An Introduction to VLBI Modelling:
a Guided Tour through the Jungle

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Overview

- What effects need modelled?
  - Geometry
  - Geophysics
  - Atmosphere

  etc.

References:

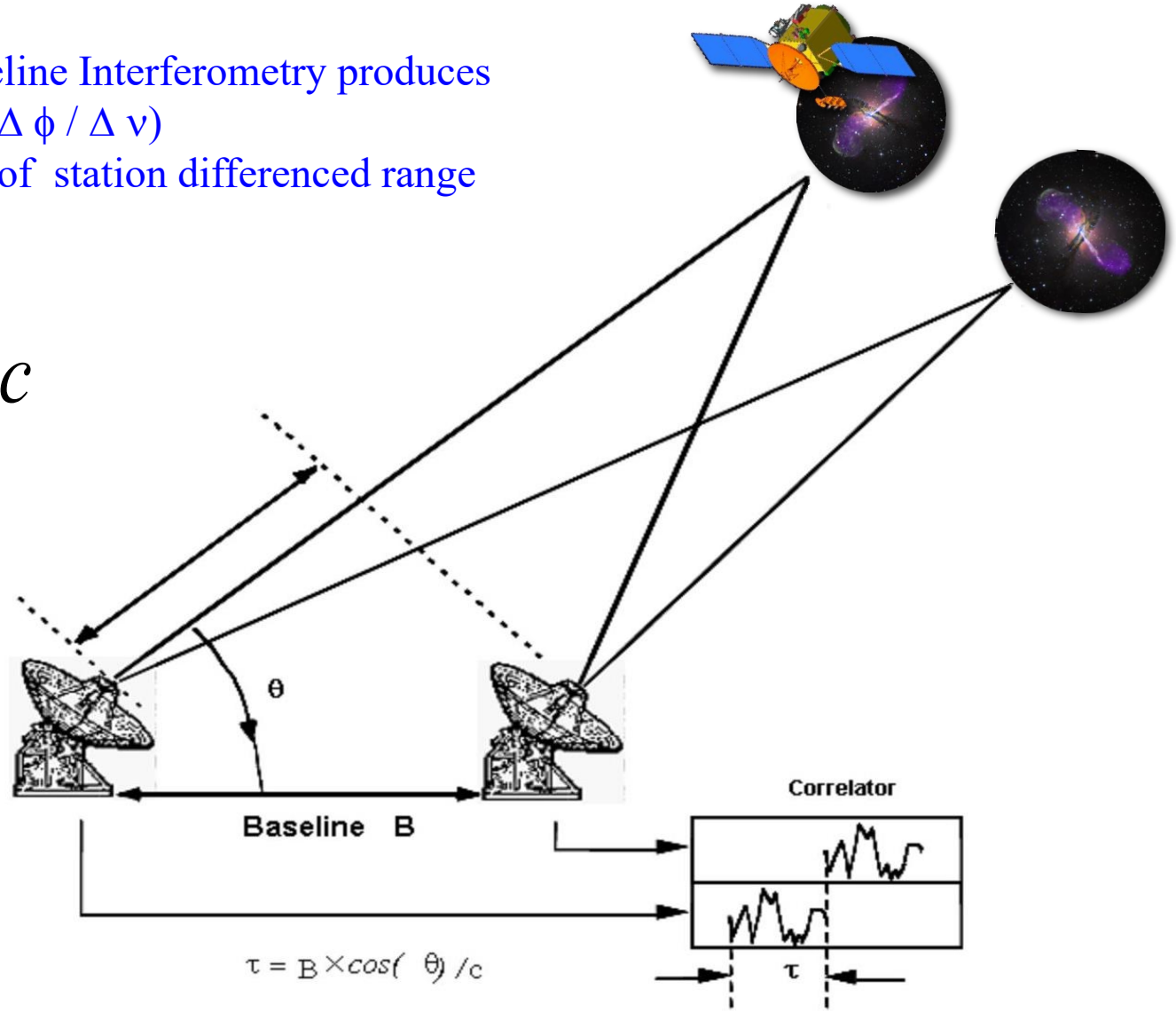
Sovers, Fanselow, Jacobs, Reviews Modern Physics, 70, 4, 1998.
http://adsabs.harvard.edu/abs/1998RvMP...70.1393S

IERS Conventions (2010)
http://iers-conventions.obspm.fr/2010officialinfo.php

VLBI Delay

Very Long Baseline Interferometry produces
a group delay ($\Delta \phi / \Delta \nu$)
which is a type of station differenced range

$$t = B \cdot s / c$$



$$\tau = \tau_{geom} + \tau_{clock} + \tau_{atmo} + \tau_{geophys} + \tau_{source} + \text{noise}$$

List of Delay Effects & uncertainties (mm)

Model component	Maximum delay	Present model uncertainty	
BASELINE GEOMETRY			
Zero-order geometric delay	6×10^9	...	Earth radius = 6378 km
Earth orbital motion	6×10^5	1	v/c = 1e-4, Diurnal v/c = 1e-6
Gravitational delay	2×10^3	2	
STATION POSITIONS			
Tectonic motion	100	1	Drift per year
Tidal motion	500	3	Semi-diurnal, diurnal
Nontidal motion	50	5	
EARTH ORIENTATION			
UTPM	2×10^4	2	
Nutation/precession	3×10^5	3	
SOURCE STRUCTURE	50	10	
ANTENNA STRUCTURE	10^4	10	
INSTRUMENTATION	3×10^5	5	
ATMOSPHERE			
Ionosphere	10^3	1	
Troposphere	2×10^4	20	2m zenith, mapping factor=10 at 6 deg elevation

Sovers et al, Rev Mod Phys, 70, 41998

Geometry

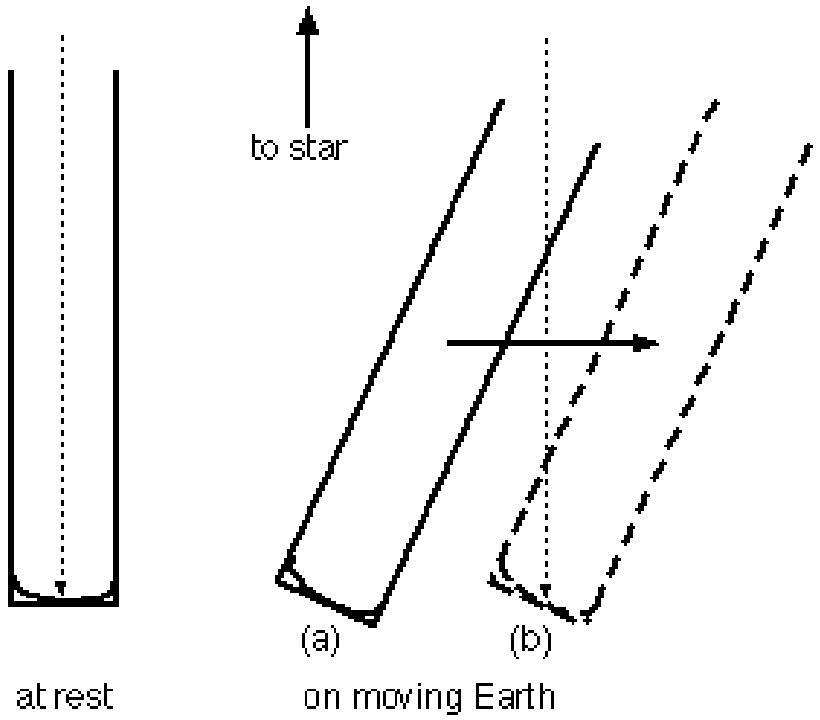
Static geometry:

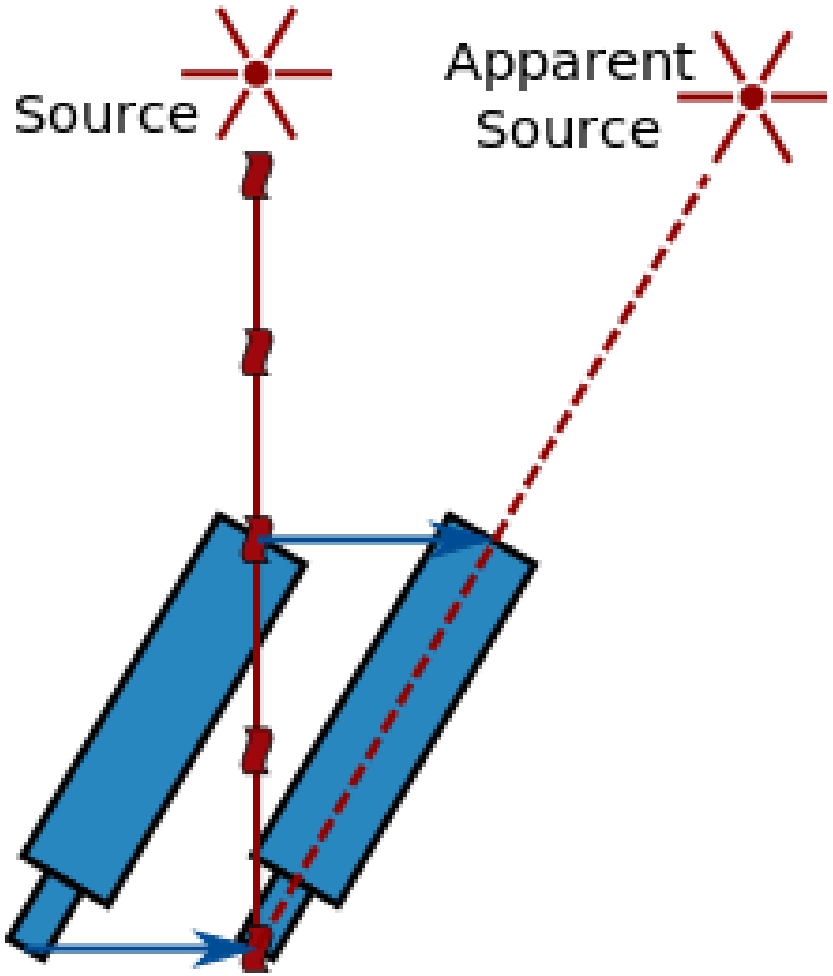
Define coordinate frame
  Geocentric, Cartesian
  origin, orientation conventions

Retardation: station move from earth rotation.
Station-2 moves after wave hits Station-1

Aberration

Credit: edu-observatory.org





Credit: wikipedia.org

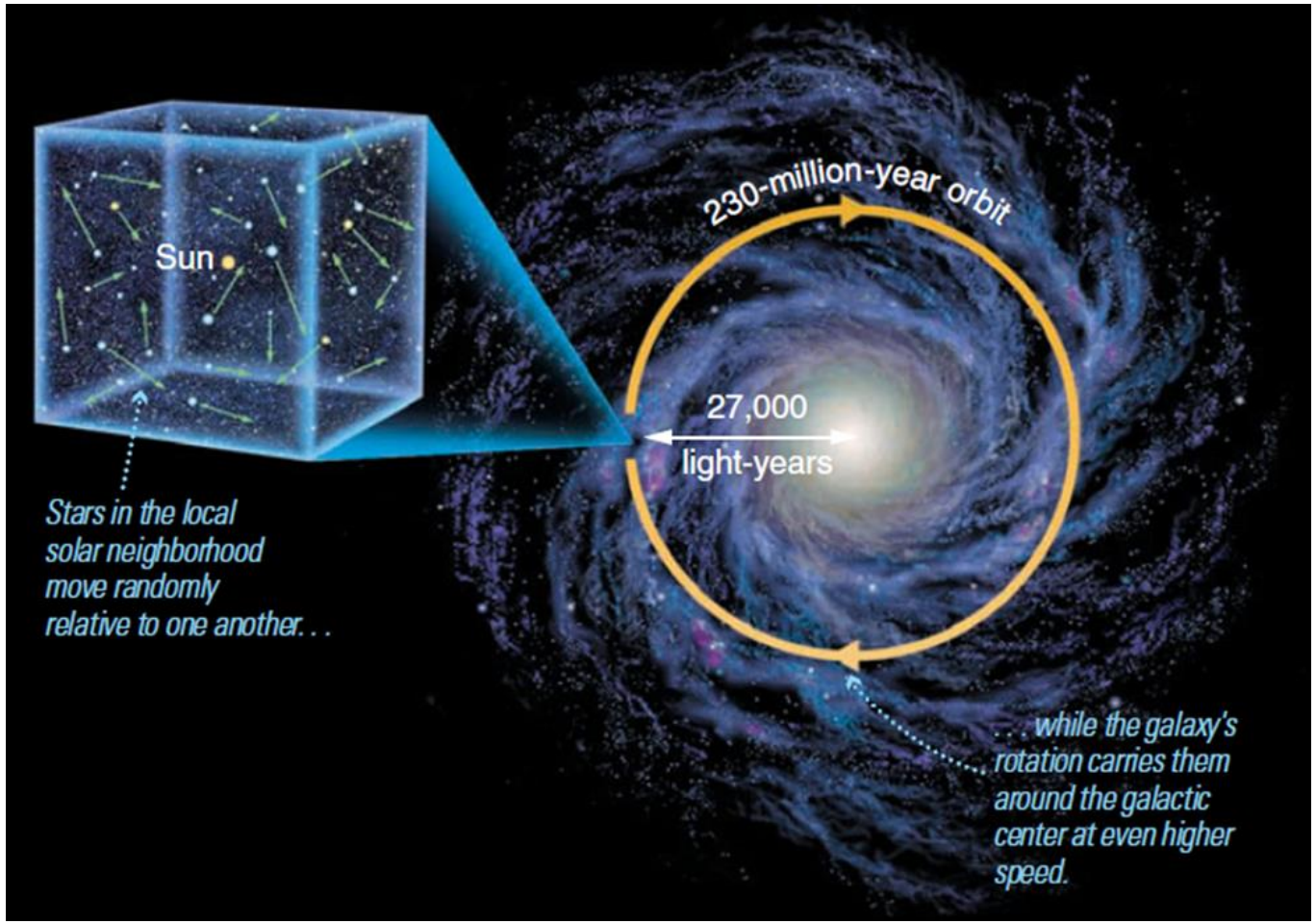
v/c ~ 1e-4 for Earth orbital motion
v/c ~ 1e-6 for Earth rotation
v/c ~ 7.e-4 for Galactic rotation.

Special Relativity

- Earth orbital motion
  v/c ~ 1e-4 (30 km/sec)

- Earth rotation
  v/c ~ 1e-6 (0.3 km/sec)

- Galactic rotation
  v/c ~ 7.3e-4 (220 km/sec)
  but period is ~230 million yr
  absorb static effect into
  RA, Dec
  Model changes of ~5 μas/year



Graphic credit: reddit.com

Special Relativity

Events are described by 4 dimensions (X, Y, Z, t).
“Space-like” means no time separation
“Time-like” means no spatial separation

Caution:

In geocentric frame baselines are “space-like”,
but troposphere, and clocks are “time-like”

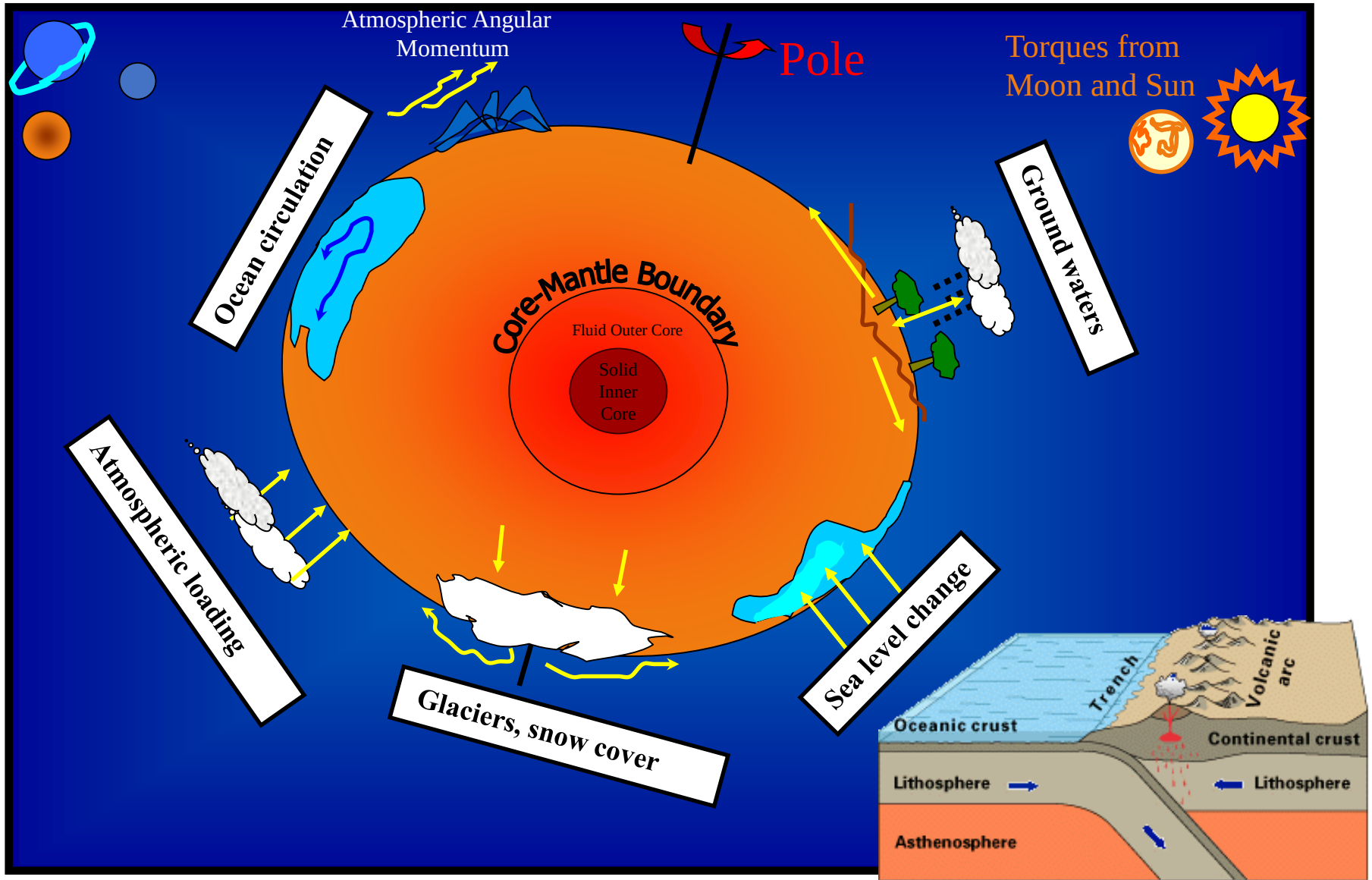
In Solar System Barycentric (SSB) frame the above does not hold.
For example, the troposphere in SSB has both space & time-like components.

General Relativity

Sun's potential: $\Phi/c^2 \sim 1e\text{-}8$
Masses e.g. sun cause not just bending, but also delay
$\tau = B \cdot s / (c/n)$
bending general relativity changes "s"
Einstein predicts (1916), Eddington measures c 1919
also changes (c/n) ala index of refraction

Model delay not bending: Shapiro delay (ref)
Planets (e.g. Jupiter) define effect at min impact param

Geophysics creates a Moving Platform: UT1, Polar Motion, Nutation, Tectonics



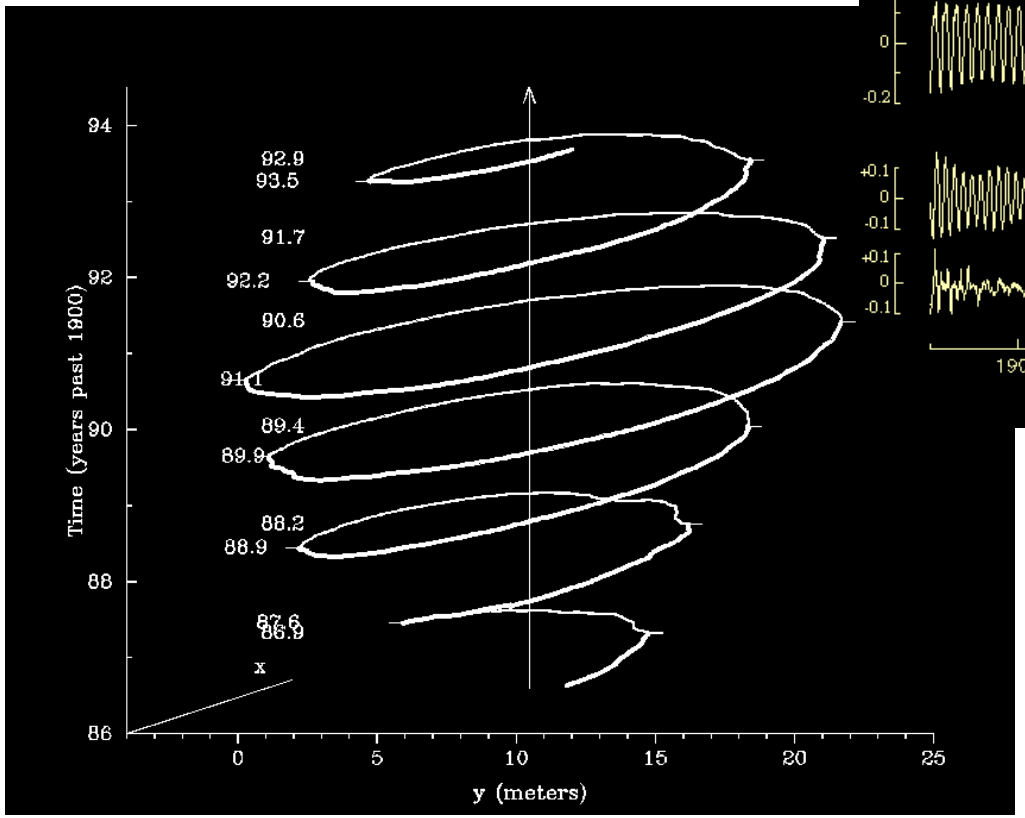
Oceanic-continental convergence

Credit:After Kurt Lambeck et al, The Earth's Variable Rotation, Cambridge, 1980

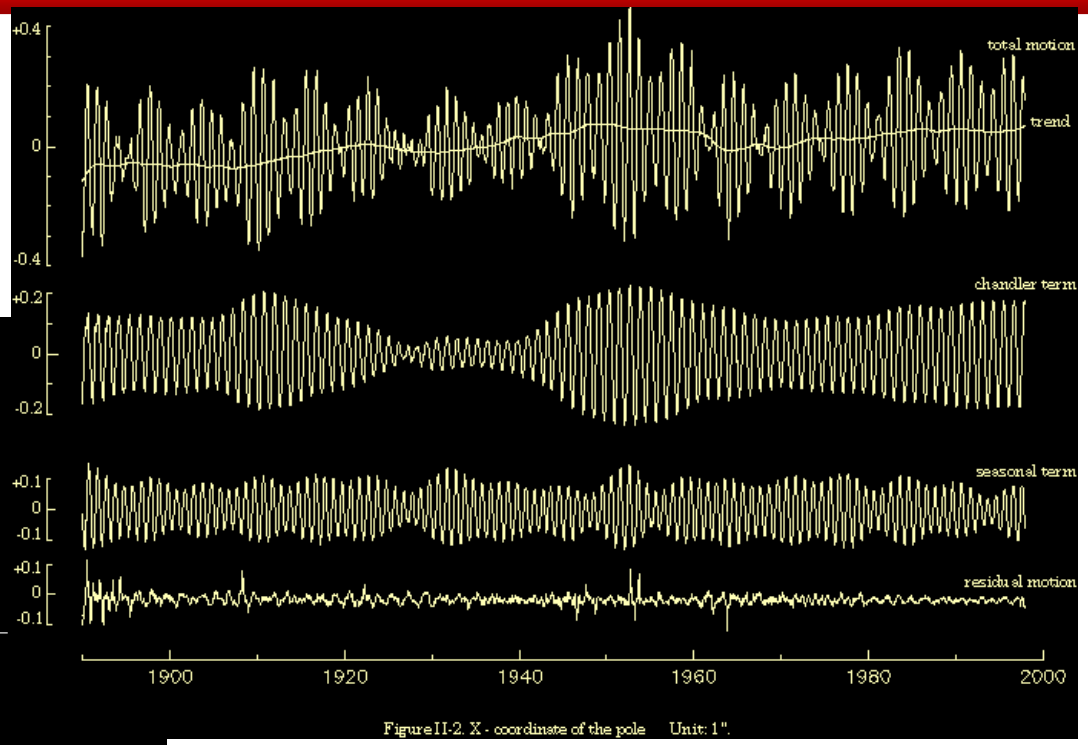
http://pubs.usgs.gov/publications/text/understanding.html

Polar Motion Spiral

Polar Motion vs. Time



Credit Sovers, Fanselow, Jacobs, Rev Mod Phys 70,, 1998.



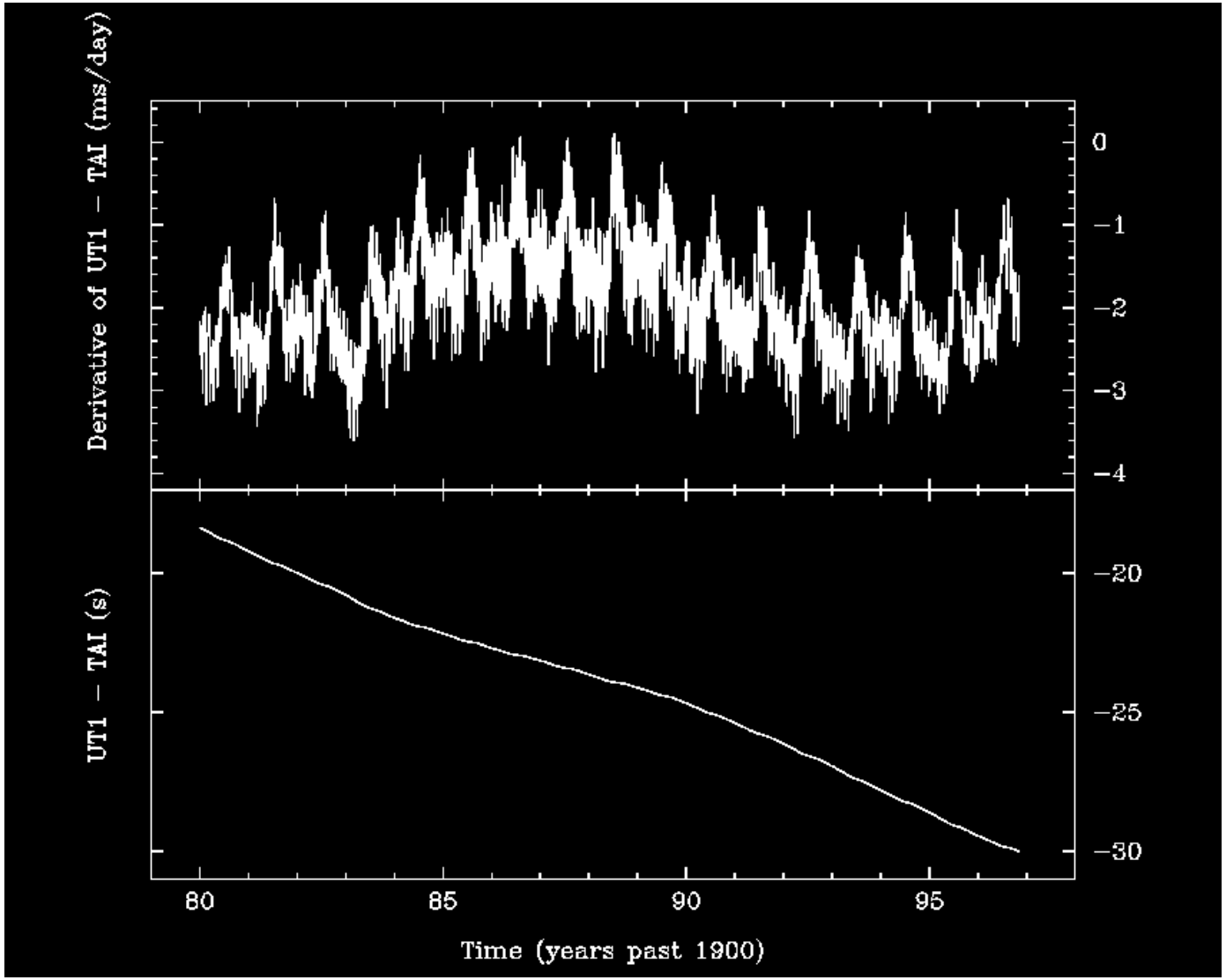
(credit http://www.iers.org/iers/earth/)

Effect ~ 10 m

Calibrated to ~1cm with

GPS + VLBI + …

Length Of Day Spectrum, UT1 vs. TAI



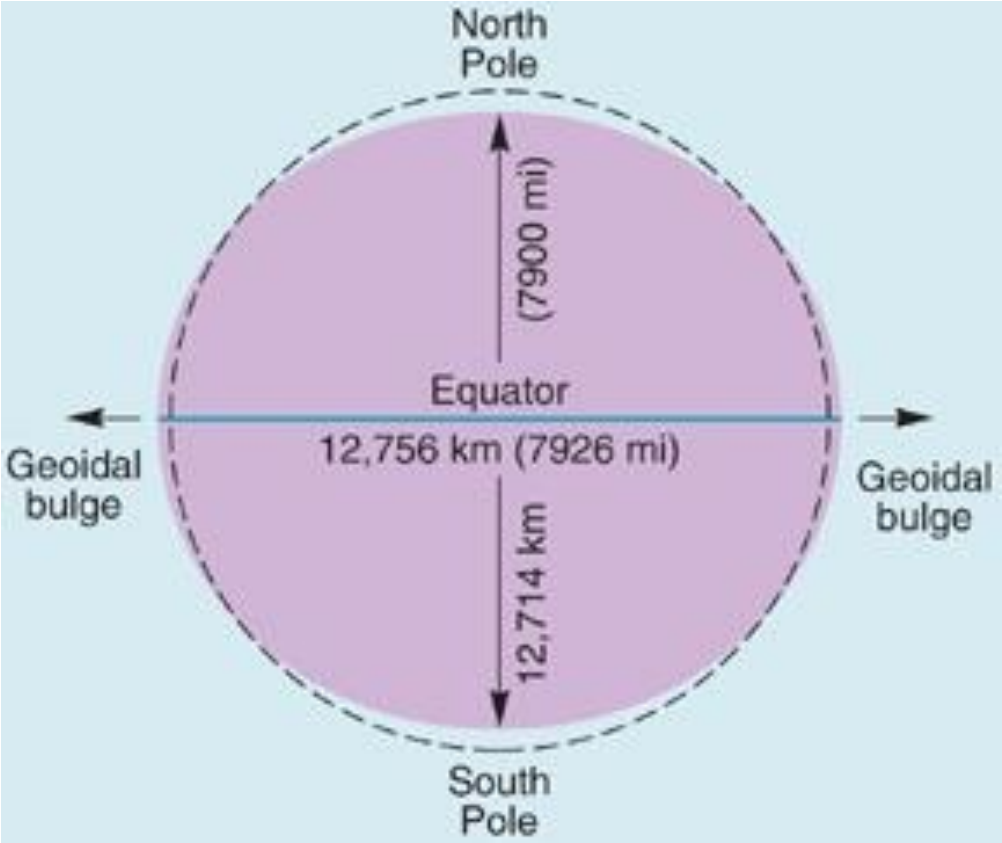
Length Of Day Spectrum (msec/day)

UT1 – TAI (sec)

Credit Sovers, Fanselow, Jacobs, Rev Mod Phys 70, 4, Oct 1998.

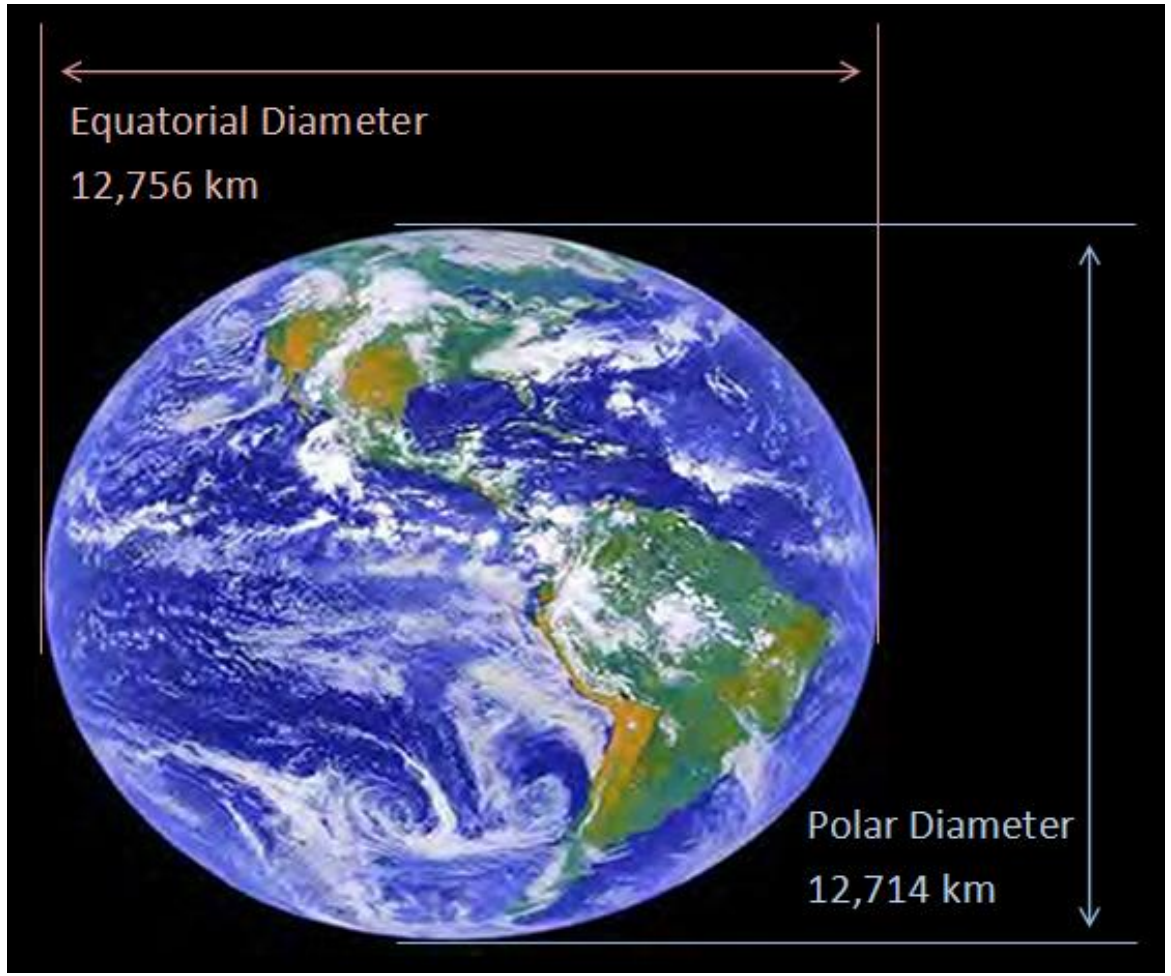
The Earth is an Oblate Spheroid

Earth non-spherical to 1 part in ~300. Allows torque



Credit: web.gccaz.edu

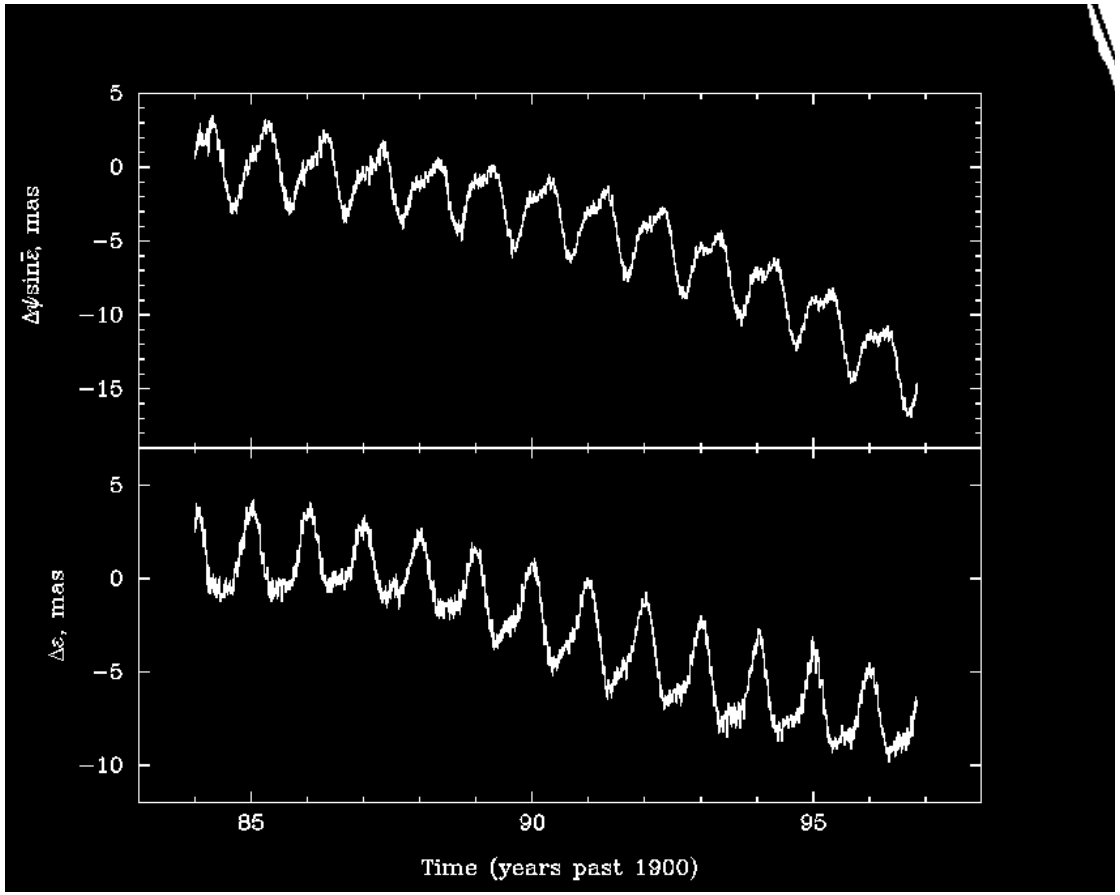
Credit: quikinformation.blogspot.com

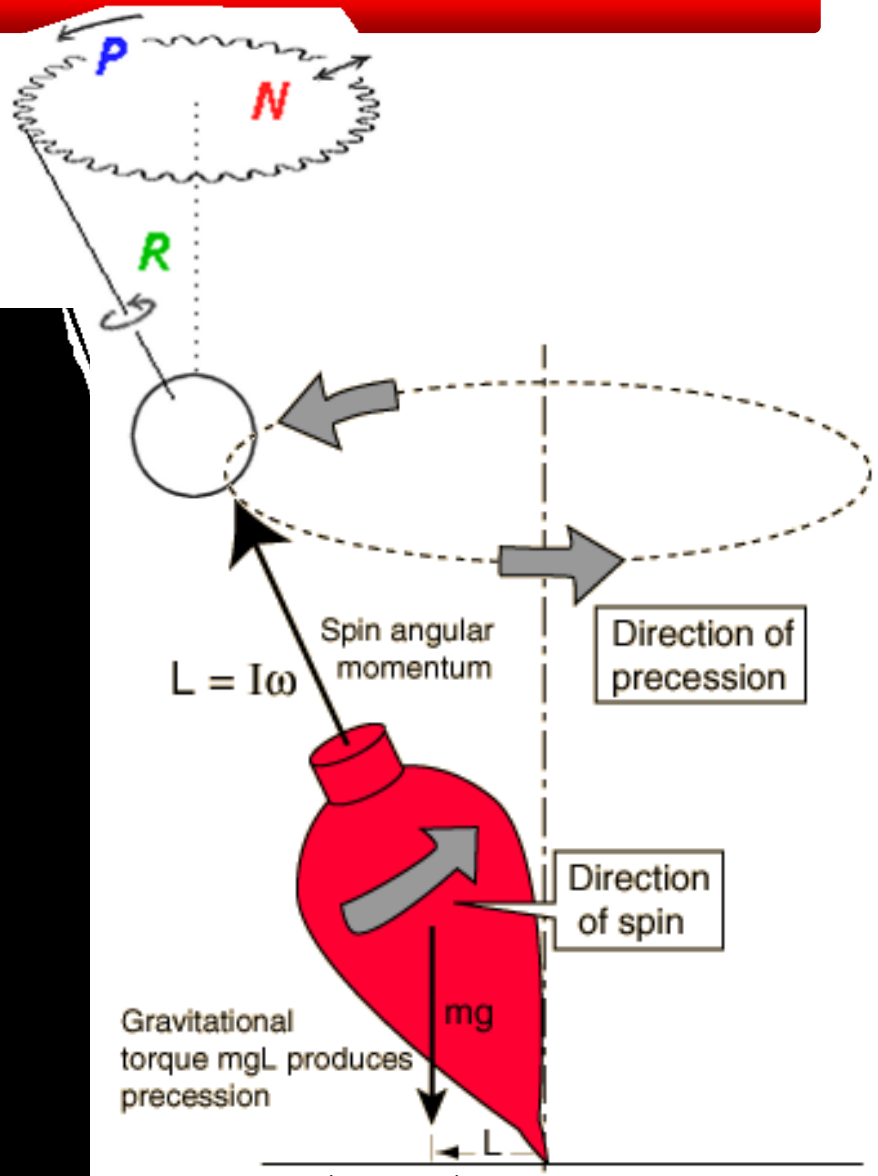


Nutation: The Earth's axis is like a Top

From German Wikipedia, by User Herbye.

1980 IAU Model vs.
VLBI observed Nutation





Credit: www.4physics.com

Credit Sovers, Fanselow, Jacobs, Rev Mod Phys 70, 4, Oct 1998.

Atmosphere

- Plasmas
  Charged (ionized) atmosphere
  ~ 350 km above Earth surface

- Neutral atmosphere 0 to ~10 km
  Troposphere
  Hydrostatic ("dry") ~ 2 m of delay
  Wet ~ 0.1 m of delay

Ionosphere

- Plasmas:Charged (ionized) atmosphere

Plasma	$\rho(e/m^3)$	ν_p (kHz)	(ν_p/ν_S)	(ν_p/ν_X)
Earth	10^{12}	8980	4×10^{-3}	10^{-3}
Interplanetary	10^{10}	898	4×10^{-4}	10^{-4}
Interstellar	10^{7}	28	1×10^{-5}	3×10^{-6}

Credit: Sovers, Fanselow,Jacobs, 1998

- Index of refraction gives changes in delay vs. vacuum

$$n=\left[1-\left(\frac{\nu_p}{\nu}\right)^2\left(1\pm\frac{\nu_g}{\nu}\cos\Theta\right)^{-1}\right]^{1/2},$$

Spitzer (1962)

- The 1/frequency squared effect can be calibrated by observing at two frequencies e.g 2.3/8.4 GHz or 8.4/ 32 GHz

Troposphere

Index of refraction > 1 yields extra delay

$N \sim a/T + b/T^2$

1/T: dry components even mixing ratio

$1/T^2$: water is polar molecule

ray trace

1/sin(elevation)

curvature corrections:

dry 8km/6400km ~ 1/800

wet 2 km/6400 ~ 1/3200

Marini form,

$$m(\varepsilon) = \cfrac{1}{\sin(\varepsilon) + \cfrac{a}{\sin(\varepsilon) + \cfrac{b}{\sin(\varepsilon) + \cfrac{c}{\cdots}}}}$$

Lanyi form: non-linear dry-wet term

$$a_1 D + a_2 W + a_3 DW + a_4 D^2 + a_5 W^2 + a_6 D^3$$

Troposphere

Index of refraction n > 1 yields extra delay

$N = (n-1) \sim a/T + b/T^2$ where T is temperature

1/T: dry components, even mixing ratio (induced dipole)

$1/T^2$: water is polar molecule (permanent diploe moment)

Mapping function relates zenith delay to delay at elevation

For flat slab atmosphere Mapping = 1/sin(elevation)

Troposphere Delay at Zenith

Sea level pressure is 1013 millibar (mb)
Zenith Hydrostatic delay: Pressure * 2.2768 mm / mb

1. Calculate zenith delay at sea level

HartRAO altitude h = 1416 m
Pressure scales as
$p(h) = p(\text{sea level}) \exp(-h/H)$
Where h is altitude, H is scale height = 8.567km

2. Calculate p(h(HartRAO))
3. Calculate zenith delay at HartRAO

Troposphere Mapping Function

Assume

1. troposphere delay depends on Elevation
   but not Azimuth (azimuthal symmetry)
2. Assume flat earth within 100km of station
   $\tau(E) = \tau(\text{zenith}) / \sin(\text{elevation})$

curvature corrections:

dry 8km / 6400 km ~ 1/ 800

wet 2 km / 6400 km ~ 1/3200

Marini mapping (1972) form

$$m(\varepsilon) = \cfrac{1}{\sin(\varepsilon) + \cfrac{a}{\sin(\varepsilon) + \cfrac{b}{\sin(\varepsilon) + \cfrac{c}{\cdots}}}}$$

Lanyi form: non-linear dry-wet term

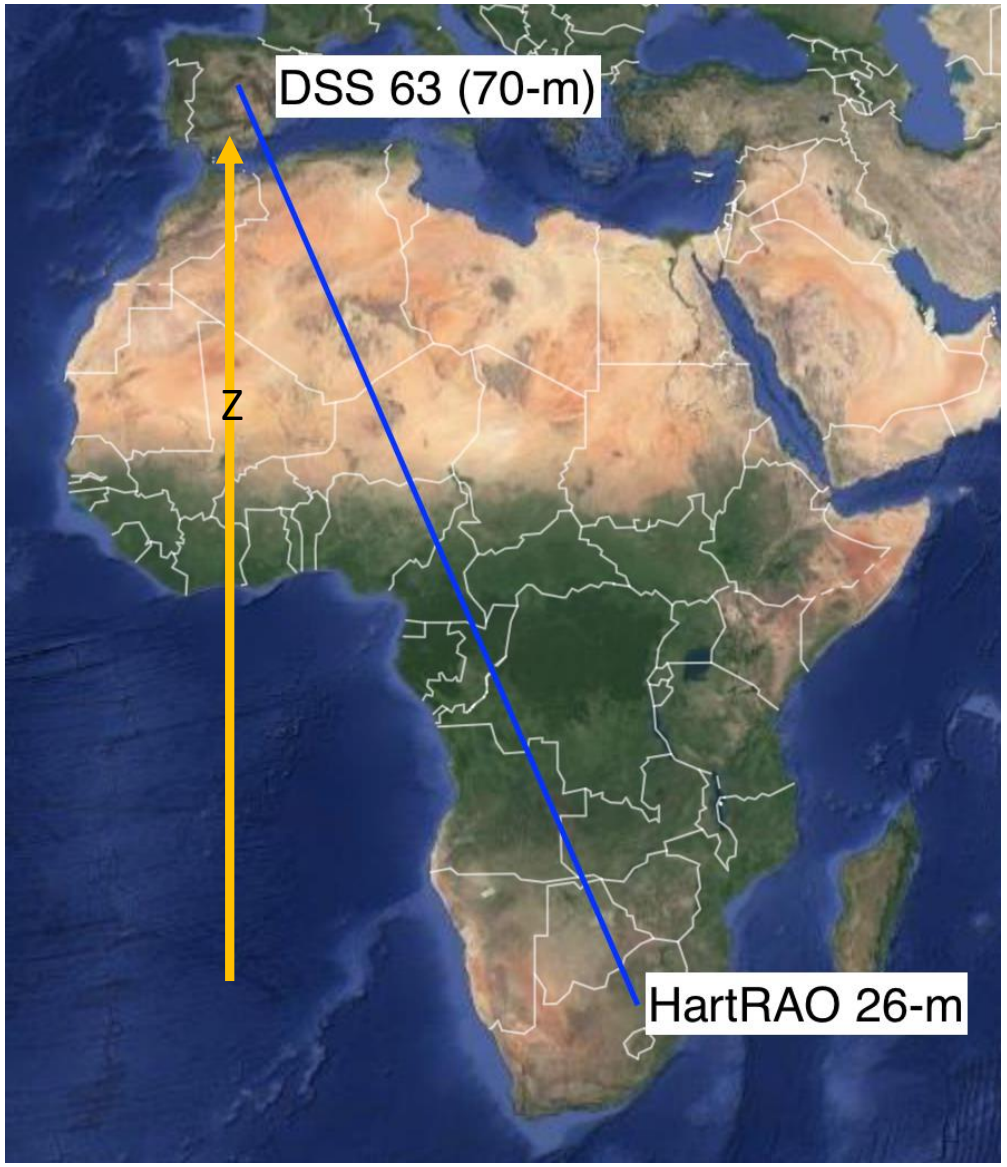
$$a_1 D + a_2 W + a_3 DW + a_4 D^2 + a_5 W^2 + a_6 D^3$$

Example: Long Baseline

HartRAO- Robledo:

What can you tell me about this baseline?

What is dominant component?
Will this help RA or Dec more?



Credit: Google maps

Example: Long Baseline

Station	X	Y	Z .
Robledo 70m	4849	-360	4115 km
HARTRAO 26m	5085	2668	-2768 km

1. a. Calculate baseline length, L
   b. North-South Component Z
   c. Equatorial component (sqrt($X^2 + Y^2$))
   d. Express in km then in seconds (distance/c)

2. Normalize Length, L by wavelength
   a. L/ λ(3.6cm)
   b. L/ λ(1.2 cm)

Example: Long Baseline

3. Calculate Resolution (radians, μas):
   λ(3.6 cm) / L
   λ(1.2 cm) / L
2. What is dominant component?
   Will this help RA or Dec more?
3. Time scale of troposphere changes?
4. Time scale of tide changes
5. Time scale of instrumental changes

Example: Long Baseline

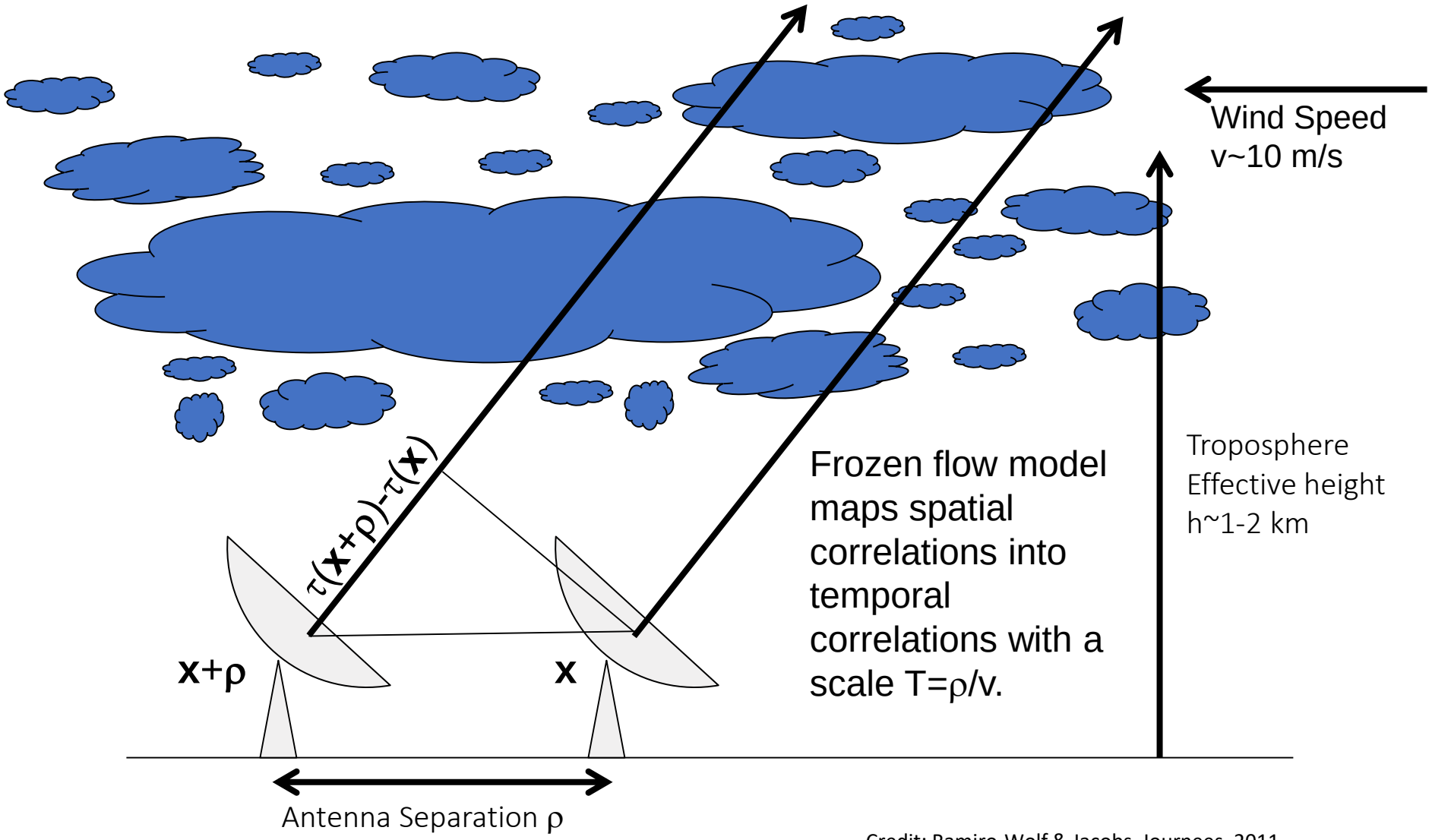
3. Calculate Resolution (radians, μas):
   λ(3.6 cm) / L
   λ(1.2 cm) / L

4. What are pros/cons of observing at 3.6cm vs. 1.2cm?
   Ionosphere? S/X dual-band simultaneous vs.
   K-band with GPS calibrations
   Troposphere?
   Resolution?
   Sensitivity?

Example: Short Baseline

HartRAO 26-meter to 15-meter baseline

Differential Troposphere Delay



Credit: Ramiro-Wolf & Jacobs, Journees, 2011

Example: Short Baseline

HartRAO 26-meter to 15-meter baseline

1. Guess baseline length, L
   Give first in km then in seconds (distance/c)
2. Antenna structure
3. To what extent do effects cancel (common mode)
4. Time scale of troposphere changes?
5. Time scale of geophysics: plate tectonics, tides
6. Time scale of instrumental changes
   Common clock?

Summary

Effects which need modelled:

- Geometry
- Geophysics
- Atmosphere

etc.

References:

Sovers, Fanselow, Jacobs, Reviews Modern Physics, 70, 4, 1998.
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