

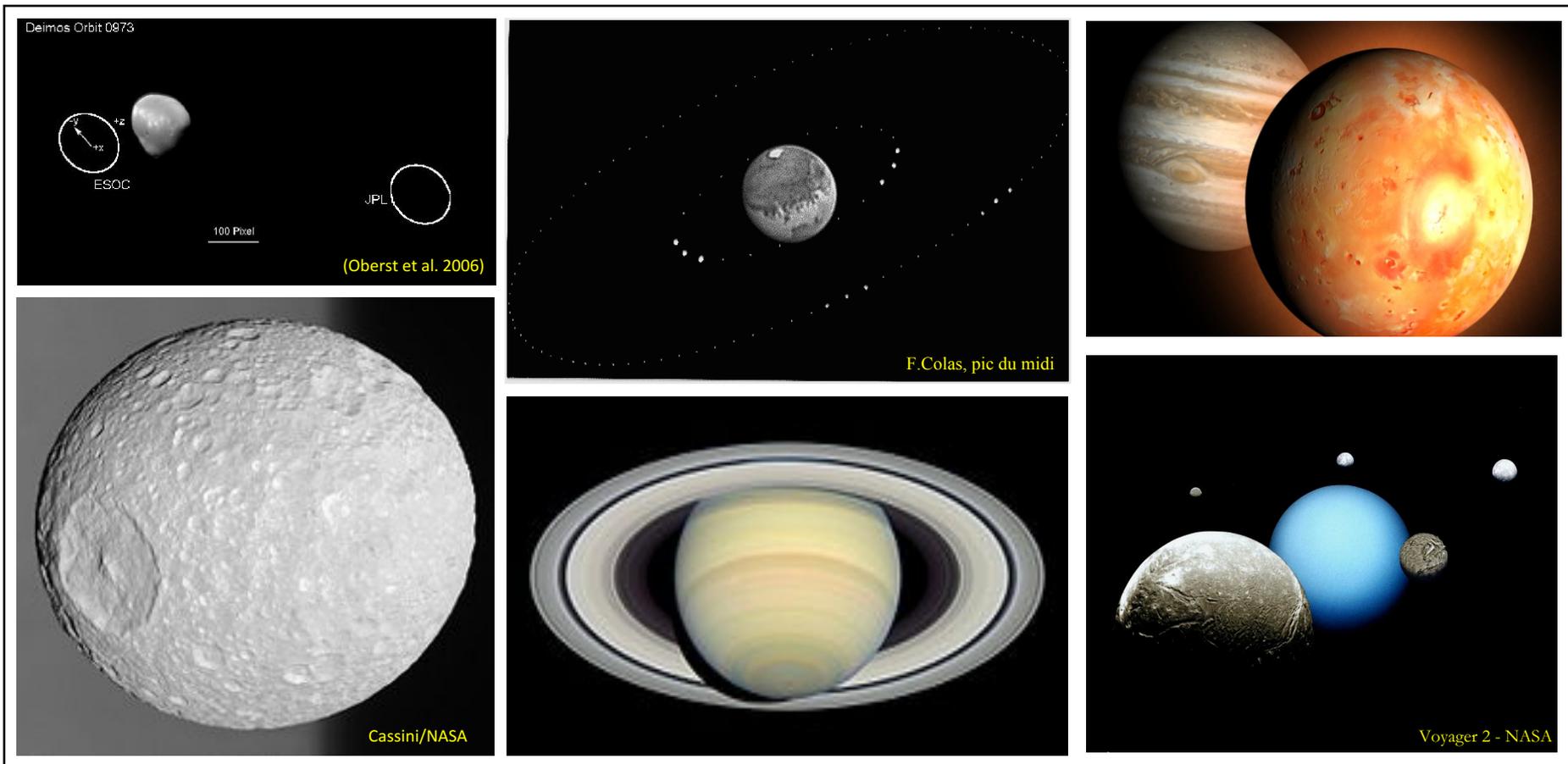
# Space geodesy vs. classical astrometry: a high complementarity



V. Lainey

Jet Propulsion Laboratory, California Institute of Technology - IMCCE

Contact: [valery.j.lainey@jpl.nasa.gov](mailto:valery.j.lainey@jpl.nasa.gov)



AGU, New Orleans, December 11<sup>th</sup> 2017

## Space geodesy and classical astrometry have much in common:

### SAME MOTIVATION:

Study the interior properties of celestial objects

Try assessing their formation and long term evolution

Provide orbit/ephemeris useful for the whole astronomic community

### SAME ASSET:

Radio-science is mandatory for the success of any mission

There always is a camera...

→ We are here anyway!

Space geodesy and classical astrometry have much in common:

### SAME METHODOLOGY:

Integration of equations of motion

$$\ddot{\vec{r}}_i = -G(m_0 + m_i) \left( \frac{\vec{r}_i}{r_i^3} - \nabla_i U_{\vec{r}_i \hat{0}} + \nabla_0 U_{\vec{r}_i \hat{0}} \right) + \sum_{j=1, j \neq i}^N Gm_j \left( \frac{\vec{r}_j - \vec{r}_i}{r_{ij}^3} - \frac{\vec{r}_j}{r_j^3} - \nabla_j U_{\vec{r}_j \hat{i}} + \nabla_i U_{\vec{r}_j \hat{i}} + \nabla_j U_{\vec{r}_j \hat{0}} - \nabla_0 U_{\vec{r}_j \hat{0}} \right)$$

$$+ \frac{(m_0 + m_i)}{m_i m_0} (\vec{F}_{\vec{r}_i \hat{0}}^T - \vec{F}_{\vec{r}_i \hat{0}}^T) - \frac{1}{m_0} \sum_{j=1, j \neq i}^N (\vec{F}_{\vec{r}_j \hat{0}}^T - \vec{F}_{\vec{r}_j \hat{0}}^T) + GR$$

Simultaneously with the variational equations

$$\frac{\partial}{\partial c_l} \left( \frac{d^2 \vec{r}_i}{dt^2} \right) = \frac{1}{m_i} \left[ \sum_j \left( \frac{\partial \vec{F}_i}{\partial \vec{r}_j} \frac{\partial \vec{r}_j}{\partial c_l} + \frac{\partial \vec{F}_i}{\partial \dot{\vec{r}}_j} \frac{\partial \dot{\vec{r}}_j}{\partial c_l} \right) + \frac{\partial \vec{F}_i}{\partial c_l} \right]$$

Unknown parameters are obtained by comparison of the modeled position with real ones using least square fit

## With few significant differences:

- **Orbit:**

S/C have polar orbit while SAT have equatorial orbits

→ SAT and S/C are sensitive to different harmonics of the primary's gravity field

- **Modeling/data treatment:**

S/C data are regularly splitted into arcs (wheel off loading, drag pressure...) while SAT= 1 arc

→ S/C will be useful for short term dynamics (gravity fields, mutual perturbations...) while SAT will be useful for long term dynamics (tidal effects...)

- **Observation accuracy is way different:**

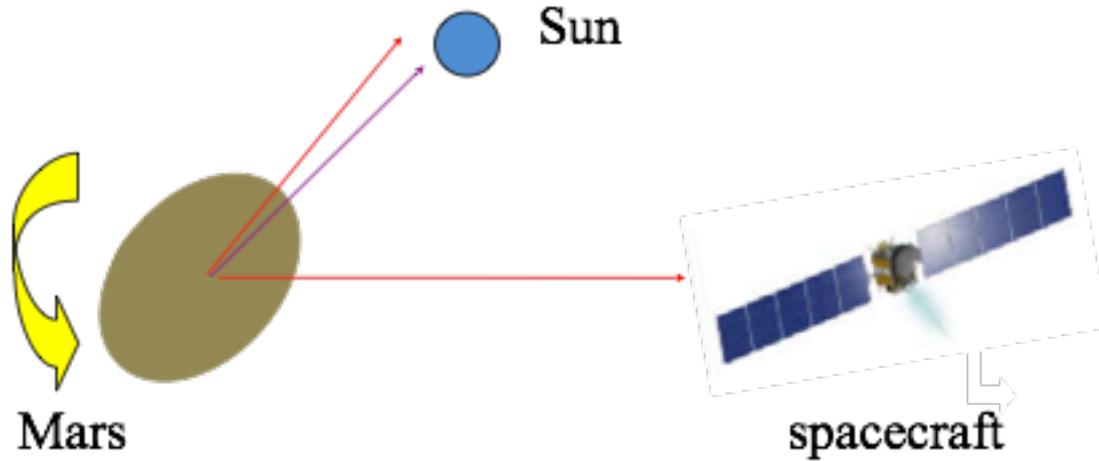
Astrometry: typically few km to few hundred of km

Geodesy: few meters to few km

**There is a huge complementarity!**

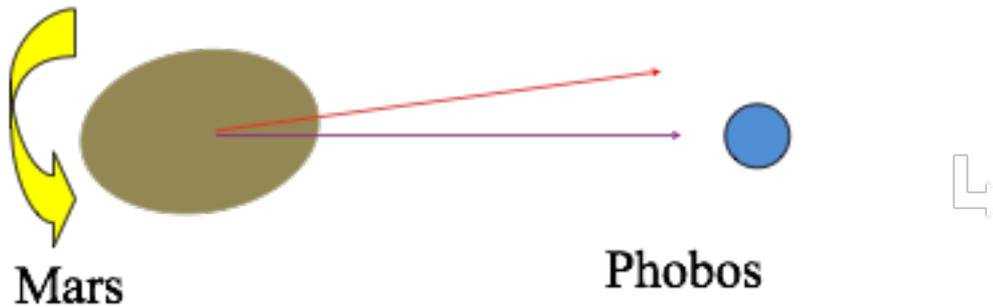
## Example of the Mars system

*Space geodesy:*



Allows the determination  
of  $k_2$

*Astrometry:*



Allows the determination  
of  $k_2/Q$

## Example of the Mars system

Estimation of Mars Love number over time (Konopliv et al. 2011):

Spacecraft	Re( $k_2$ ), Im( $k_2$ ) = 0	Re( $k_2$ ), Im( $k_2$ ) = 0.01	Re( $k_2$ ), Im( $k_2$ ) est.	Notes
MGS	0.173 ± 0.009	0.168 ± 0.009	0.159 ± 0.016 0.023 ± 0.025	Best overall solution this paper Im( $k_2$ ) = 0, 5× formal $\sigma$
MGS	0.153 ± 0.017			Data to April 14, 2002, Yoder et al. (2003)
MGS	0.166 ± 0.011			Data to December 5, 2004, Konopliv et al. (2006)
ODY	0.172 ± 0.014	0.185 ± 0.014	0.167 ± 0.025 -0.004 ± 0.016	Without arcs affected by dust, best Odyssey solution Im( $k_2$ ) = 0, 10× formal $\sigma$
ODY	0.104 ± 0.013		0.015 ± 0.021 -0.076 ± 0.014	All arcs but with no dust model
ODY	0.161 ± 0.013	0.173 ± 0.013	0.131 ± 0.022 -0.025 ± 0.014	All arcs but with dust model to 30–40 km
ODY	0.172 ± 0.013	0.184 ± 0.013	0.154 ± 0.022 -0.015 ± 0.014	All arcs but with dust model to 30–50 km
MRO	0.175 ± 0.010	0.175 ± 0.010	0.176 ± 0.010 0.036 ± 0.040	10× formal $\sigma$
<i>Other determinations</i>				
MGS	0.201 ± 0.059			Bills et al. (2005)
	0.163 ± 0.056			
MGS	0.176 ± 0.041			Lemoine et al. (2006)
MGS	0.130 ± 0.030			Balmino et al. (2005) (see Marty et al., 2009)
MGS + ODY	0.120 ± 0.003			Marty et al. (2009)

Konopliv et al. (2011)  
provides  $k_2=0.164 \pm 0.009$

Estimation of Phobos tidal acceleration over time (Jacobson 2010):

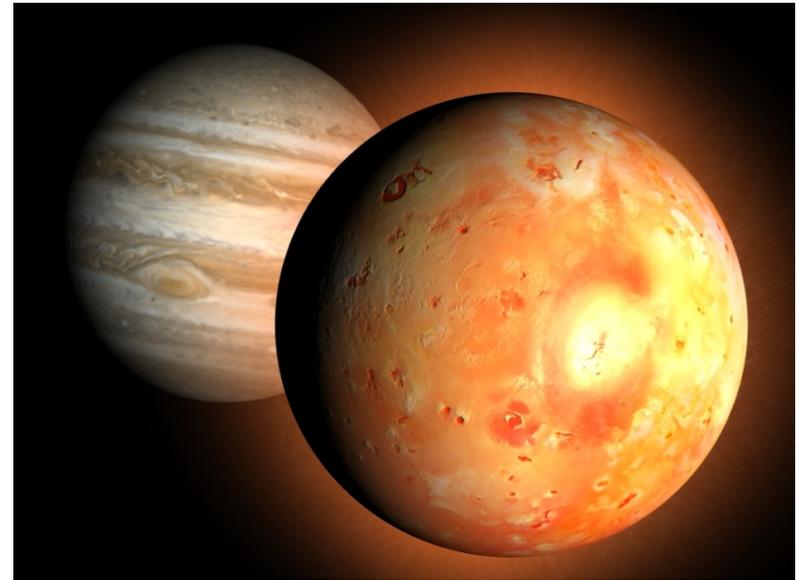
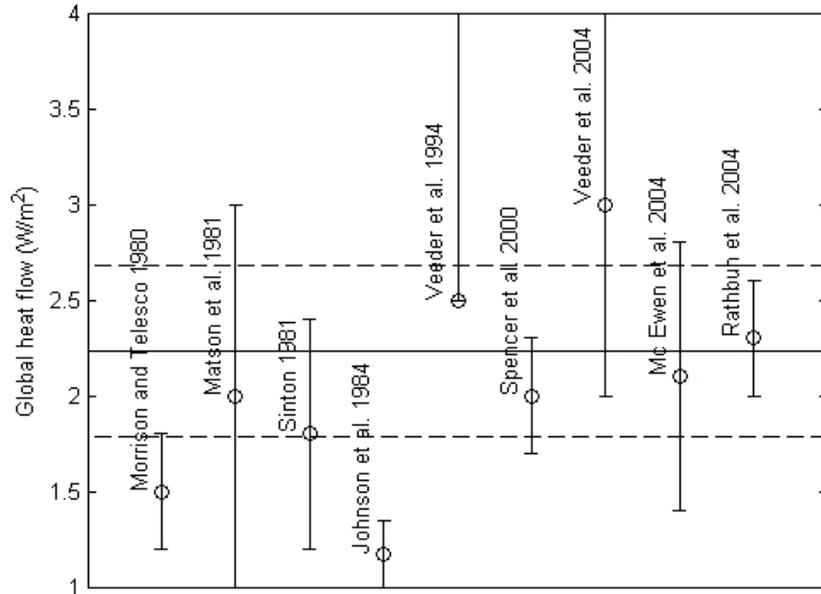
Reference	$s \times 10^{-3}$ (deg yr <sup>-2</sup> )	$\kappa_2$	$Q$	$\gamma$ (deg)
Sharpless (1945)	1.882 ± 0.171			
Shor (1975)	1.427 ± 0.147			
Sinclair (1978)	1.326 ± 0.118			
Jacobson et al. (1989)	1.249 ± 0.018			
Chapront-Touzé (1990)	1.270 ± 0.008			
Emelyanov et al. (1993)	1.290 ± 0.010			
Bills et al. (2005)	1.367 ± 0.006	0.163	85.6 ± 0.4	0°3346 ± 0°0014
Rainey & Aharonson (2006)	1.334 ± 0.006	0.153	78.6 ± 0.8	0°3645 ± 0°0039
Lainey et al. (2007)	1.270 ± 0.015	0.152	79.9 ± 0.7	0°3585 ± 0°0031
Current	1.270 ± 0.003	0.152	82.8 ± 0.2	0°3458 ± 0°0009

Pretty good agreement since decades!

## Example of the Jovian system:

Lainey et al. 2009 determined Io's tidal dissipation to be:  $k_2/Q = 0.015 \pm 0.003$

One can compare this value with the ones derived from IR emission

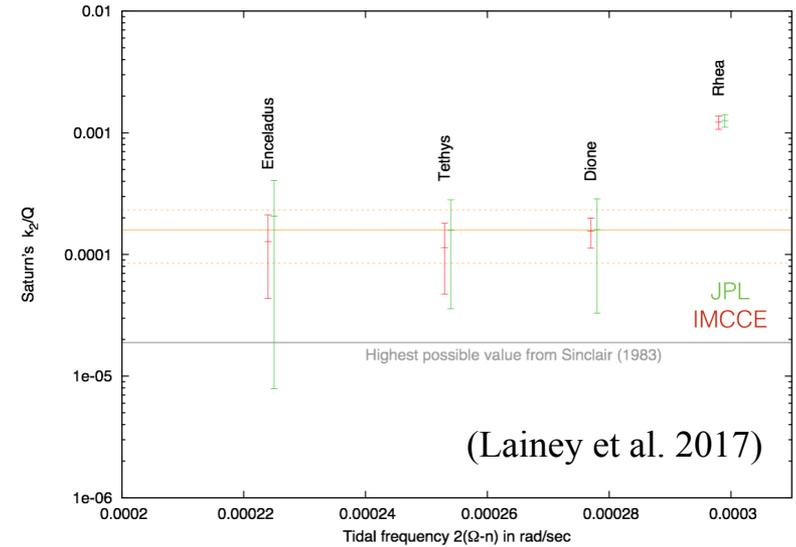


Dirkx et al. 2016, 2017 showed that astrometry of Io will be mandatory to properly benefit of Juice tracking data when in orbit around Ganymede (Laplace resonance issue)

# Example of the Saturnian system:

## Determination of Saturn's $k_2/Q$

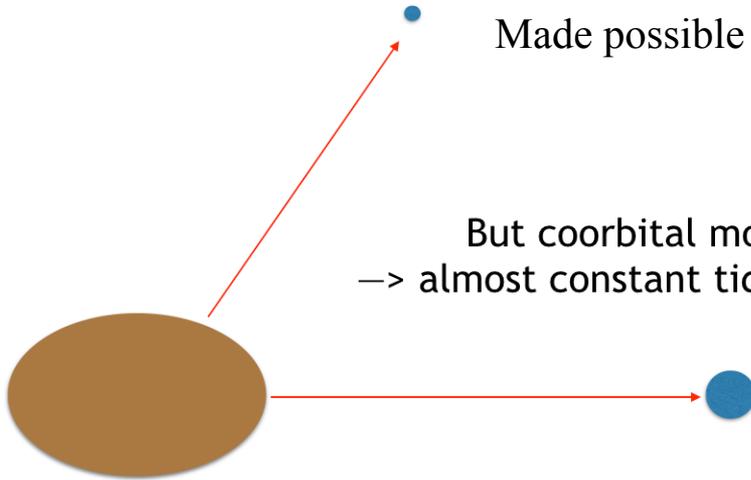
Made possible thanks to astrometric long time span!



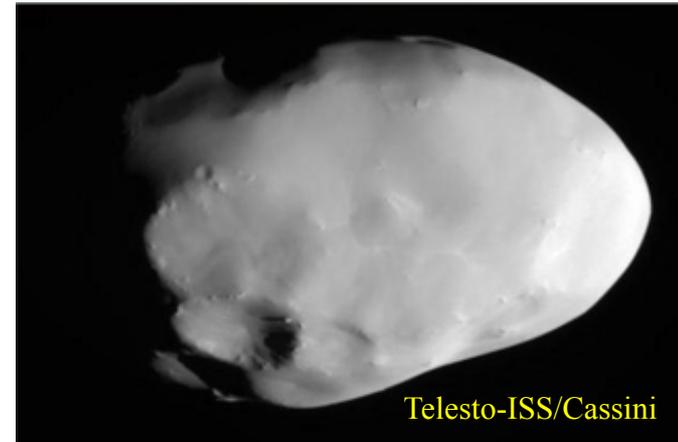
## Determination of Saturn's $k_2$

Made possible thanks to the presence of four Lagrangian satellites!

But coorbital moons  
→ almost constant tidal angle!!



$$k_2 = 0.390 \pm 0.024 \text{ (Lainey et al. 2017)}$$



## Conclusion:

- Space geodesy and classical astrometry are extremely complementary
- Both discipline evolve fast with significant technological improvement
- Cassini mission provided excellent results with astrometry as this was part of the mission right from the beginning
- Increasing scientific exchanges, especially in the context of further space mission would benefit to everyone!

