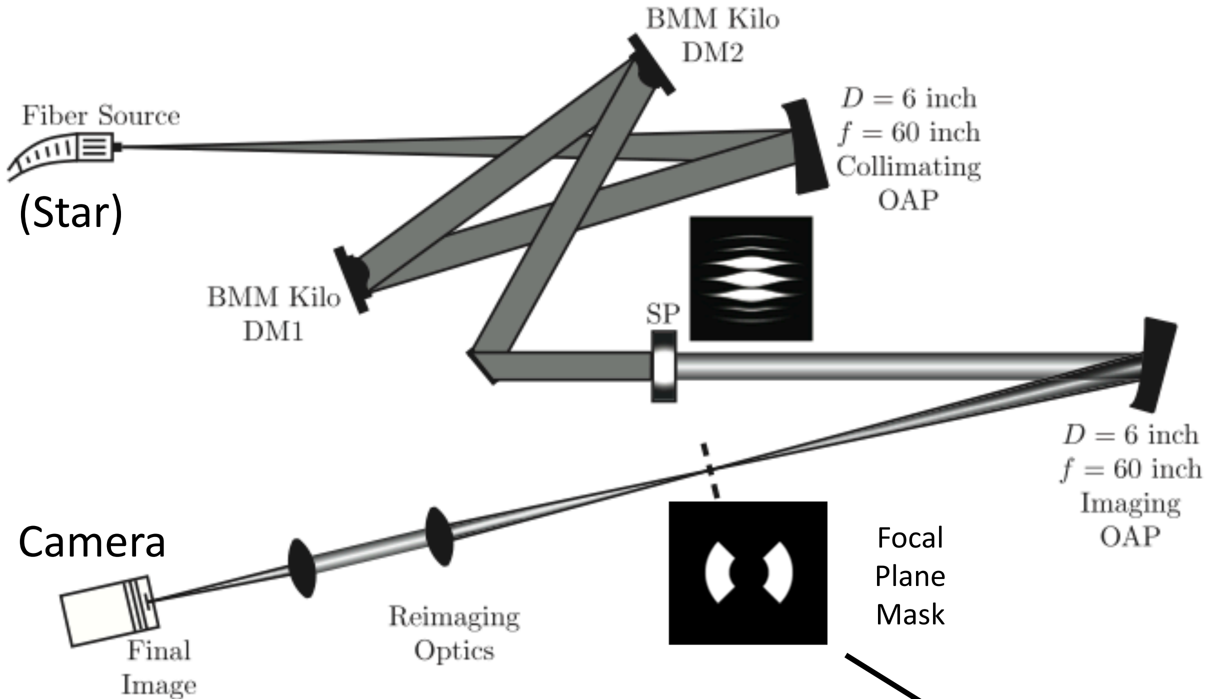
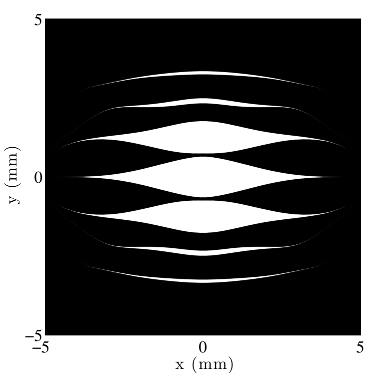


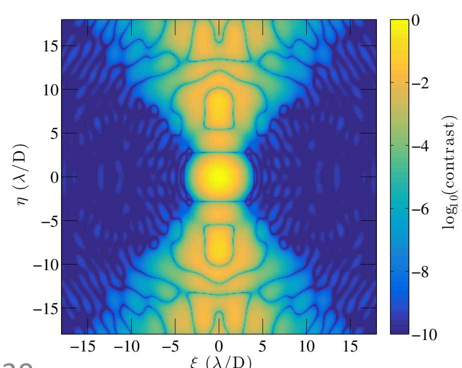
Image Credit:
Groff & Kasdin 2013

Image Credit:
Riggs et al. 2016

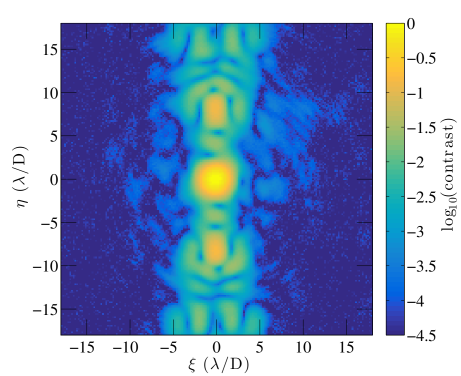
Shaped Pupil



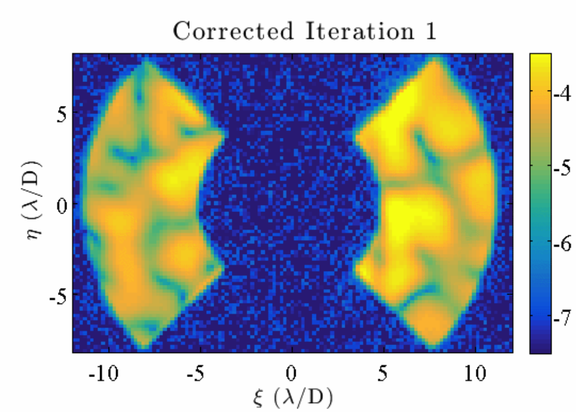
Ideal PSF



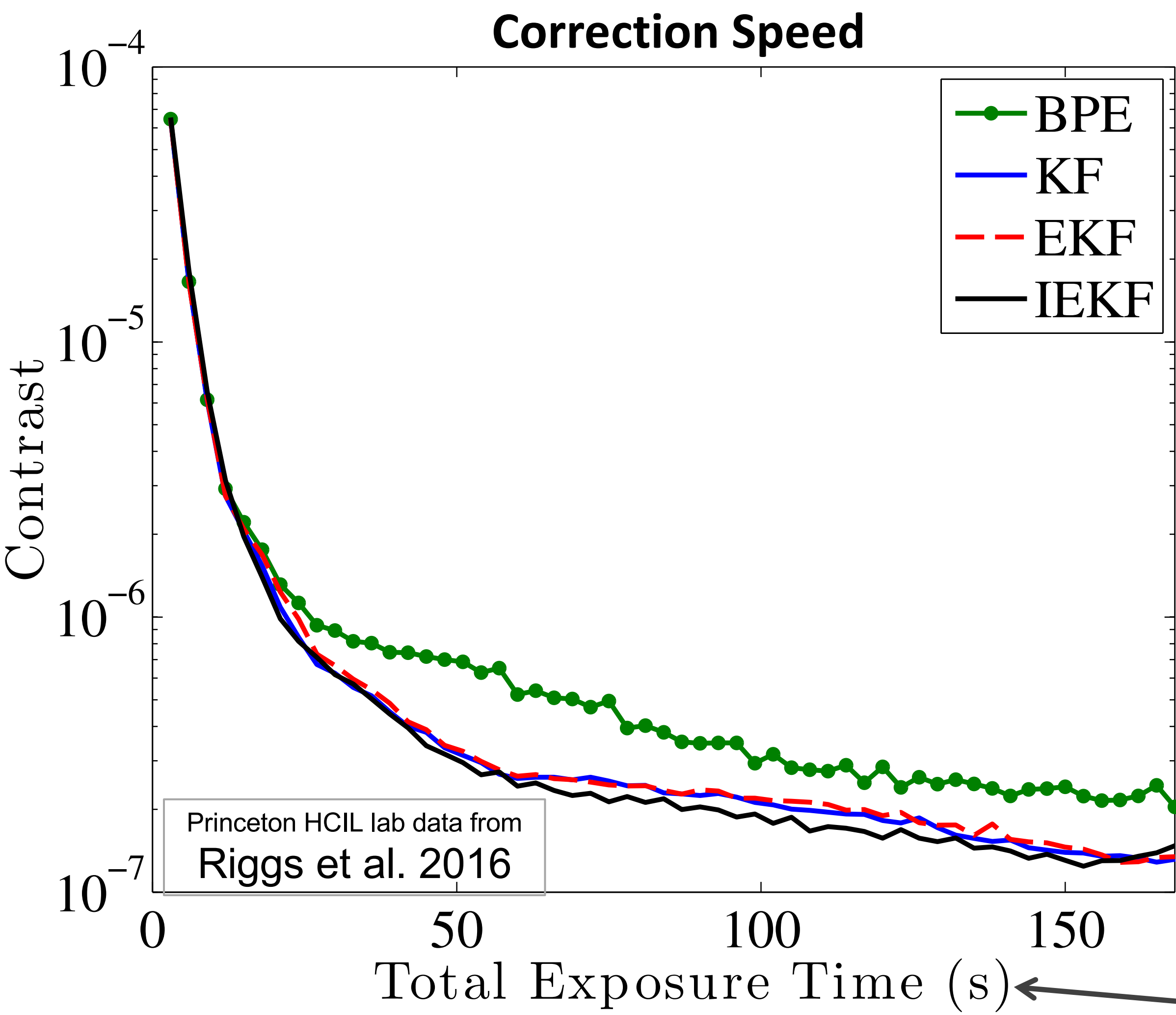
Measured Initial PSF

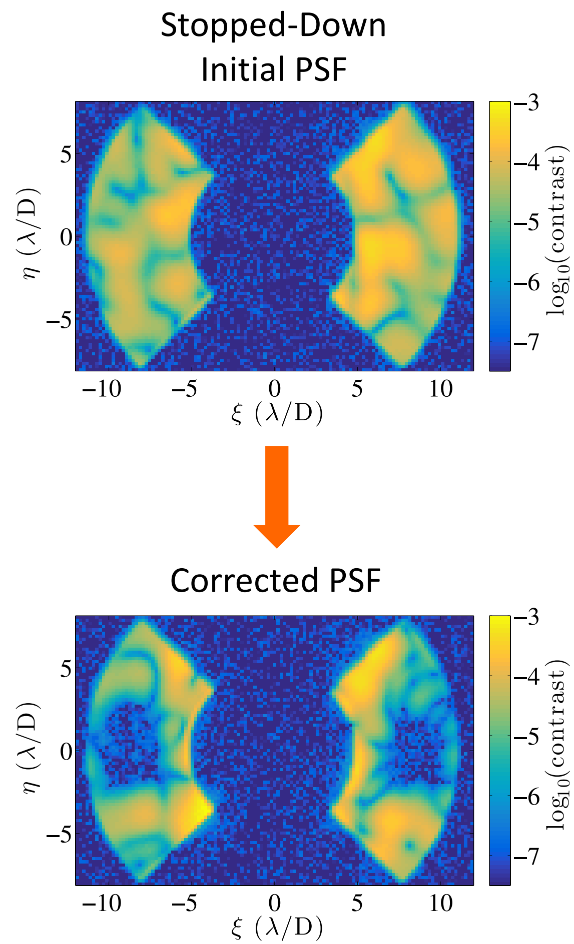


Stopped-Down PSF



IEKF Validation at Princeton





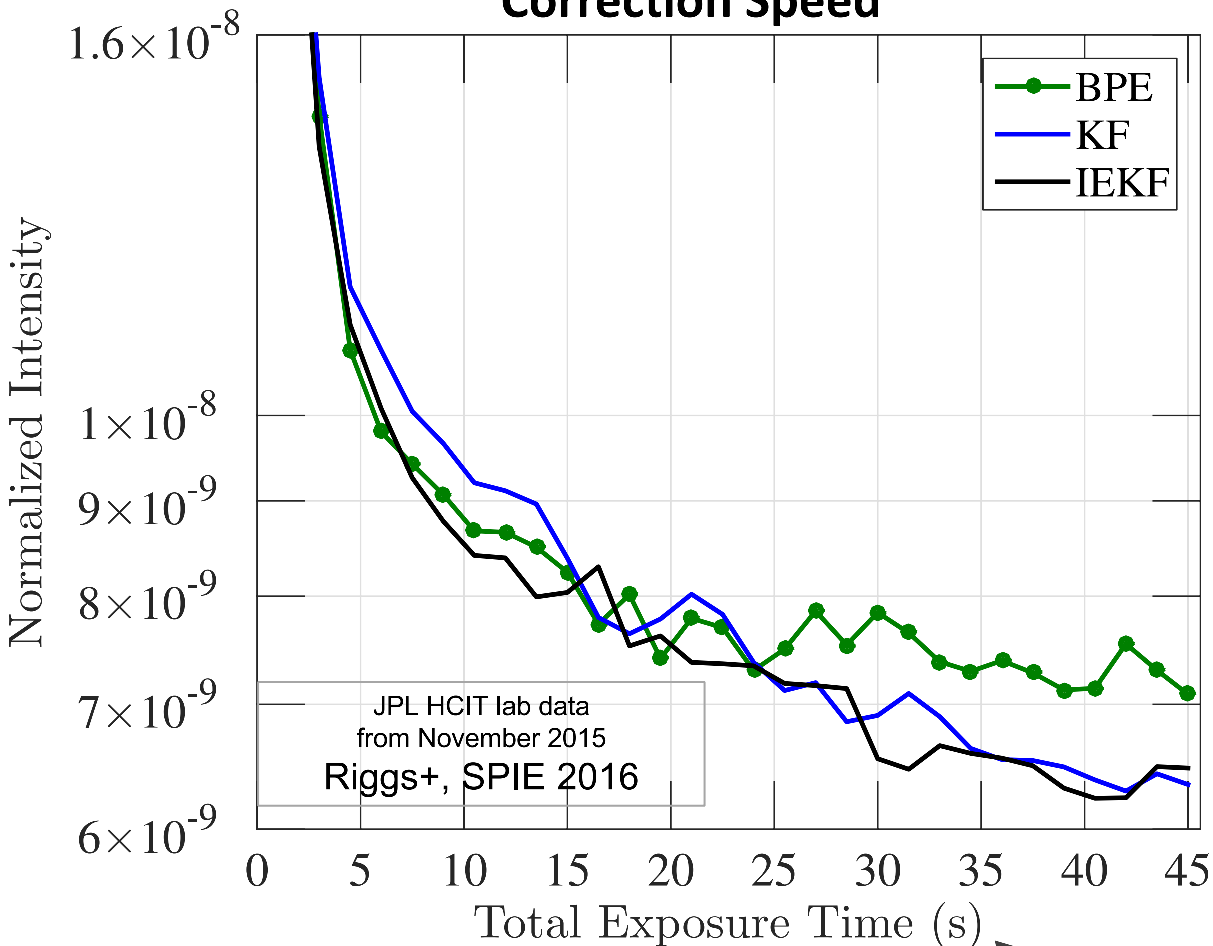
(Lab time for laserlight. Real starlight will require much longer exposures.)

Takeaways:

- EKF & IEKF as fast as KF
- All Kalman filter types are faster and achieve better contrast than BPE.

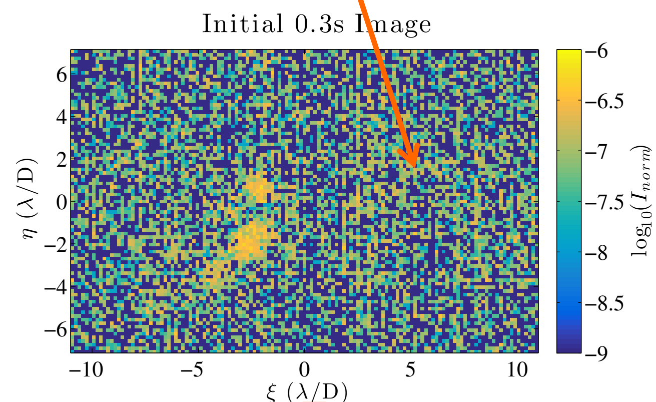
IEKF Validation at JPL

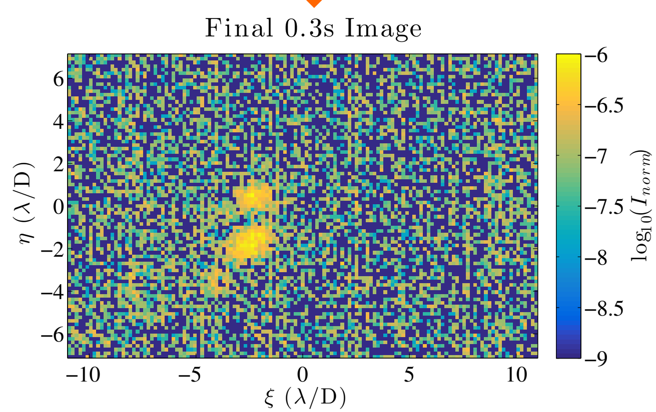
Correction Speed



(Normalized Intensity ≈ Contrast/1.3)

- WFIRST SPLC design
- NEC = $2x10^{-8}$
- 1-sided dark hole

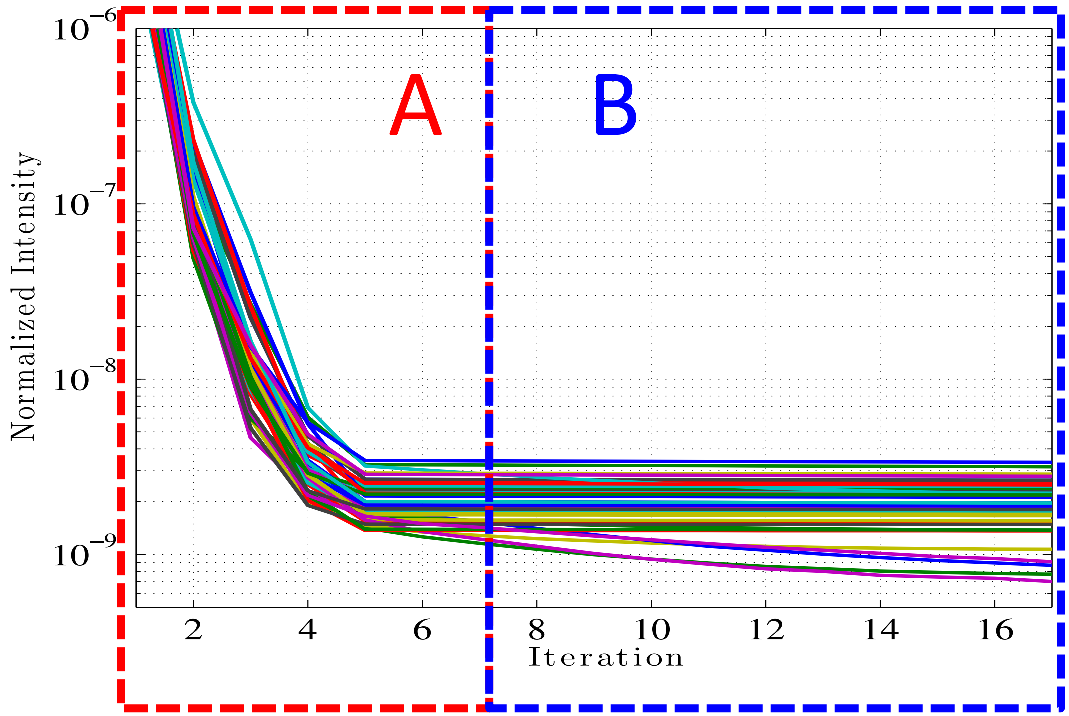




(Lab time for laserlight. Real starlight will require much longer exposures.)

Takeaways:
- ➢ IEKF as fast as KF
- ➢ KF & IEKF are **faster** and achieve **better contrast** than BPE.

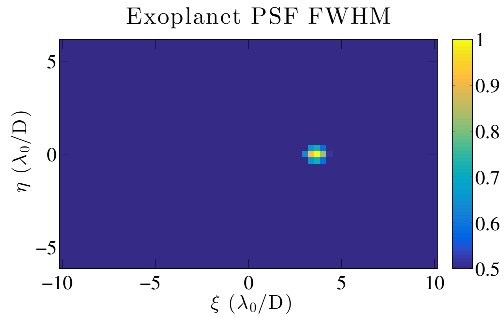
Wavefront Correction Scheme



- For initial testing: "science star" correction starts when dark hole already exists.
- Two phases of correction:
  - Stage A: "**Bright star**" correction: Dig dark hole on bright star. No planet present yet.
  - Stage B: "**Science star**" correction: Planet (or no planet) included in incoherent signal

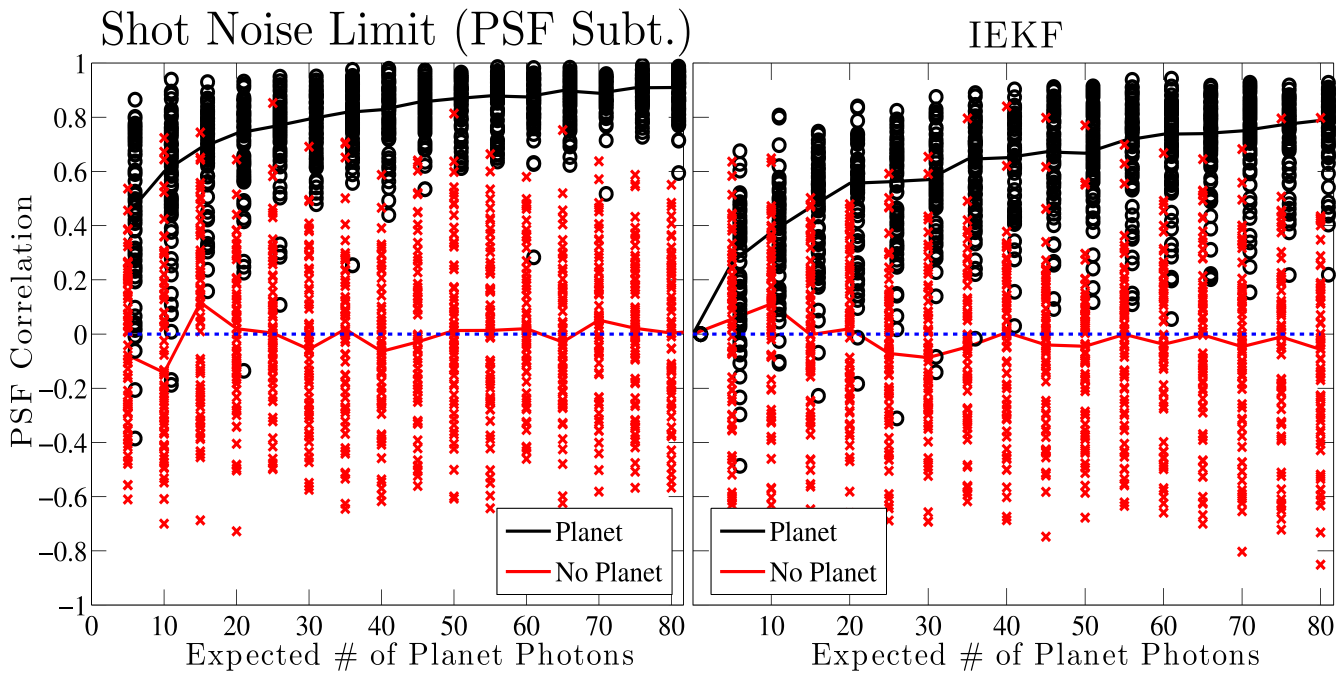
Planet PSF Correlation

Detection Metric: Normalized **2-D PSF correlation** between planet's template PSF and IEKF's incoherent intensity estimate



Model-Based Template PSF

- 11 pixels within FWHM



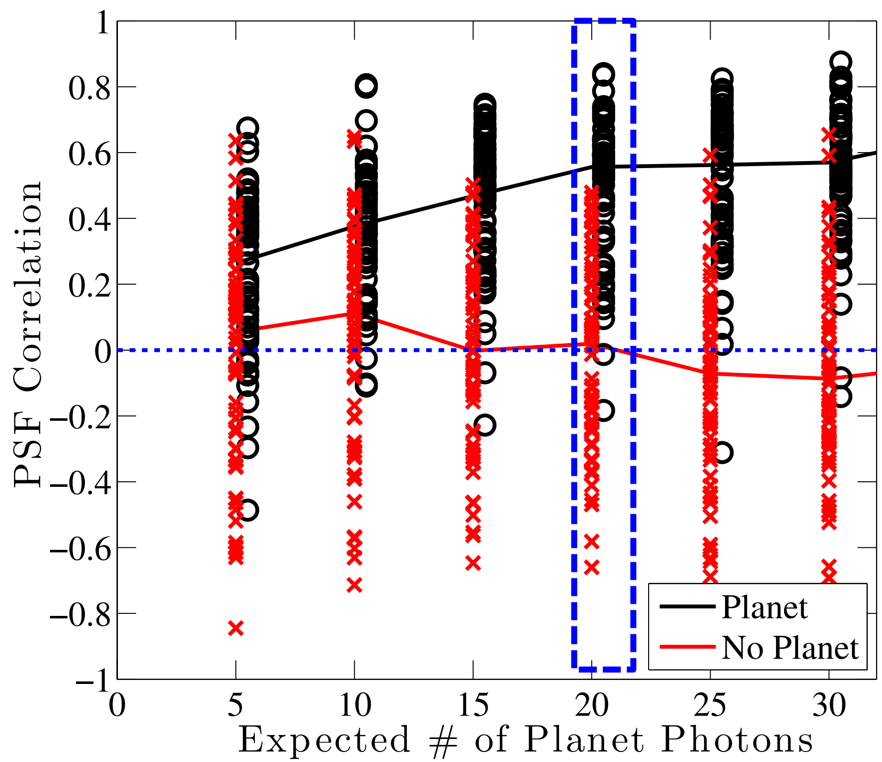
➢ PSF correlation increases with exposure time if planet is present

Receiver Operating Characteristic (ROC)

ROC curve: Plots **tradeoff** between **probability of detection** & **probability of false alarm**

Probability of detection = Fraction of all true planets counted (***black*** *points above threshold*)
Probability of false alarm = Fraction of spurious signals counted as planets (***red*** *points above threshold*)

- 1 ROC curve per time step
- Parametrizes the PSF Correlation estimates
- Built by setting minimum PSF correlation value (threshold)



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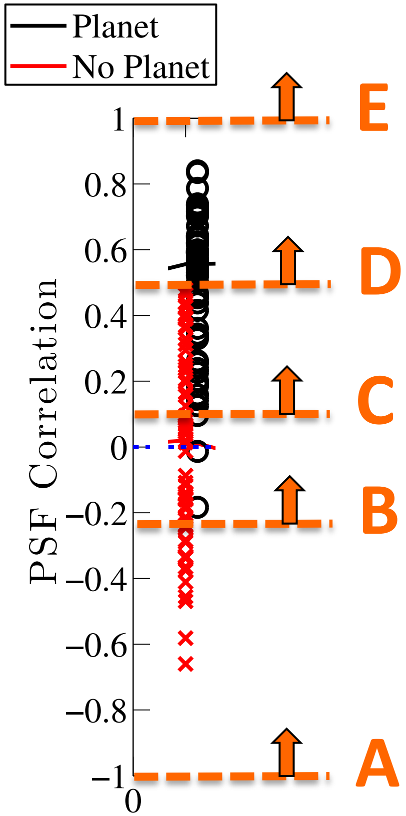
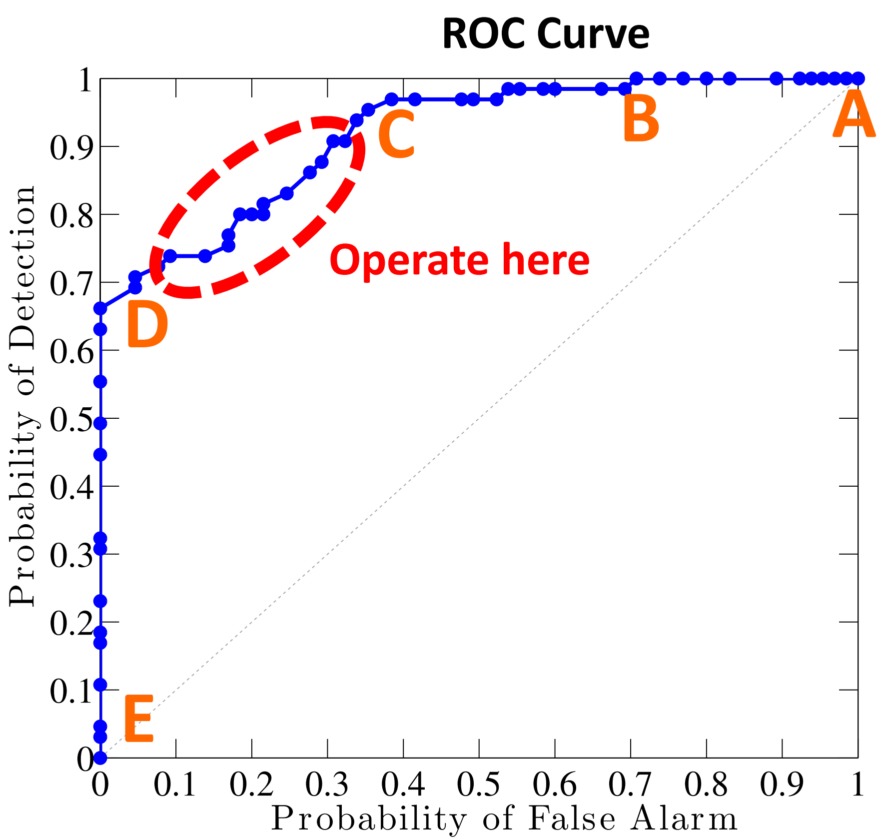




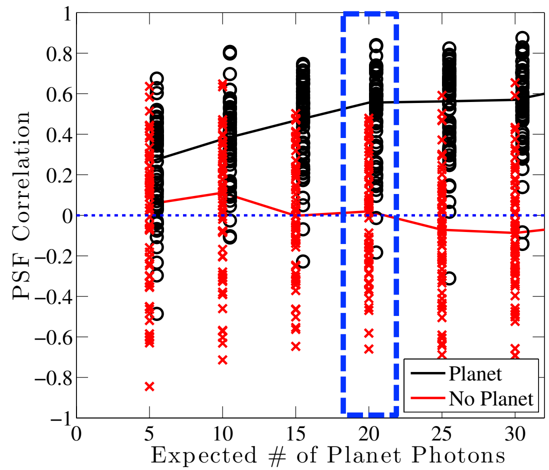
Jet Propulsion Laboratory
California Institute of Technology

ROC Curve Construction

Receiver Operator Characteristic (ROC) Curve:

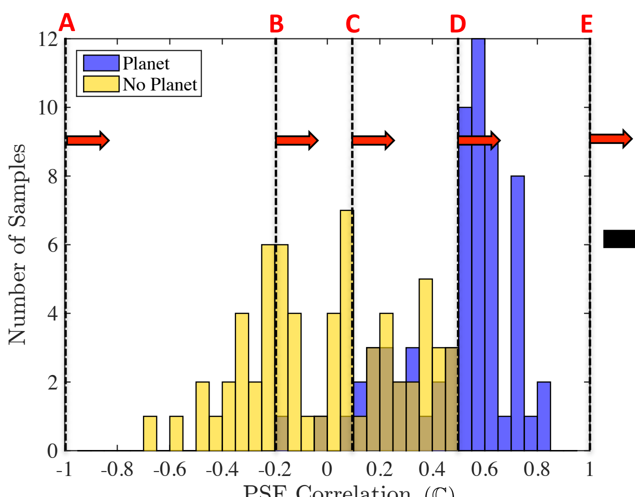
Plots **probability of detection** vs **false alarm rate**

- One ROC curve per time step
- Built by setting minimum PSF correlation value (threshold)



Probability of detection =
- Fraction of all true planets counted

False alarm rate =
- Fraction of spurious signals counted as planets



ROC Curve