

# *Poincare*

## A Multi-Body, Multi-System Trajectory Design Tool in MONTE

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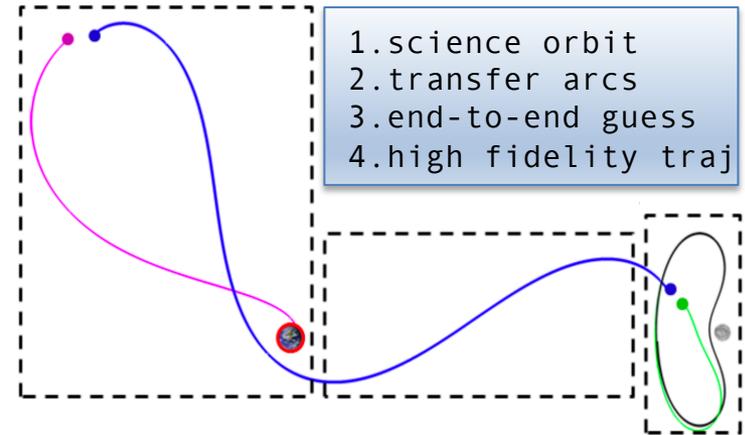
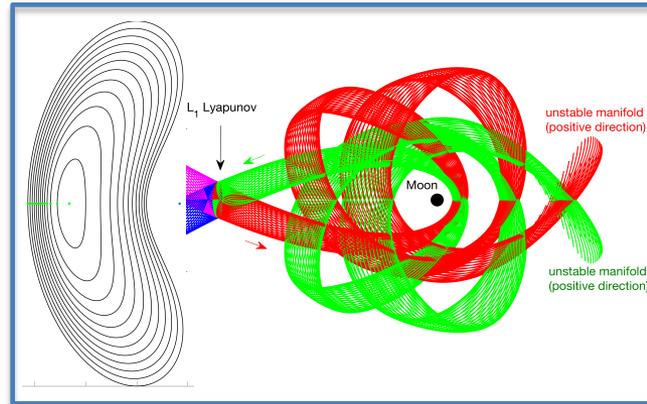


# Poincare

## WHAT Poincare is

- Trajectory design tool based on a catalog of three-body science orbits and a differential corrector to compute connecting transfer arcs between orbits in multi-body systems

mission to an  
Earth-Moon  
libration  
point orbit



## WHY we developed Poincare

- Existing common orbit design approach used in our section is limited and far from intuitive
- Design usually relies on the use of personal code or highly specialized code
- This code generally produces point solutions for a few basic orbital families
- Poincare offers an "all-in-one" integrated search within one interface and setup in MONTE

## HOW we did it

- MONTE Tier2 code with unit tests and formal docs
- Focused on low energy trajectories
- Simple, modular design, easy to maintain, update, and expand



# Orbital Catalog



Libration Point Orbits	P2-Centered Orbits	Resonant Orbits	3B Systems	Stored Parameters
L <sub>1</sub> , L <sub>2</sub> , L <sub>3</sub> Lyapunov	Distant Retrograde Orbits	1:1 resonance	Sun-Earth	primary/secondary body
L <sub>1</sub> , L <sub>2</sub> , L <sub>3</sub> Halo	Distant Prograde Orbits	1:2 resonance	Earth-Moon	gravitational constant mu
L <sub>1</sub> , L <sub>2</sub> , L <sub>3</sub> Vertical	Low Prograde Orbits	1:3 resonance	Saturn-Titan	Initial state
L <sub>1</sub> , L <sub>2</sub> , L <sub>3</sub> Axial		1:4 resonance	Saturn-Enceladus	period
L <sub>4</sub> , L <sub>5</sub> Short Period		2:1 resonance	Jupiter-Europa	jacobi constant
L <sub>4</sub> , L <sub>5</sub> Long Period		2:3 resonance	Sun-Mars	eigenvalues
L <sub>4</sub> , L <sub>5</sub> Vertical		3:1 resonance	Mars-Phobos	stability index
L <sub>4</sub> , L <sub>5</sub> Axial		3:2 resonance		$v = \frac{1}{2} \left( \lambda_{max} + \frac{1}{\lambda_{max}} \right)$
Butterfly		3:4 resonance		number of patch points
Dragonfly		4:1 resonance		diff corrector constraints
		4:3 resonance		propagation error and tol

## How orbital families are calculated

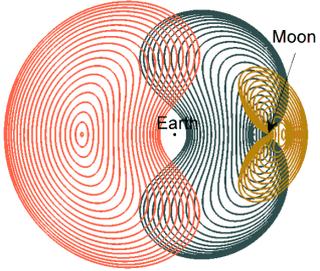
- Multiple shooting differential corrections algorithm based on a generalized constraint and free variable method within pseudo-arclength continuation scheme
- One algorithm for all – easy to implement and maintain
- 36 families per three-body system, over 800,000 orbits currently stored in catalog
- Easy access and search for orbits in standalone SQLite database
- Current plan to make the database publicly available in JPL's Horizons website



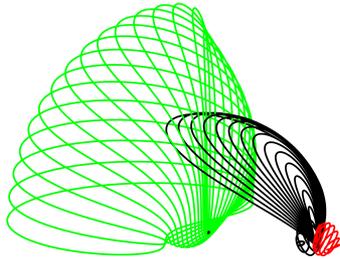
# Orbital Catalog



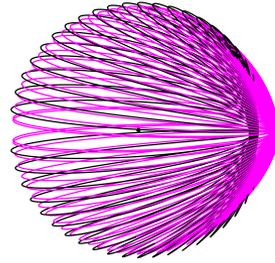
Lyapunov



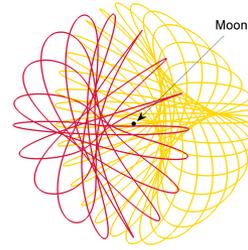
Halo



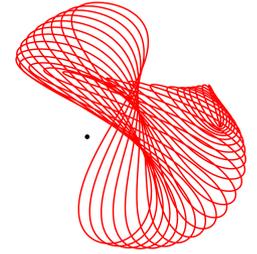
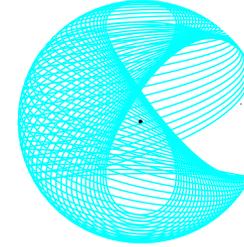
Vertical



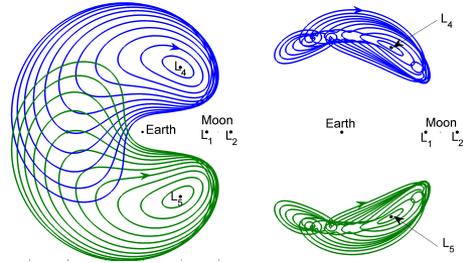
Axial



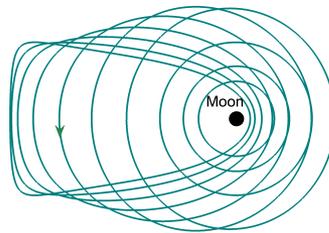
Triangular Vertical/Axial



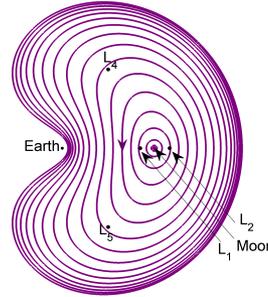
Short/Long Period



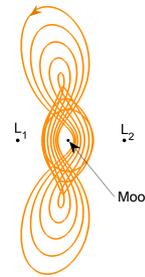
LPO



DR0



DPO



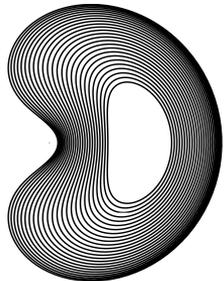
Dragonfly



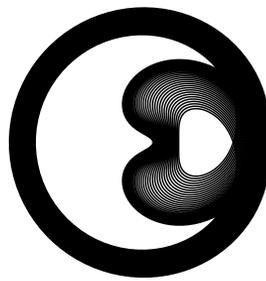
Butterfly



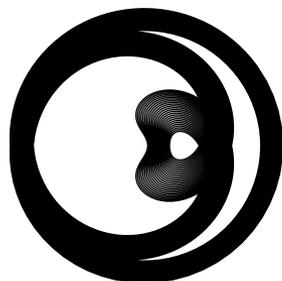
1:1



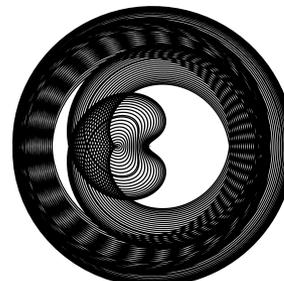
1:2



1:3



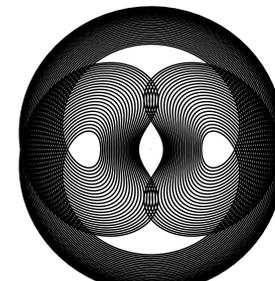
1:4



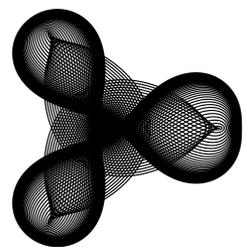
2:1



2:3



3:1





# Dynamical Model

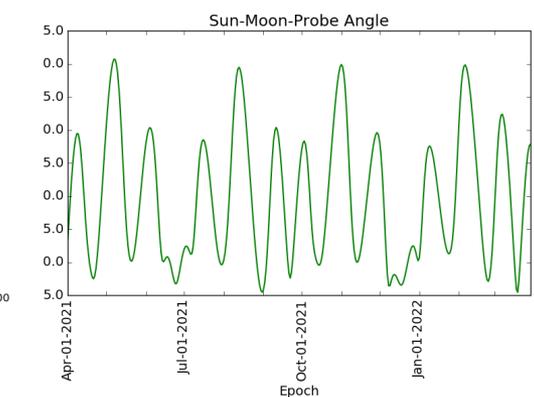
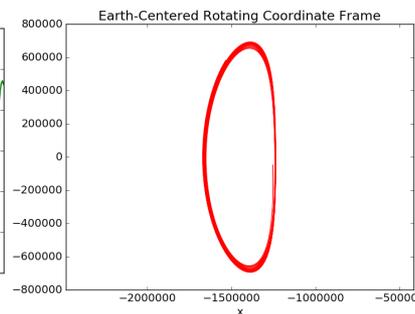
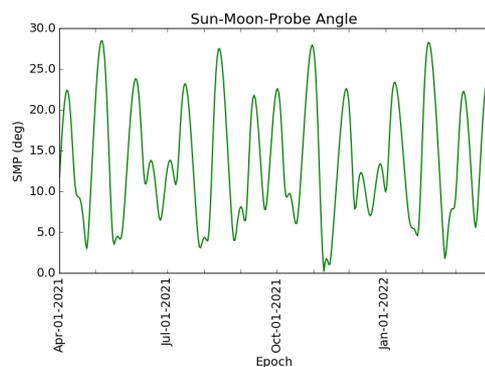
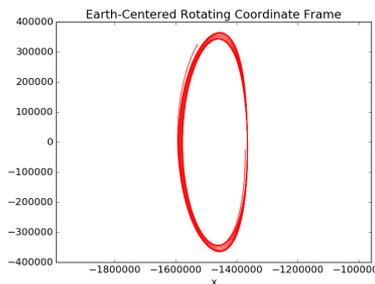


## Circular Restricted Three-Body Model BOA universe

- Provides independent MONTE python packages:
  - set up a three-body universe  $\rightarrow$  `poincare.makeCR3BP()`
  - access and search for orbits in catalog  $\rightarrow$  `poincare.queryDB()`
  - parameterize units if needed  $\rightarrow$  `poincare.makeDimensional()`
  - easy state propagator interface  $\rightarrow$  `poincare.propagate()`
- Visualize and save trajectory in a boa, then utilize full MONTE capabilities

## Common Applications

- Almost one-million multi-body periodic orbits readily available
- Fast search and selection based on parameters like orbital energy or stability
- Ideal for quick science orbit trade-off studies

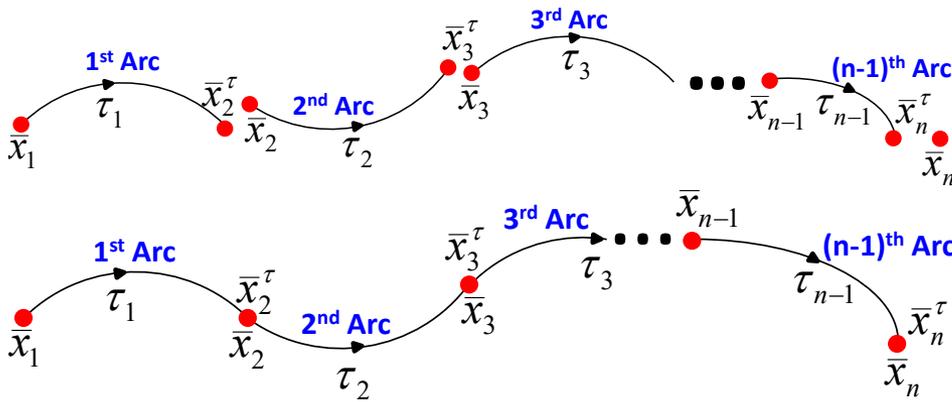




# Differential Corrector



- Expands the use of the periodic orbit catalog
- Straightforward free variables and constraints implementation in MONTE Tier-2



```
poincare.corrector.makePatchPoints()
poincare.Corrector()
poincare.corrector.solve()
poincare.addClosedOrbitConstraints()
poincare.corrector.addConstraint[Cartesian.y()]
```

## Common Uses

- Compute a specific periodic orbit by specific Jacobi constant, period, or amplitude
- Compute quasi-periodic orbits starting from a periodic orbit by selecting the number of revolutions or amplitude (e.g. Lissajous orbits)
- Compute quasi-periodic orbits in a full ephemeris system from a periodic orbit in the CR3BP

```
7 Cosmic/CORRECTOR/P1/X -1.151972689405265e+04 *km -1.151972689405265e+04 *km -1.1519726894052646e+03 *km
15 Cosmic/CORRECTOR/P2/Y 9.604395582072197e+03 *km 1.955322770694432e+04 *km 1.960439558207220e+04 *km
-----
9 P0->P1 State: Y -1.00000000000000e+00 *km 1.026111093939107e+00 *km 1.00000000000000e+00 *km
11 P0->P1 State: DX -1.00000000000000e-05 *km/sec -1.111131090636430e-05 *km/sec 1.00000000000000e-05 *km/sec
15 P1->P2 State: Y -1.00000000000000e+00 *km 1.063062499204534e+00 *km 1.00000000000000e+00 *km
20 P2->P3 State: X -1.00000000000000e+00 *km -1.005207620703004e+00 *km 1.00000000000000e+00 *km
23 P2->P3 State: DX -1.00000000000000e-05 *km/sec -1.104945924335077e-05 *km/sec 1.00000000000000e-05 *km/sec
24 P2->P3 State: DY -1.00000000000000e-05 *km/sec -1.009549871732254e-05 *km/sec 1.00000000000000e-05 *km/sec
-----
Problem: CORRECTOR
Optimizer: DBLSE
Status: NOT_SET
Iterations: 5
-----
TOTAL( SUM ) 0.00000000000000e+00
-----
7 Cosmic/CORRECTOR/P1/X -1.151972689405265e+04 *km -1.151972689405265e+04 *km -1.1519726894052646e+03 *km
15 Cosmic/CORRECTOR/P2/Y 9.604395582072197e+03 *km 1.955334137018752e+04 *km 1.960439558207220e+04 *km
-----
17 P1->P2 State: DX -1.00000000000000e-05 *km/sec -1.000001326710749e-05 *km/sec 1.00000000000000e-05 *km/sec
23 P2->P3 State: DX -1.00000000000000e-05 *km/sec -1.000002003226363e-05 *km/sec 1.00000000000000e-05 *km/sec
-----
Problem: CORRECTOR
Optimizer: DBLSE
Status: CONVERGED
Iterations: 6
-----
TOTAL( SUM ) 0.00000000000000e+00
-----
7 Cosmic/CORRECTOR/P1/X -1.151972689405265e+04 *km -1.151972689405265e+04 *km -1.1519726894052646e+03 *km
15 Cosmic/CORRECTOR/P2/Y 9.604395582072197e+03 *km 1.955334136972732e+04 *km 1.960439558207220e+04 *km
```

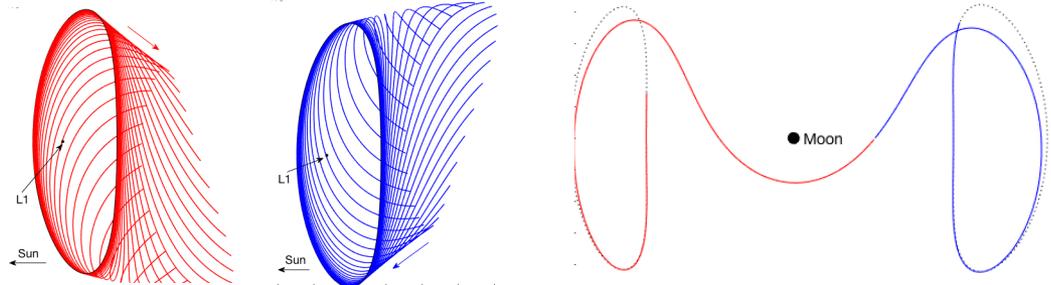


# Manifold Generator



- Manifold trajectories are efficient mechanisms for transport between orbits

**unstable** → depart the orbit  
**stable** → approach the orbit



- Defined parameters include:
  - number of stable and unstable manifold trajectories to be integrated
  - propagation time for each manifold trajectory
  - perturbation parameter
- Optional algorithm to systematically determine the step size for each periodic orbit
- Output includes:
  - dynamical system set-up
  - initial conditions associated with each propagated manifold trajectory
- Fast and efficient on-the-fly computation
- Output BOA file is small-sized and contains all information needed for re-integration or post-processing of manifold trajectories



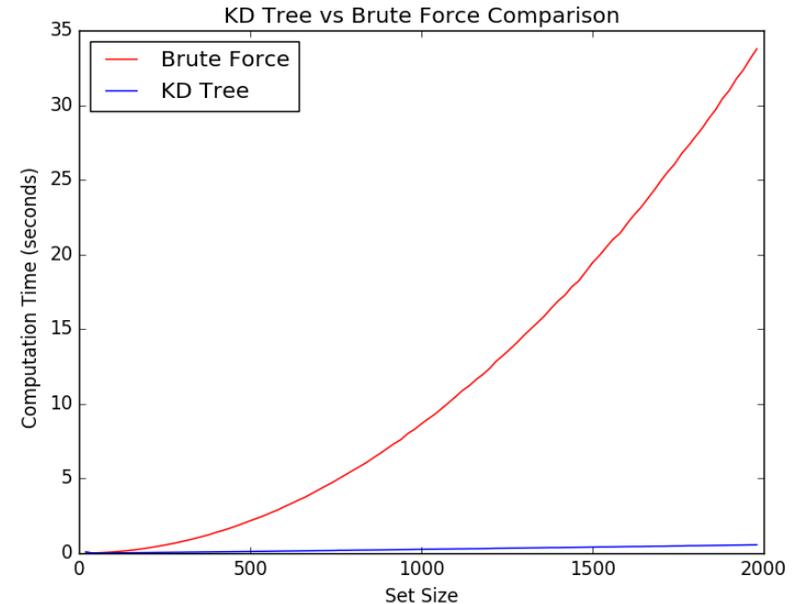
# Map Generator



- Current prototype available, final delivery expected in 2019
- Real-time computation of orbit connections via stable and unstable manifolds
- Allows user to generate and interact with Poincaré Maps
- Available types of maps include any user-specified surfaces of section, periapsis, apoapsis, and close-pass (or flybys) maps.
- Hooks exist to easily define any other type of map
- Efficient nearest neighbor search available for systematic detection of connections:
  - Similar to a Binary Search
  - Implemented in SciPy
  - $O(\log n)$

## Preliminary results based on current prototype

Step	Time Taken
Manifold trajectory propagation (10k)	10 Minutes
Map generation (event search)	2 Minutes
KD tree search	3 Minutes



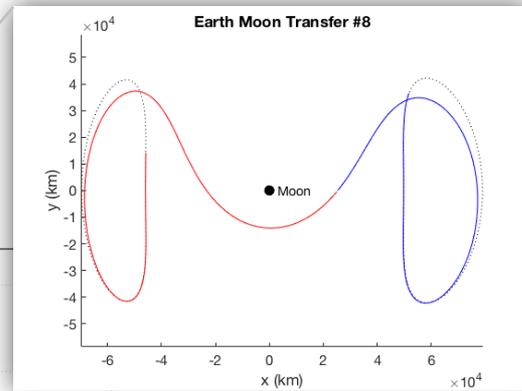
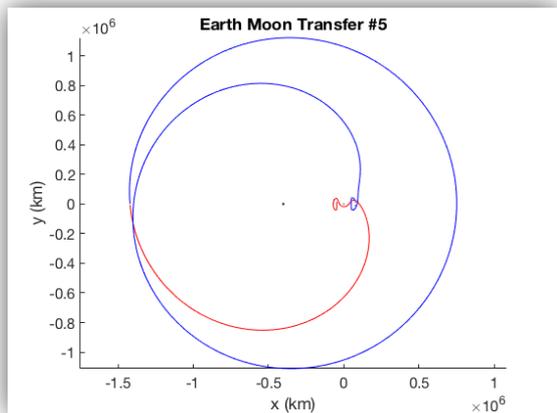


# Optimal Transfer Finder Prototype



Red Departs  
Blue Approaches

Hyperplane Poincare Map

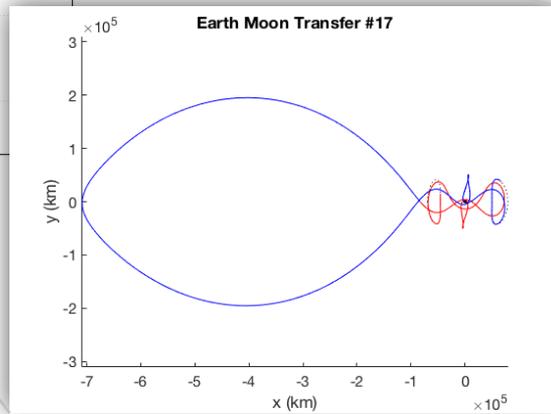
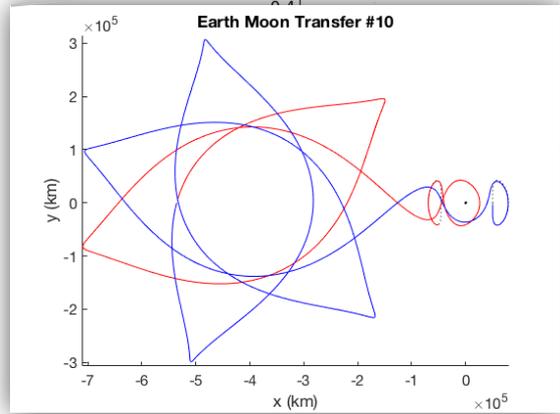


Rank: 6  
Distance: 67.90 km  
TOF: 123.84 days  
 $\Delta V$ : 0.521 m/s

Rank: 8  
Distance: 3.30 km  
TOF: 30.49 days  
 $\Delta V$ : 0.578 m/s

Rank: 19  
Distance: 31.06 km  
TOF: 86.91 days  
 $\Delta V$ : 0.917 m/s

Rank: 10  
Distance: 43.34 km  
TOF: 130.87 days  
 $\Delta V$ : 0.656 m/s





# Poincare in a Nutshell

Theory regarding computation and connection of multi-body orbits is pre-existing and well-understood; however, it was not available in a user-friendly way and within an all-in-one environment in MONTE



Allows mission designers to explore destinations, to or from a science orbit, that fit within time of flight and propellant constraints by leveraging low energy multi-body dynamical techniques to produce low-cost, or even  $\Delta V$ -free transfers

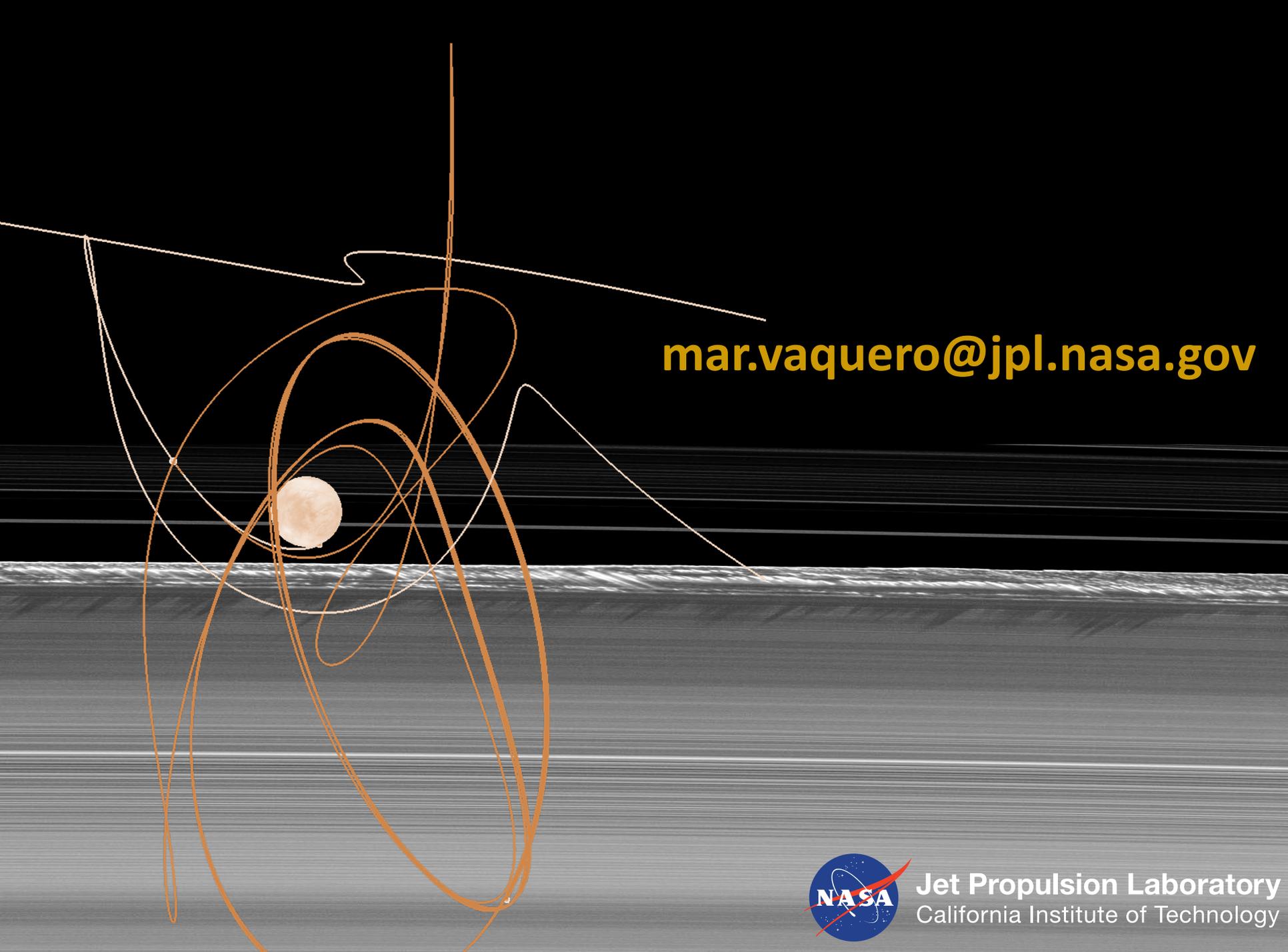


extensive multi-body orbital catalog

- User-friendly, modular design
- Easy maintenance and highly adaptable to changes in core scripts

robust differential corrector

manifold and map generators

A complex orbital diagram showing the Moon's path around Earth. The Earth is represented by a small yellow sphere in the center. Multiple overlapping orange and white lines represent various orbital paths, including elliptical orbits and a highly eccentric path that extends far into space. The background is a dark gradient with a horizontal band of light gray and white, suggesting a horizon or a specific orbital plane.

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