Comparison of EKF and UKF for Robotic Missions to Mars

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Mission Design & Navigation
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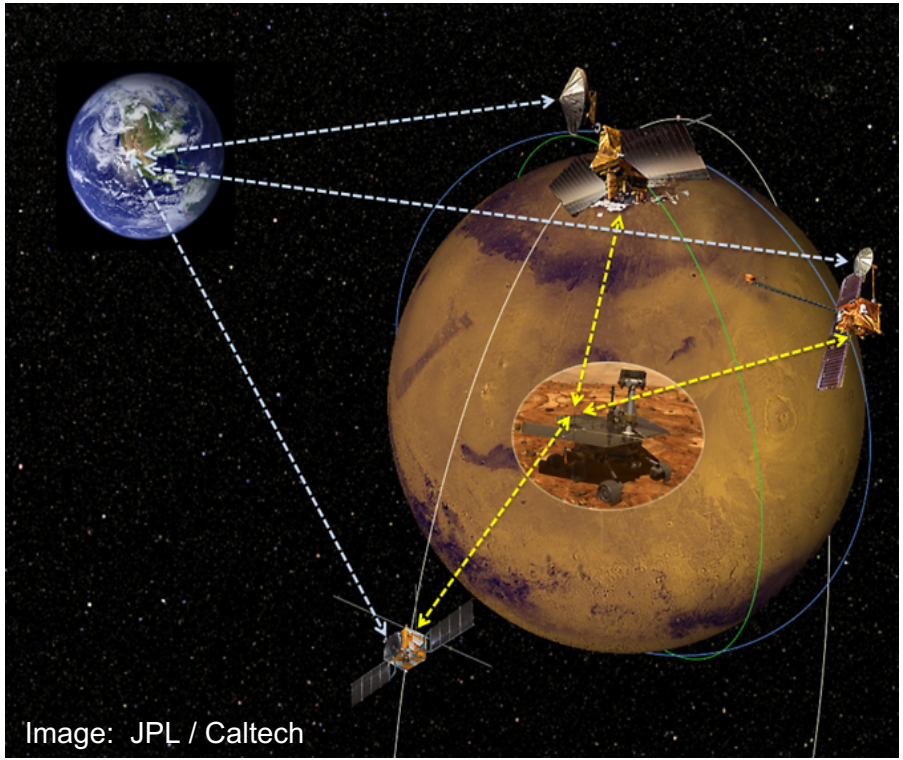
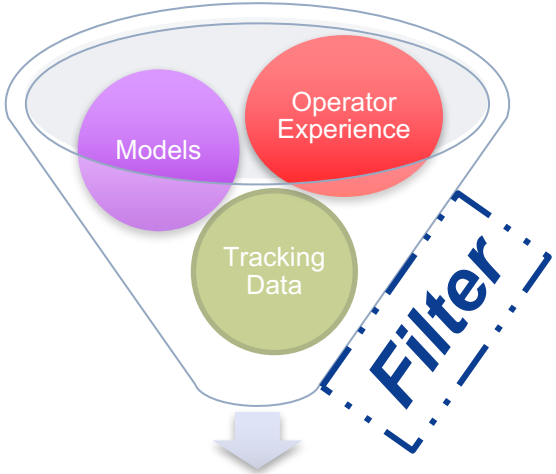
Mars Robotic Exploration

A rich history of scientific discovery and operations experience



Orbit Determination



Updated predictions (state, etc.)

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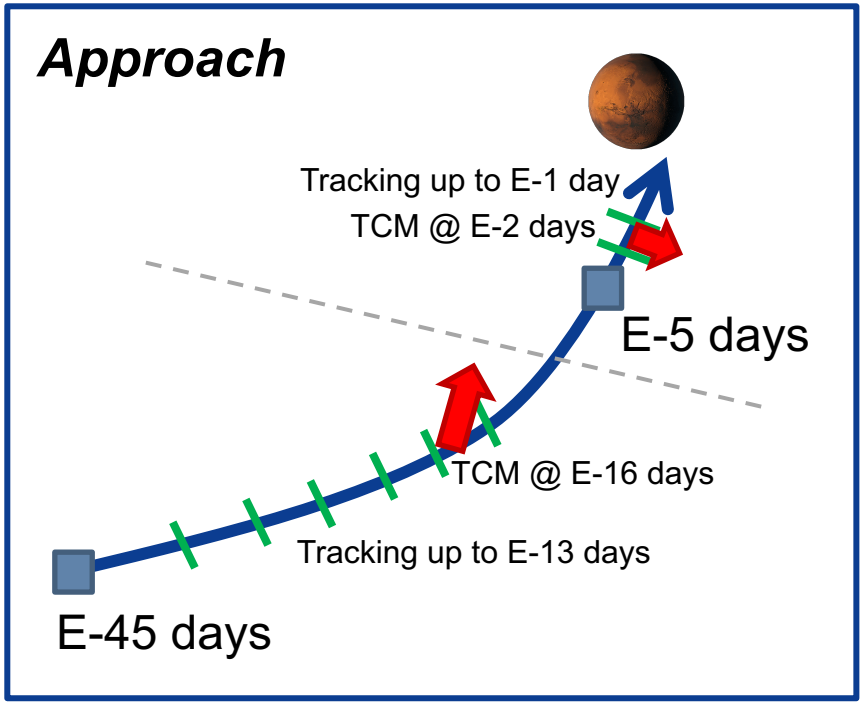
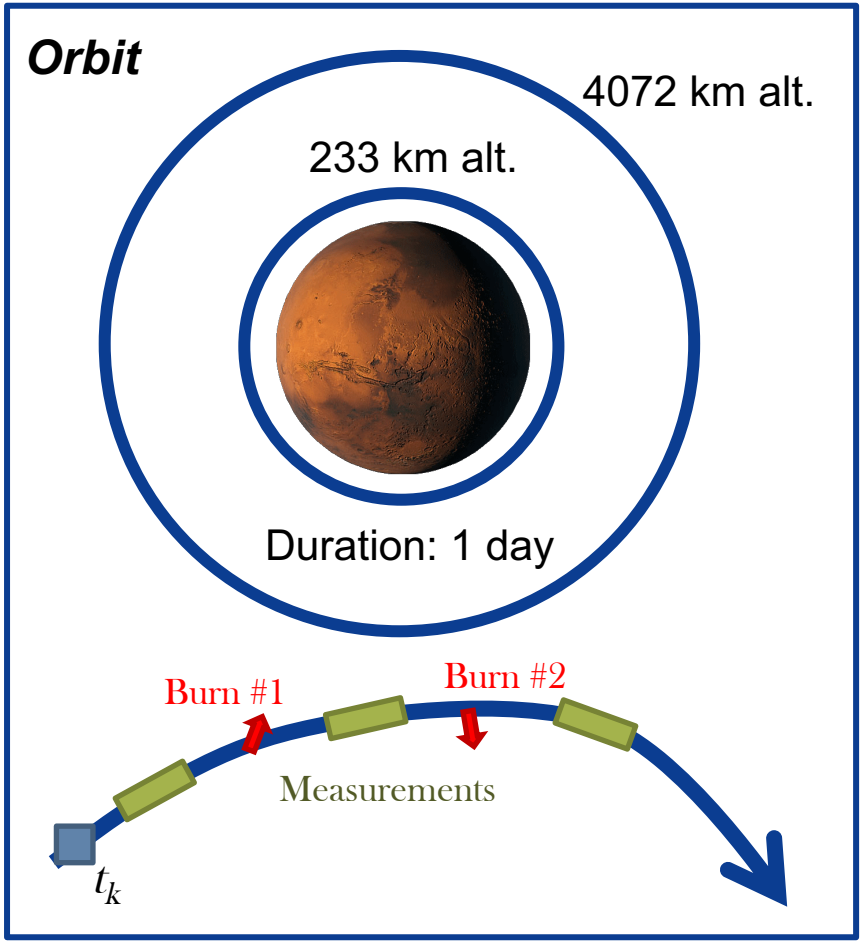
Trajectory Scenarios

Different phases of a Mars mission





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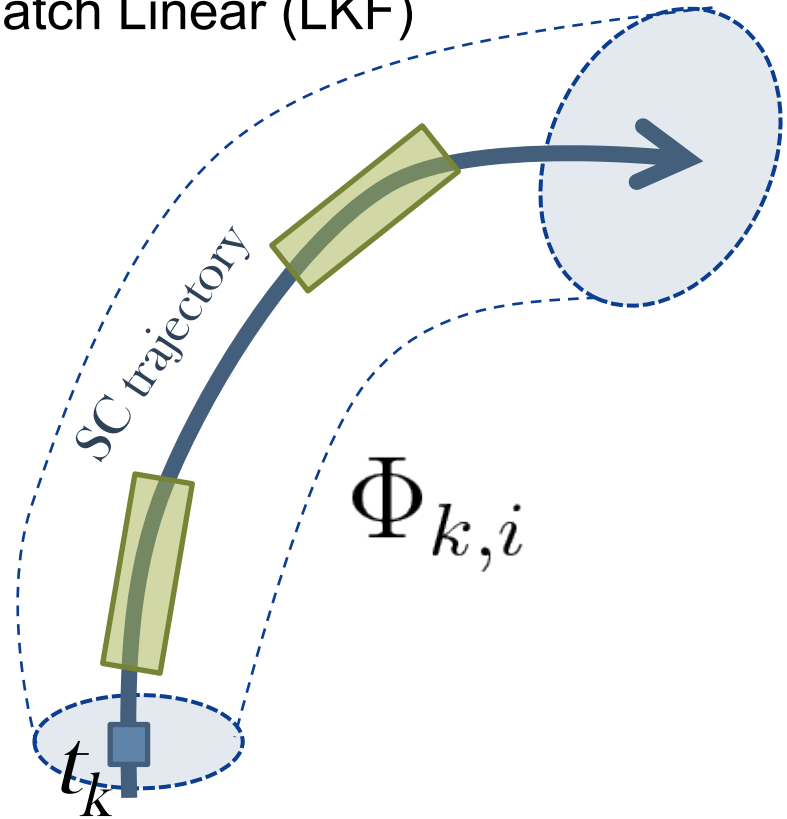
Kalman Filtering

Batch Linear (LKF)



Reality

Parameters $\vec{x}_{k+1} = \vec{f}(\vec{x}_k, \vec{\mathrm{w}}_k, t_k)$

Measurements $\vec{y}_k = \vec{h}(\vec{x}_k, t_k) + \vec{\mathrm{v}}_k$

Linearized Model

$$\delta\dot{\vec{x}}_k = \vec{F}_k \delta\vec{x}_k + \vec{\mathrm{w}}_k$$

$$\delta\vec{y}_k = \vec{H}_k \delta\vec{x}_k + \vec{\mathrm{v}}_k$$

Parameter Update

$$\hat{x}_k^+ = K^{-1}\left(H_k^T R_k^{-1} \Delta\tilde{y}_k + (P_k^-)^{-1}\hat{x}_k^-\right)$$

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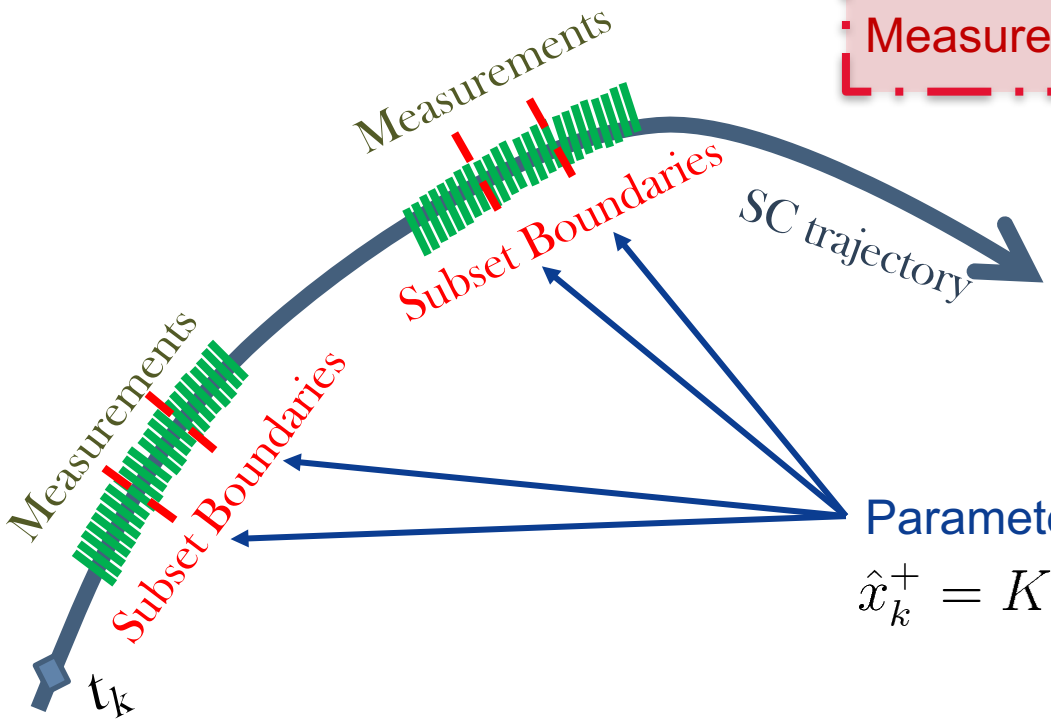
Kalman Filtering

Batch Extended (EKF)

Reality

Parameters $\vec{x}_{k+1} = \vec{f}(\vec{x}_k, \vec{\mathrm{w}}_k, t_k)$

Measurements $\vec{y}_k = \vec{h}(\vec{x}_k, t_k) + \vec{\mathrm{v}}_k$



Linearized Model

$$\delta\dot{\vec{x}}_k = \vec{F}_k \delta\vec{x}_k + \vec{\mathrm{w}}_k$$

$$\delta\vec{y}_k = \vec{H}_k \delta\vec{x}_k + \vec{\mathrm{v}}_k$$

Parameter Updates

$$\hat{x}_k^+ = K^{-1}\left(H_k^T R_k^{-1} \Delta\tilde{y}_k + (P_k^-)^{-1}\hat{x}_k^-\right)$$

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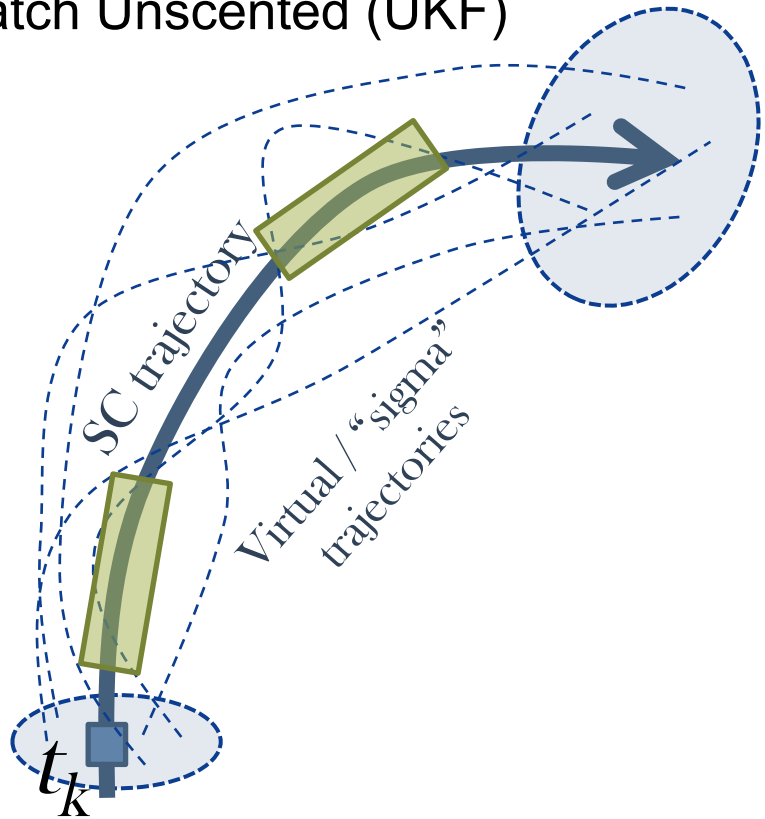
Kalman Filtering

Batch Unscented (UKF)



Reality

Parameters $\vec{x}_{k+1} = \vec{f}(\vec{x}_k, \vec{\mathrm{w}}_k, t_k)$

Measurements $\vec{y}_k = \vec{h}(\vec{x}_k, t_k) + \vec{\mathrm{v}}_k$

Parameter Update

$$\hat{x}_k^+ = \hat{x}_k^- + \bar{P}_k^{xy}(\bar{P}_k^{yy})^{-1}\Delta\tilde{y}_k$$

Correlation Matrices

$$\bar{P}_k^{xy} = \sum_{j=0}^{2M} W_s^{(j)}(\bar{\chi}_{j,k} - \hat{x}_k)(\tilde{\gamma}_{j,k} - \bar{y}_k)^T$$

$$\bar{P}_k^{yy} = \sum_{j=0}^{2M} W_s^{(j)}(\tilde{\gamma}_{j,k} - \bar{y}_k)(\tilde{\gamma}_{j,k} - \bar{y}_k)^T + R_k$$

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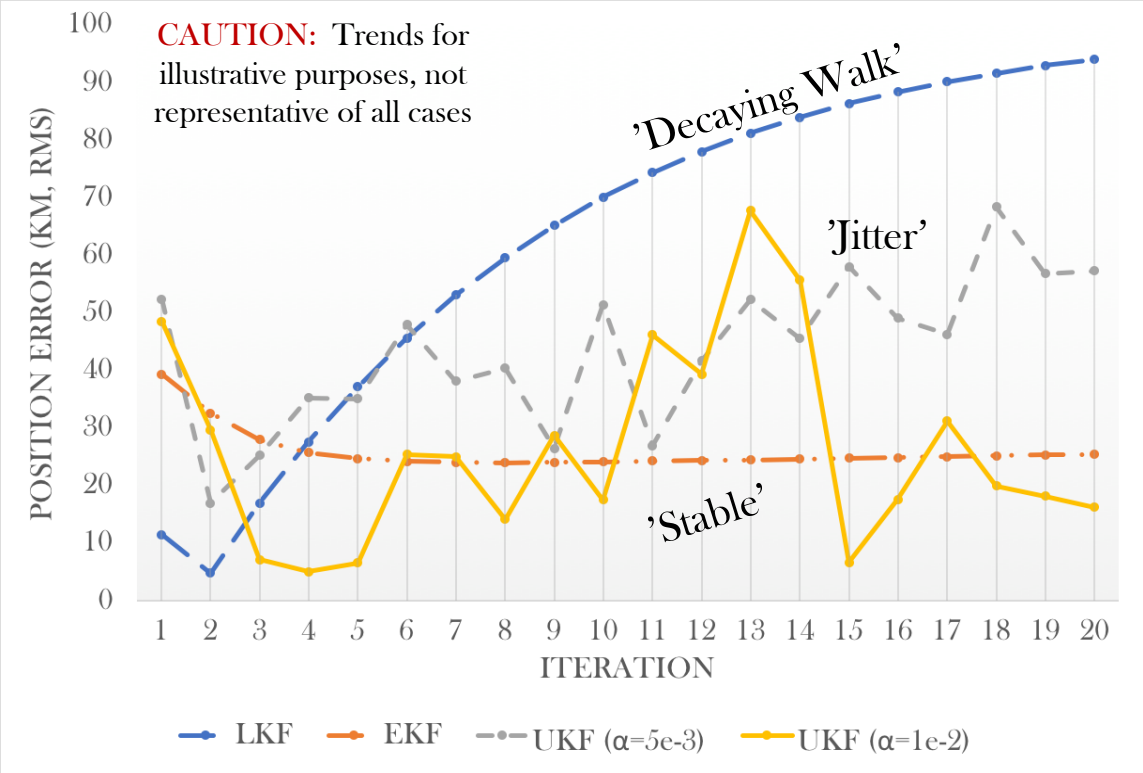
Example Trends

Sometimes exhibited, sometimes not…



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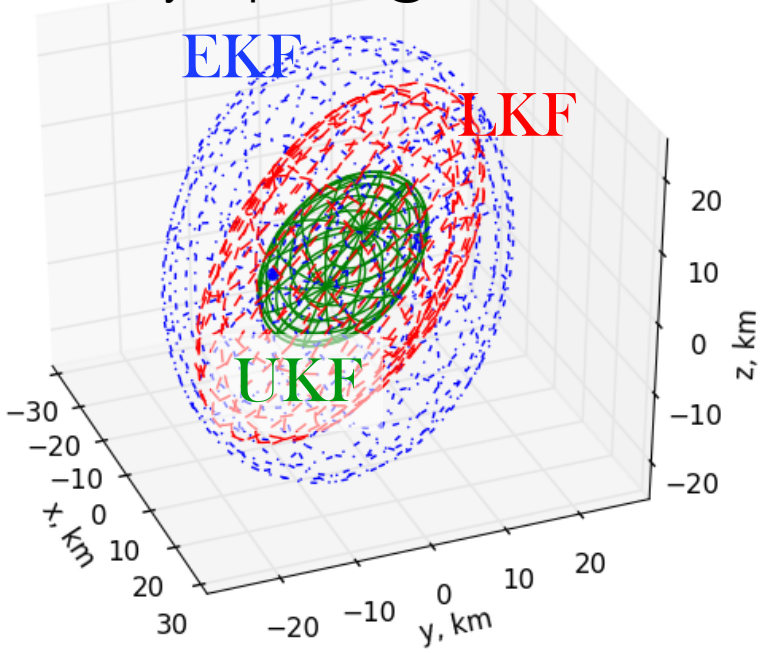
Approach Results: E-45 days

100-km, 1-m/s errors, Doppler pass every 3 days

Uncertainty ellipsoids @ Mars



Pass Dur.	Pos. Err. (km, RMS)	Iter.	Time per iter. (s)
Linear			
2 hrs	84.6	9*	0.7
4 hrs	71.2	9*	1.1
6 hrs	48.7	8*	1.4
Extended			
2 hrs	57.6	3*	0.9
4 hrs	39.9	4	1.5
6 hrs	15.8	5	2.1
Unscented			
2 hrs	55.3	5	8.6
4 hrs	14.4	3	19.7
6 hrs	9.6	3	39.4

*Case exhibits 'decaying walk'

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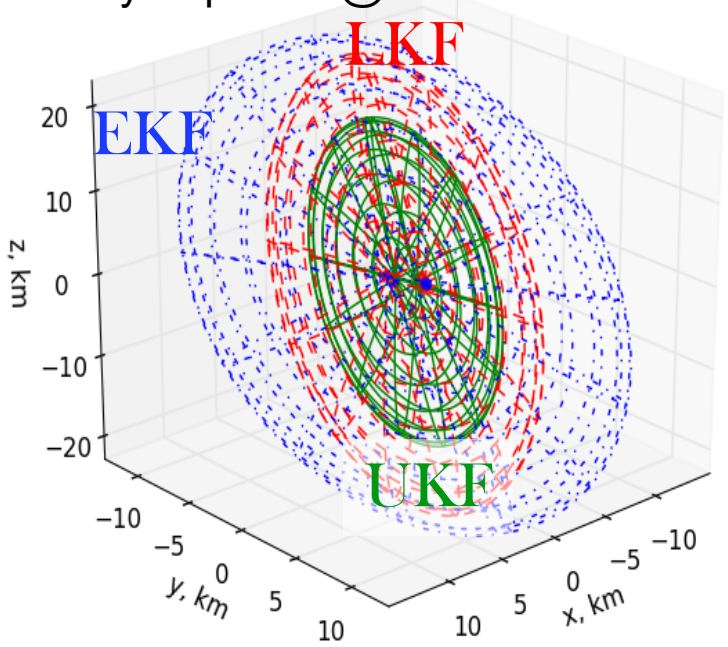
Approach Results: E-5 days

6-hr Doppler passes every 2 days

Uncertainty ellipsoids @ Mars



State Error	Pos. Err. (km, RMS)	Iter.	Time per iter. (s)
Linear			
500 km 5 m/s	59.4	3	0.5
100 km 1 m/s	56.7	4	0.5
50 km 0.5 m/s	60.5	4	0.5
Extended			
500 km 5 m/s	57.0	4	0.8
100 km 1 m/s	58.3	4	0.8
50 km 0.5 m/s	59.1	5	0.8
Unscented			
500 km 5 m/s	37.4	7	7.3
100 km 1 m/s	21.8	6	7.3
50 km 0.5 m/s	22.0	3	7.3

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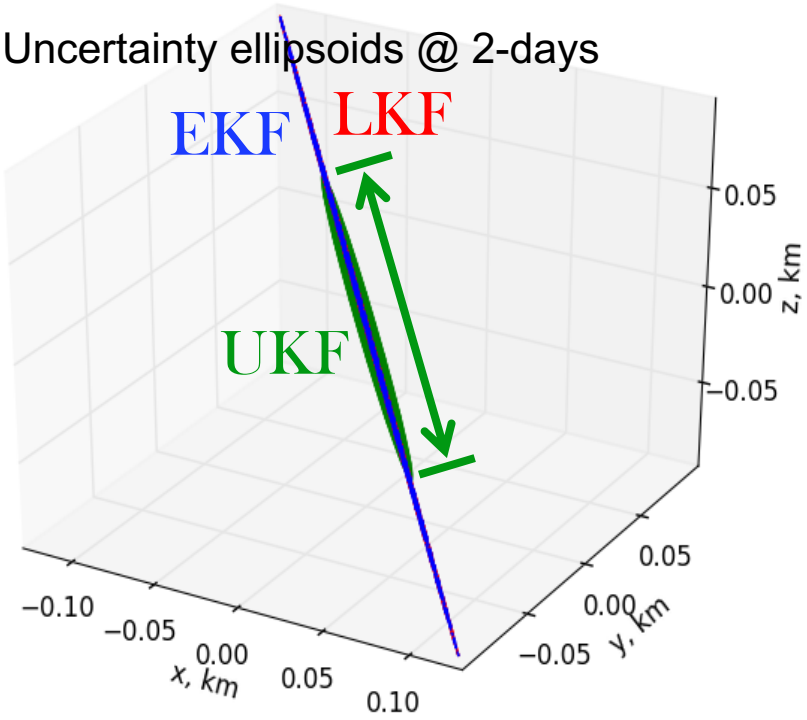
Orbit Results: High Alt.

1-km, 10-cm/s errors

Uncertainty ellipsoids @ 2-days



Pass Dur.	Pos. Err. (m, RMS)	Iter.	Time per iter. (s)
Linear			
0.5 hrs	252	40**	0.9
2 hrs	232	4	1.0
Cont.	12.1	6	1.4
Extended			
0.5 hrs	167	40**	1.0
2 hrs	206	2*	1.3
Cont.	12.6	3	2.2
Unscented			
0.5 hrs	187	3	7.9
2 hrs	46.5	4	9.5
Cont.	21.5	9	16.0

*Case exhibits 'decaying walk'

**Case reached maximum iterations

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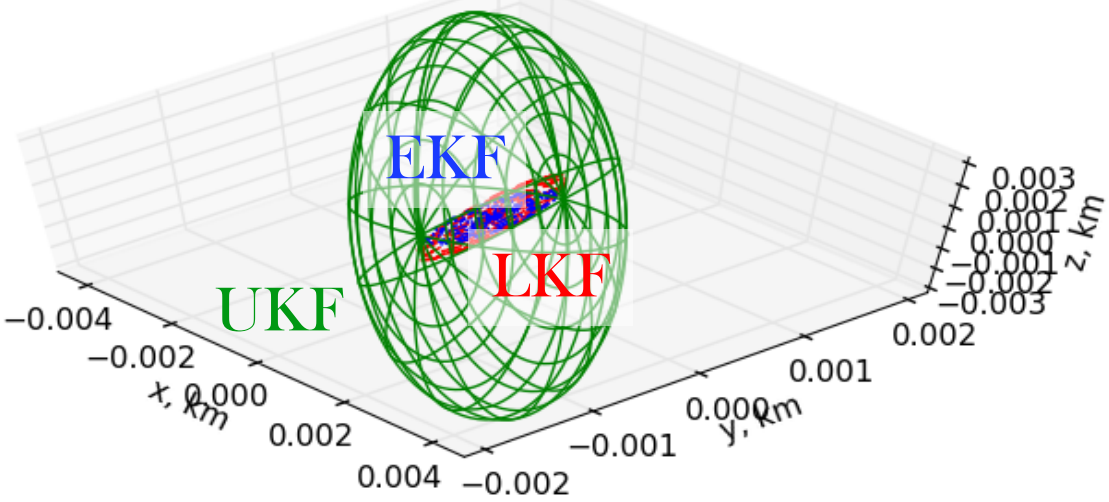
Orbit Results: Low Alt.

2-hour Doppler passes

Uncertainty ellipsoids @ 2-days



State Error	Pos. Err. (m, RMS)	Iter.	Time per iter. (s)
Linear			
10 km 1 m/s	–	–	–
1 km 10 cm/s	0.4	3	8.1
100 m 5 cm/s	1.2	4	8.7
Extended			
10 km 1 m/s	–	–	–
1 km 10 cm/s	3.5	7	12.6
100 m 5 cm/s	1.1	8	12.8
Unscented			
10 km 1 m/s	6.6	5	41.5
1 km 10 cm/s	1.9	3	42.3
100 m 5 cm/s	0.6	5	41.5

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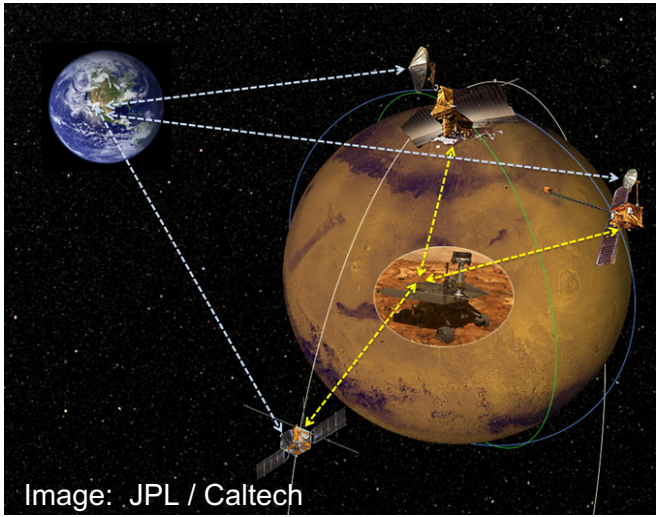
Conclusions

Observations & Recommendations



- Batch Linear KF
  - “Small” errors, “short” time spans
  - Very powerful with operator experience
- Batch Extended KF
  - Improves on LKF (usually)
  - Be careful with measurement subsets
- Batch Unscented KF
  - Generally more robust (errors, sparse measurements, nonlinearities)
  - No need for partial derivatives
  - Improved covariance assessments

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