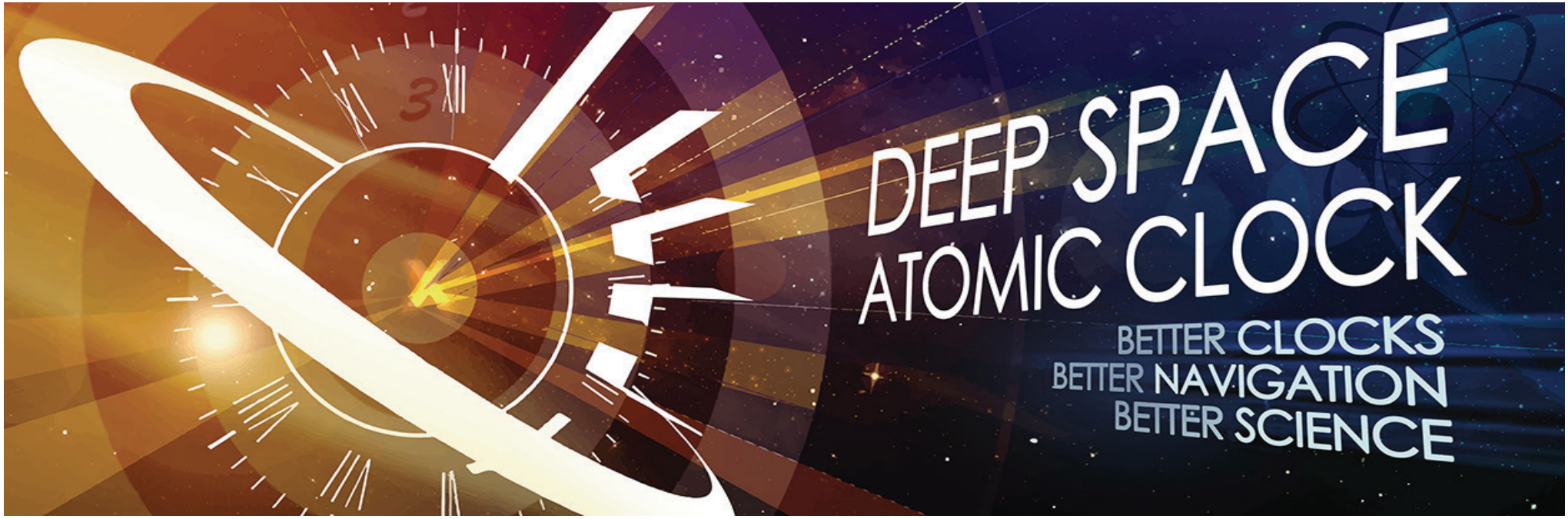



# Batch Sequential Estimation with Non-uniform Measurements and Non-stationary Noise

Todd Ely and Jill Seubert

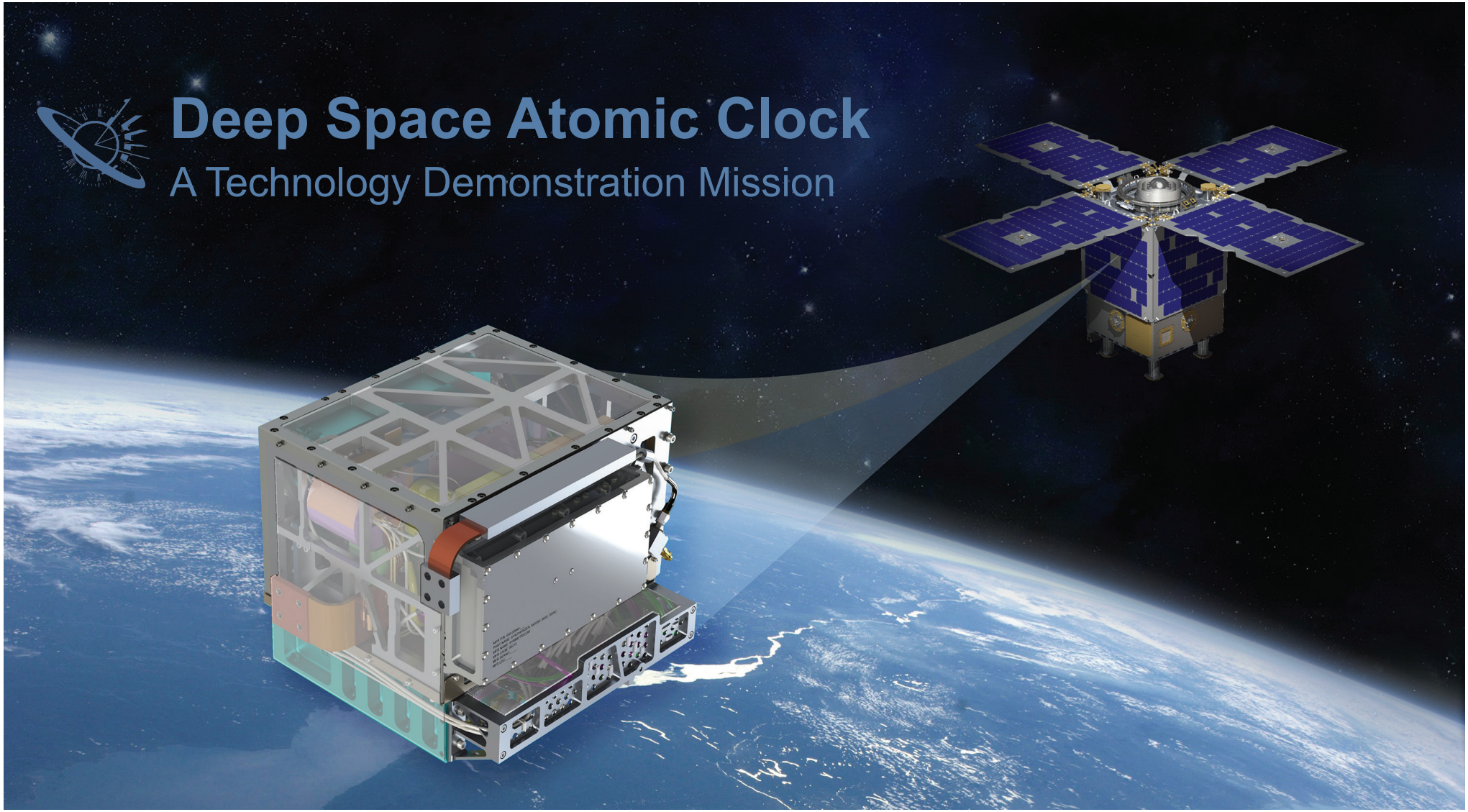
August 2017

© 2017 California Institute of Technology. Government sponsorship acknowledged.

Jet Propulsion Laboratory
California Institute of Technology

# Deep Space Atomic Clock

## A Technology Demonstration Mission



NASA

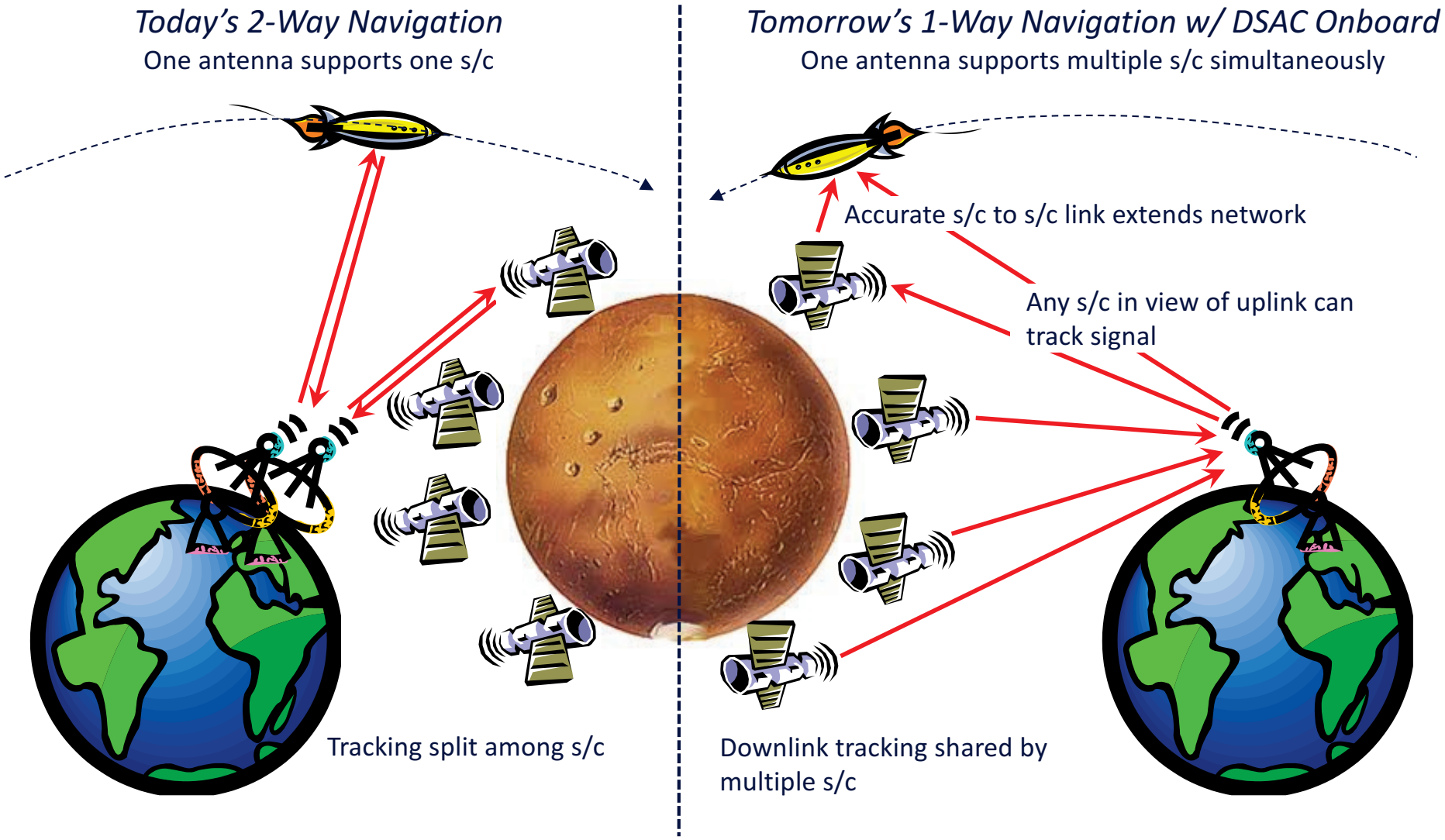
TDM

SPACE COMMUNICATIONS SCaN AND NAVIGATION

JPL
Jet Propulsion Laboratory
California Institute of Technology

SURREY
SATELLITE TECHNOLOGY US

# DSAC Enables a Scalable DSN Tracking Architecture



# Standard Kalman Filter Scenario

- Standard assumption is the epoch of the state estimate and measurements are coincident

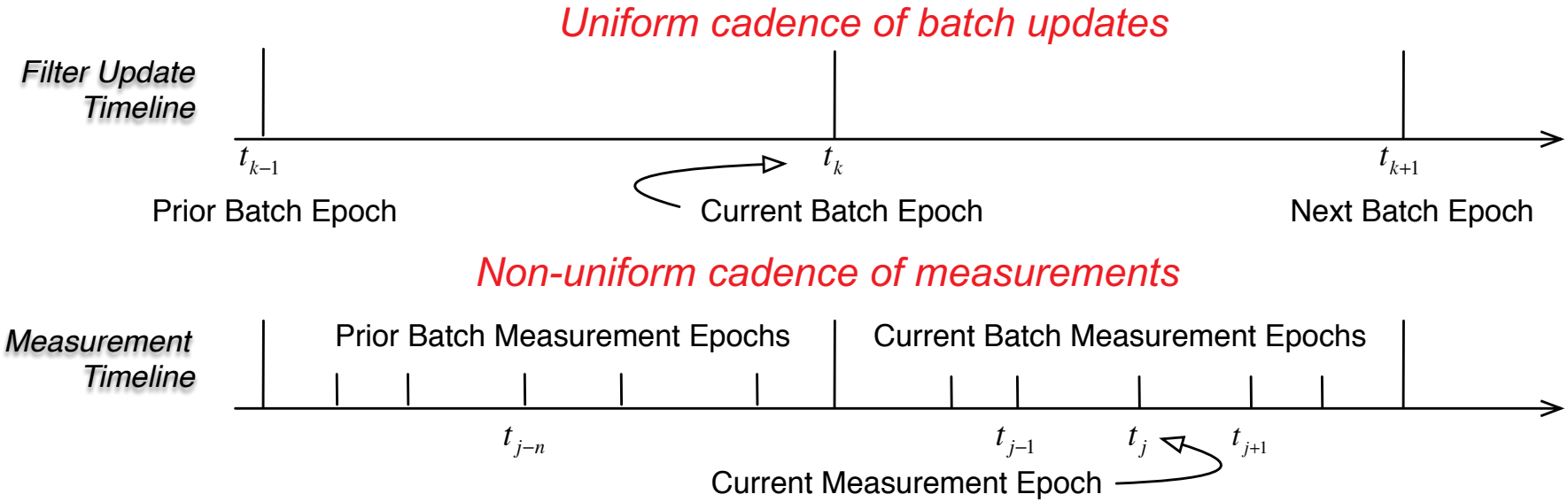
$$\hat{\mathbf{x}}(t_k) = E[\mathbf{x}_k | \mathbf{Z}_k = \{\mathbf{z}_1, \dots, \mathbf{z}_k\}]$$

$$\mathbf{x}_k = \mathbf{\Phi}_{k,k-1}\mathbf{x}_{k-1} + \mathbf{w}_{k,k-1}$$

$$\mathbf{z}_k = \mathbf{H}_k\mathbf{x}_k + \mathbf{v}_k$$

- What are the implications when these epochs are not coincident?

# Generalized Measurement Model




$$\mathbf{x}_{k+1} = \mathbf{\Phi}_{k+1,k}\mathbf{x}_k + \mathbf{w}_{k+1,k}$$

$$\mathbf{z}_j = \mathbf{H}_j\mathbf{x}_j + \mathbf{v}_j$$

$$\hat{\mathbf{x}}(t_k) = E\left[\mathbf{x}_k | \mathbf{Z}_j = \{\mathbf{z}_1, \dots, \mathbf{z}_j\}\right]$$

- The measurements arrive after the epoch of the state estimate of interest, that is $t_j > t_k$

# Batch Sequential Filter

- Effect 'pulls forward' process noise into the measurement model

$$\mathbf{z}_j = \mathbf{H}_j \boldsymbol{\Phi}_{j,k} \mathbf{x}_k + \mathbf{H}_j \mathbf{w}_{j,k} + \mathbf{v}_j \triangleq \mathbf{H}_{j,k} \mathbf{x}_k + \mathbf{v}'_{j,k}$$

$$\boxed{\boldsymbol{R}'_{j,k} \triangleq E\left[\mathbf{v}'_{j,k} \mathbf{v}'^{\mathrm{T}}_{j,k}\right] = \mathbf{H}_j \boldsymbol{Q}_{j,k} \mathbf{H}_j^{\mathrm{T}} + \boldsymbol{R}_j}$$

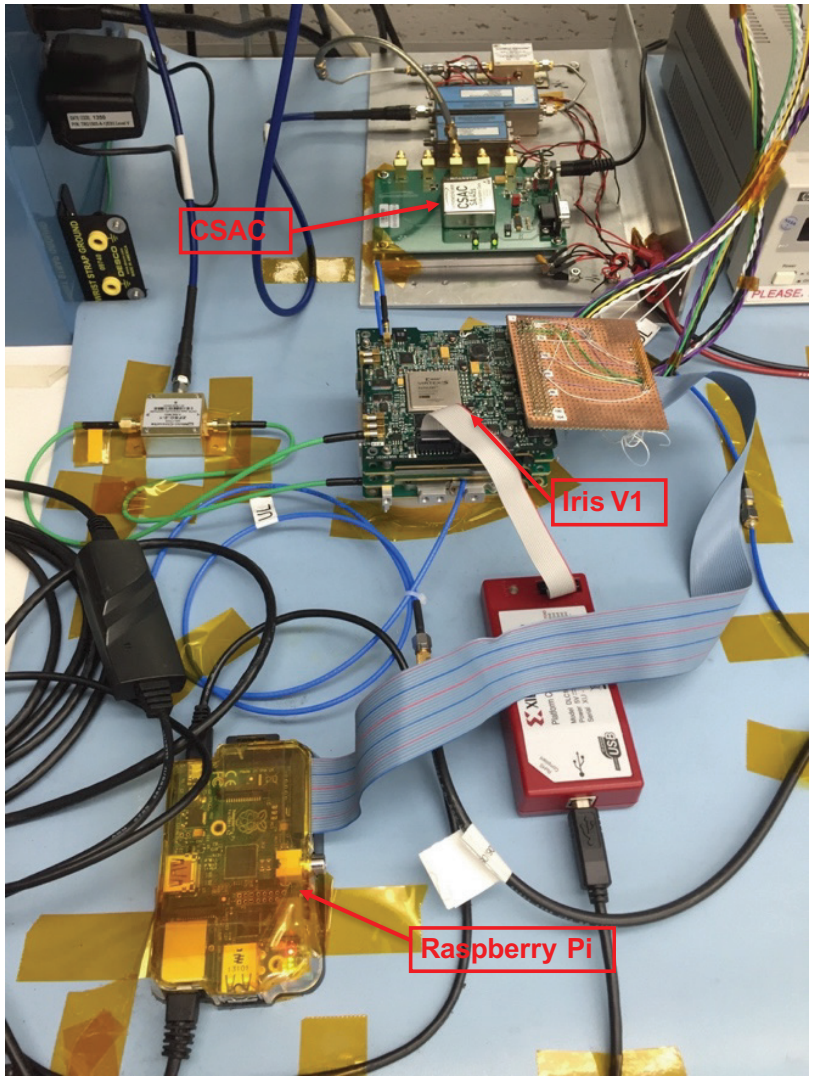
- The Kalman Filter equations are updated to a more general form that admits correlations between measurement and process noise

$$\hat{\mathbf{x}}_k^+ = \hat{\mathbf{x}}_k^- + \mathbf{K}_j\left[\mathbf{z}_j - \mathbf{H}_{j,k}\hat{\mathbf{x}}_k^-\right] \forall \left\{\mathbf{z}_j \middle| t_k \leq t_j < t_{k+1}\right\}$$

$$\hat{\mathbf{x}}_{k+1}^- = \boldsymbol{\Phi}_{k+1,k}\hat{\mathbf{x}}_k^+$$

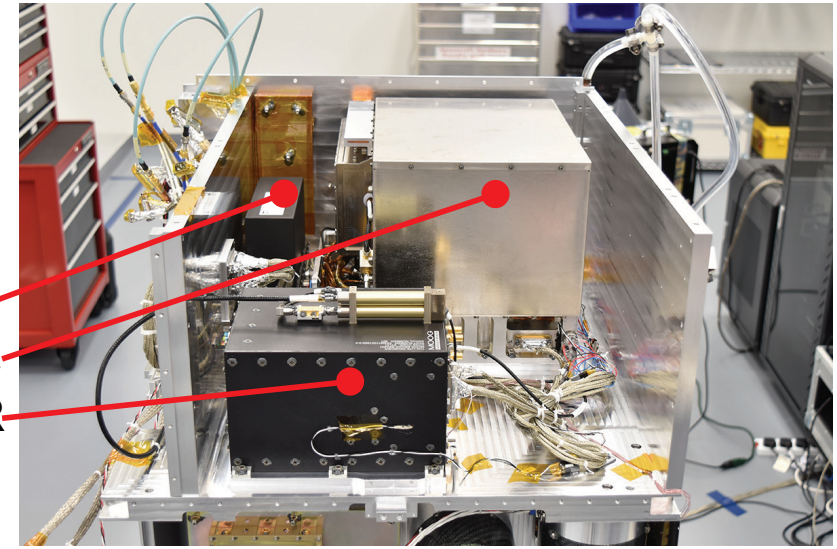
$$\mathbf{P}_k^+ = \left(\mathbf{I} - \mathbf{K}_j\mathbf{H}_{j,k}\right)\mathbf{P}_k^-\left(\mathbf{I} - \mathbf{K}_j\mathbf{H}_{j,k}\right)^{\mathrm{T}} + \mathbf{K}_j\boldsymbol{R}'_{j,k}\mathbf{K}_j^{\mathrm{T}} - \left(\mathbf{I} - \mathbf{K}_j\mathbf{H}_{j,k}\right)\mathbf{S}_{j,k-1}\mathbf{K}_j^{\mathrm{T}} - \mathbf{K}_j\mathbf{S}_{j,k-1}^{\mathrm{T}}\left(\mathbf{I} - \mathbf{K}_j\mathbf{H}_{j,k}\right)^{\mathrm{T}}.$$

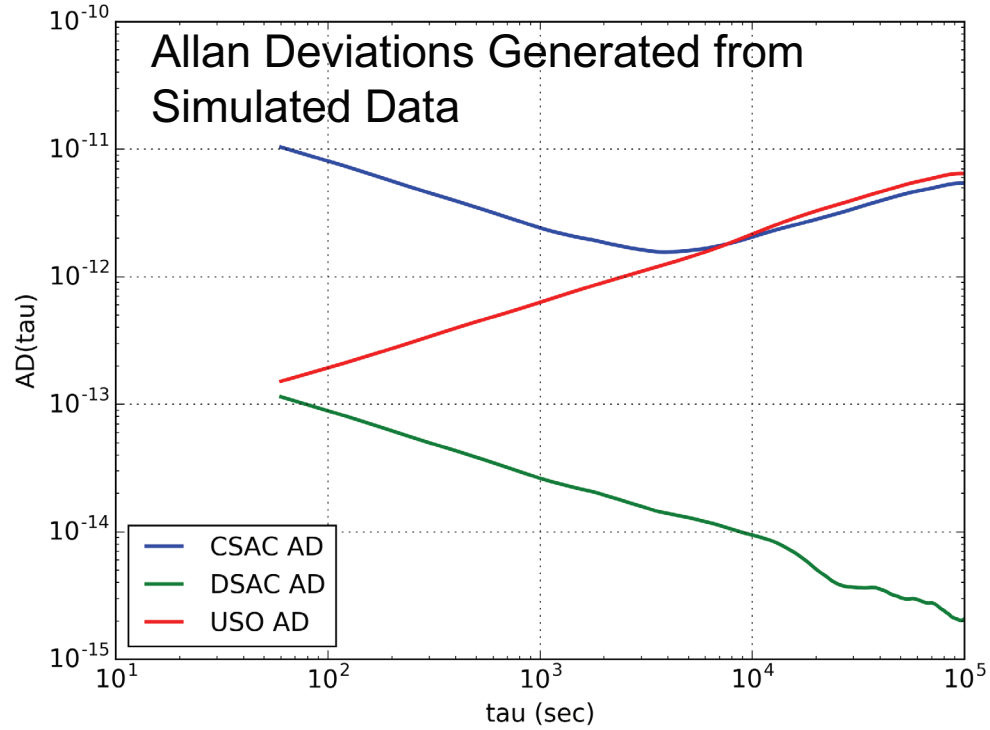
# Space Clocks: CSAC, USO, and DSAC




Iris V1 and CSAC Test Set Up

DSAC Payload
- USO
- DSAC
- GPSR





# Simplified 2-D Clock Model with Phase and Phase Difference Measurements

- Clock phase error and rate error dynamics

$$x_j = x_k + \Delta t_{j,k} y_k + w_{j,k}^1$$
$$y_j = y_k + w_{j,k}^2$$

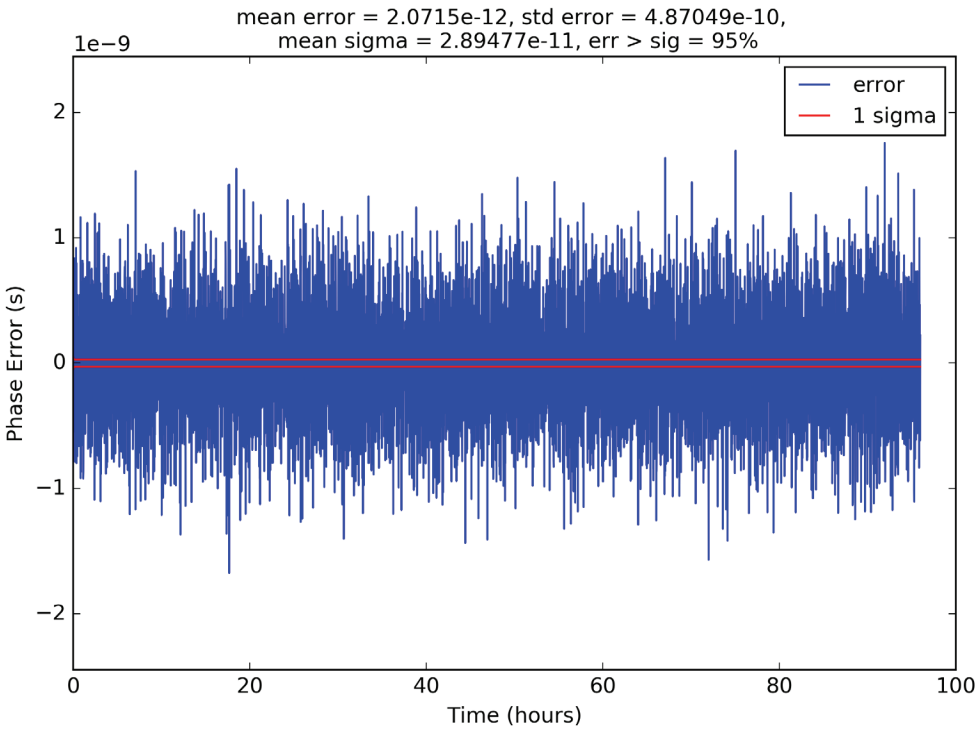
- Correlated process noise

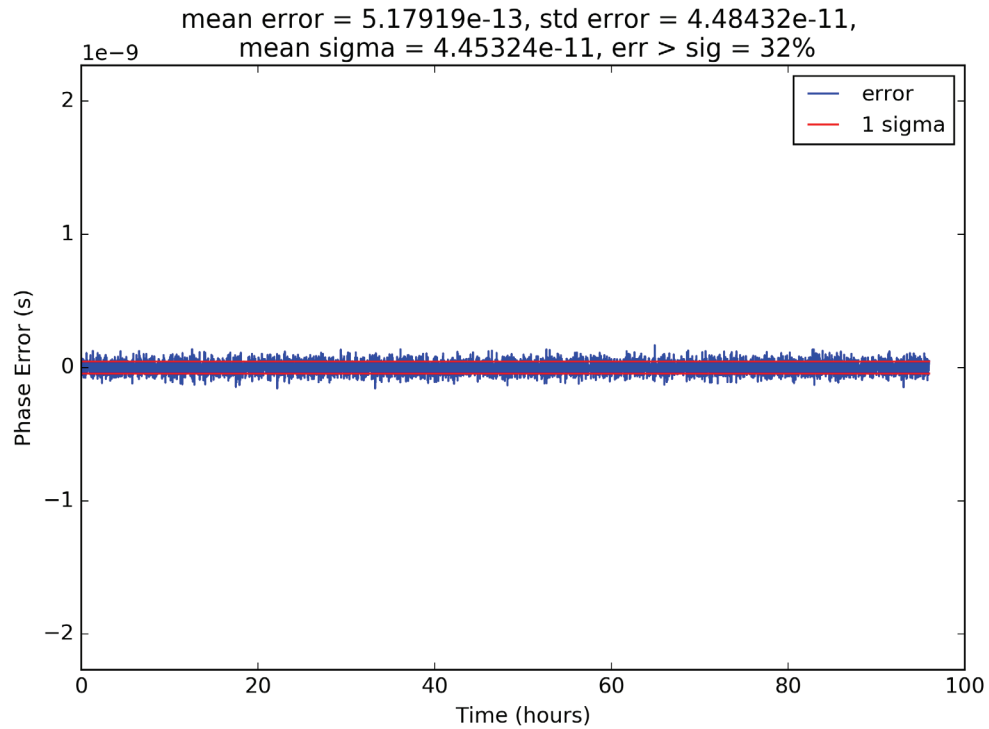
$$\boldsymbol{Q}_{j,k} = \begin{bmatrix} \sigma_1^2 \Delta t_{j,k} + \frac{1}{3}\sigma_2^2 \Delta t_{j,k}^3 & \frac{1}{2}\sigma_2^2 \Delta t_{j,k}^2 \\ \frac{1}{2}\sigma_2^2 \Delta t_{j,k}^2 & \sigma_2^2 \Delta t_{j,k} \end{bmatrix}$$

- Clock phase and phase difference measurements

$$\begin{bmatrix} z_j^1 \\ z_j^2 \end{bmatrix} \triangleq \begin{bmatrix} x_j + v_j^1 \\ x_j + v_j^2 - \left(x_{j-1} + v_{j-1}^2\right) \end{bmatrix}$$

# CSAC Clock Sim – Naïve Filter (Standard KF) vs Proper Measurement Weight Filter




Suboptimal solution with naïve filter

- Errors exceed 1-sigma uncertainty 95% >> 33%

Optimal solution with proper measurement deweighting

- Filter sample statistics consistent with 1-sigma uncertainty
- Lower magnitude noise

# Deep Space One-Way Radiometric Measurements

- Phase: $$\Phi(t) = \Delta r(t) + c[x_R(t) - x_T(t-\tau)] - I(t) - S(t) + T(t) + PW(t) + M^{\Phi}(t) + b_R^{\Phi}(t) + b_T^{\Phi} + N + v(t)$$

- Doppler: $$F(t) \triangleq \frac{\Phi(t) - \Phi(t-T)}{T}$$

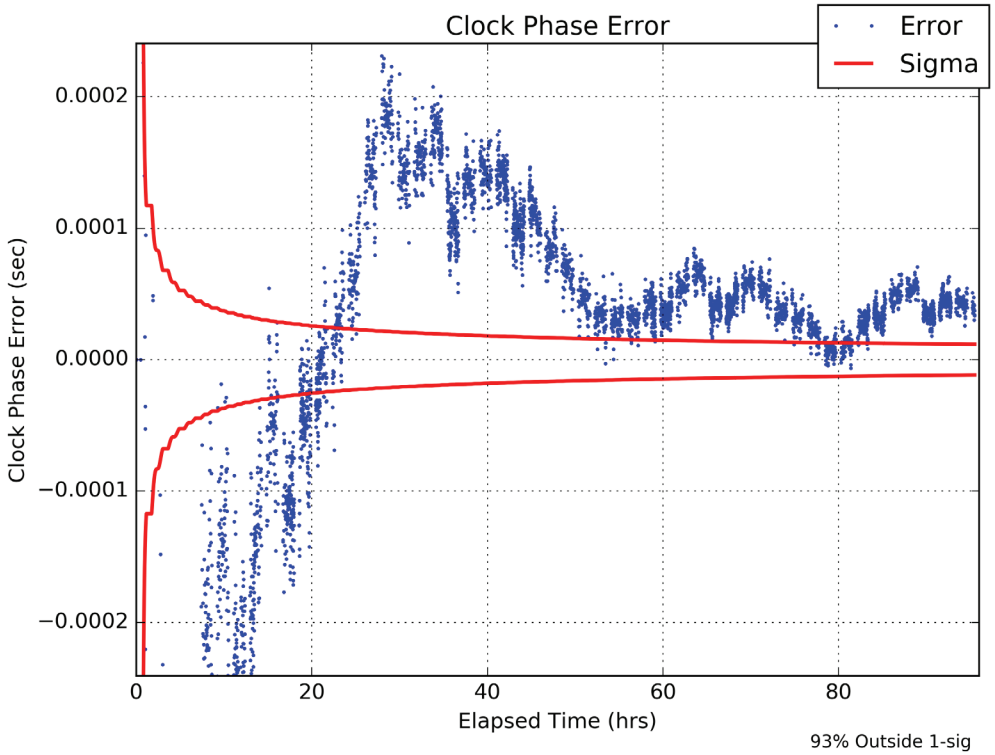
  $\frac{\sqrt{2}\sigma_v(t)}{T} \sim$ 0.1 mm/s with T = 60 sec → AD ~ 3.E-13

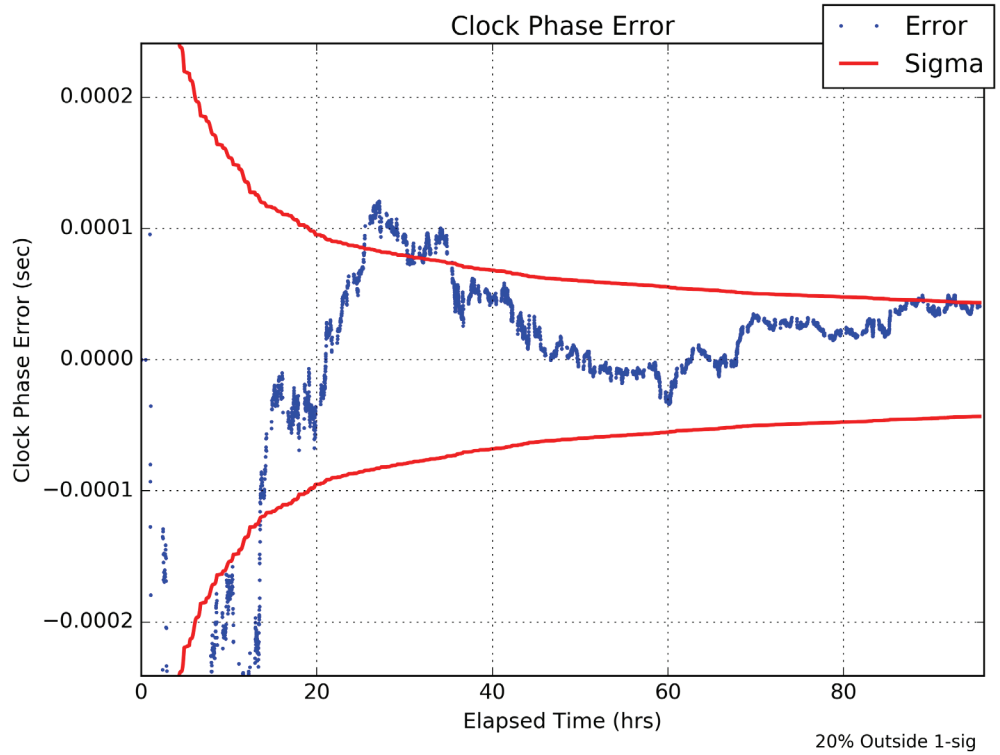
- Range: $$\mathrm{R}(t) = \Delta r(t) + c[x_R(t) - x_T(t-\tau)] + I(t) + S(t) + T(t) + M^{\mathrm{R}}(t) + b_R^{\mathrm{R}}(t) + b_T^{\mathrm{R}} + \varepsilon(t)$$

  $\sigma_\varepsilon(t) \sim$ 1 – 2 m → AD ~ 1.E-10 on a 60 sec interval

- Clocks with AD > 3.E-13 at 60 sec a significant error source and will require proper deweighting for optimal solutions

# Mars OD Sim – Naïve Filter (Standard KF) vs Proper Measurement Weight Filter – Clock Only with CSAC



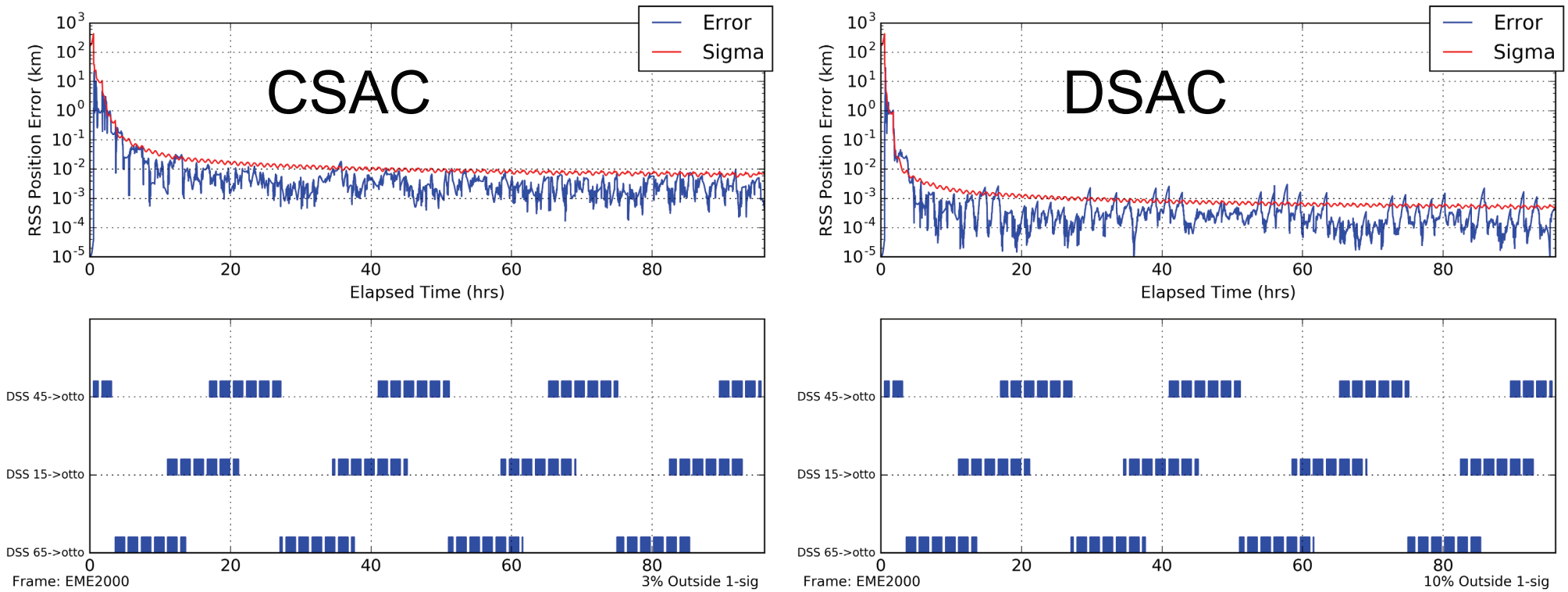

Suboptimal solution with naïve filter

- Errors exceed 1-sigma uncertainty 93% >> 33%

Optimal solution with proper measurement deweighting

- Filter sample statistics consistent with 1-sigma uncertainty
- Lower magnitude short-term noise

# Mars OD Sim – Proper Measurement Weight Filter – All Errors – CSAC and DSAC cases




- Naïve filter results (not shown) yield divergent OD solutions
- Expectedly, DSAC OD on par with traditional OD using 2-Way data types, CSAC OD about an order of magnitude worse
- Proper measurement deweighting required

# Conclusions

- Batch sequential filtering with non-uniform measurement frequency requires an alternate KF approach
  - Naïve use of the standard KF algorithms yields divergent solutions when measurement error sources that participate in the filter state are significant relative to instrument noise
  - Modifying the measurement weight to include portion of process noise in weight solves problem

- Issues with correlated measurement noise or correlations with process noise not as significant of an issue

- Investigation into other significant error sources such as solar plasma (most significant error for two-way data types)

Jet Propulsion Laboratory
California Institute of Technology

jpl.nasa.gov