Nonlinear Kinetic Inductance

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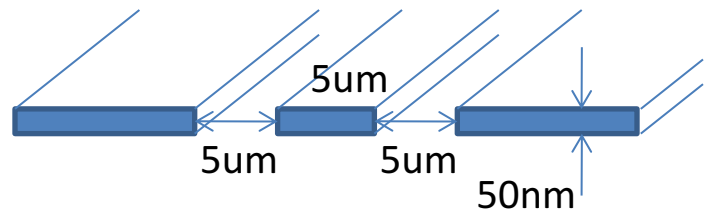
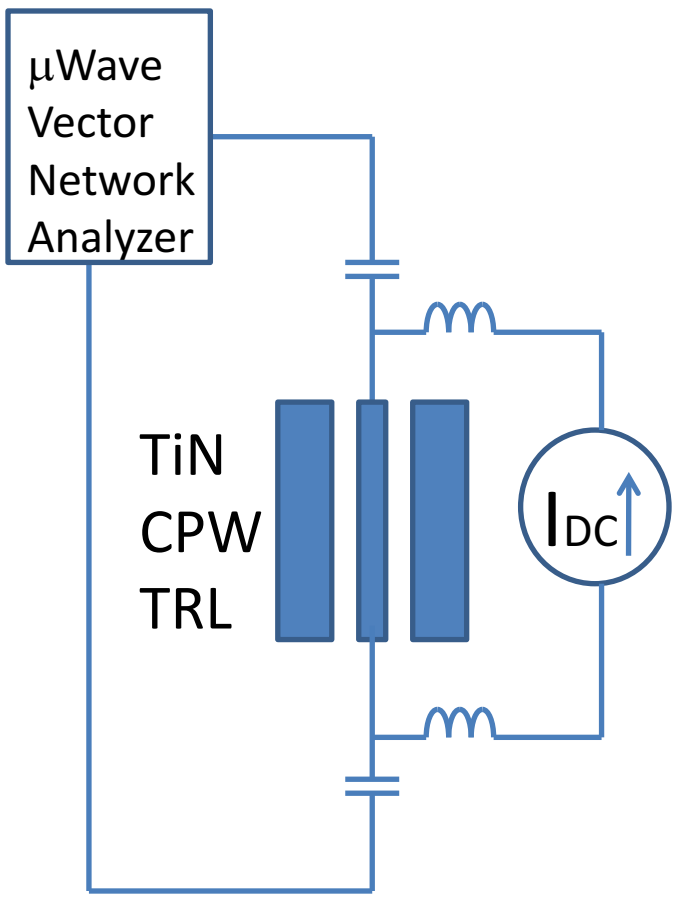
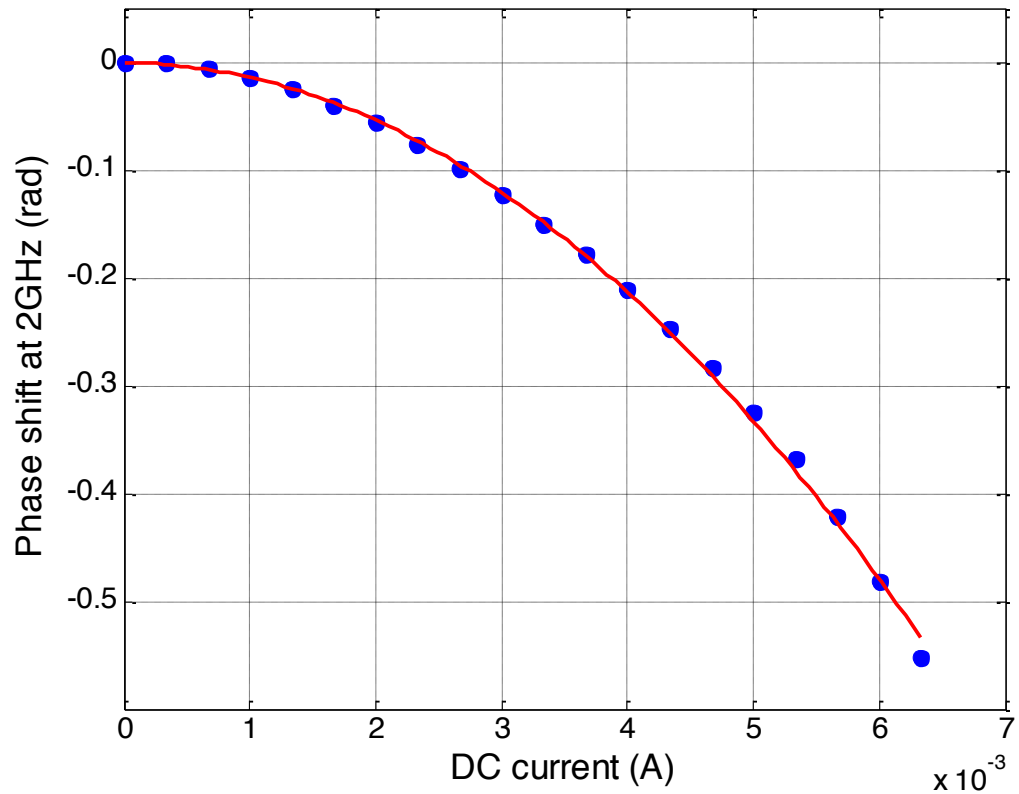
Nonlinear kinetic inductance

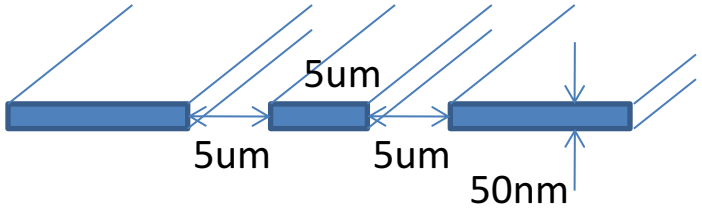
0.1 meter long CPW line







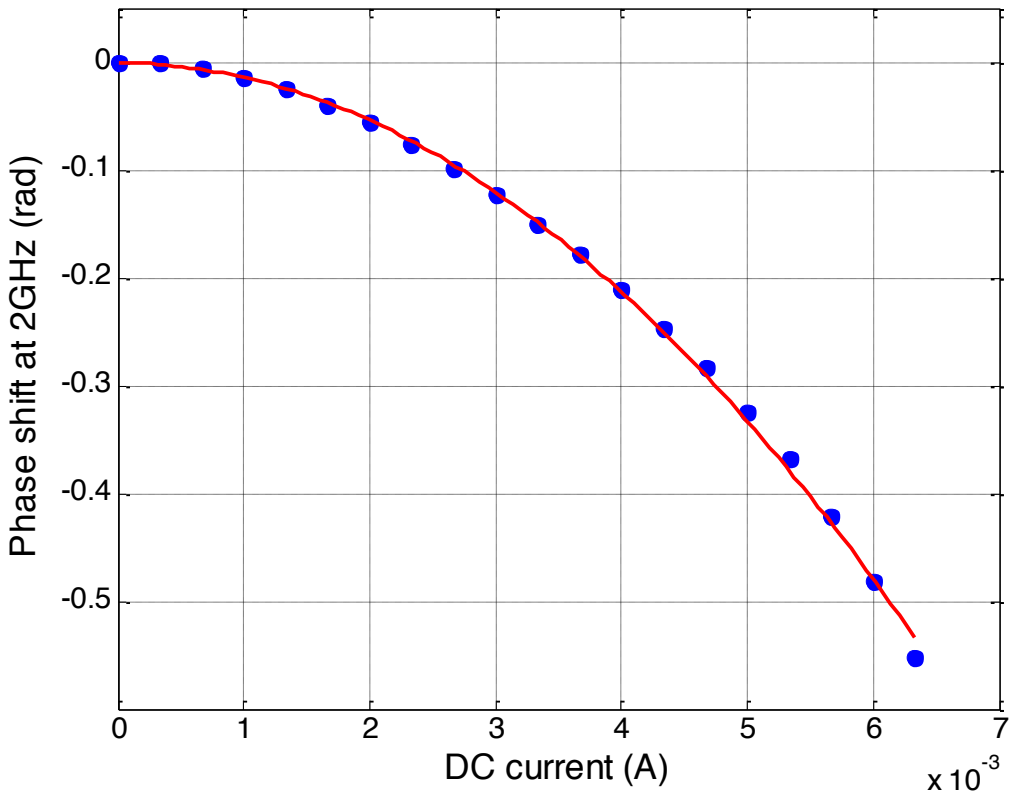
Nonlinear kinetic inductance



- $\Delta\theta \sim I^2$
- $L = L_0 (1 + I^2/I_*^2)$
- $\delta v_{ph} \sim I^2 \sim P$
- Kerr medium
- Line length =0.1m

-> 21 radians

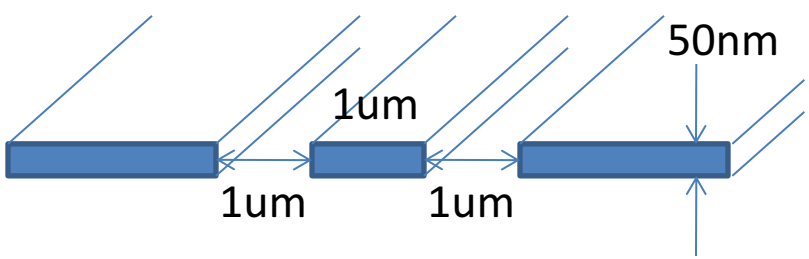
- $\Delta l/l \sim 2.5\%$



Superconducting Nitrides – large kinetic inductance

- $\lambda \approx 105$ nm x $(\rho_n\ [\mu\Omega.\text{cm}] / T_c\ [\text{K}])^{1/2}$
  - Nitrides: $\rho_s \approx 100\ \mu\Omega.\text{cm}$
  - λ (TiN, NbTiN) ≈ 500 nm

Example: TiN CPW line



$L_s / L_{tot} \approx 0.94$

$v_{ph} \approx 0.1\ c$

$Z0 \approx 220\ \Omega$

Kinetic Inductance Non-linearity

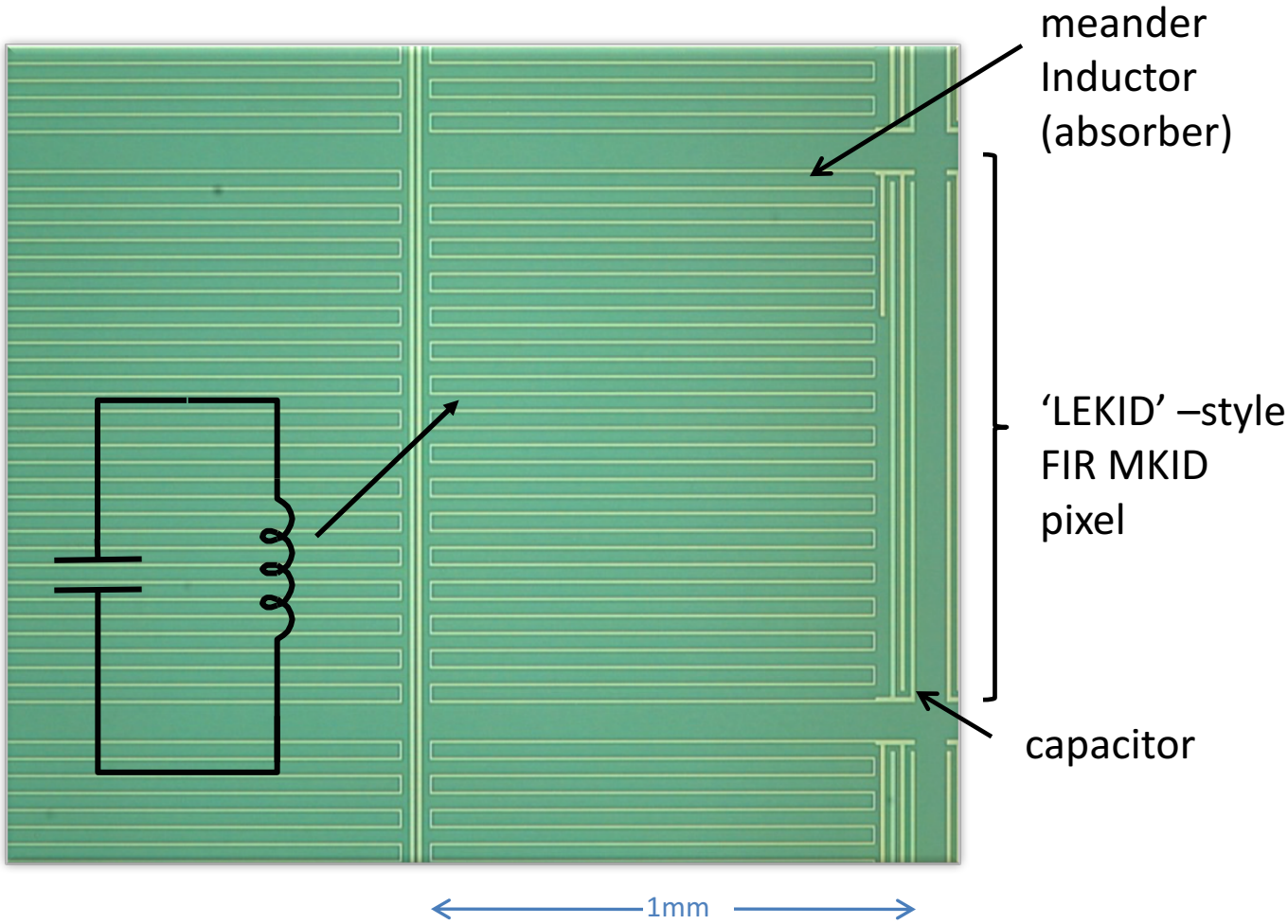
- Ginsberg-Landau theory
  - $L_s(I)$, $\lambda(I)$

$$\frac{\delta v_{ph}}{v_{ph}(I = 0)} = \frac{-L_{kin}}{4\, L_{tot}} \frac{\mu_0\, \lambda^2\, J_s^2}{\mu_0\, H_c^2}$$

Supercurrent kinetic energy

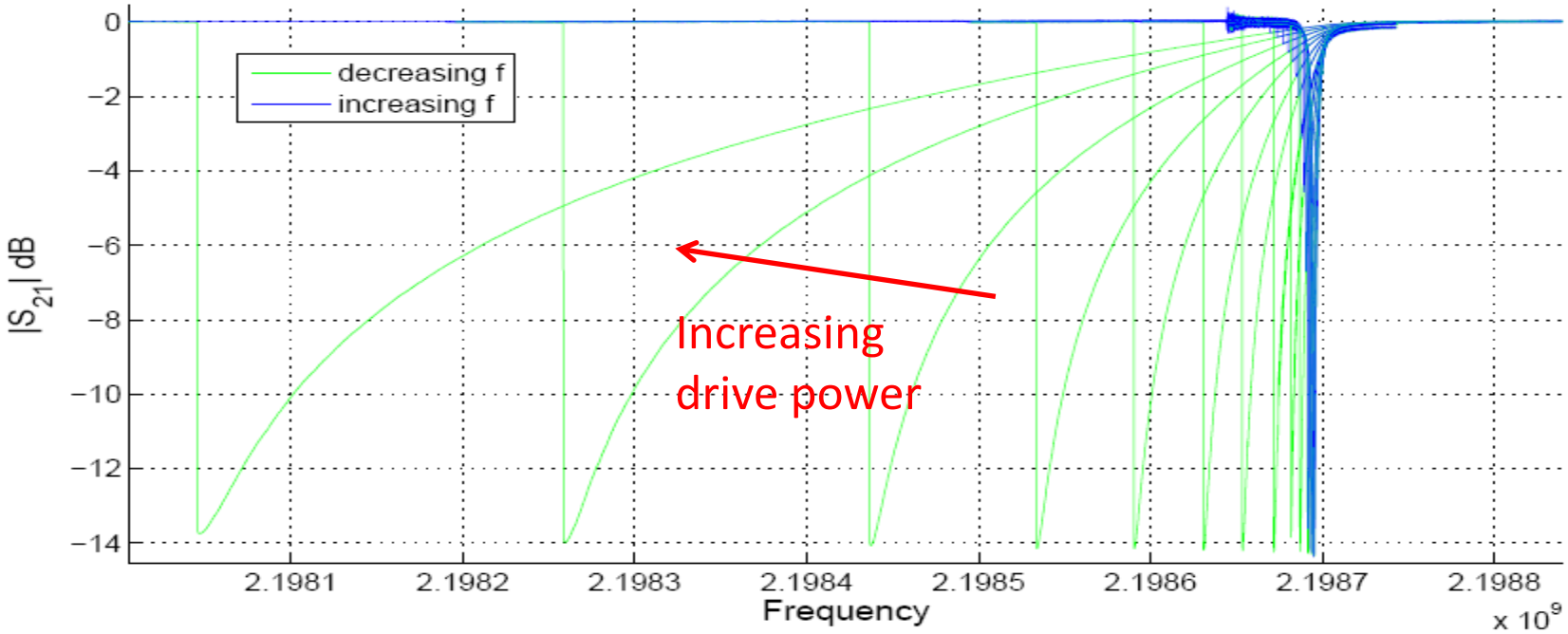
Condensation energy

TiN Resonator measurements

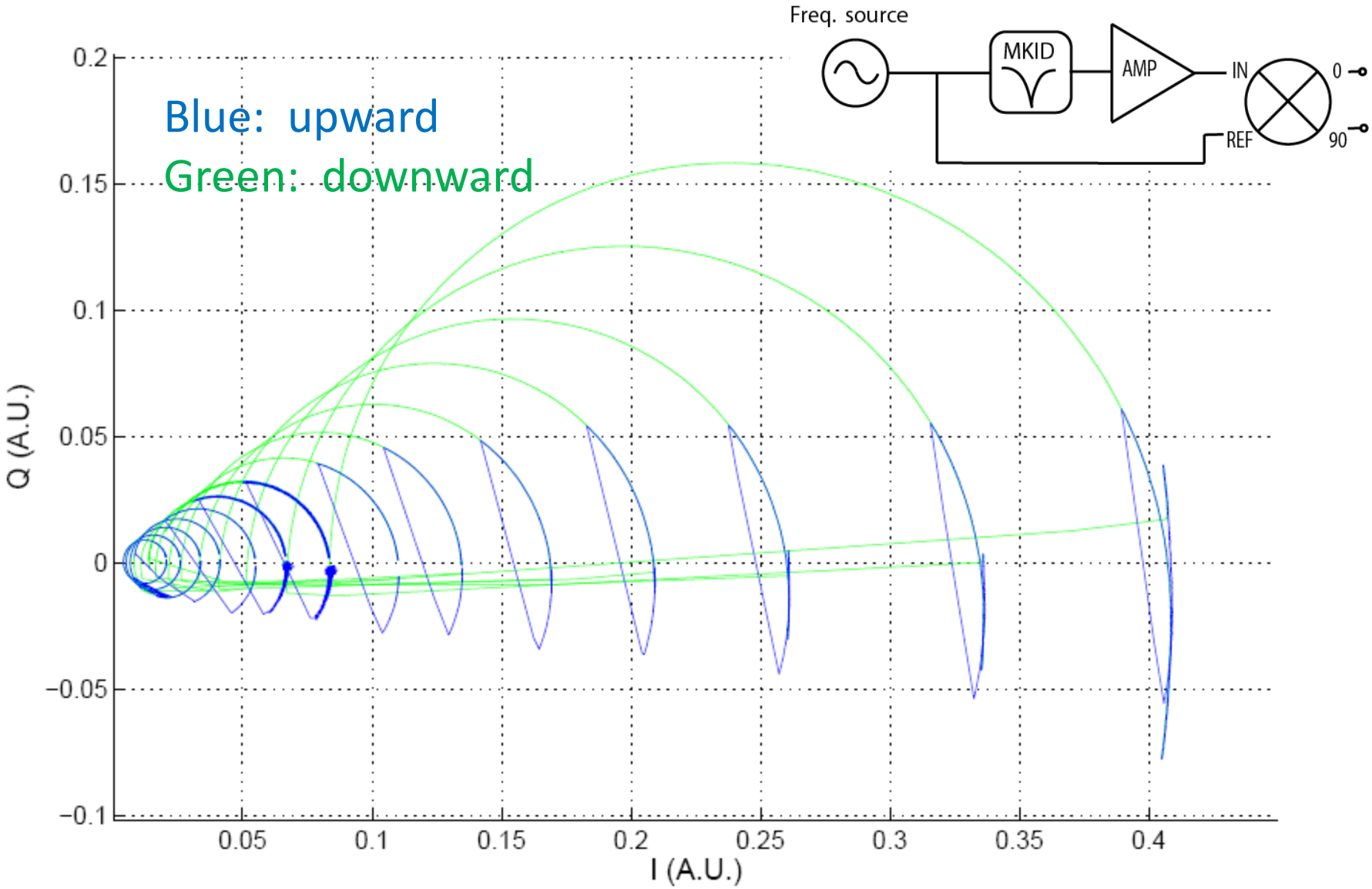


Nonlinear “Duffing” oscillator

- Resonance frequency depends on resonator current
- Hysteretic resonance curves:



Complex transmission



Non-linear resonator model

$$\begin{aligned} \delta f / f_0 &= -k_f I_{res}^2 \\ \delta Q_i^{-1} &= k_Q I_{res}^2 \end{aligned}$$

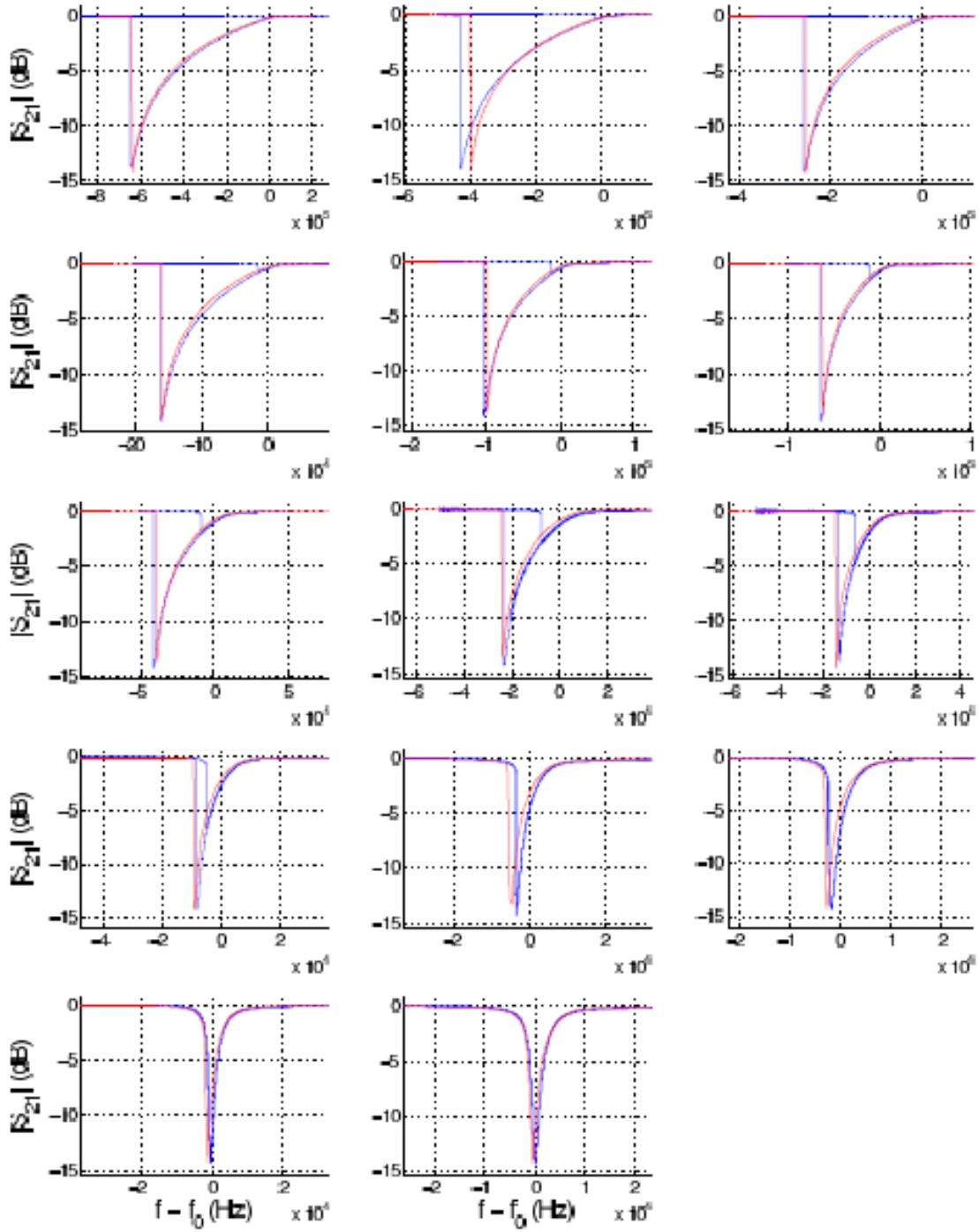
See Yurke and Buks (2008); Dahm and Scalapino (1997)

$$S_{21} = 1 - \frac{Q_t/Q_c}{1 + 2iQ_t[f_0 + \delta f_0(f) - f]/f_0}$$

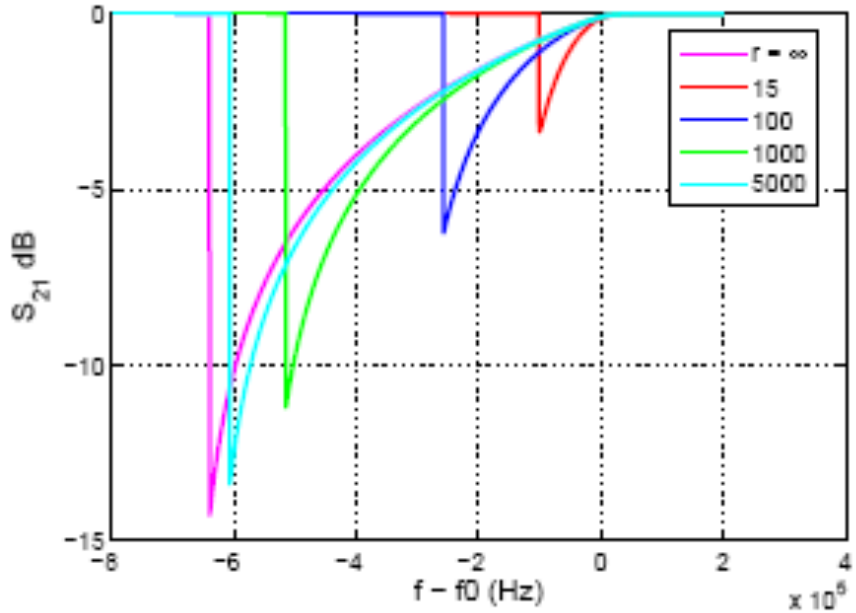
$$\frac{1}{Q_t} = \frac{1}{Q_c} + \frac{1}{Q_i}$$

$$I_{res}^2 = \frac{Q_c|1 - S_{21}|^2 P_{feedline}}{Z_0}$$

- Fits using NL resonator model
- $K_Q = 0$!

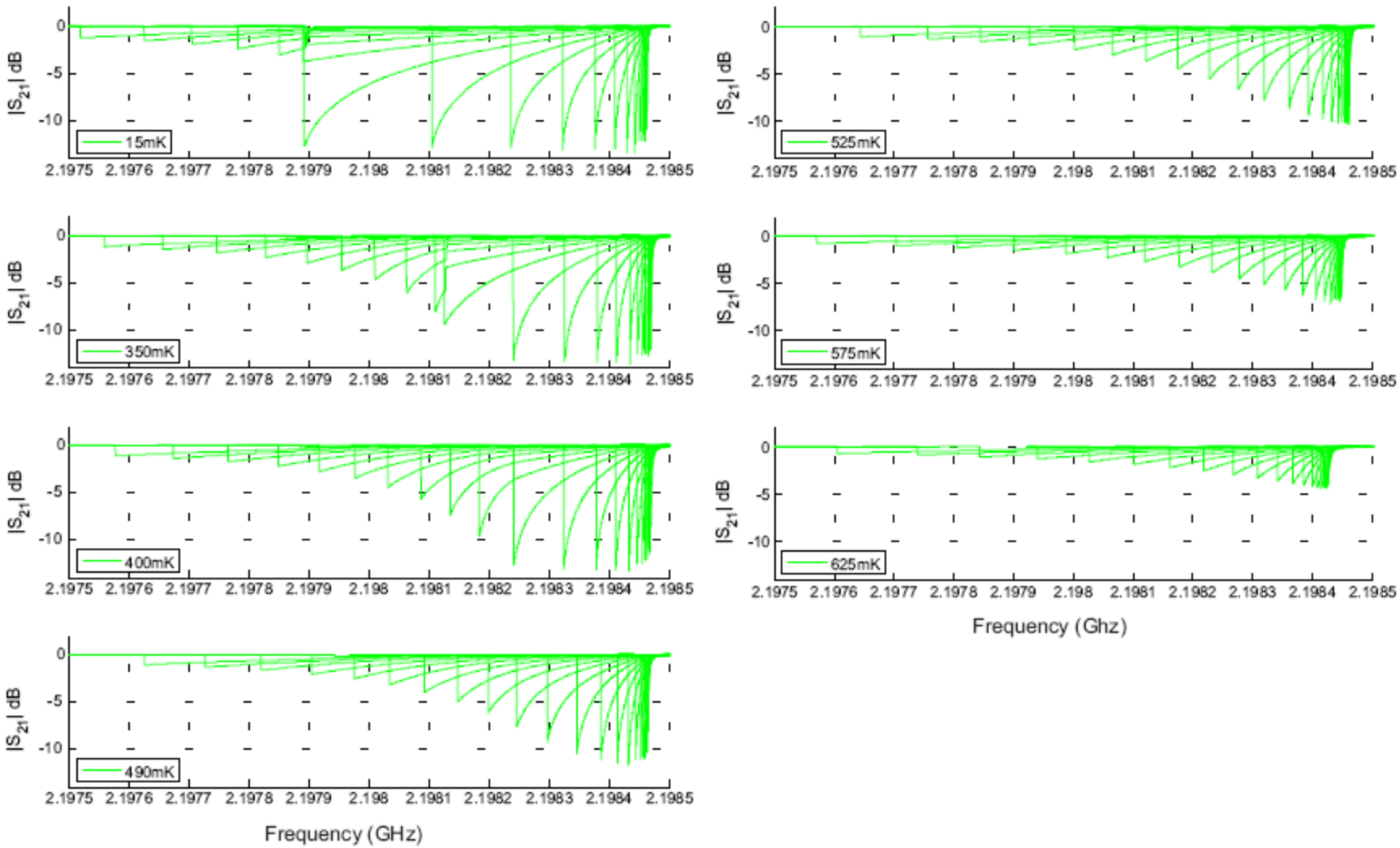


- Model results for different ratios of inductive to dissipative response

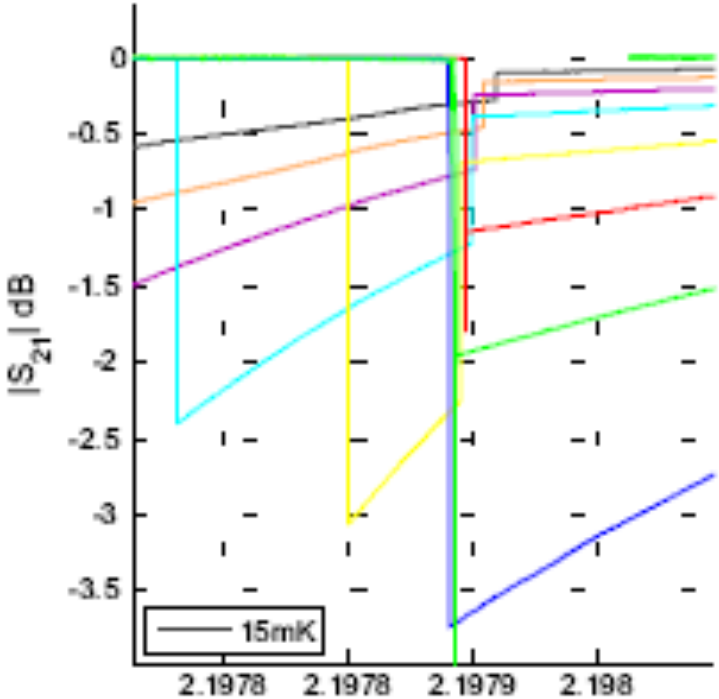


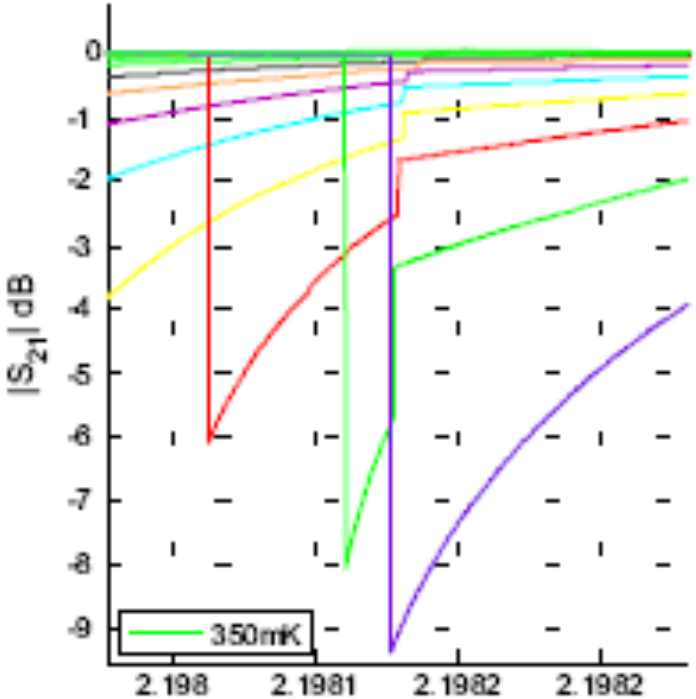
- δ resistance/ δ reactance < 2 x 10^{-4}

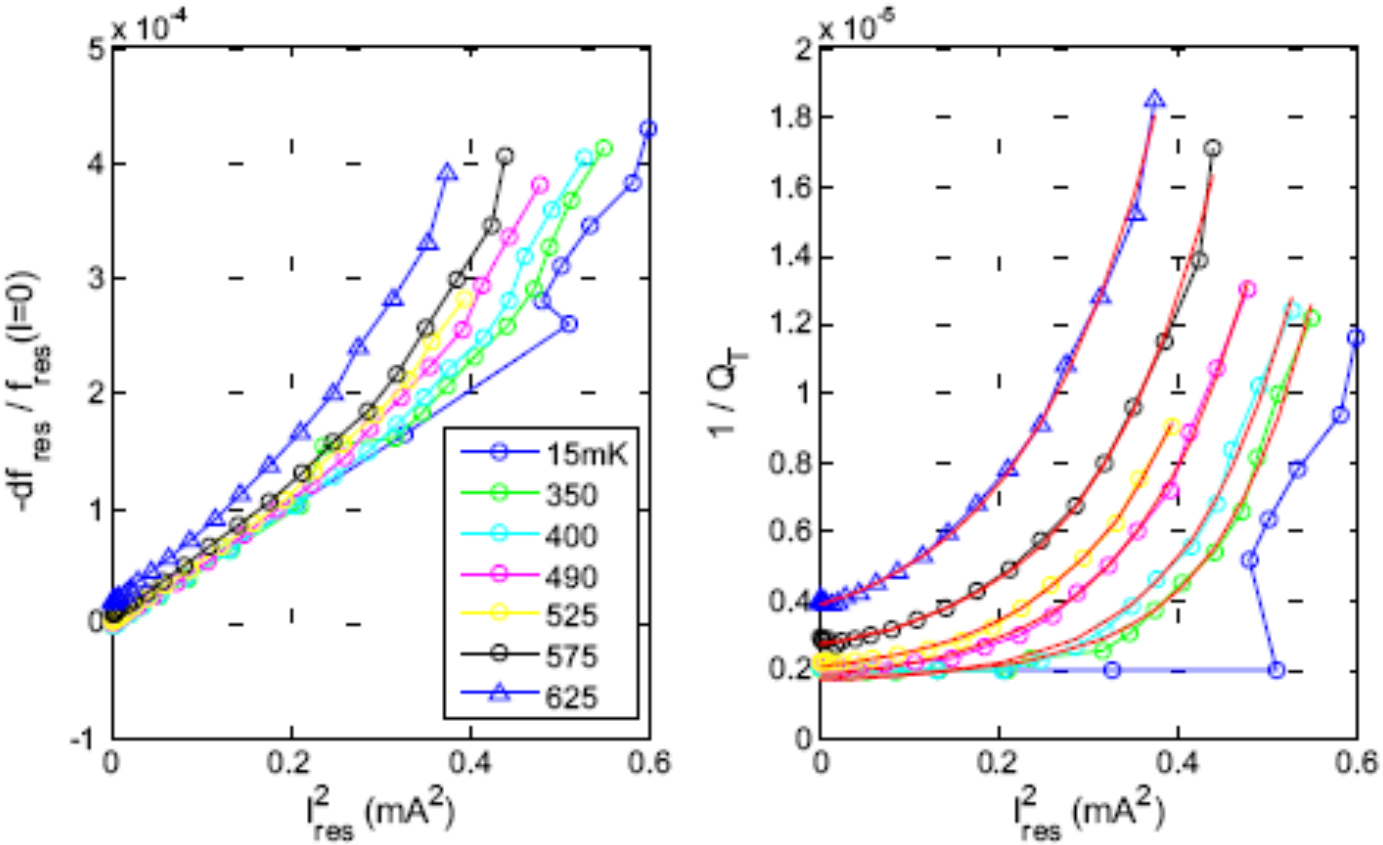
Measurements at elevated T



Transition to dissipative regime






x 10^-4
-df_res / f_res(I=0)
I^2_res (mA^2)
15mK
350
400
490
525
575
625
x 10^-6
1 / Q_T
I^2_res (mA^2)

Kinetic Inductance Parametric Up-converter (KPUP)

Objective

- Investigate new techniques for multiplexed TES bolometer array readout
  - enable factor of 10 increase in array sizes
  - alternative to SQUID based methods
  - adapt readout techniques developed for MKIDs to TES arrays

Kinetic Inductance Nonlinearity

- Superconducting film surface inductance:
  - $L_s = L_0 (1 + I^2/I_*^2 + \ldots)$
  - Eg. Ginsberg Landau theory:

$$|\psi|^2 = \psi_\infty^2 \left[1 - \left(\frac{\xi m^* v_s}{\hbar}\right)^2\right]$$

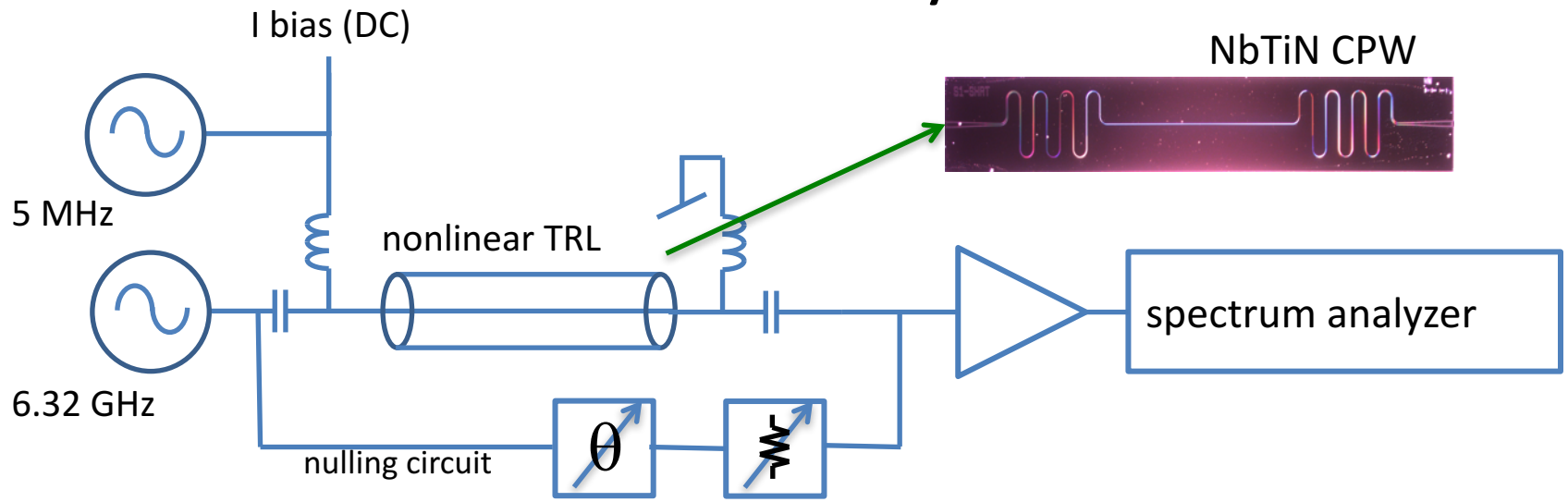
$$\frac{n_s^*}{n_{s,0}^*} = 1 - \frac{1}{2}\frac{\mu_0 \lambda_{eff}^2 J_s^2}{\mu_0 H_c^2}$$

Kinetic energy of supercurrent

Condensation energy

$$\frac{\delta L_s}{L_s} = \frac{\delta \lambda_{eff}}{\lambda_{eff}} = \frac{1}{2}\frac{\delta n_s^*}{n_{s,0}^*}$$

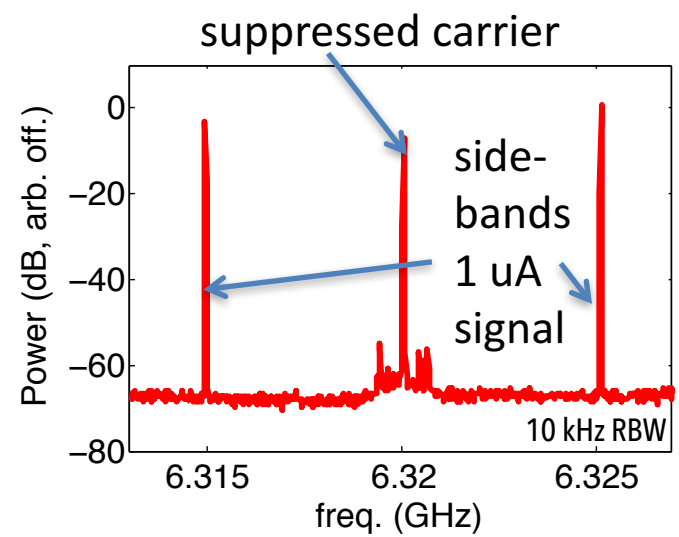
Up-conversion via kinetic inductance nonlinearity



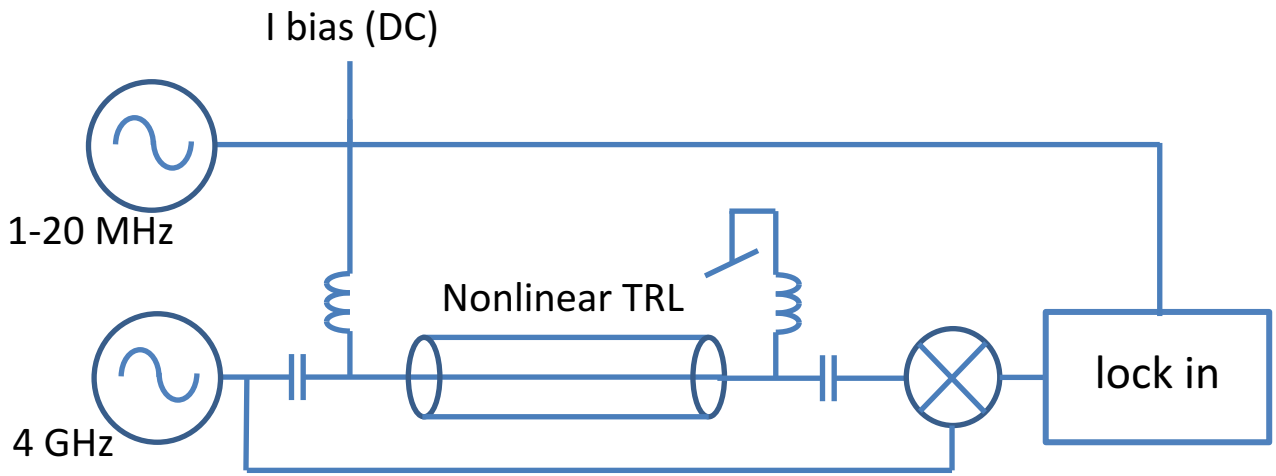
- Sidebands produced through phase modulation:

$$Ae^{i\omega_c t + i\xi I \sin \Omega t} \approx$$

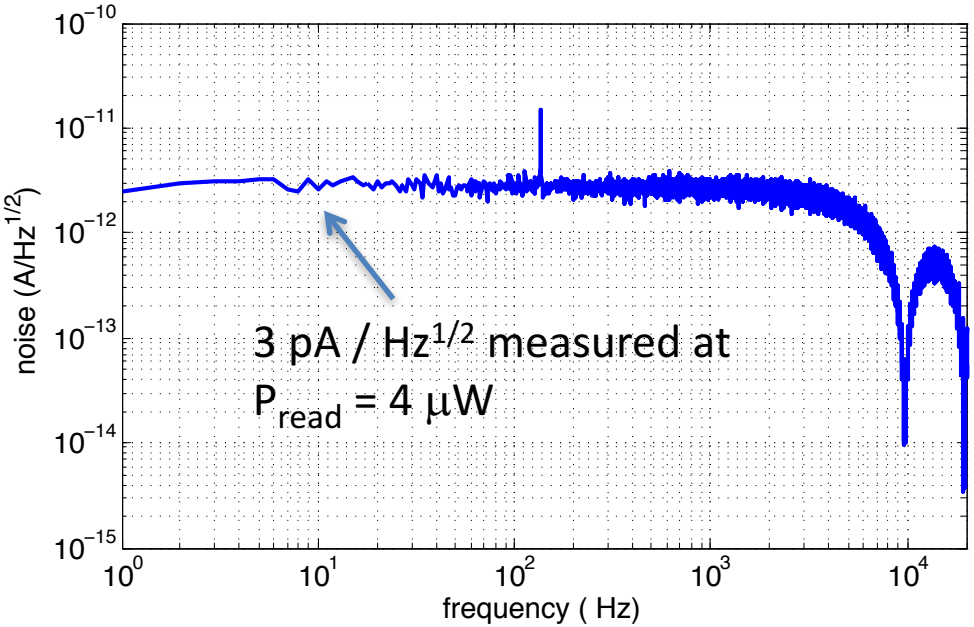
$$Ae^{i\omega_c t} + \frac{A\xi I}{2} e^{i(\omega_c + \Omega)t} - \frac{A\xi I}{2} e^{i(\omega_c - \Omega)t}$$



Current noise measurement

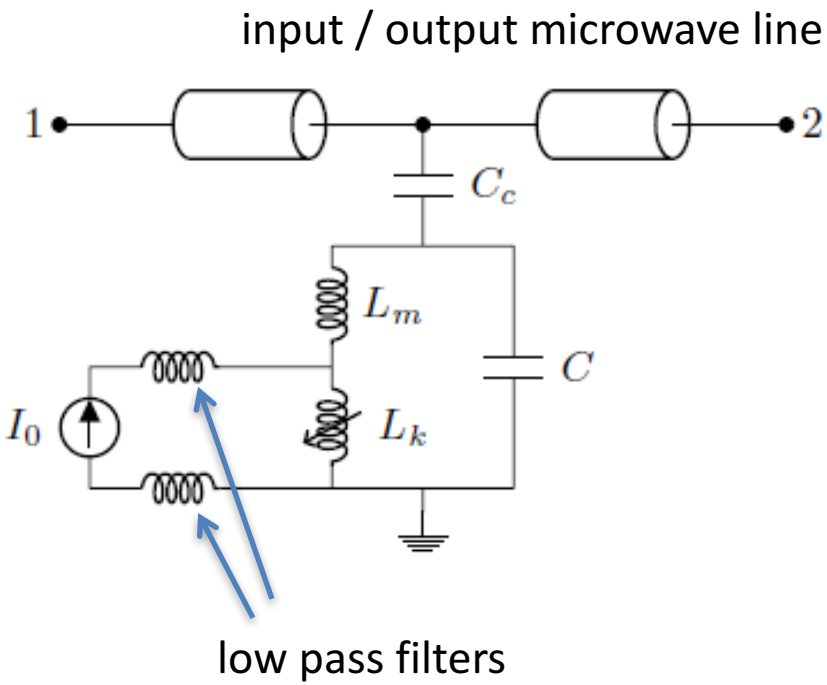


- $(S_\theta^{amp})^{1/2} = \sqrt{\frac{k_b T_N}{2P_{read}}}$
- for Pread = 1μW, T_N = 5K,
  - > $(S_I^{amp})^{1/2}$ ~ 6 pA / $Hz^{1/2}$
- Note: carrier tone power need not be dissipated at base temp



KPUP device concept

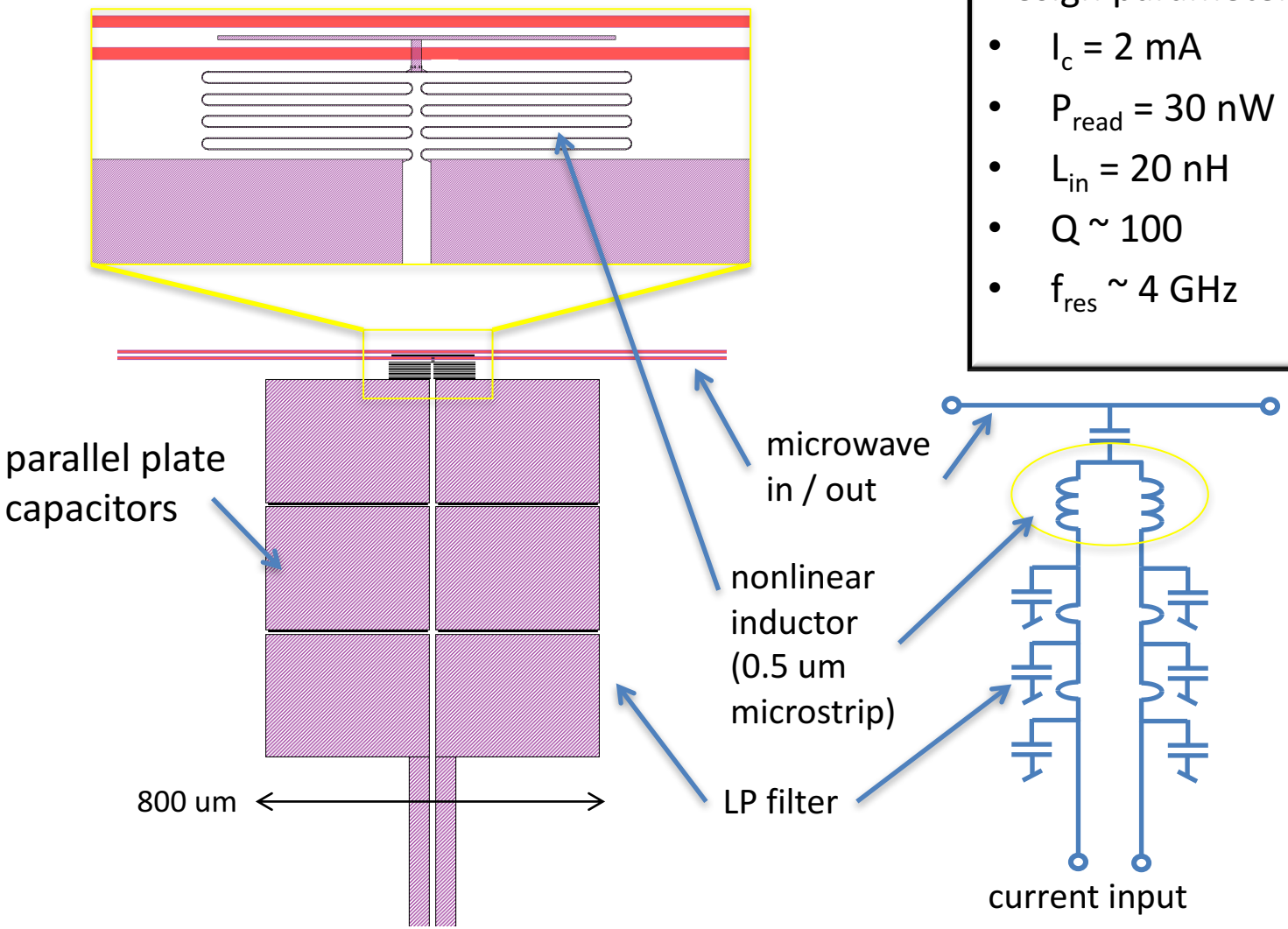
- low frequency current modulates resonator frequency
- compared to non-resonant TRL:
  - phase response increased by Q
  - but max readout power decreased by Q^2
  - smaller input inductance



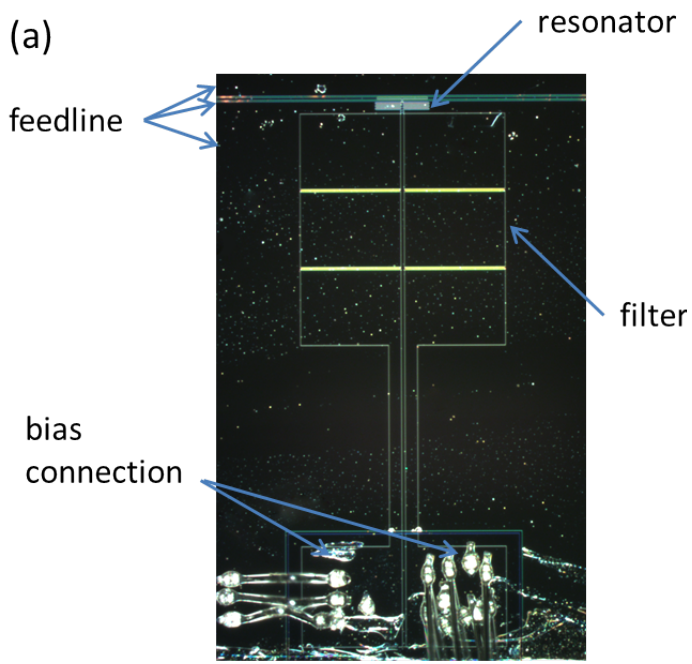
KPUP design

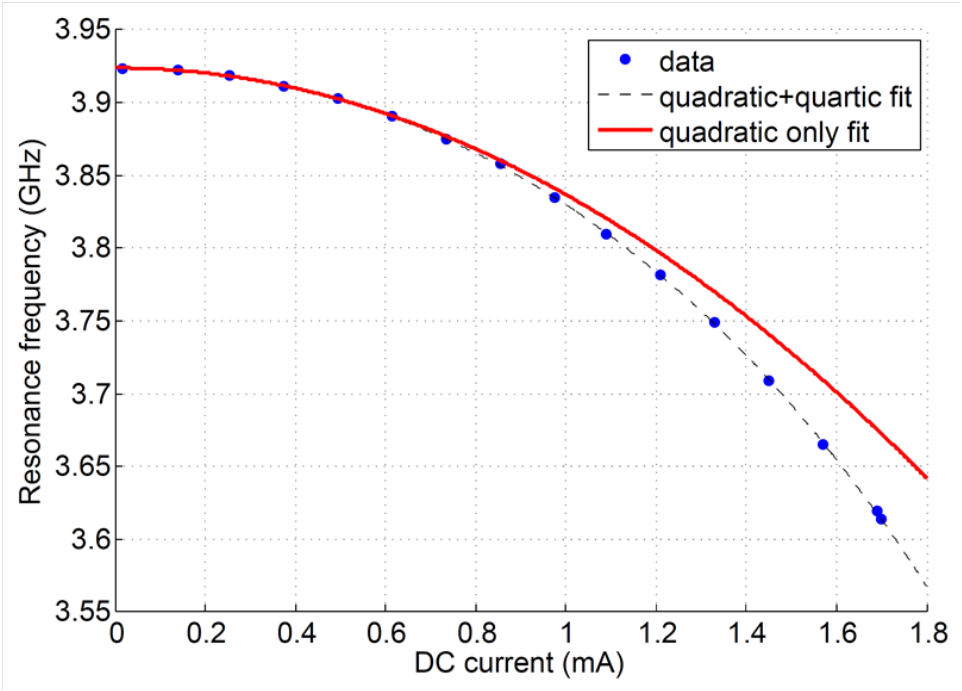
Design parameters:

- I_c = 2 mA
- P_{read} = 30 nW
- L_{in} = 20 nH
- Q ~ 100
- f_{res} ~ 4 GHz



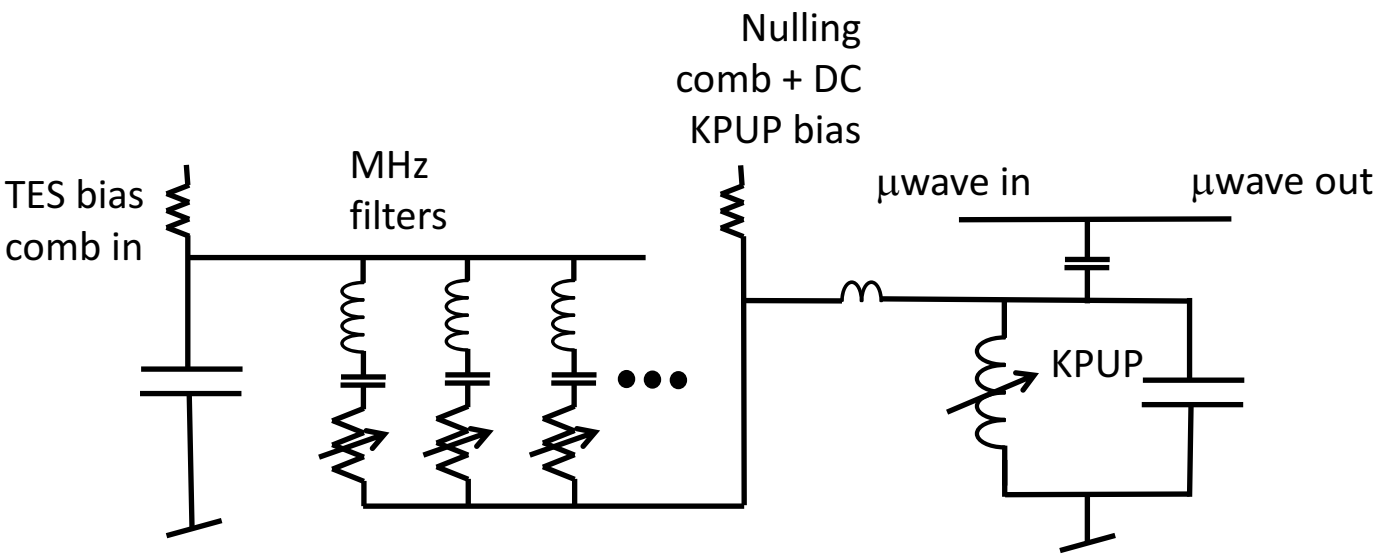
- Frequency shift measurement





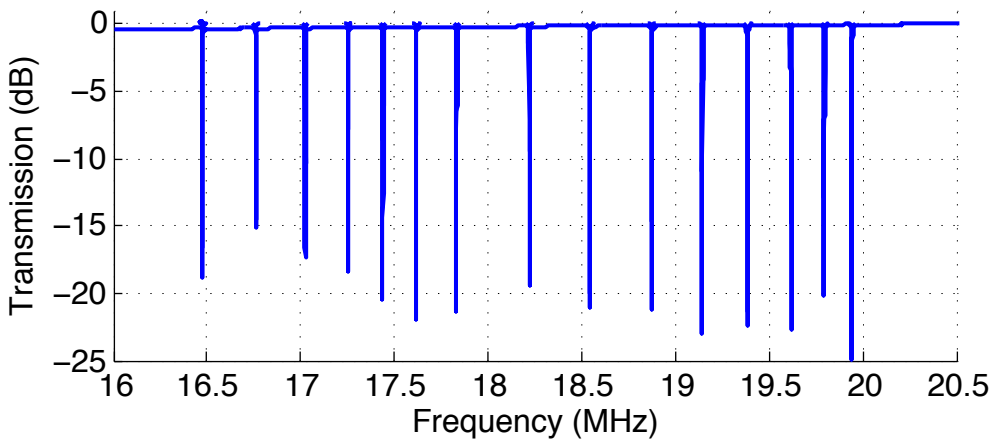
TES readout circuit

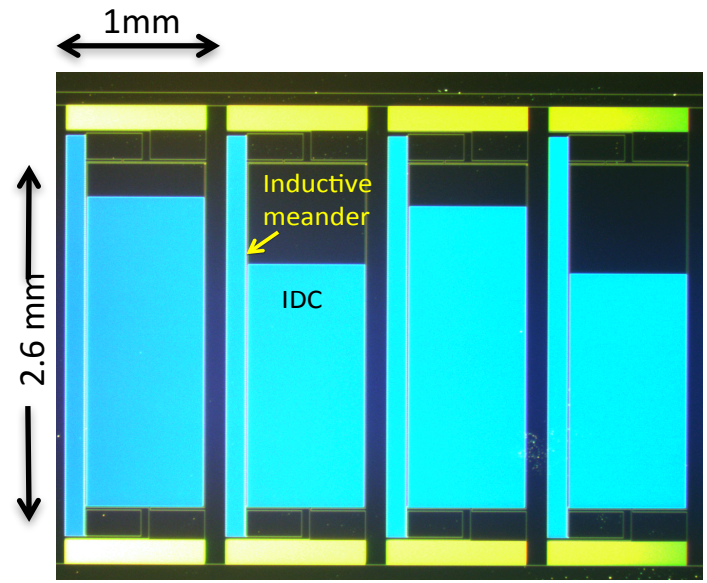
- Several TES signals combined at input to one KPUP
- Similar to circuit used for AC SQUID MUX

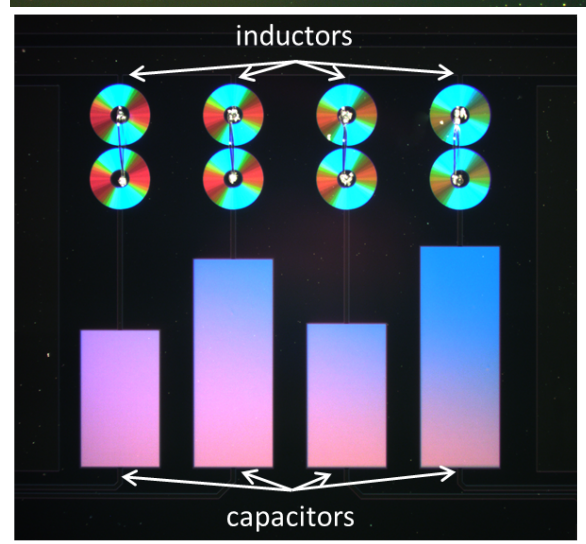


AC first stage MUX

- L-C filters
  - single layer NbTiN
  - interdigital capacitors
  - inductance is mainly kinetic
  - L ~ 4 μH, C ~ 20 pF
  - new version uses spiral inductors







AC first stage MUX

- TES array setup

BICEP2 type tile

KPUP chip

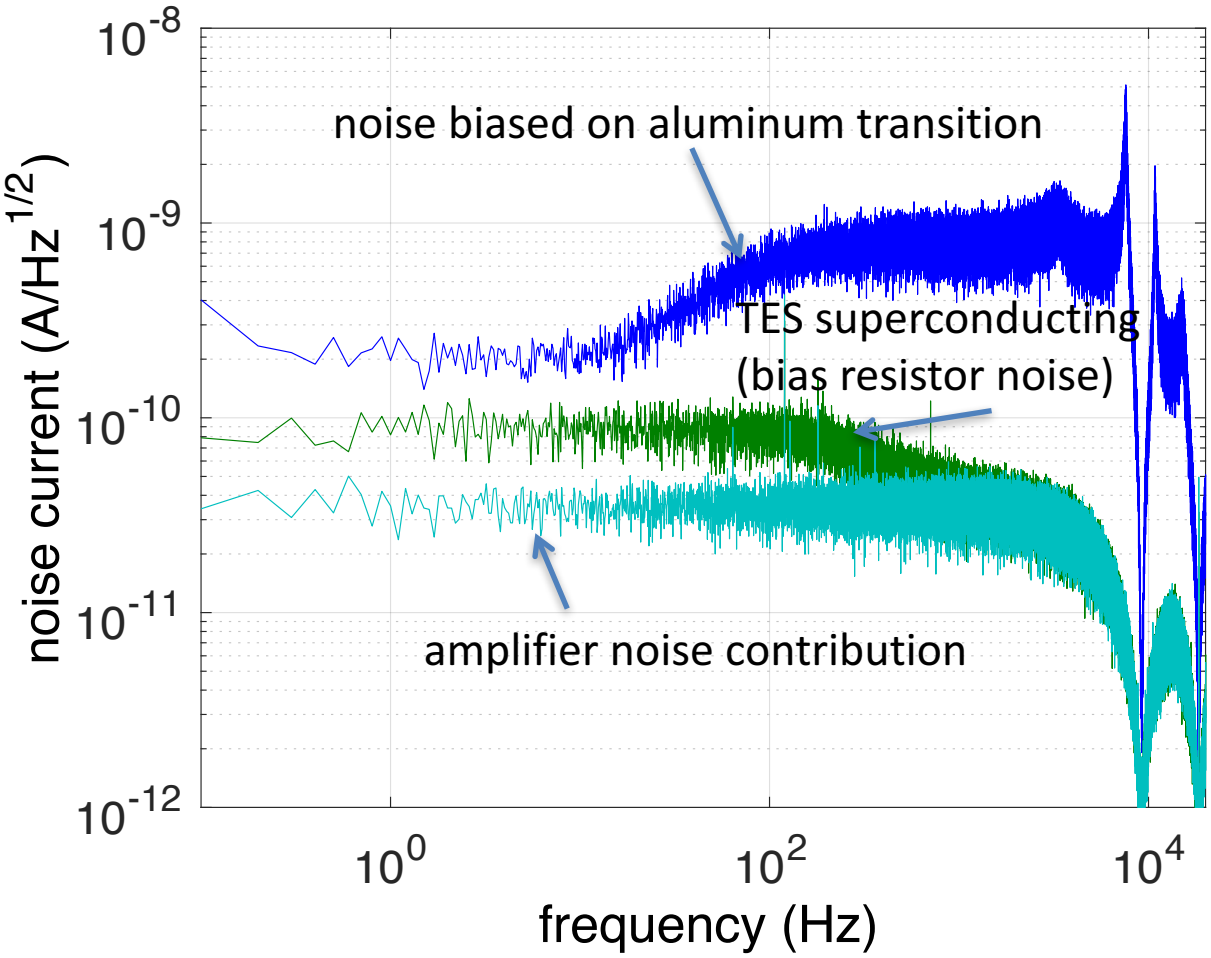
filter chip
(16 channels)

Bias comb in

niobium
box

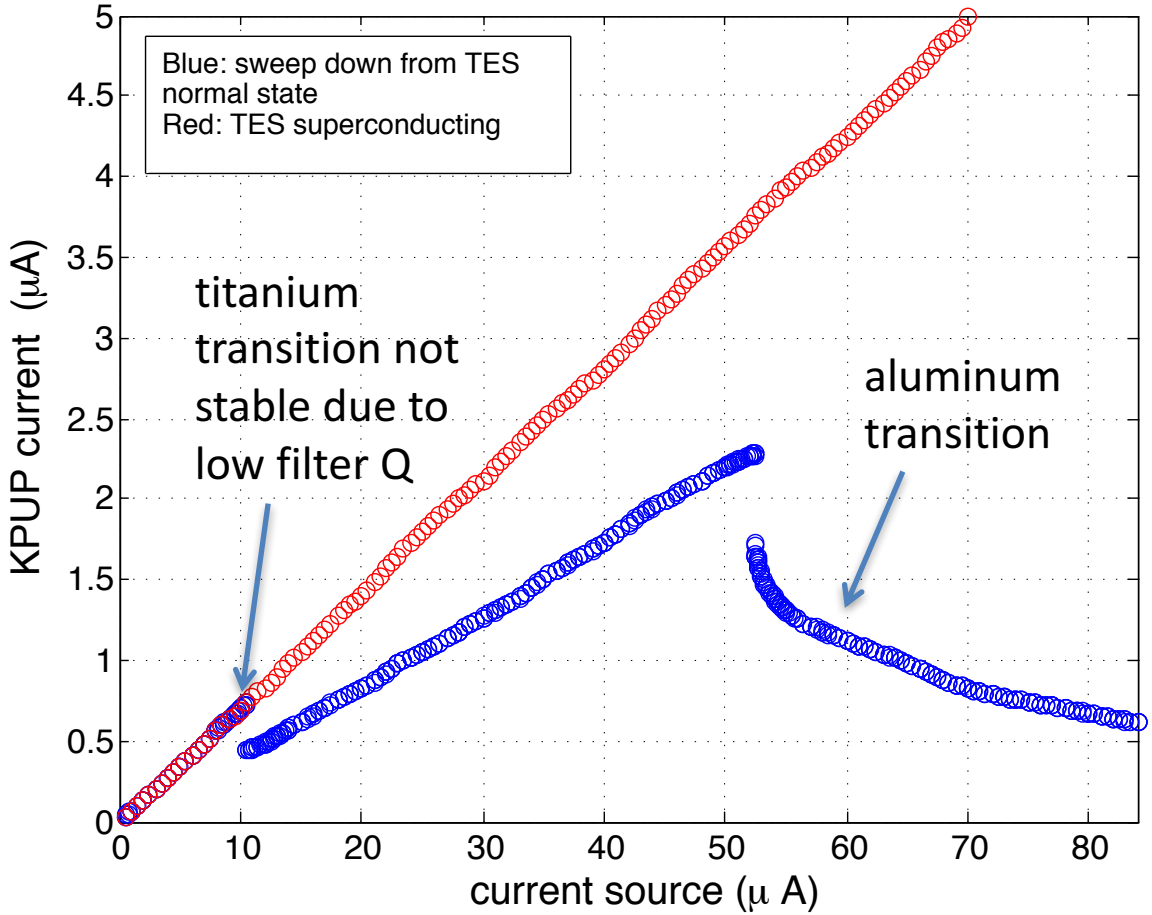
microwave in/out

Noise

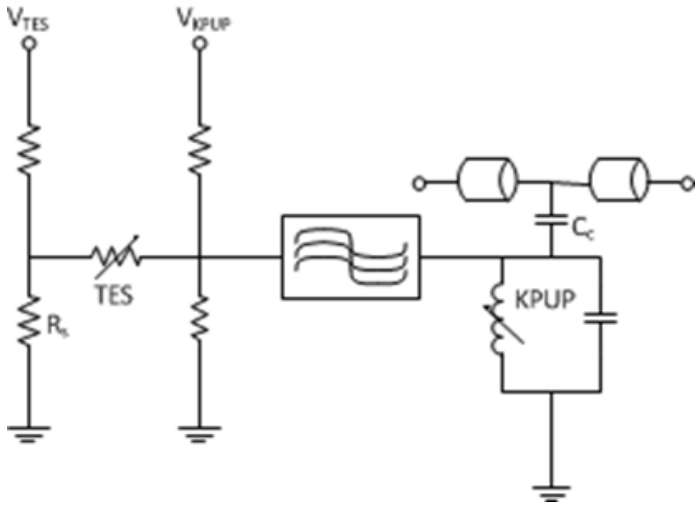

noise biased on aluminum transition
TES superconducting
(bias resistor noise)
amplifier noise contribution
noise current (A/Hz^1/2)
frequency (Hz)
10^-8
10^-9
10^-10
10^-11
10^-12
10^0
10^2
10^4

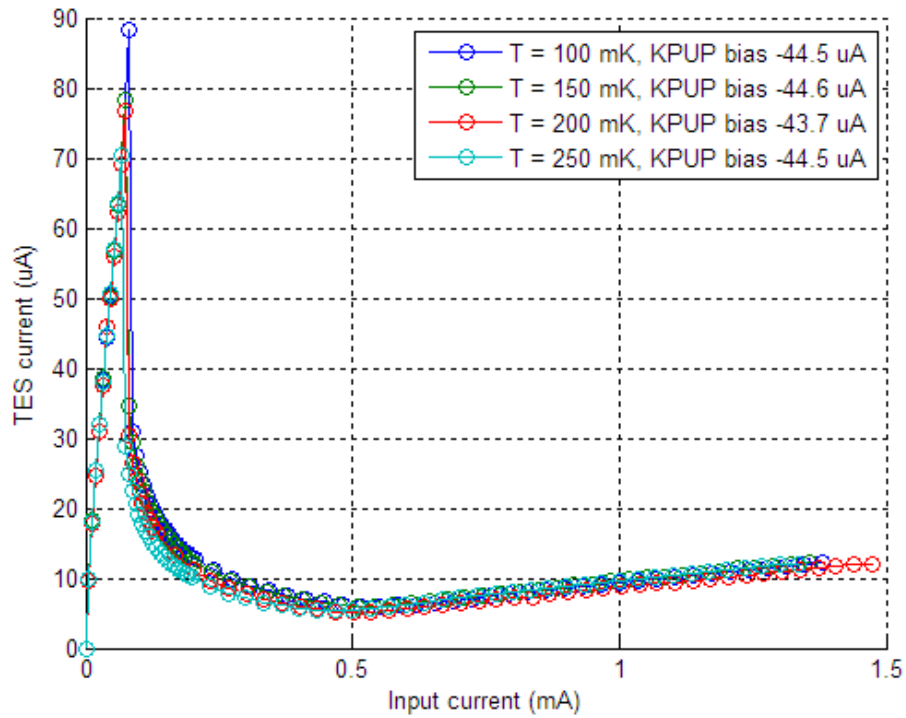
Bias Curve

- Measured without “nulling” of current into the KPUP
- Improving filter Q for stable operation on Ti transition



Bias curves from an older device





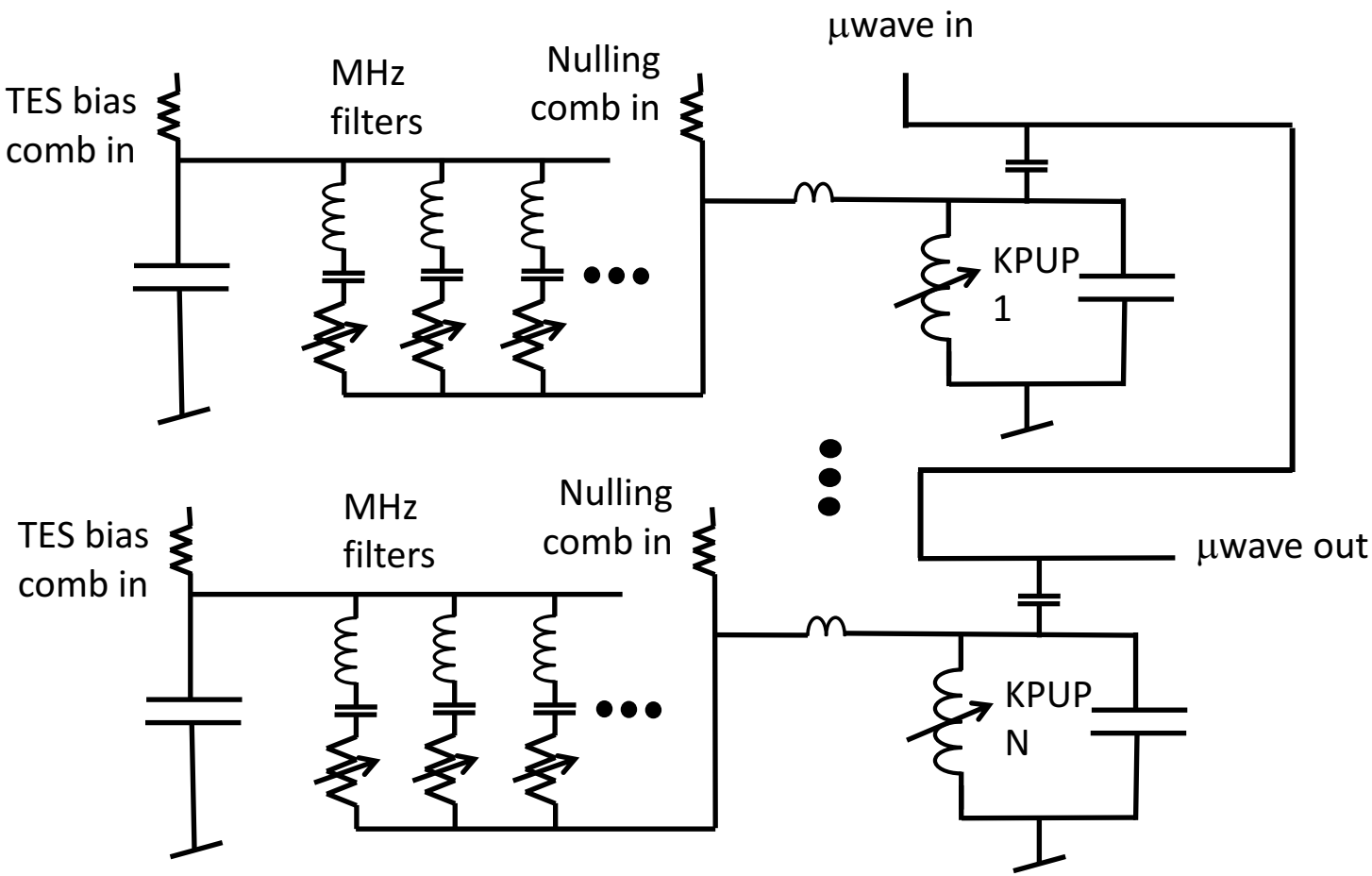
TES parameters:
Tc ~ 350 mK
g ~ 300 pW / K

$$\text{phonon noise} \approx 8\,\frac{\text{pA}}{\sqrt{\text{Hz}}}$$

Compared to SQUID array

- Input inductance is somewhat smaller ~ 19 nH
  - compared to ~100 nH for SQUID array
  - could be decreased further
- Bias power is ~ 30 nW
  - Only a fraction of that is dissipated in KPUP
  - KPUP can be located on cold stage
  - Wiring inductance between TES and KPUP can be small
- Microwave readout allows for multiplexing of several KPUPs
- Output is monotonic, rather than periodic

MUX circuit

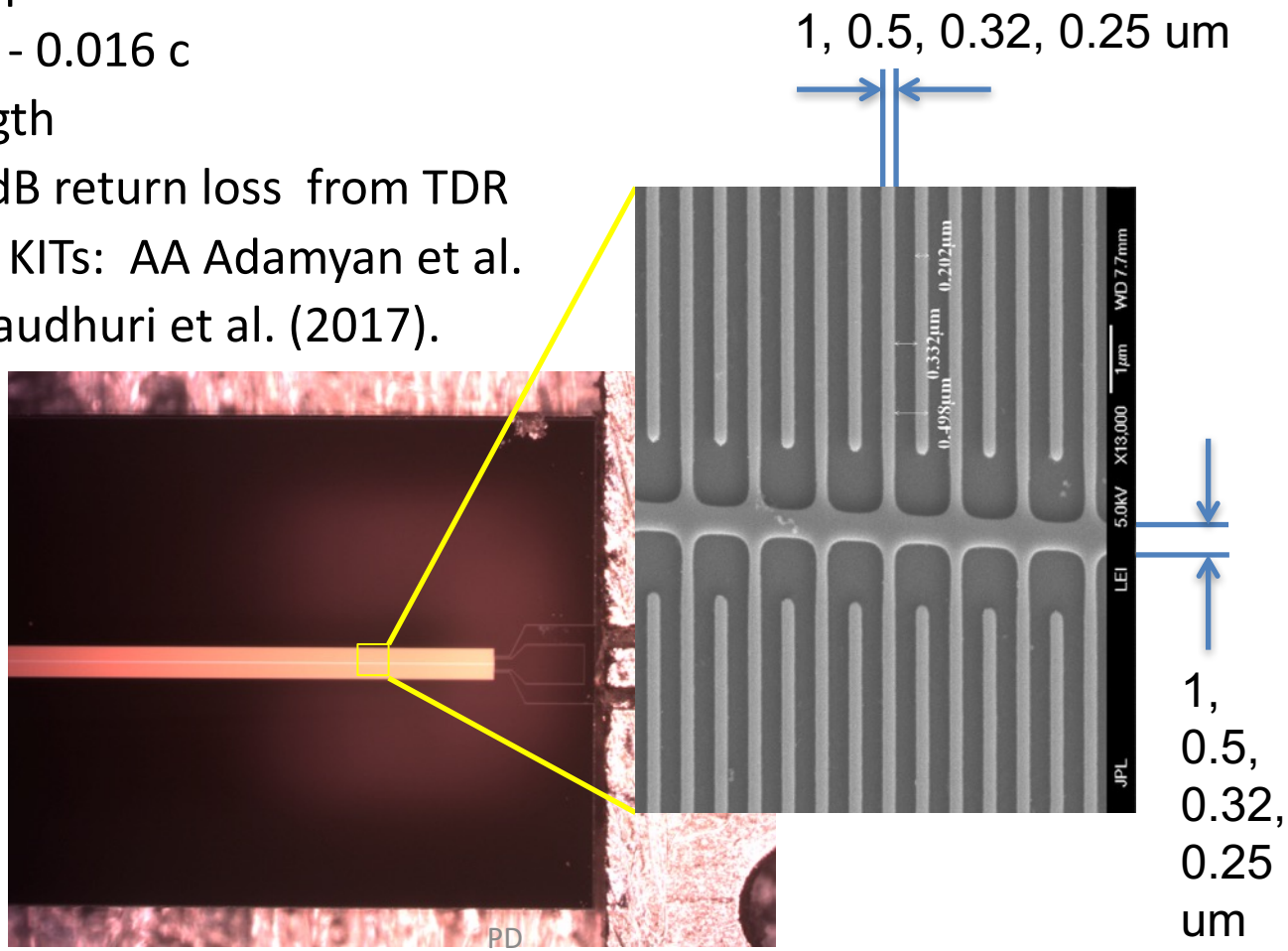


- Question: can the TES bias comb inputs be shared between sub arrays (how uniform can they be made?)

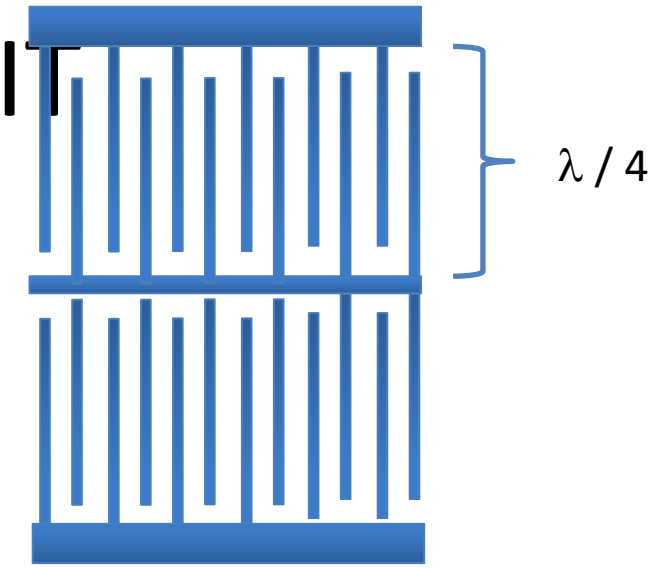
50Ω Nanowire KIT

50 ohm KIT

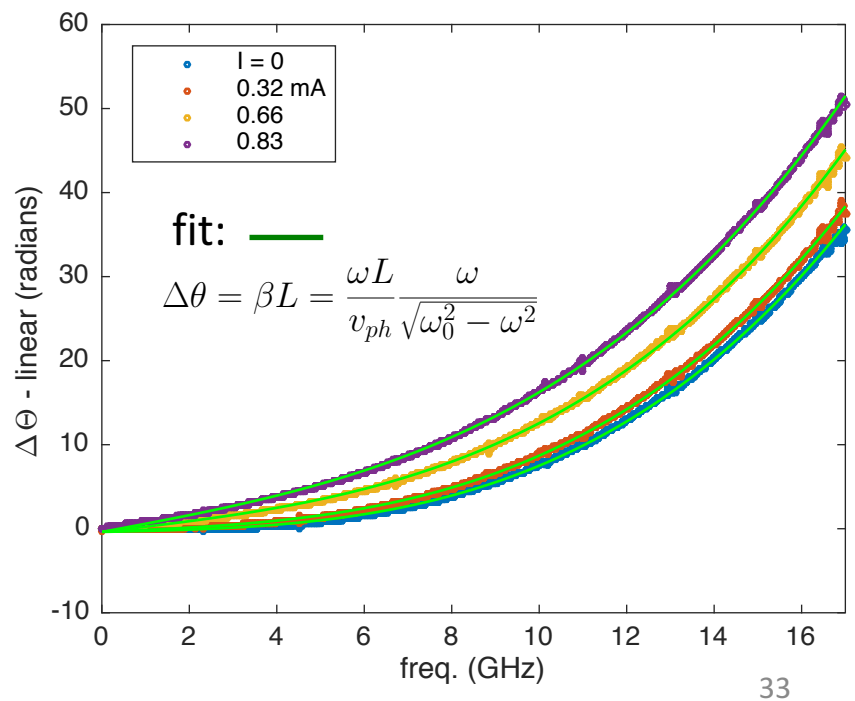
- Z_0 = 200-300 ohms for CPW KIT (due to large L_{kin})
- Increase capacitance with IDC
- v_{ph} = 0.004 - 0.016 c
- 2.5 cm length
- -18 to -22 dB return loss from TDR
- Other 50Ω KITs: AA Adamyan et al. (2016), Chaudhuri et al. (2017).



50 ohm KIT

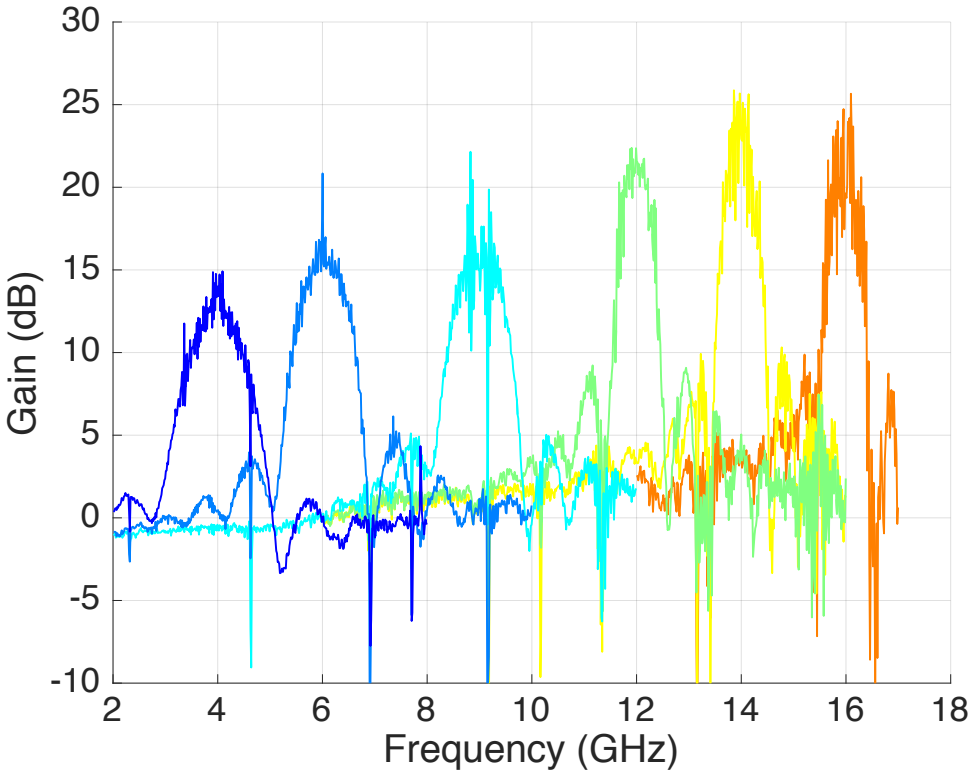


- Lines are dispersive
  - λ / 4 resonance from IDC finger length
  - f_{res} = 60 GHz for 0.5 um line width
  - Pump harmonics suppressed due to phase mismatch
    - no additional stop bands needed
  - Effect on bandwidth, phase matching



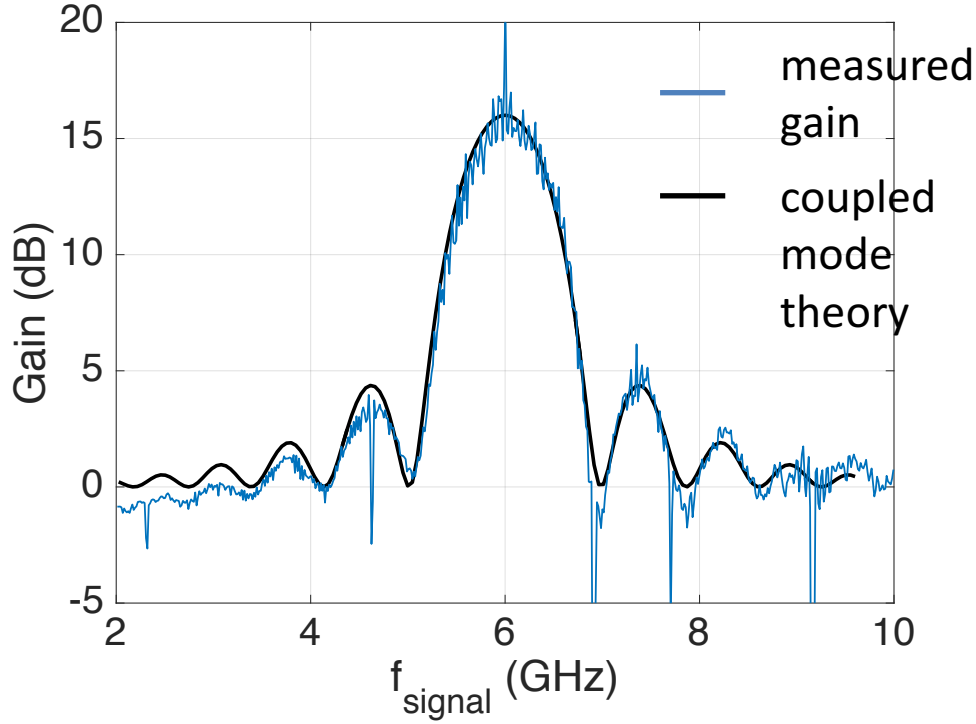
4WM in 0.25 micron linewidth devices, 2.5 cm length

- Non-phase matched 4 wave mixing
- Pump frequency is not constrained
  - No dispersion engineering
  - not phase matched
- 15 – 20 dB
- Pump power ~ 5 uW
- relatively narrow bandwidth due to dispersion



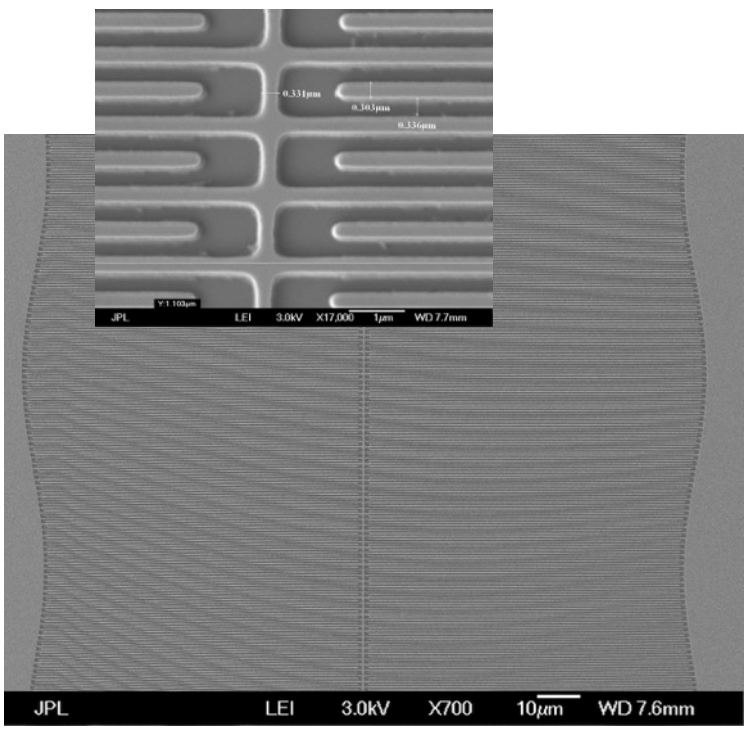
Gain measurement compared to theory

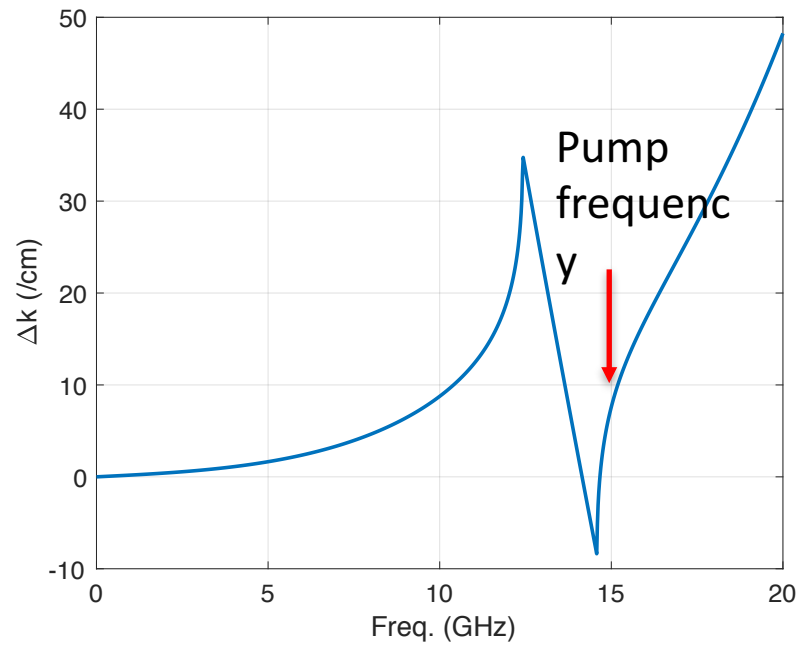
- Coupled mode theory including harmonic generation and measured dispersion
  - Single parameter fit : $(I / I_*)^2 = 0.08$
  - Max nonlinearity agrees with phase shift vs. dc current measurements



3WM: Engineering dispersion

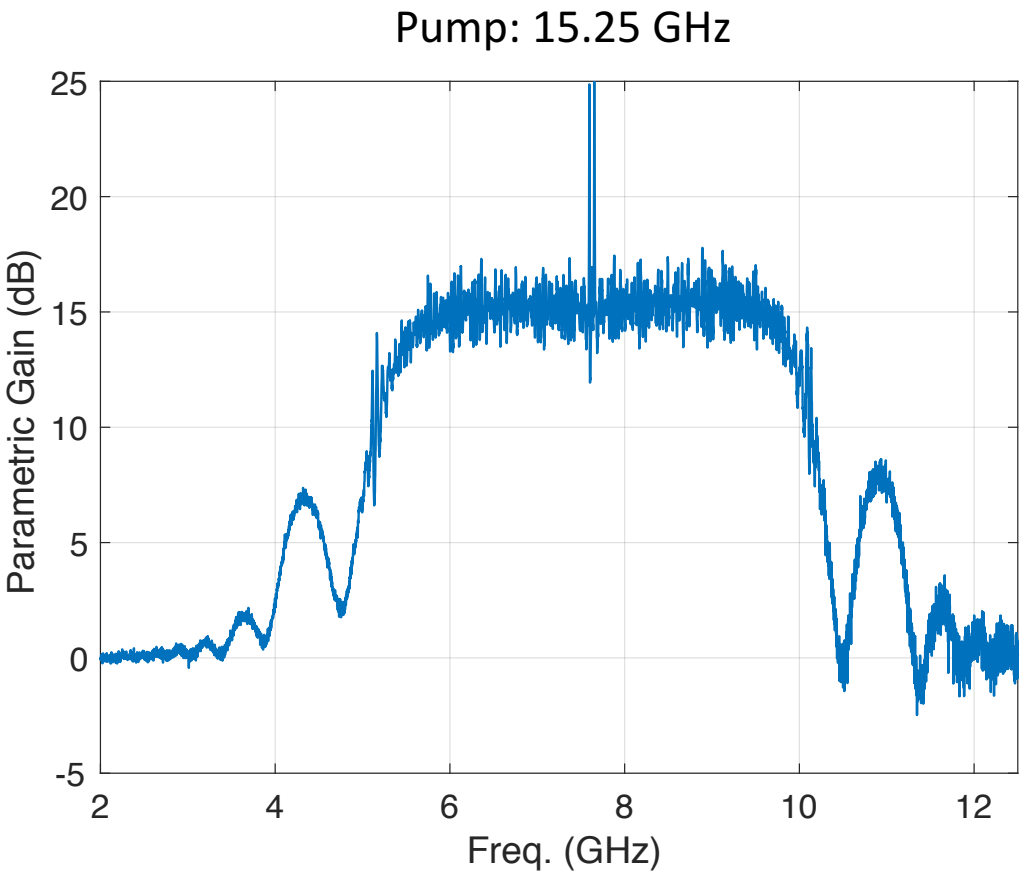
- Dispersive effect of capacitor finger self-resonance produces phase mismatch between pump and signal/idler
  - Can compensate by adding periodic modulation





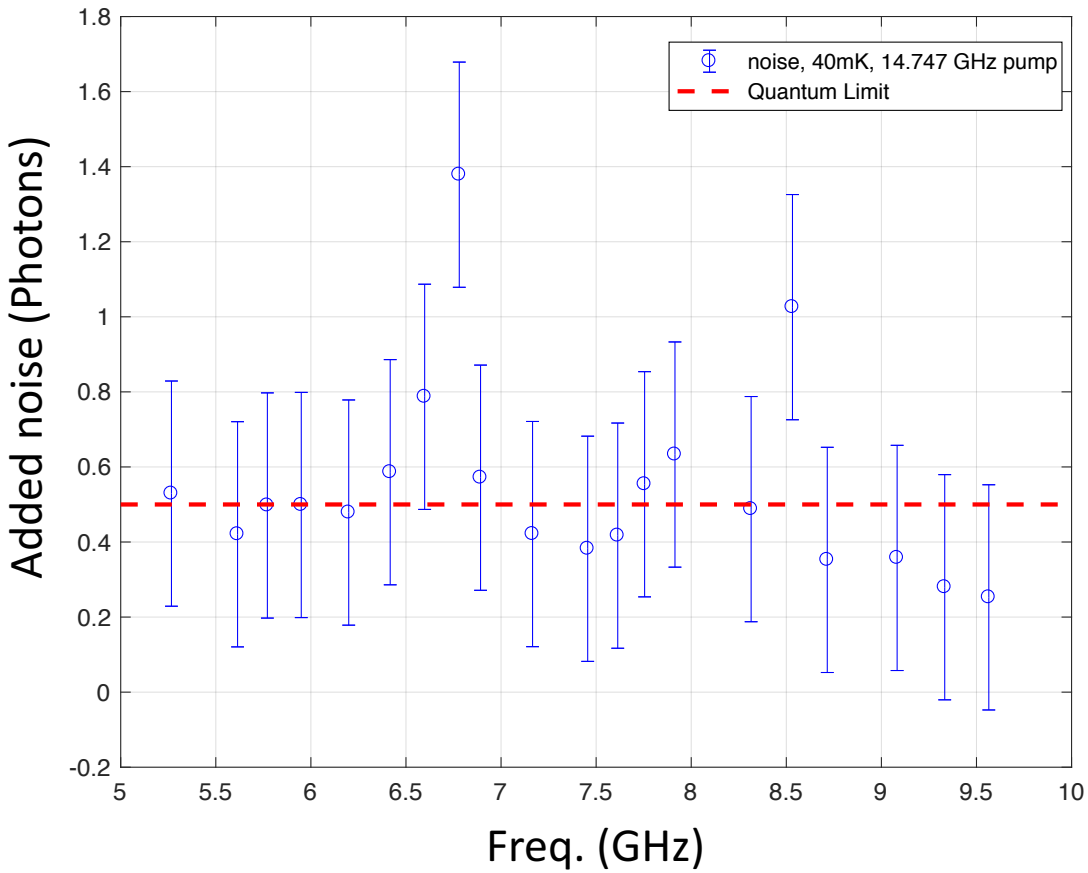
Gain

- 320 nm linewidth
- 2.5 cm total length
- Pump on / pump off transmission is plotted
  - There is some dissipation at 4K which disappears at low temperature
- Pump power in range 200 nW to 2 μW
  - Depends on applied DC current



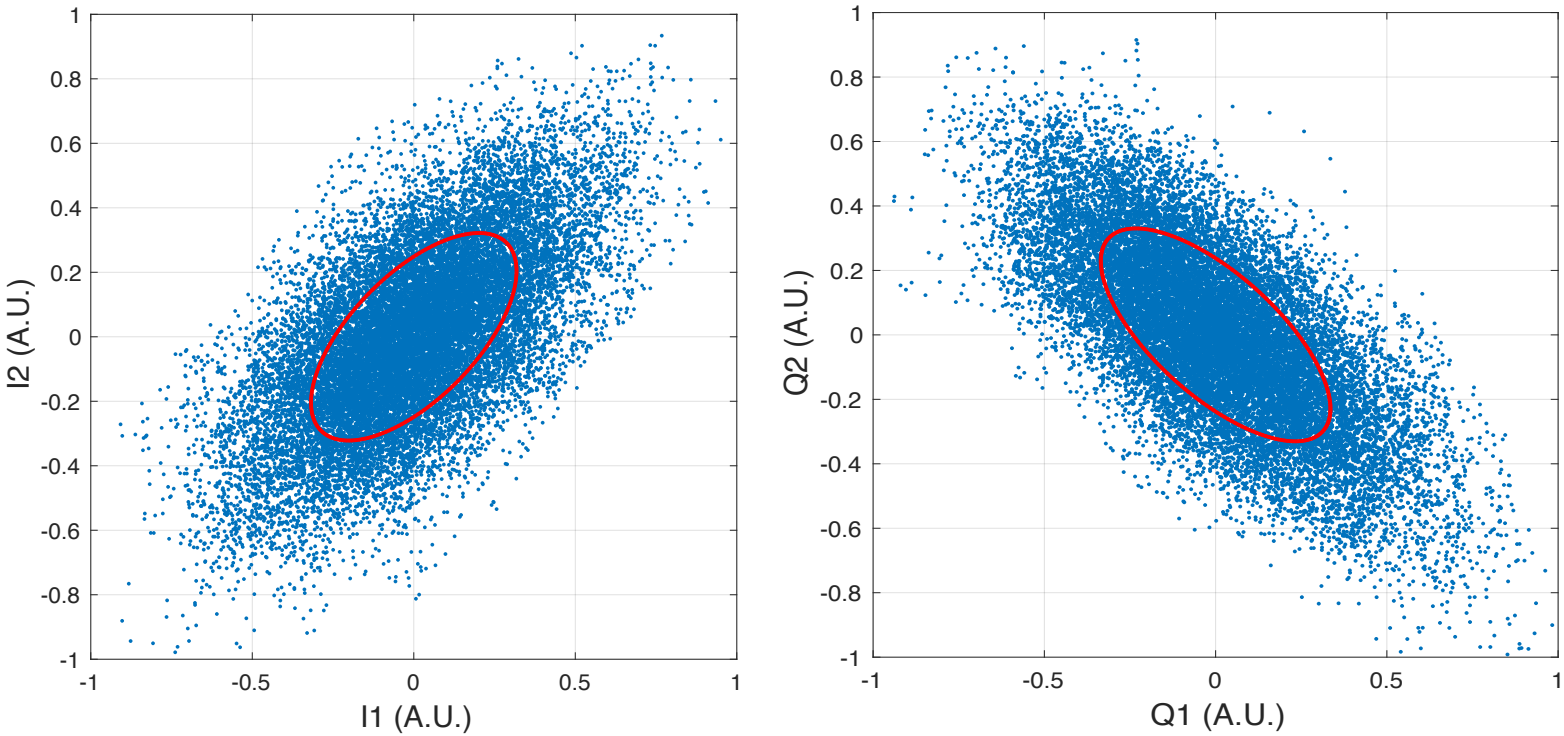
Noise

- Y-factor noise measurement



Signal-idler noise correlation

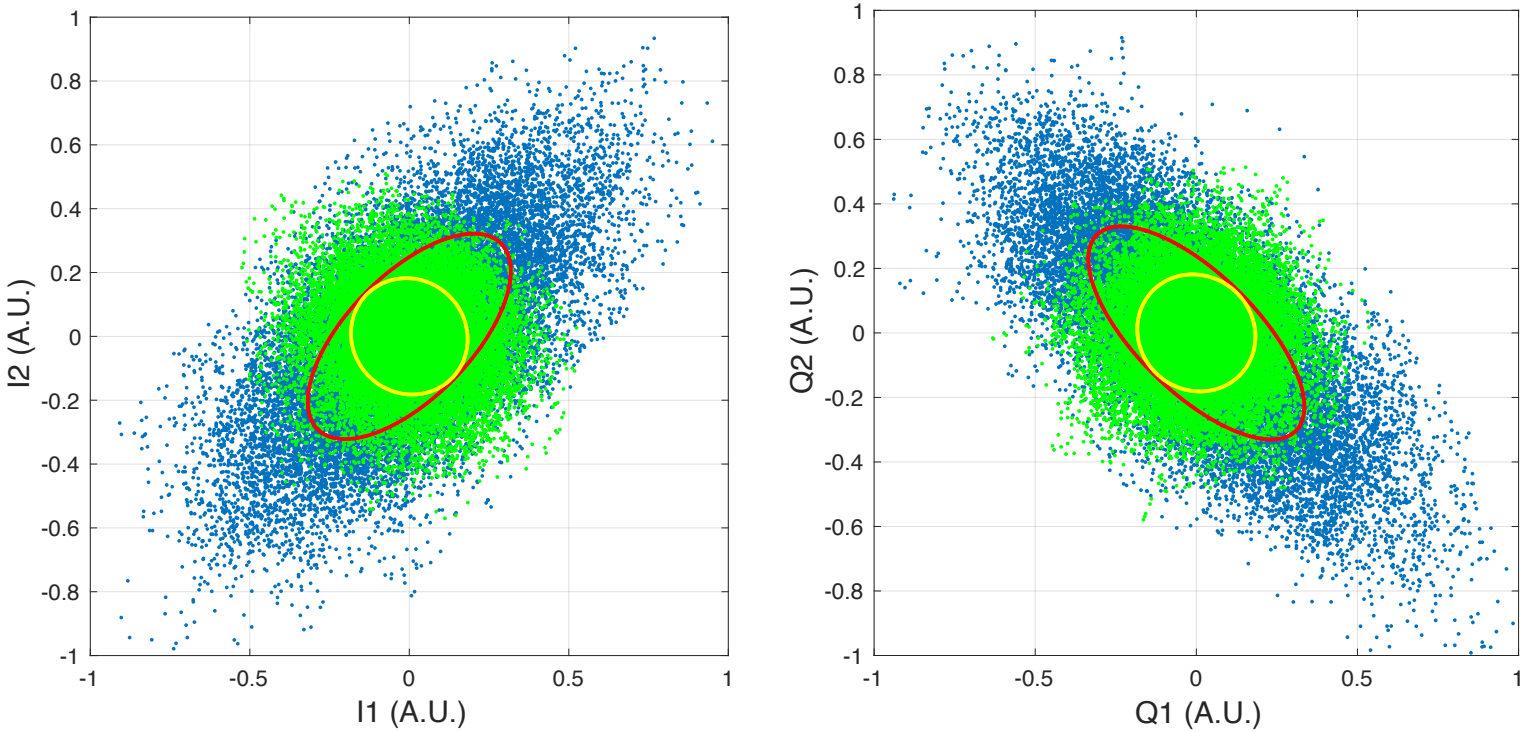
- Noise at frequencies symmetrically located around $f_{pump}/2$: 7.1384 GHz (I1, Q1), 7.6086 GHz (I2, Q2)



Pump on

Signal-idler noise correlation

- Noise at frequencies symmetrically located around $f_{pump}/2$: 7.1384 GHz (I1, Q1), 7.6086 GHz (I2, Q2)



Pump on
Pump off

Conclusions

- Kinetic Inductance Nonlinearity
  - Important parameter for MKID detectors
  - Similar to Josephson Effect - Can be used to make interesting devices
    - Up-converter/ Current sensor: KPUP
    - Parametric Amplifiers