Coronagraph Design Optimization for Segmented Aperture Telescopes

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August 8, 2017

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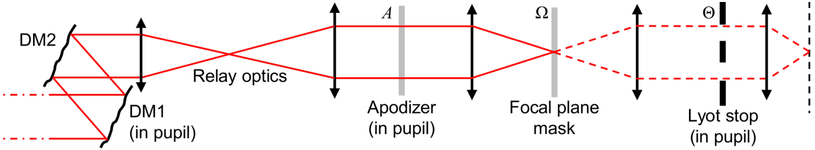
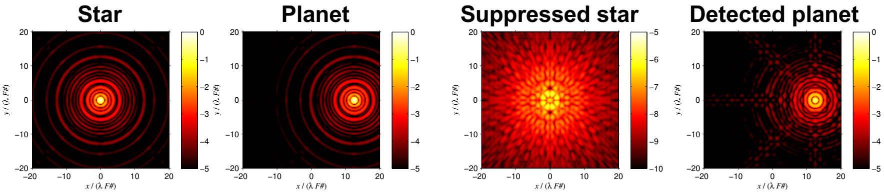
Introduction

- **Our Goal:** Directly image Earth-like planets in the habitable zone.
- **Requires an optimal tradeoff between planet light throughput and diffracted starlight suppression**
  - We have excellent coronagraph designs for uniform apertures
  - Challenge - Find corrections (phase control or apodization) which achieve 10^{-10} diffracted starlight suppression while maximizing throughput
- **How can we find the best feasible coronagraphs?**
  - For this talk: Feasible coronagraphs are comprised of 2 DM's, a focal plane mask, and Lyot stop
  - **A non-convex problem with nonlinear dependence on parameters!**
  - **Algorithmic Strategy :** find feasible coronagraphs which best match "Input-Output" of theoretically ideal CG's for arbitrary apertures.
- **Application to vortex coronagraphs for use with segmented apertures**
  - Entrance Pupil Apodization (gray-scale) masks
  - Beam shaping with phase control
- **Summary and Ongoing work**

Three-mask coronagraph concept

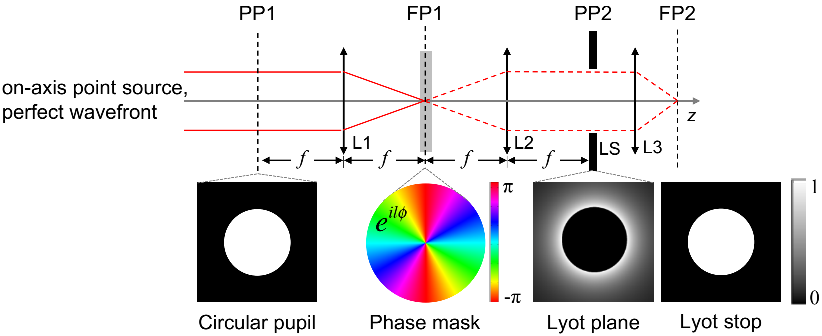




Coronagraphs are designed to:

1. **Passively suppress starlight**
2. **Maximize signal from planets**

Vortex coronagraph for unobscured telescopes



Light from point source rejected by Lyot stop for even (nonzero) charges.

Example of a unitary operator "rotating" an input mode (uniform circle) to an output mode in the Lyot stop null space.

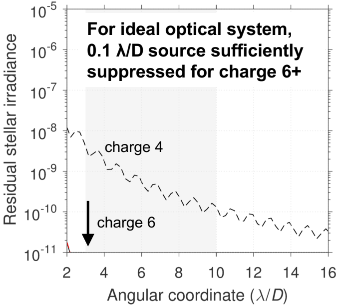
We will see this strategy again in this talk!

Mawet et al., *ApJ* **633** 1191 (2005)

Performance with *unobscured* telescopes

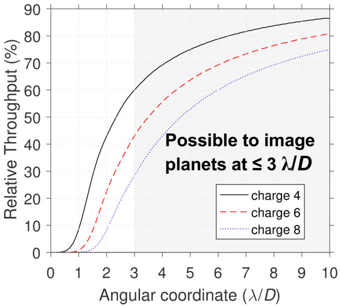
1. Insensitivity to finite size of star (and jitter).
2. High throughput at small angular separations.

Residual starlight (ideal)



Stellar irradiance is azimuthally averaged and normalized to the peak of the telescope PSF.

"Planet" throughput



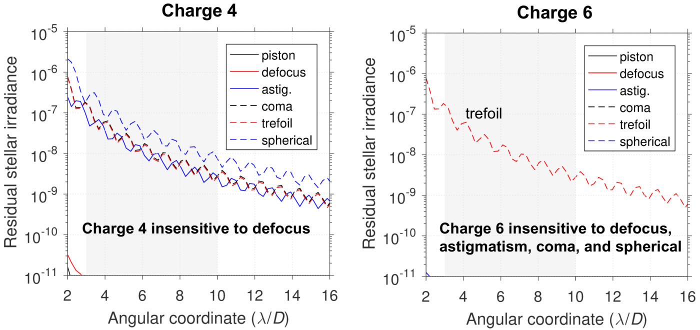
Throughput is defined as energy within 0.7 λ/D of the source position, normalized to that of the telescope.

Mawet et al., *Proc. SPIE* 773914 (2010)

Performance with *unobscured* telescopes

3. Insensitivity to low order aberrations.

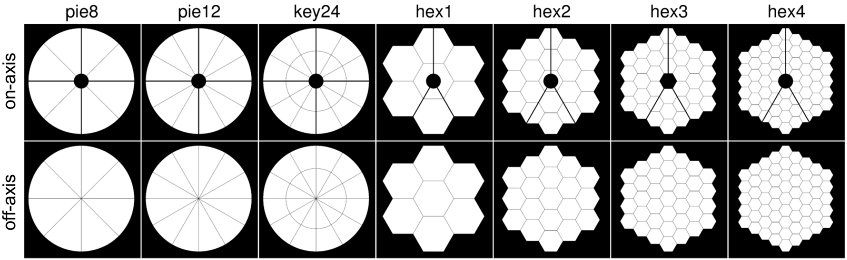
Residual starlight with λ/1000 rms wavefront error



Stellar irradiance is azimuthally averaged and normalized to the peak of the telescope PSF.

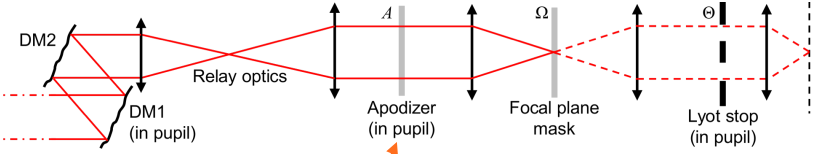
Mawet et al., *Proc. SPIE* 773914 (2010)

Can we take advantage of these benefits on segmented apertures?



SCDA study, led by Stuart Shaklan (JPL), supported by the Exoplanet Exploration Program (ExEP).

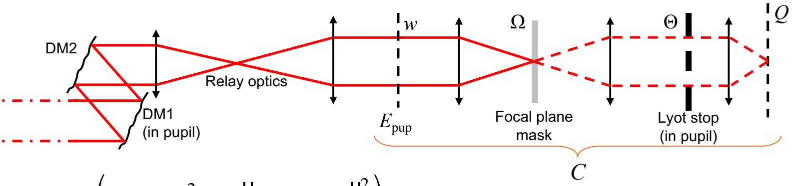
Two Approaches to Suppression of Diffracted Starlight



Option #1: Apodizing pupil mask (e.g. Shaped pupil, APLC, APP, etc.)

Option #2: Beam shaping with DMs (e.g. WFIRST Lyot coronagraph)

Auxiliary field optimization: beam shaping



$$\min_{w}\left(\left\|QCw\right\|^2 + b\left\|w - E_{\text{pup}}\right\|^2\right)$$

Algorithm:

1. Solve for pupil field that will create the specified dark hole:

$$w = \left(bI + C^{\dagger}QC\right)^{-1} bE_{\text{pup}}$$

2. Determine the DM surfaces that achieve the best match between E_{pup} and the target field w.

3. Repeat steps 1 and 2 until sufficient starlight suppression is obtained.

C – coronagraph propagation operator
Q – dark hole region
w – "auxiliary" field
b – regularization parameter
E_{pup} – current pupil field

Aux. field optimization algorithm developed by Jeff Jewell, JPL

Auxiliary Field Optimization - Algorithm

- Our original optimization problem for the DM's has a *nonlinear* dependence on parameters:

$$\{\Gamma, \Phi\} \quad = \quad \min_{\Gamma,\Phi} \left(\left\| QC \left(P_\lambda^\dagger e^{i(\lambda_0/\lambda)\Gamma} P_\lambda e^{i(\lambda_0/\lambda)\Phi} \right) A \right\|^2 \right) \tag{1}$$

- Common to linearize the DM terms and solve linear problems, then repeat ...
- New approach - ask the coronagraph *what (conjugate pupil) field would give a dark hole? Then try to adjust the DM's to hit that target field*.
  - Set of dark hole fields such that $\|QCW\|^2 \le \delta$ for some small δ. Which field is relevant?
  - The one that is closest to the actual field!
  - Leads to a parametric family of fields (depending on the current DM phases and parameter 'b')

$$\hat{W} \quad = \quad \min_{W} \left(\|QCW\|^2 + b \left\| W - \left(P_\lambda^\dagger e^{i(\lambda_0/\lambda)\Gamma} P_\lambda e^{i(\lambda_0/\lambda)\Phi} \right) A \right\|^2 \right) \tag{2}$$

  - With the target fields W, we then solve

$$\{\Gamma, \Phi\} \quad = \quad \min_{\Gamma,\Phi} \left(\left\| W - \left(P_\lambda^\dagger e^{i(\lambda_0/\lambda)\Gamma} P_\lambda e^{i(\lambda_0/\lambda)\Phi} \right) A \right\|^2 \right) \tag{3}$$

Auxiliary Field Optimization - Convergence

- The iteration is really a form of "expectation-maximization"
- We imagine our objective is "promoted to a probability on DM solutions along with auxiliary variables"

$$P[\Gamma, \Phi, W] \quad \propto \quad e^{-\beta \; \|QCW\|^2 - b\|W - U(\Gamma,\Phi)A\|^2} \tag{4}$$

- This form is *designed* so that analytic integration over W leads to $-\log P \rightarrow_{b \gg 1} \beta \; \|QCU(\Gamma, \Phi)\|^2$.
- The conditional $P[W|\Gamma, \Phi]$ is Gaussian with a "peak" at the field

$$\max_W \left(-\log P[W|\Gamma, \Phi]\right) \quad = \quad \left(bI + C^\dagger QC\right)^{-1} bU(\Gamma, \Phi)A \tag{5}$$

- Jensen's inequality gives

$$\begin{aligned}
\log\left(\frac{P[\Gamma, \Phi]}{P[\Gamma_n, \Phi_n]}\right) \quad &\geq \quad \int dW \;\; P[W|\Gamma_n, \Phi_n] \;\; \log\left(\frac{P[\Gamma, \Phi|W]}{P[\Gamma_n, \Phi_n|W]}\right) \\
&= \quad -\int dW \;\; P[W|\Gamma_n, \Phi_n] \;\; \left(\|W - U(\Gamma, \Phi)A\|^2 - \|W - U(\Gamma_n, \Phi_n)A\|^2\right)
\end{aligned}$$

- Maximizing the right-hand side never results in a worse DM solution
- For the Gaussian (conditioned on (Γ_n, Φ_n)) maximizing the right-hand side equivalent to

$$\{\Gamma_{n+1}, \Phi_{n+1}\} \quad = \quad \min_{\Gamma, \Phi}\left(\left\|\hat{W}(\Gamma_n, \Phi_n) - \left(P_\lambda^\dagger e^{i(\lambda_0/\lambda)\Gamma} P_\lambda e^{i(\lambda_0/\lambda)\Phi}\right) A\right\|^2\right) \tag{6}$$

with $\hat{W}(\Gamma_n, \Phi_n)$ the mean field given the past DM solutions.

Auxiliary Field Optimization - Numerical Details

- We solve for the auxiliary field in a conjugate pupil plane

$$\begin{aligned}\hat{W} &= \min_{W}\left(\|QCW\|^2 + b\left\|\hat{W} - U(\Gamma,\Phi)A\right\|^2\right)\\ &= \left(bI + C^{\dagger}QC\right)^{-1} bU(\Gamma,\Phi)A \end{aligned} \tag{7}$$

- Notice the "internal" dark hole dependence Q - this is a small region ...
- Convenient to use the Sherman-Woodbury form

$$\hat{W} = \left[I - C^{\dagger}Q\left(bI + QCC^{\dagger}Q\right)^{-1}QC\right] U(\Gamma,\Phi)A \tag{8}$$

The internal matrix now lives in the focal plane!

- We explicitly pre-compute this matrix (for N_{DH} pixels in the Dark Hole, an $N_{DH} \times N_{DH}$ matrix).
- The $b \to 0$ limit gives a field which *vanishes in the dark hole!*
  - There exist focal plane eigenmodes of $QCC^{\dagger}Q$

$$QCC^{\dagger}Q|f_j\rangle = |\alpha_j|^2|f_j\rangle \tag{9}$$

  - Associated set of orthogonal pupil plane eigenmodes!

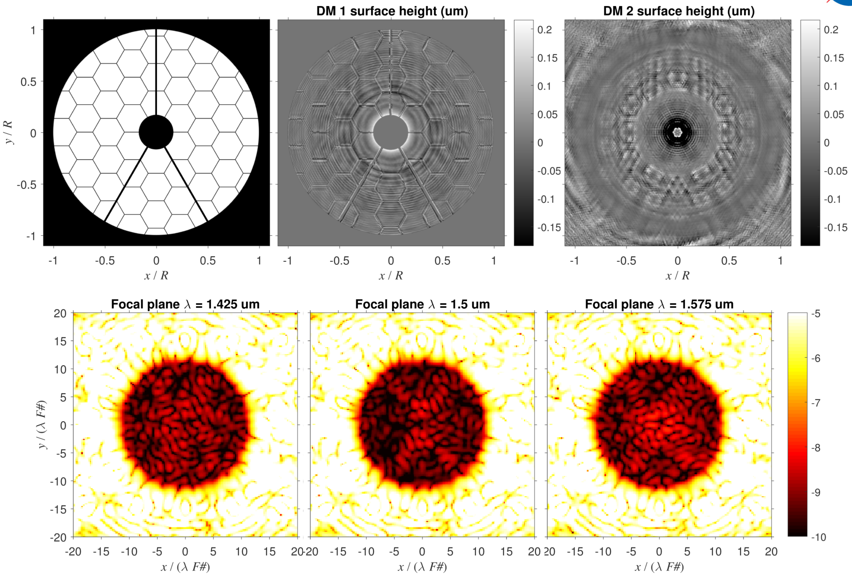
$$C^{\dagger}Q|f_j\rangle = \alpha_j|g_j\rangle \tag{10}$$

  (such that $QC|g_j\rangle = \alpha_j^*|f_j\rangle$, i.e. QC maps pupil modes $|g_j\rangle$ to focal plane modes $|f_j\rangle$)).
  - The solution for the aux-field given explicitly by

$$|\hat{W}\rangle = |U(\Gamma,\Phi)A\rangle - \left(\sum_j |g_j\rangle\left(\frac{|\alpha_j|^2}{b+|\alpha_j|^2}\right)\langle g_j|U(\Gamma,\Phi)A\rangle\right) \tag{11}$$

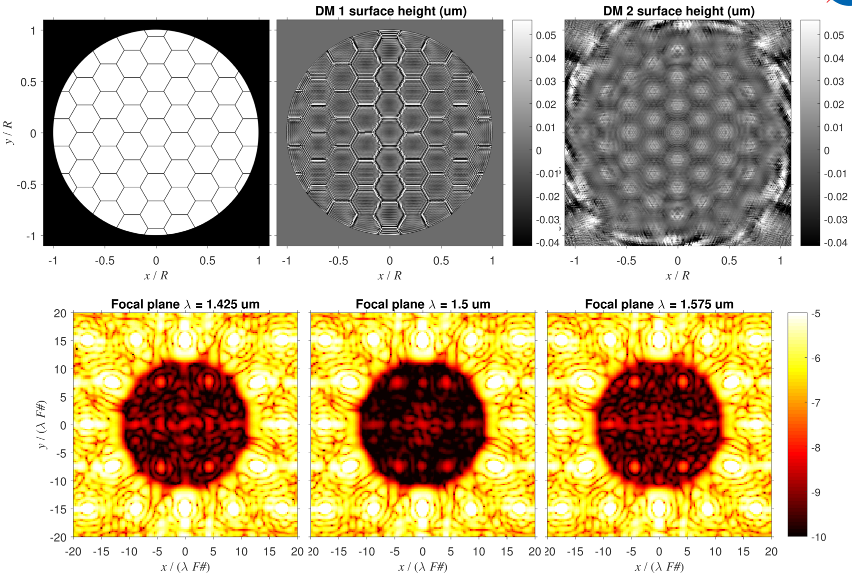
  - As $b \to 0$, the above perfectly vanishes for all the pupil modes $|g_j\rangle$ that comprise the dark hole (once passed through the downstream coronagraph).

Beam shaping with central obscuration



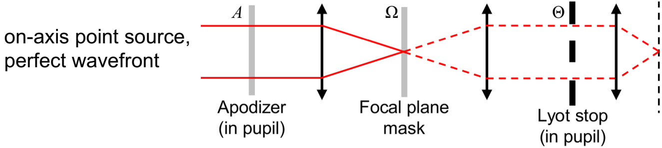
Solution obtained via “Auxiliary Field Optimization” (Jewell et al., in prep.)

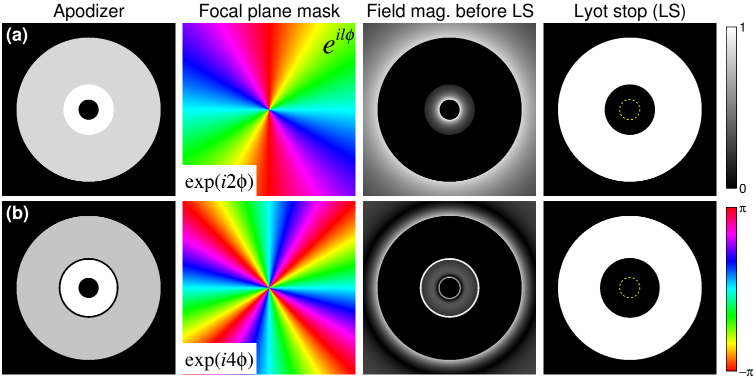
Beam shaping without central obscuration



Solution obtained via "Auxiliary Field Optimization" (Jewell et al., in prep.)

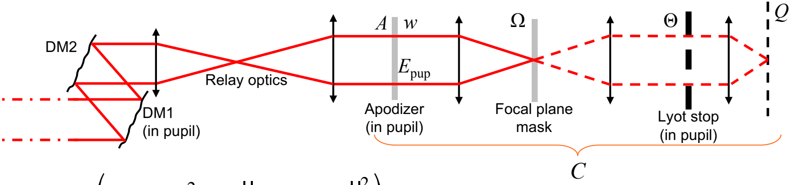
Grayscale apodized vortex coronagraph





Mawet et al., *ApJS* **209** 7 (2013)

Auxiliary field optimization: gray-scale apodizers



$$\min_{w}\left(\|QCw\|^2 + b\|w - E_{\text{pup}}\|^2\right)$$

Algorithm:

1. Solve for pupil field that will create the specified dark hole:

$$w = \left(bI + C^{\dagger}QC\right)^{-1} bE_{\text{pup}}$$

2. Apply constraints set by optical system to $A = |w|$:

$$0 \le A \le 1$$

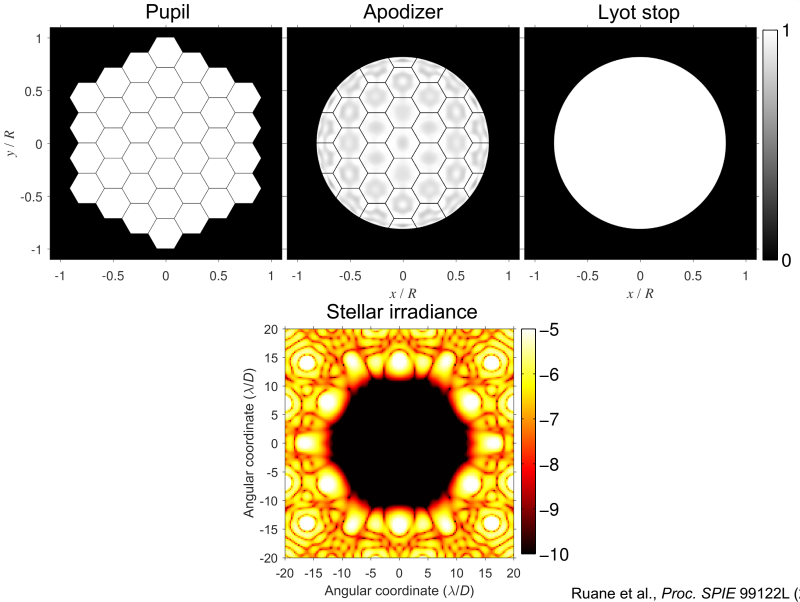
$$\text{supp}\{A\} = \text{supp}\{P\}$$

3. Set $E_{\text{pup}} = PA$, and repeat

C – coronagraph propagation operator
Q – dark hole region
w – auxiliary field
b – regularization parameter
E_{pup} – current pupil field
A – gray-scale apodizer
P – original pupil field

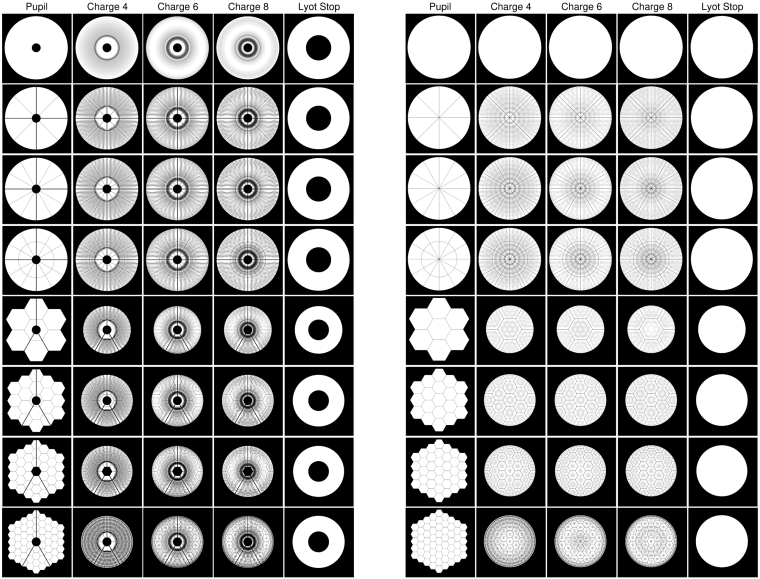
Aux. field optimization algorithm developed by Jeff Jewell, JPL

Grayscale apodized vortex coronagraph



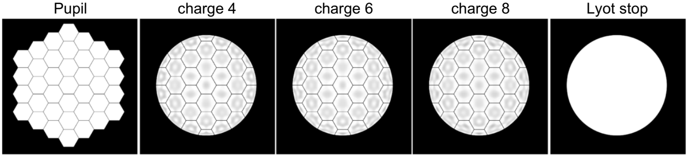
Ruane et al., *Proc. SPIE* 99122L (2016)

A family portrait of apodizer designs

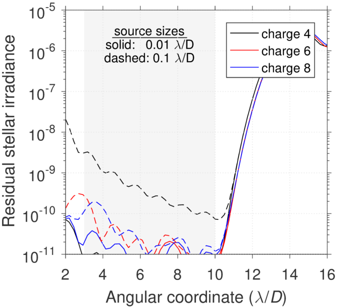


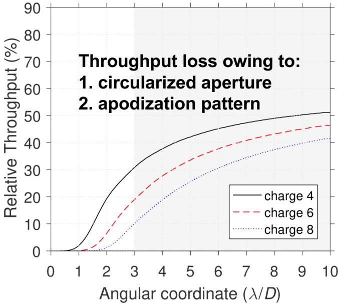
Performance for off-axis segmented telescopes

hex3: 3-ring hexagonally segmented primary (37 segments)



1. Insensitivity to finite size of star and jitter maintained.
2. Apodizer introduces a throughput loss.





Performance for off-axis segmented telescopes

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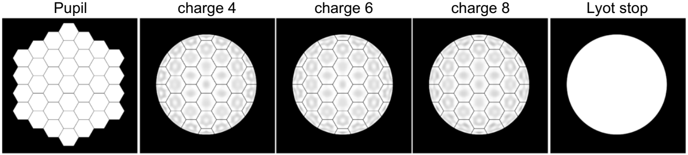
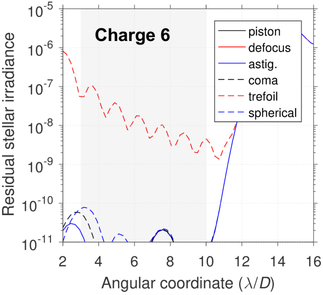


3. Insensitivity to Zernike aberrations maintained.

Residual starlight with λ/1000 rms wavefront error



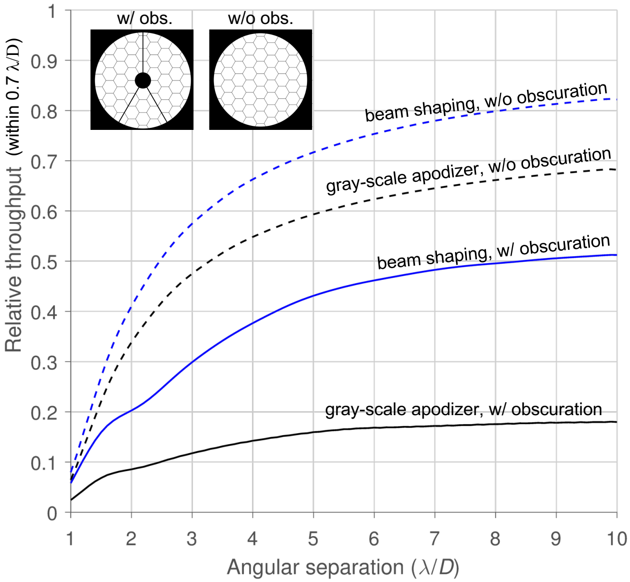
Improving designs for on-axis telescopes

- **Compounding issues with current on-axis designs:**
  1. Decreased throughput.
  2. More sensitivity to the finite size of the star.
  3. Large D means λ/D is smaller with respect to the star.

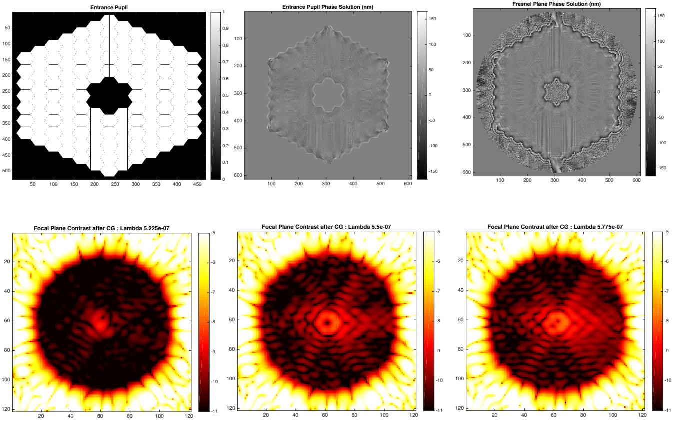
- **Updating optimization procedure to combat these effects.**

- **Several approaches have yet to be considered:**
  - Gray-scale apodizers with updated metrics
  - Lyot stop optimization
  - Focal plane mask optimization
  - Beam shaping with DMs

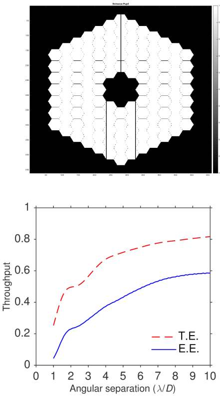
Throughput comparison



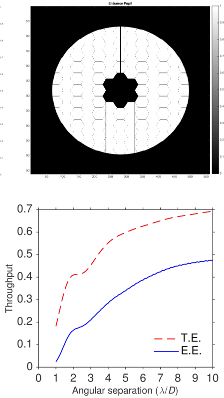
Coronagraph Baseline – (Pixel) DM1 and DM2 Phase, 10% Band (550 nm)
Science Focal Plane Dark Holes



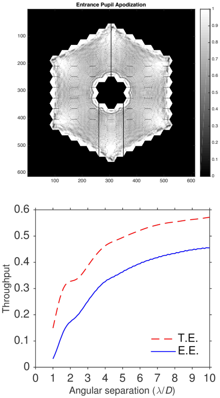
Throughput Comparisons



Beam-shaping,
Original Aperture



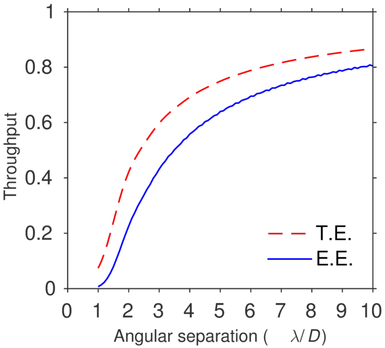
Beam-shaping,
Circularized Aperture

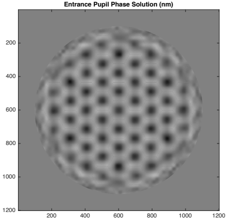


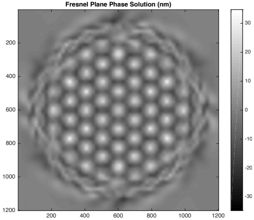
Apodization with
Beam-shaping

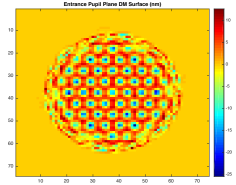
SCDA Hex Unobscured Aperture – DM Solutions

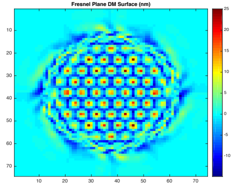
- Charge 6 vortex in focal plane, circular Lyot stop (outer radius 0.99 of entrance pupil)
- 10% Band (550 nm center wavelength)
- 74 x 74 DM array (actuator pitch 400e-6 meters)
- *Solution achieves 2.8e-10 broadband contrast*



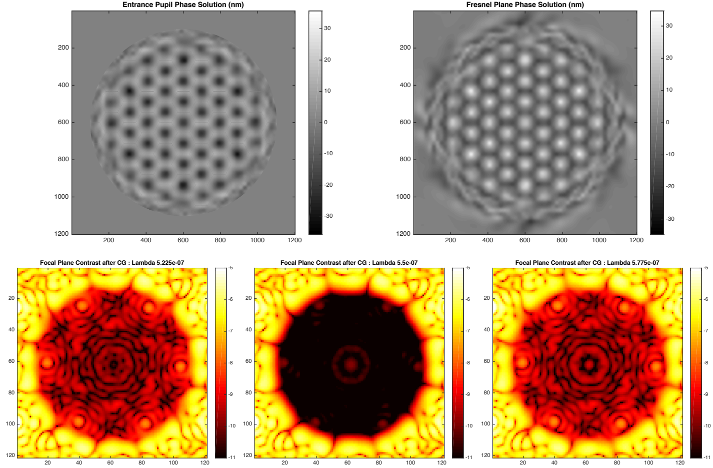








SCDA Hex Unobscured Aperture – DM Solutions



Summary and Ongoing Work

- We have developed an "auxiliary field" algorithm for coronagraph designs with segmented apertures
  - Capable of finding phase solutions for segmented apertures to "rotate" the on-axis source into a fixed null space (i.e. the downstream vortex coronagraph)
  - Capable of adjusting the end to end null space with pupil apodization
- Exploring designs for robustness to finite stellar disc with a central obscuration
  - Auxiliary field iterative algorithm can be applied directly for phase only focal plane masks departing from the vortex (for the uniform circular aperture)
  - We also have a generalized form of the algorithm to explore general phase and amplitude focal plane masks (we introduce target off-axis modes into the ideal "Input-Output response")
- Extending the phase solutions to work broadband with DM's to reclaim apodization throughput loss
  - Current broadband phase solutions are "pixel" based
  - Monochromatic phase solutions with DM influence functions also completed
  - Moving to faster GPU based machines to complete the broadband DM solutions

This work was partially performed at the Jet Propulsion Laboratory, California Institute of Technology , under contract with the National Aeronautics and Space Adminstration. Government sponsorship acknowledged.