White Noise & stability study for the CMB Probe, TDM option

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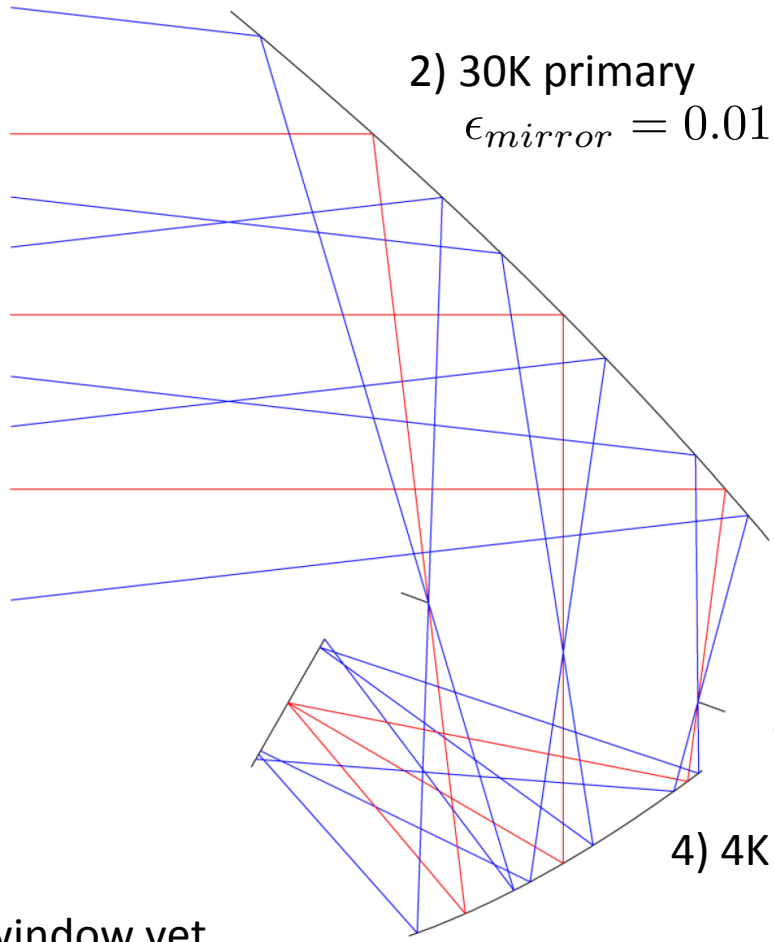
Outline

1. Optical Loading
2. Bolometer Gs
3. NEP & NEI vs audio frequency
4. Aliasing
5. Science channel white noise NEP totals
6. Vetting of model against BICEP2/Keck
7. 1/f stability

Assumptions on Optical Loading[1]

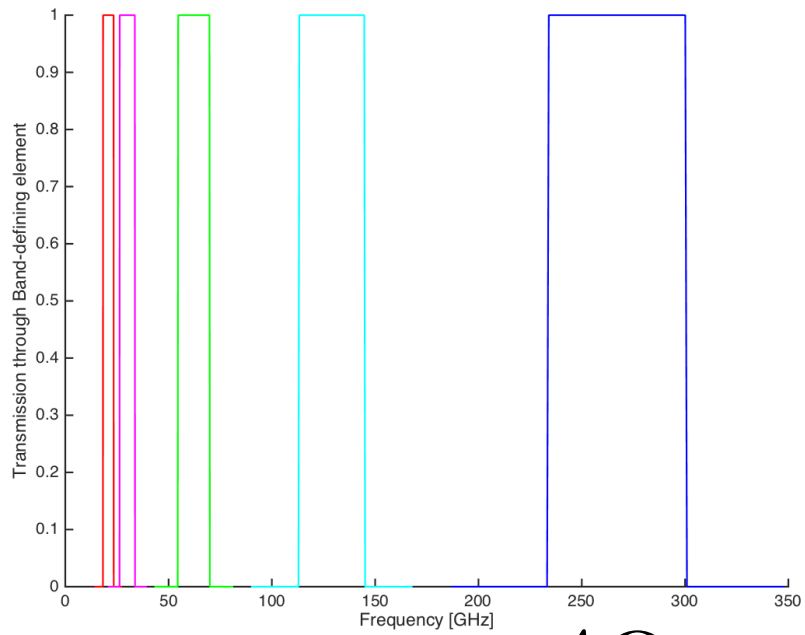


- No filters, lenses, or window yet

$$P_i = \int d\nu \frac{h\nu^3/c^2}{e^{h\nu/k_B T_i} - 1} \left[\eta_{det} B(\nu)\right] \epsilon_i(\nu) \prod_{j>i} (1 - \epsilon_i(\nu))$$

Assumptions on Optical Loading[1]

- 25% bandwidth, top-hat profiles
- Five illustrative bands (21 total):



- Antenna-coupled: single moded $A\Omega = \lambda^2$
- Detector efficiency $\eta_{\mathrm{det}} = 0.7$

Optical Loading

Channel	Cent_Freq [GHz]	P_cmb	P_mirror1	P_stop	P_mirror2	P_total	T_rj [K]
1	20.8	0.1000	0.0073	0.0174	0.0010	0.1257	2.50
2	25.0	0.1152	0.0096	0.0203	0.0013	0.1464	2.42
3	30.0	0.1317	0.0126	0.0236	0.0016	0.1695	2.34
4	36.0	0.1510	0.0166	0.0276	0.0020	0.1972	2.27
5	43.2	0.1664	0.0214	0.0312	0.0025	0.2215	2.12
6	51.8	0.1875	0.0287	0.0362	0.0032	0.2556	2.04
7	62.2	0.1971	0.0364	0.0395	0.0039	0.2769	1.84
8	74.6	0.2072	0.0473	0.0436	0.0047	0.3027	1.68
9	89.6	0.2112	0.0614	0.0471	0.0055	0.3252	1.50
10	107.5	0.2093	0.0805	0.0502	0.0064	0.3465	1.33
11	129.0	0.1934	0.1025	0.0508	0.0071	0.3538	1.14
12	154.8	0.1698	0.1317	0.0501	0.0077	0.3593	0.96
13	185.8	0.1381	0.1684	0.0471	0.0080	0.3615	0.81
14	222.9	0.1020	0.2142	0.0416	0.0077	0.3655	0.68
15	267.5	0.0682	0.2739	0.0347	0.0070	0.3839	0.59
16	321.0	0.0380	0.3389	0.0255	0.0057	0.4081	0.53
17	385.2	0.0186	0.4309	0.0174	0.0042	0.4712	0.51
18	462.2	0.0072	0.5210	0.0100	0.0027	0.5408	0.48
19	554.7	0.0021	0.6204	0.0048	0.0014	0.6286	0.47
20	665.6	0.0005	0.7336	0.0019	0.0006	0.7365	0.46
21	798.7	0.0001	0.8468	0.0006	0.0002	0.8477	0.44

- All power units are pW
- P_total sums contributions from 4 sources
- $T_{RJ} = P_{tot}/(k_B \eta \Delta\nu)$

Bolometer Properties: G

- Bolometer saturates at a safety factor of SF=2.5 above total loading[2] $P_{sat} = P_{tot} + P_{Joule} = SF * P_{tot}$
- Thermal Conductivity at Transition[1]:

$$P_{\rm sat} = G_c T_c \frac{1-(T_o/T_c)^{\beta+1}}{\beta+1}$$

- Can rescale to other temps (e.g. Bath T_o, 450mK)

$$G_* = G_c (T_*/T_c)^{\beta}$$

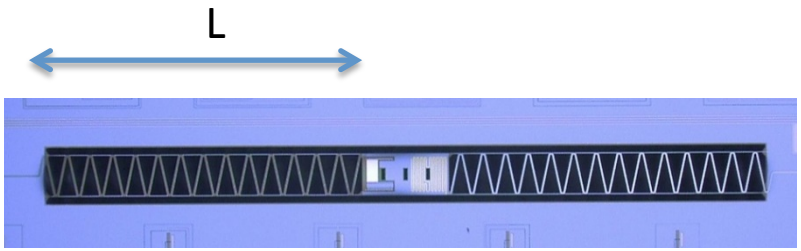
- $\beta \sim 2$ because leg thickness is less than a phonon wavelength[2,3]
- Thermal Time-constant, sans feedback[4]: $\tau_o = C/G$
- Loop gain from electrothermal feedback[3,4] $\mathcal{L} = \frac{\alpha P_{\rm Joule}}{G_c T_c} \sim 17$
- α describes changes in TES *R* vs *T*- typically ~100
- Thermal Time-constant, with loop gain[4]: $\tau = \frac{\tau_o}{\mathcal{L}-1}$

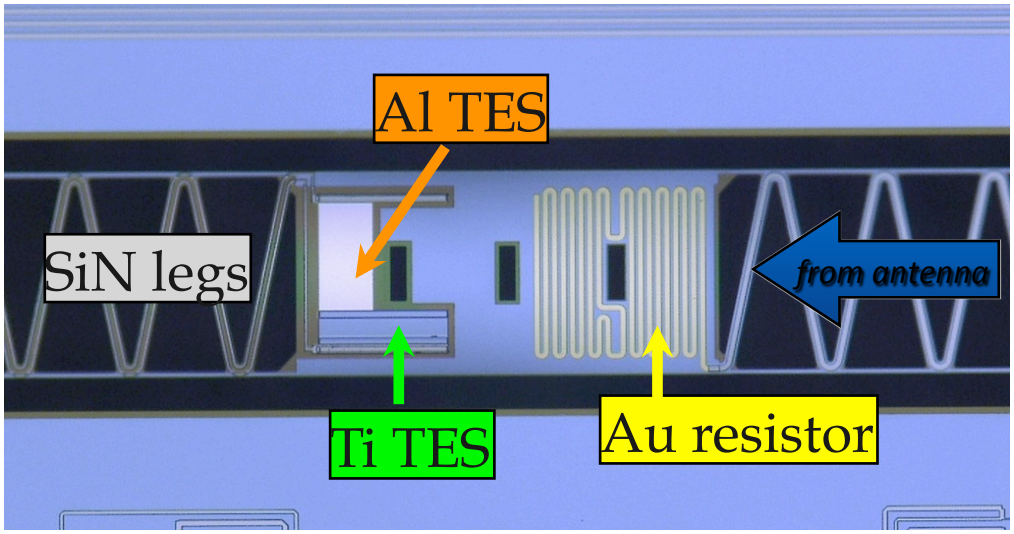
Bolometer Properties: G &C

Channel	Cent_Freq [GHz]	P_total	G0	Gc	G450	tau [ms]
1	20.8	0.13	0.54	2.10	1.73	59.21
2	25.0	0.15	0.62	2.44	2.02	50.82
3	30.0	0.17	0.72	2.83	2.34	43.90
4	36.0	0.20	0.84	3.29	2.72	37.72
5	43.2	0.22	0.94	3.70	3.06	33.58
6	51.8	0.26	1.09	4.27	3.53	29.10
7	62.2	0.28	1.18	4.62	3.82	26.87
8	74.6	0.30	1.29	5.05	4.18	24.57
9	89.6	0.33	1.38	5.43	4.49	22.88
10	107.5	0.35	1.48	5.79	4.78	21.47
11	129.0	0.35	1.51	5.91	4.88	21.03
12	154.8	0.36	1.53	6.00	4.96	20.71
13	185.8	0.36	1.54	6.04	4.99	20.58
14	222.9	0.37	1.56	6.10	5.04	20.35
15	267.5	0.38	1.63	6.41	5.30	19.38
16	321.0	0.41	1.74	6.81	5.63	18.23
17	385.2	0.47	2.01	7.87	6.50	15.79
18	462.2	0.54	2.30	9.03	7.46	13.76
19	554.7	0.63	2.68	10.50	8.67	11.84
20	665.6	0.74	3.14	12.30	10.16	10.10
21	798.7	0.85	3.61	14.15	11.70	8.78

- T_o=250mK, T_c=500mK (Ti)
- These Gs will be easier to achieve with low T_o~100mK than 250mK (see next slide)
- I've presumed C~1pJ/K, achievable with few um thick Gold at T_c~500mK. By thinning/omitting the gold, we can speed some of these up. (Or drop the T_c)

Bolometer Design Considerations





- G_c~2pW/K (T_c~500mK) **is challenging** for existing JPL recipes.
- Scaling from lowest JPL achieved ~15pW/K (SPIDER 95GHz[5]) detector, a 20GHz CMB-probe would use L~10mm long leg, zig-zagged to be~ 40mm long.
- Largest device we have made has L~ 1mm long legs
- If we change bath temp from 250mK ->100mK, then G drops by (100/250)^2 for a given geometry.
- So we would need L~1.5mm to achieve G_c~2pW/K or G_o~0.54pW/K.

G &C for T_o=100mK

Channel	Cent_Freq [GHz]	P_total	G0	Gc	tau [ms]
1	20.8	0.13	1.29	5.17	24.16
2	25.0	0.15	1.51	6.02	20.74
3	30.0	0.17	1.74	6.97	17.91
4	36.0	0.20	2.03	8.11	15.39
5	43.2	0.22	2.28	9.11	13.70
6	51.8	0.26	2.63	10.52	11.87
7	62.2	0.28	2.85	11.39	10.96
8	74.6	0.30	3.11	12.46	10.03
9	89.6	0.33	3.34	13.38	9.33
10	107.5	0.35	3.56	14.26	8.76
11	129.0	0.35	3.64	14.56	8.58
12	154.8	0.36	3.70	14.78	8.45
13	185.8	0.36	3.72	14.87	8.40
14	222.9	0.37	3.76	15.04	8.30
15	267.5	0.38	3.95	15.79	7.91
16	321.0	0.41	4.20	16.79	7.44
17	385.2	0.47	4.85	19.38	6.44
18	462.2	0.54	5.56	22.25	5.61
19	554.7	0.63	6.47	25.86	4.83
20	665.6	0.74	7.58	30.30	4.12
21	798.7	0.85	8.72	34.88	3.58

- T_o=100mK, T_c=200mK (AlMn, Hf)
- Strong driver for lower temperature

Detector NEP, no aliasing yet[1,6]

$$NEP^2_{\gamma,shot} = 2h\nu P_{tot}$$

$$NEP^2_{\gamma,bose} = 2\nu P^2_{tot}/\sqrt{\Delta\nu}$$

Photon Noise

- ν refers to optical frequencies: 10GHz-1THz

$$NEP^2_{TES} = 4K_B T_c R_o I_o^2 \frac{1}{\mathcal{L}}(1 + (2\pi f \tau_o)^2)$$

$$NEP^2_{shunt} = 4k_B T_{sh} R_{sh} I_o^2 \frac{(\mathcal{L}-1)^2}{\mathcal{L}^2}(1 + (2\pi f \tau)^2)$$

Johnson Noise

- f refers to audio time-stream frequencies: 10mHz-10kHz

$$NEP^2_G = 4\gamma(T_o, T_c) k_B G_c T_c^2$$

$$\gamma(T_o, T_c) \equiv \frac{1+\beta}{2\beta+3} \frac{(T_o/T_c)^{3+2\beta} - 1}{(T_o/T_c)^{1+\beta} - 1}$$

Phonon (Leg) Noise

Time Constants of Modes[6]

- Detector performance governed by linearized 1st-order differential equations for $(\delta I, \delta T)$
- Modes have eigenvalues of:

$$\frac{1}{\tau_\pm} = \frac{1}{2\tau_{el}} + \frac{1-\mathcal{L}}{2\tau_o} \pm \sqrt{\left(\frac{1}{\tau_{el}} - \frac{1-\mathcal{L}}{\tau_o}\right)^2 - 4\frac{R_o\mathcal{L}(2+\beta_I)}{\tau_o}}$$

- Where the electrical time constant is:

$$\tau_{el} = \frac{L}{R_{sh} + R_o(1+\beta_I)}$$

- β_I describes changes in TES resistance vs current- normally 0.
- The responsivity rolls off as single poles with time-constants $\tau_\pm$

Current Noise and SQUID Noise[6]

- Responsivity:

$$S(f) = -\frac{(1-\tau_+/\tau)(1-\tau_-/\tau)}{I_o R_o(2+\beta_I)} \frac{1}{\sqrt{(1+(2\pi f\tau_+)^2)(1+(2\pi f\tau_-)^2)}}$$

- In the lab, we often approximate this to:

$$S(f) \approx -1/V_{bias}$$

- Units: A/W. This interrelates NEI and NEP:

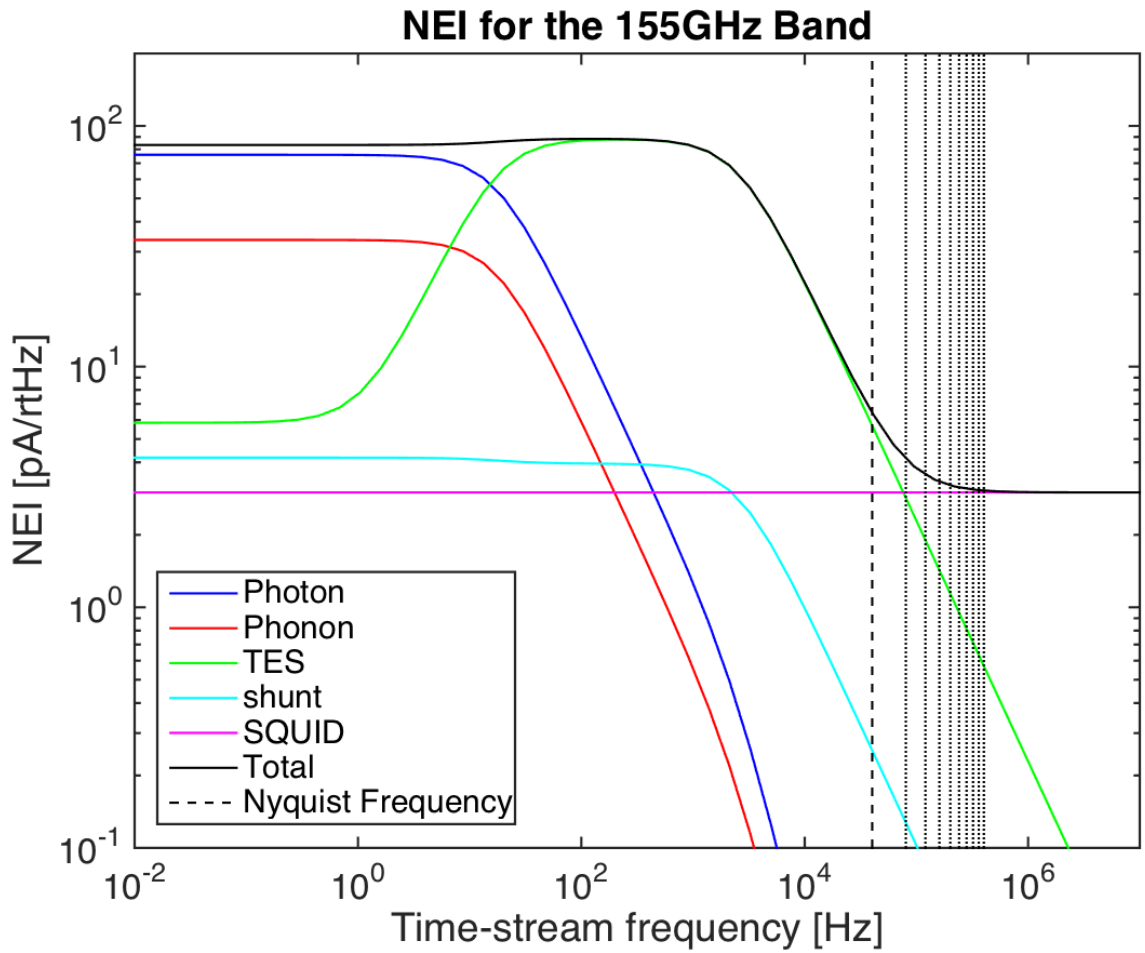
$$NEI_*^2 = S(f)^2 NEP_*^2$$

- The SQUID has white broad-band current noise. Most measured to be NEI~3pA/√Hz
- Responsivity still applies, but with a transfer function bearing an additional pole of:

$$\frac{1}{\sqrt{1+(2\pi f L/R)^2}}$$

- R is the dynamic resistance of the SQUID, ~5Ω
- L is the inductance, including the Nyquist Inductor to limit aliasing, typically ~1-2uH.

NEI frequency dependence



- Curves for the 155GHz channel (Channel no 12)
- Science channel at left, probably 0.01Hz-10Hz
- No flicker noise yet
- Aliasing will fold current noise at frequency multiples of f_{Nqy} (faint dashed lines) into the in-band noise.

Aliased Noise[7]

- We roll-off high frequency electrical noise with LR circuits to mitigate aliasing. L_{nqy}~2uH in balloon and ground experiments.
- N_{MUX}=32 and 64 for existing systems, could increase to 128. I use 32 here.
- We must sample fast enough to detect high frequency noise set by stray inductance (~0.5uH):

$$f_{Nyq} = \frac{1}{2\pi} \frac{1}{2N_{MUX} L_{stray}/R_{SQ}}$$

- but not sample so fast that detectors don't have time to settle!
- For a given audio frequency *f*, we can sum aliased contributions to current noise in added multiples of the Nyquist frequency.

$$NEI^2_{*+aliasing} = NEI^2_* + \sum_{j=1}^{j_{max}} \left[NEI^2_*(2jf_{Nyq} - f) + NEI^2_*(2jf_{Nyq} + f) \right]$$

- We terminate the sum once the sum converges
  - j_{max} only needs to be 10 for detector noise because of low frequency poles.
  - j_{max} for SQUIDs needs to be much higher (1000) because the *NEI* is white. Sum increases in-band SQUID noise by ~50%.

Detector White NEPs

Channel	Cent_Freq [GHz]	NEP_photon	NEP_phonon	NEP_TES	NEP_shunt	NEP_det_alias	NEP_SQUID	NEP_total	NET_per_det [uK.rts]
1	20.8	3.09	2.36	0.41	0.29	0.35	2.12	4.47	72.17
2	25.0	3.43	2.54	0.44	0.32	0.38	2.12	4.89	66.14
3	30.0	3.80	2.74	0.48	0.34	0.41	2.12	5.34	60.69
4	36.0	4.26	2.95	0.51	0.37	0.44	2.12	5.87	55.51
5	43.2	4.68	3.13	0.54	0.39	0.47	2.12	6.34	51.42
6	51.8	5.27	3.36	0.58	0.42	0.50	2.12	7.00	47.13
7	62.2	5.73	3.50	0.61	0.44	0.52	2.12	7.47	44.34
8	74.6	6.32	3.66	0.64	0.46	0.55	2.12	8.07	41.69
9	89.6	6.95	3.79	0.66	0.47	0.57	2.12	8.67	39.76
10	107.5	7.66	3.91	0.68	0.49	0.58	2.12	9.35	38.53
11	129.0	8.29	3.95	0.69	0.49	0.59	2.12	9.90	38.85
12	154.8	8.99	3.98	0.69	0.50	0.59	2.12	10.52	40.81
13	185.8	9.76	3.99	0.69	0.50	0.60	2.12	11.19	45.78
14	222.9	10.65	4.02	0.70	0.50	0.60	2.12	11.99	56.39
15	267.5	11.89	4.12	0.72	0.51	0.61	2.12	13.16	77.94
16	321.0	13.38	4.24	0.74	0.53	0.63	2.12	14.58	130.01
17	385.2	15.71	4.56	0.79	0.57	0.68	2.12	16.90	256.84
18	462.2	18.40	4.89	0.85	0.61	0.73	2.12	19.58	645.15
19	554.7	21.70	5.27	0.92	0.66	0.79	2.12	22.87	2152.14
20	665.6	25.70	5.70	0.99	0.71	0.85	2.12	26.86	9477.52
21	798.7	30.17	6.12	1.06	0.76	0.91	2.12	31.31	59048.89

- Unless otherwise noted, units are aW/rtHz
- T_o=250mK, T_c=500mK
- *Not* Background limited for lowest 5 channels. Could assume more aggressive SF or lower T_c.
- Single Detector NET at far right. NOTE: explodes above 460GHz- need to check.

$$NET = NEP/(\sqrt{2}dP/dT)$$

- Aliasing penalty is small: 5% at most (lowest channel)

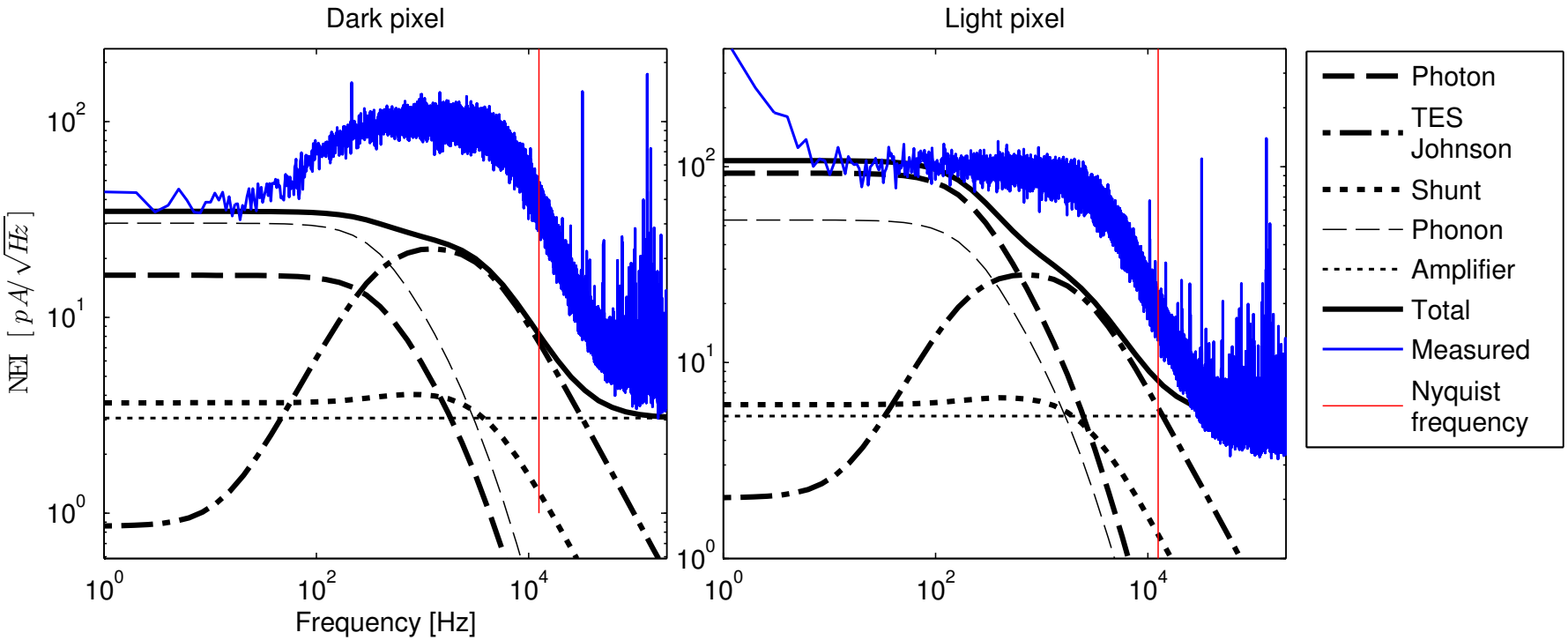
Detector White NEPs, lower temp

Channel	Cent_Freq [GHz]	NEP_photon	NEP_phonon	NEP_TES	NEP_shunt	NEP_det_alias	NEP_SQUID	NEP_total	NET_per_det
1	20.8	3.09	2.36	0.41	0.29	0.35	2.12	4.47	72.17
2	25.0	3.43	2.54	0.44	0.32	0.38	2.29	4.89	66.14
3	30.0	3.80	2.74	0.48	0.34	0.41	2.46	5.34	60.69
4	36.0	4.26	2.95	0.51	0.37	0.44	2.65	5.87	55.51
5	43.2	4.68	3.13	0.54	0.39	0.47	2.81	6.34	51.42
6	51.8	5.27	3.36	0.58	0.42	0.50	3.02	7.00	47.13
7	62.2	5.73	3.50	0.61	0.44	0.52	3.14	7.47	44.34
8	74.6	6.32	3.66	0.64	0.46	0.55	3.29	8.07	41.69
9	89.6	6.95	3.79	0.66	0.47	0.57	3.41	8.67	39.76
10	107.5	7.66	3.91	0.68	0.49	0.58	3.52	9.35	38.53
11	129.0	8.29	3.95	0.69	0.49	0.59	3.55	9.90	38.85
12	154.8	8.99	3.98	0.69	0.50	0.59	3.58	10.52	40.81
13	185.8	9.76	3.99	0.69	0.50	0.60	3.59	11.19	45.78
14	222.9	10.65	4.02	0.70	0.50	0.60	3.61	11.99	56.39
15	267.5	11.89	4.12	0.72	0.51	0.61	3.70	13.16	77.94
16	321.0	13.38	4.24	0.74	0.53	0.63	3.82	14.58	130.01
17	385.2	15.71	4.56	0.79	0.57	0.68	4.10	16.90	256.84
18	462.2	18.40	4.89	0.85	0.61	0.73	4.39	19.58	645.15
19	554.7	21.70	5.27	0.92	0.66	0.79	4.74	22.87	2152.14
20	665.6	25.70	5.70	0.99	0.71	0.85	5.13	26.86	9477.52
21	798.7	30.17	6.12	1.06	0.76	0.91	5.50	31.31	59048.89

- T_o=100mK, T_c=200mK, same SF=2.5
- Background limited for all channels

TDM Noise Stability[2]

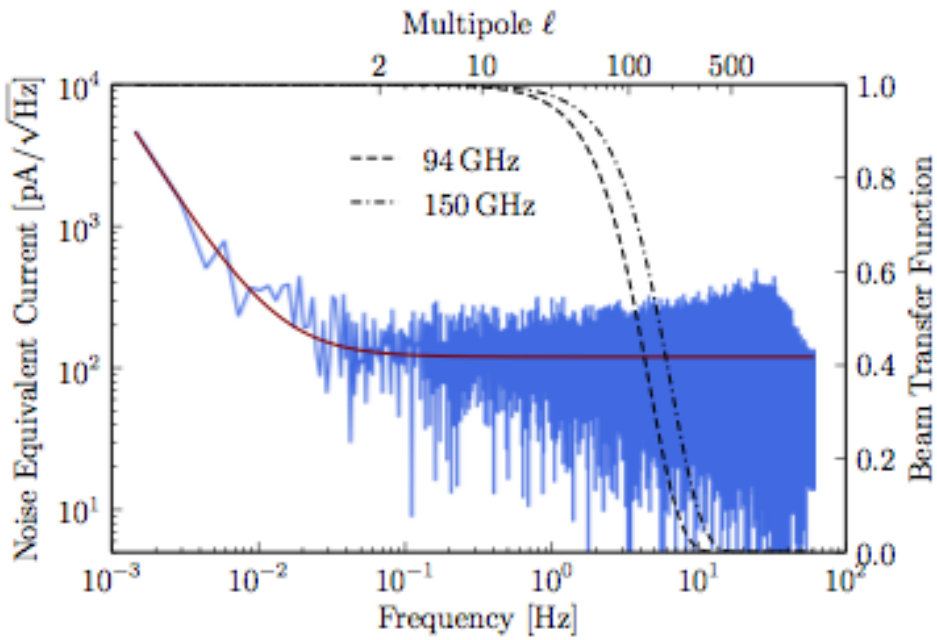
- Measured White Noise in Keck Array 150GHz: Dark vs light

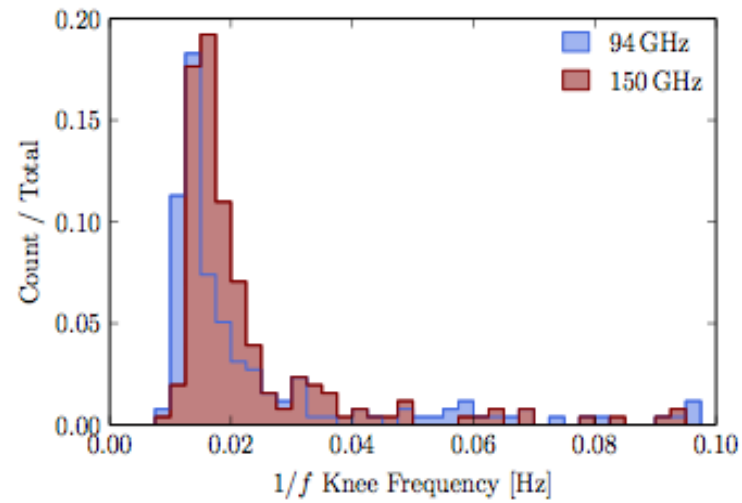


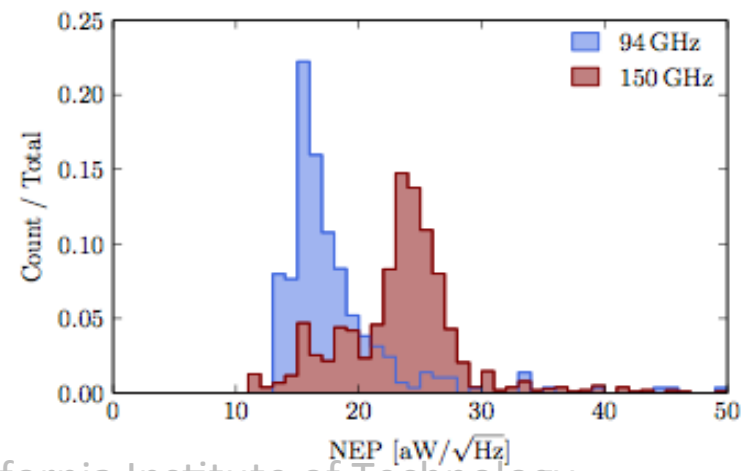
- Our model makes no attempts to model "excess noise" above the science channels
- Good Agreement between model and data for low-frequency white noise
- 1/f (atmospheric) ~ 5Hz

TDM Noise Stability[5]

- Pre-flight SPIDER Noise Characterization (in the lab) with an internal cold load:



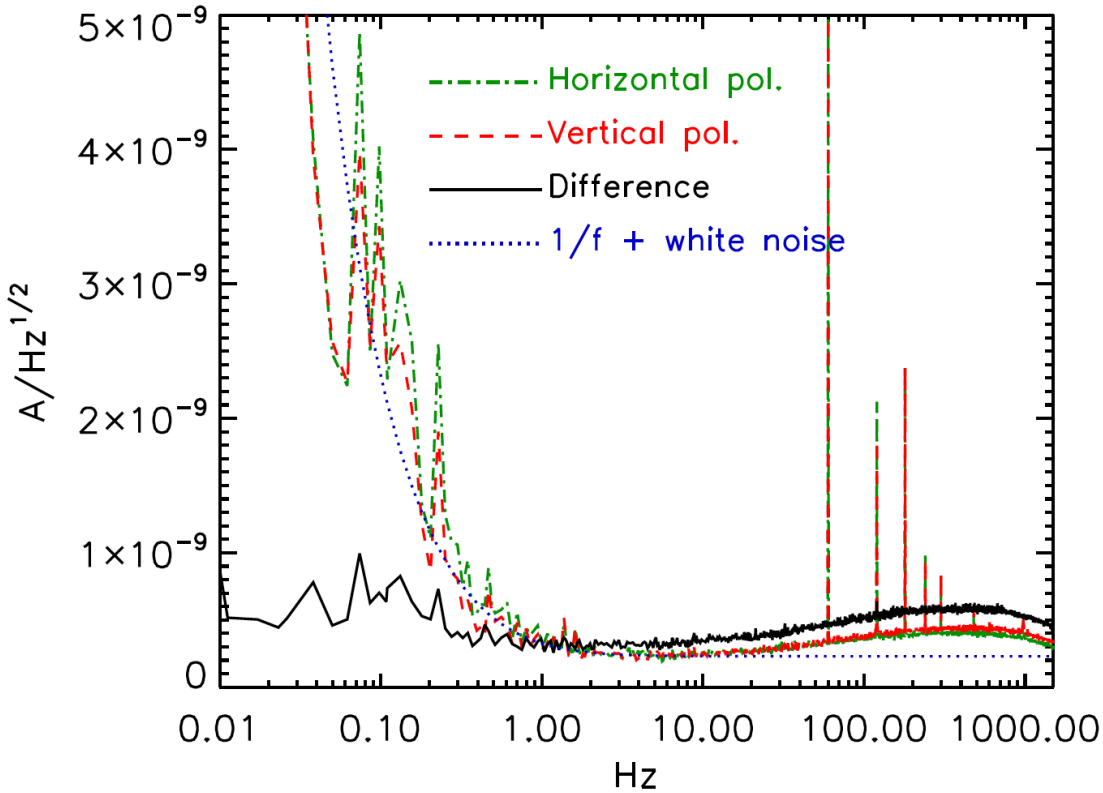




- 1/f knee 15-20 mHz, non pair differencing.

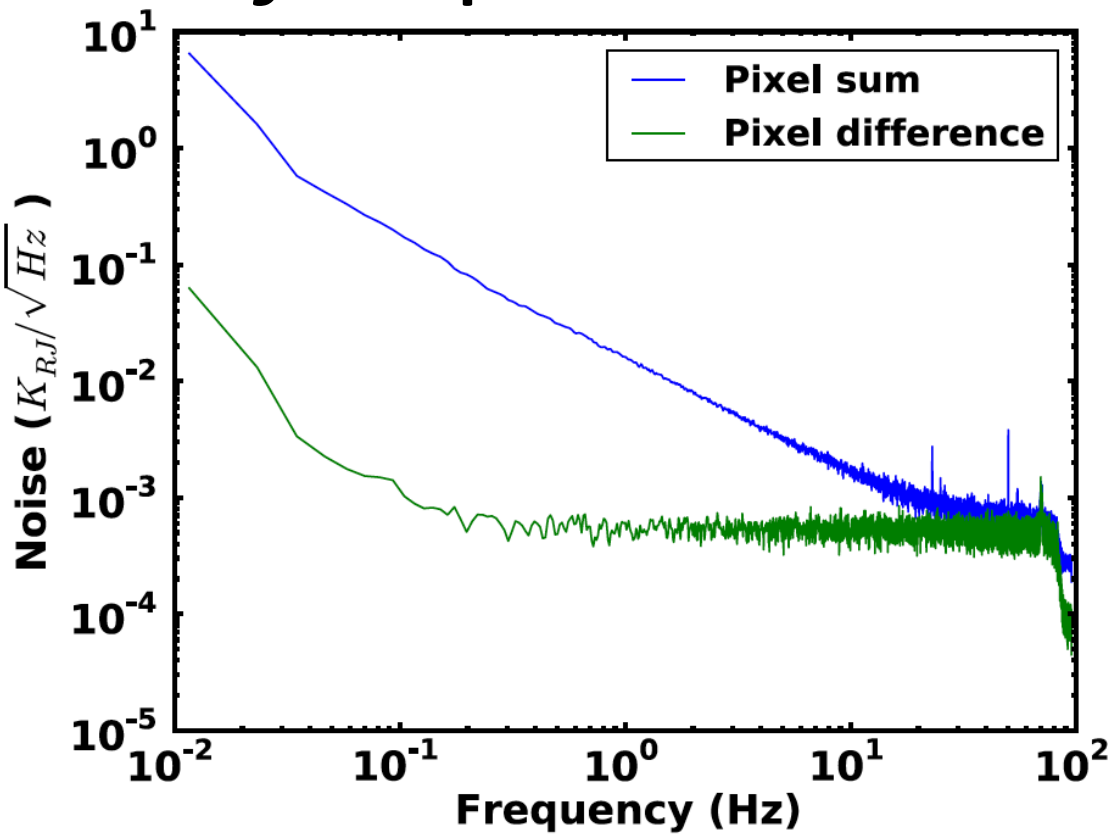
TDM Noise Stability[8]

- Lab Characterization at NIST, dark tests with TES bolometers, so bound to be flattering



- 1/f knee at ~1Hz
- Noise is Common mode between pairs- Difference <10mHz
- Consistent with SPIDER's experiences

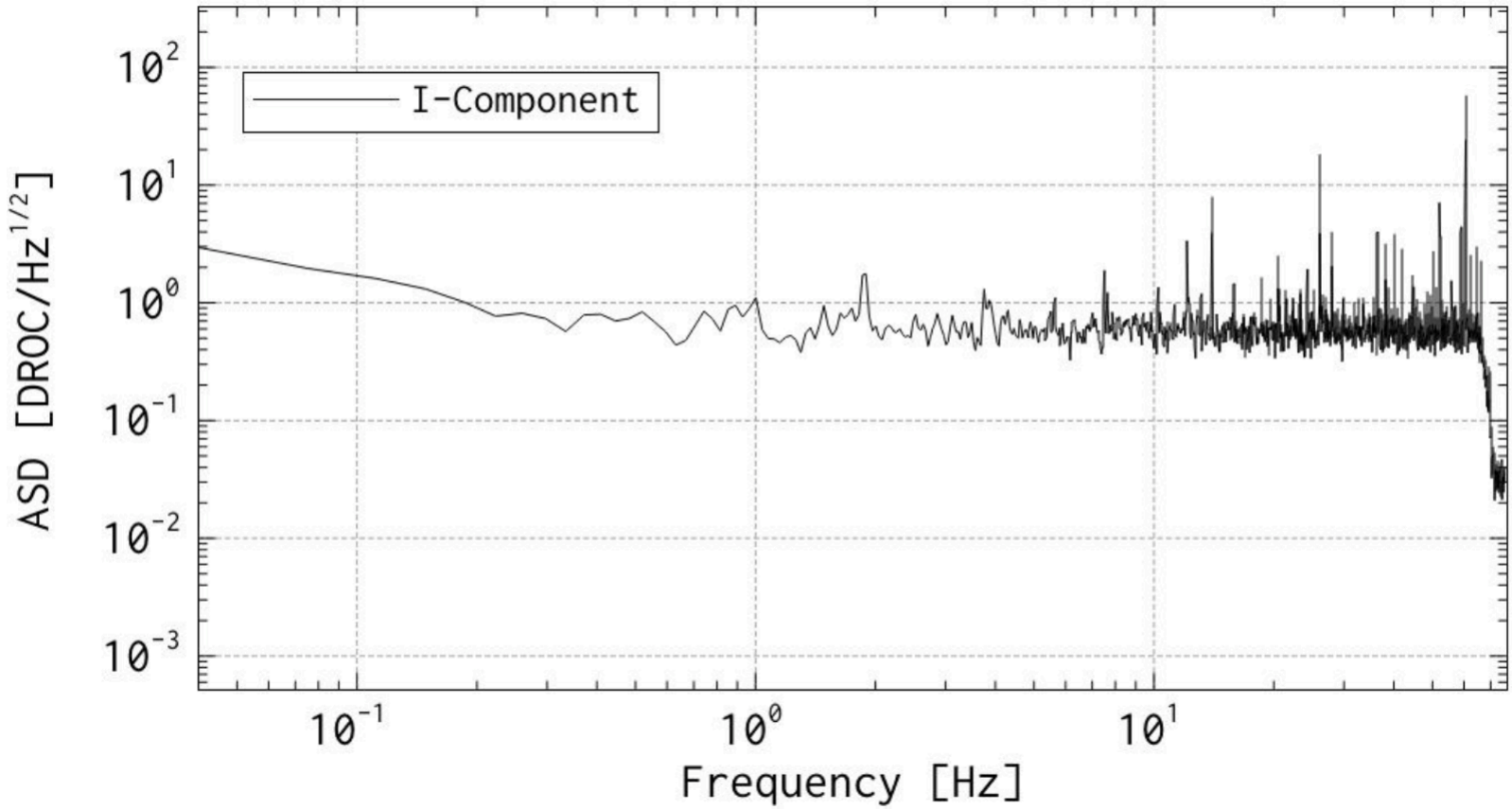
An *unfair* peak at FDM 1/f



- Pair-differenced and summed detectors from Polarbear using FDM[9]: 1/f~10Hz before differencing (atmospheric fluctations?) and ~100mHz after.
- "Unfair" because it dates to (2012), includes atmosphere, and the Polarbear team was not optimizing for this. I presume FDM can do better.

So this should not be the figure of record for FDM. It should be replaced.

A *more flattering* peak at FDM 1/f



- Measurement at McGill of a dark over-biased detector[10], so:
  - very low responsivity, no photon noise
  - Shunt noise, but little G-noise or TES noise
- 1/f knee ~100mHz?

References

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