Review of Multi-Agent Algorithms for Collective Behavior: a Structural Taxonomy

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Abstract: In this paper, we review multi-agent collective behavior algorithms in the literature and classify them according to their underlying mathematical structure. For each mathematical technique, we identify the multi-agent coordination tasks it can be applied to, and we analyze its scalability, bandwidth use, and demonstrated maturity. We highlight how versatile techniques such as artificial potential functions can be used for applications ranging from low-level position control to high-level coordination and task allocation, we discuss possible reasons for the slow adoption of complex distributed coordination algorithms in the field, and we highlight areas for further research and development.

Keywords: Autonomous mobile robots, Agents, Distributed Control, Decentralized Control

1. INTRODUCTION

Multi-agent robotic systems hold promise to enable new classes of missions in aerospace, terrestrial, and maritime applications, delivering higher resilience and adaptability at lower cost compared to existing monolythic systems. In particular, in the aerospace domain, multi-agent systems hold great promise for applications including multi-UAV patrolling, satellite formations for astronomy and Earth observation, and multi-robot planetary exploration. A number of algorithms have been proposed to control the collective behavior of such systems, ranging from low-level position control to high-level motion planning and task allocation algorithms.

Many excellent surveys of algorithms for collective behavior exist in the literature; however, such papers generally focus either on single applications (e.g., formation control (Oh et al., 2015) or coverage (Schwager et al., 2009)) or on specific control techniques (e.g., consensus (Garin and Schenato, 2010; Cao et al., 2013)). Several works study the fundamental limitations of performance of multi-agent systems: e.g., Martinez et al. (2007) and Rossi and Pavone (2014) explore time and communication complexity in synchronous and asynchronous systems respectively, and Gupta et al. (2006) studies robustness to agent failures. However, these works only survey the performance of a limited number of applications and algorithms. In contrast, in this paper, we survey the *general* family of collective behavior algorithms for multi-agent systems and classify them according to their underlying mathematical *structure*, without restricting our focus to specific tasks or individual classes of algorithms. In doing so, we aim to capture fundamental mathematical properties of algorithms (e.g. scalability with respect to the number of agents and bandwidth use) and to show how the same algorithm or family of algorithms can be applied to multiple tasks and missions. In particular, the goal of this paper is threefold:

- to act as a guide to practitioners in the selection of control algorithms for a given task or application;
- to highlight how mathematically similar algorithms can be used for a variety of tasks, ranging from low-level control to high-level coordination;
- to explore the state-of-the-art in the field of control of multi-agent systems and identify areas for future research.

Tasks in multi-agent systems can be broadly categorized into the following classes (Brambilla et al., 2013):
(1) **Spatially-organizing behaviors**, where agents coordinate to achieve a given spatial configuration and have negligible interactions with the environment. These tasks can be further classified into: (a) *Aggregation:* converging to one location. (b) *Pattern Formation:* achieving a desired formation. (c) *Coverage:* covering an area.
(2) **Collective explorations**, where agents interact with the environment but have minimal interaction among themselves. These tasks can be classified into: (a) *Area Exploration:* exploring the environment for mapping or surveillance. (b) *Goal Searching:* searching for targets.
(3) **Cooperative decision making**, where agents both coordinate among themselves and interact with the environment to accomplish complex tasks. These tasks can be further classified into: (a) *Task Allocation:* distributing tasks among agents. (b) *Collective Transport:* coordinating to transport large objects. (c) *Motion Planning:* finding paths in cluttered environments. (d) *Distributed Estimation:* estimating the state of one or multiple targets.
These simple tasks are the fundamental building blocks of many complex multi-agent applications.

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[1] Part of this research was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration. Federico Rossi and Marco Pavone were partially supported by the Office of Naval Research, Science of Autonomy Program, under Contract N00014-15-1-2673.

	Aggregation	Pattern Formation	Coverage	Area Exploration	Goal Searching	Task Allocation	Collective Transport	Motion Planning	Distributed Estimation	High Scalability	Low Bandwidth Use	Maturity
Consensus	✓	✓	✓						✓	✓	✓	H
Artificial Potential Functions (APF)	✓	✓	✓	✓		✓	✓	✓		✓	✓	F
Distributed Feedback Control	✓	✓							✓	✓	✓	F
Geometric Algorithms												
Voronoi-based Algorithms	✓		✓	✓				✓		✓	✓	H
Circumcenter Algorithms	✓	✓								✓	✓	S
Bearing-only Algorithms	✓	✓								✓	✓	H
Maze Searching Algorithms								✓		✓	✓	S
Leader-Follower (LF) Algorithms		✓								✓	✓	S
Velocity Obstacle (VO) based Algorithms								✓		✓	✓	F
State Machines and Behavior Composition												
Automata-based Algorithms						✓			✓	✓	✓	S
Behavior Composition						✓	✓					H
Petri Networks						✓					-	H
Game Theory based Algorithms						✓					-	S
Resource Allocation Systems								✓		✓	-	S
Bio-Inspired Algorithms												
Kilobot Self-Assembly Algorithm		✓								✓	✓	H
Optimotaxis Source-Searching Algorithm					✓					✓	✓	S
Beeclust Foraging Algorithm				✓						✓	✓	S
Shepherding Algorithm	✓									✓	✓	S
Termite-Inspired Collective Construction Algorithm						✓	✓			✓	✓	H
Fish-inspired Goal Searching Algorithms		✓			✓					✓	✓	H
Gillespie Self-Assembly Algorithm		✓								✓	✓	H
Mergeable Modular Robots		✓								✓	✓	H
Density based Control												
Markov Chain-based Algorithms		✓	✓			✓				✓	✓	H
Smoothed Particle Hydrodynamics (SPH)		✓	✓							✓	✓	H
Optimal Transport based Algorithm		✓						✓		✓	✓	S
Distributed Optimization Algorithms												
Distributed Linear Programming		✓				✓				✓	✓	S
Distributed Convex Optimization		✓				✓			✓	✓	✓	S
Distributed Dynamic Programming						✓		✓				H
Sequential Convex Programming								✓		✓	✓	H
Distributed Auction						✓				✓	✓	H
Local Optimization Algorithms for Global Behavior												
Decentralized Model Predictive Control (DMPC)		✓						✓		✓		H
Formal Methods								✓		✓	✓	S
Sampling-based Motion-Planning Algorithms								✓				H
Centralized Optimization Algorithms												
MILPs and MINLPs		✓				✓		✓	✓		-	H
Linear and Convex Optimization						✓		✓	✓	✓	-	S
Markov Decision Processes (MDP)						✓		✓			-	H
Multi-Agent Traveling Salesman Problems			✓	✓	✓	✓					-	H
Multi-Armed Bandits				✓	✓	✓			✓	✓	-	S
Direct Methods for Optimal Control			✓	✓				✓			-	F
Multiagent Reinforcement Learning				✓		✓					-	H
Frontier Techniques				✓	✓						-	F
Network Flow Algorithms						✓		✓		✓	-	S
Combinatorial Motion Planning								✓		✓	-	S

Table 1. Categorization of collective behavior algorithms according to their mathematical structure and applicability of each algorithm to common multi-agent tasks. The scalability, bandwidth use, and level of demonstrated maturity of each algorithm (formally defined in Section 1) are also reported.

Communication structure In **centralized** algorithms, all agents share their information with a central node, which computes and issues a joint set of control actions. In **distributed** algorithms, agents can only explicitly share information with their neighbors. Centralized algorithms can be implemented in a distributed fashion with a **shared-world** approach, discussed in Section 2.10.

Methodology We performed a thorough review of papers on multi-agent systems in major controls and robotics journals and conferences. It is not feasible to cite all existing works on control of multi-agent systems; accordingly, in this paper, we focus on identifying and classifying the key mathematical *structures* and *techniques* that drive coordination algorithms, as opposed to individual contributions. We refer the reader to the extended version of this paper (Rossi et al., 2018a) for a more thorough literature review and a detailed discussion of the mathematical formulation and properties of each technique.

We classify mathematical techniques according to their: (1) **Scalability:** Highly scalable algorithms have been demonstrated on systems with more than 50 agents (in simulations or hardware). (2) **Bandwidth use:** In low bandwidth algorithms, agents only communicate with their physical neighbors and do not exchange large messages. (3) **Maturity:** The three classes of algorithms are: (i) only demonstrated in 'simulation' (S) (ii) demonstrated in 'hardware' (H) either in the lab or in technology demonstration missions (iii) demonstrated in 'field' (F) deployments (excluding technology demonstrator missions). A formal description and discussion of these metrics is provided in the extended version (Rossi et al., 2018a).

Organization Our key contribution is Table 1, which reports the proposed taxonomy of mathematical techniques for collective behavior, highlights the tasks that each mathematical technique can achieve, and lists relevant performance metrics. In Sections 2.1–2.10 we provide a synthetic description of the classification and relevant references. Finally, in Section 3 we draw conclusions and suggest directions for future research.

2. A STRUCTURAL TAXONOMY OF MULTI-AGENT COLLECTIVE BEHAVIOR ALGORITHMS

2.1 Consensus algorithms

Consensus is among the oldest and most widely used distributed algorithms. Each agent shares and averages its state with its neighbors (Tsitsiklis et al., 1986; Ren et al., 2007). Applications include synchronization (Li and Rus, 2006), flocking (Tanner et al., 2007; Olfati-Saber, 2006), formation flying (Chung et al., 2013), and distributed estimation (Rabbat and Nowak, 2004). In *gossip algorithms* (Boyd et al., 2006), each agent communicates with a single randomly-selected neighbor at each step. In *cyclic pursuit algorithms* (Marshall et al., 2004), the consensus algorithm is executed on a directed ring communication topology.

2.2 Artificial Potential Functions (APF)

APF algorithms synthesize agents' control inputs using the gradient of a suitably-defined potential function (Khatib, 1986). These algorithms are very popular due to their simplicity, scalability, and ability to adapt to a number of tasks. Applications include pattern formation (Sepulchre et al., 2007) , flocking (Zavlanos et al., 2007), path planning (Koditschek and Rimon, 1990), and task allocation (Weigel et al., 2002).

2.3 Distributed Feedback Control

Each agent is endowed with a feedback controller that uses the agent's and its neighbors' states as the input (Bamieh et al., 2002; Feddema et al., 2002). In particular, tools for synthesis of **distributed LQG control** are available that can adapt to noisy communication links (Sahai and Mitter, 2006), and packet losses (Liu and Goldsmith, 2004), with applications to formation flying (Ogren et al., 2002) and distributed estimation.

2.4 Geometric Algorithms

In geometric algorithms, agents leverage their neighbors' location and speed information to perform spatially organizing tasks and path planning. **Voronoi algorithms** compute Voronoi partitions for coverage (Cortés et al., 2004), path planning (Zhou et al., 2017), and task allocation problems (Pavone et al., 2011). Other geometric algorithms include **circumcenter algorithms** for rendezvous (Cortés et al., 2006), **bearing-only algorithms** for formation control (Fredslund and Mataric, 2002) and rendezvous (Yu et al., 2008), **maze searching algorithms** for path planning (Lumelsky and Harinarayan, 1997), **leader-follower algorithms** for formation flying (Mesbahi and Hadaegh, 1999), and **velocity obstacles** for collision avoidance (van den Berg et al., 2008).

2.5 State Machines and Behavior Composition

Automata-based algorithms leverage complex state machines and message-passing among agents to establish communication graphs and elect leaders for task allocation (Lynch, 1997; Rossi and Pavone, 2014). **Behavior composition** algorithms rely on composition of elementary behaviors for collective transport (Rus et al., 1995). **Petri networks** (King et al., 2003) and **game theory** (Arslan et al., 2007) algorithms are used for centralized task allocation. **Resource allocation systems** are used for multi-agent motion planning (Reveliotis and Roszkowska, 2011).

2.6 Bio-Inspired Algorithms

Bio-inspired algorithms mimic the behavior of swarms of animals such as insects and fish. We present a non-exhaustive list: the **Kilobot algorithm** achieves complex two-dimensional shapes and was demonstrated on a thousand-agent testbed (Rubenstein et al., 2014); the **Optimotaxis source-searching algorithm** is inspired by the run and tumble behaviors of bacteria (Mesquita et al., 2008); the **Beeclust foraging algorithm** is inspired by the behavior of honey bees (Hereford, 2011); **Shepherding algorithms** enable control of large numbers of uncontrolled agents with few controlled agents (Strömbom et al., 2014); a **Termite-inspired algorithm** generates low-level rules for construction of complex structures (Werfel et al., 2014); a **Fish-inspired goal-searching algorithm** switches between individual and collective behavior based on confidence level (Wu and Zhang, 2012); the **Gillespie self-assembly algorithm** leverages chemical kinetics; **Mergeable modular robots** connect to form larger bodies or split into separate bodies, with self-healing properties (Mathews et al., 2017).

2.7 Density based Control

As opposed to the agent-based *Lagrangian* framework, density-based algorithms adopt an *Eulerian* framework by treating agents as a continuum and controlling their density. **Markov chain** based algorithms partition the workspace into disjoint cells and control the transition probabilities between cells for pattern formation and goal searching applications (Açıkmeşe and Bayard, 2015; Bandyopadhyay et al., 2017b). **Smoothed particle hydrodynamics (SPH)** (Zhao et al., 2011) and **optimal transport** (Bandyopadhyay et al., 2014) based algorithms are also used for swarm formation control.

2.8 Distributed Optimization Algorithms

Distributed optimization algorithms allow agents to jointly solve optimization problems through information exchange and local computations. **Distributed linear programming** (Bürger et al., 2012) is used for pattern formation and task allocation; **distributed convex optimization** can encode richer convex constraints (Boyd et al., 2011). **Distributed dynamic programming** (Bertsekas, 1982) is used for task allocation and motion planning. **Sequential Convex Programming** can solve non-convex motion planning problems through local convexification and iteration (Morgan et al., 2016). The above algorithms can also be used in a **distributed model-predictive control** framework (Scattolini, 2009). Market-based protocols like **distributed auction** (Gerkey and Mataric, 2002), mechanism design (Dias, 2004), and coalition formation (Shehory and Kraus, 1998) are widely used for task allocation.

2.9 Local optimization algorithms for global behavior

In local optimization algorithms, each agent solves an optimization problem; while the resulting behavior is not generally optimal for the entire system, favorable global properties such as collision avoidance can be guaranteed. In **decentralized model predictive control (DMPC)** each agent employs a local model-predictive control algorithms; inter-agent communication is used to coordinate the agents' plans (Richards and How, 2007). Distributed MPC has been used for flocking and motion planning (Dunbar and Murray, 2002; Schouwenaars et al., 2006). **Formal methods** are used in concert with low-level control primitives for multi-agent motion planning with guaranteed collision avoidance (Kress-Gazit et al., 2008). Decentralized multi-agent **sampling-based motion planning algorithms** have enjoyed significant practical success because of their simplicity, ability to handle higher-dimensional spaces, and probabilistic completeness (Bandyopadhyay et al., 2017a; Desaraju and How, 2012).

2.10 Centralized optimization algorithms

Mixed-integer linear programs (MILPs) and mixed-integer convex programs (MICPs), can solve simultaneous task allocation and path planning (Bellingham et al., 2003), tracking (Xu et al., 2013), formation flying (Richards et al., 2002), and defend-the-flag problems (Earl and D'Andrea, 2002). **Linear and convex optimization** problems can also be used to solve task allocation problems (Bertsekas, 1998; Turpin et al., 2014) with collision avoidance constraints (Açıkmese et al., 2006), and for distributed estimation and target tracking (Aslam et al., 2003). **Markov decision processes (MDPs)** and partially observable MDPs capture the stochastic nature of the environment and model the agents' *coordination mechanism* (Boutilier, 1999). POMDPs have been used for multi-agent path planning (Omidshafiei et al., 2015) and task allocation. Several approximation algorithms are available to solve the **m-vehicle traveling salesman problem (TSP)** and the team orienteering problem, building blocks for spatial task allocation, persistent monitoring, and information-gathering problems (Yu et al., 2014). **Multi-agent multi-armed bandit** problems (Gittins, 1979) capture the trade-off between exploration and exploitation: they have been employed for task allocation (Le Ny et al., 2008) , goal searching, and tracking applications (Landgren et al., 2016). **Direct methods for trajectory optimization** (Von Stryk and Bulirsch, 1992) are used for area coverage, goal searching, and motion planning (Leonard et al., 2010). **Multi-agent reinforcement learning (MARL)** has been used for exploration (Chalkiadakis and Boutilier, 2003) and task allocation (Liu and Nejat, 2016). **Frontier techniques** (Burgard et al., 2000) are used for urban search-and-rescue, reconnaissance (Olson et al., 2012) and sample collection (Eich et al., 2014). **Network flow** formulations have been proposed for Air Traffic Control (Menon et al., 2004) and for control of autonomous vehicles offering on-demand transportation (Pavone et al., 2012; Rossi et al., 2018b). Several **cooperative combinatorial motion planning** algorithms have been proposed for multi-agent systems: we refer the reader to (Sharon et al., 2015) for a thorough review. Centralized optimization algorithms can be implemented in a distributed fashion with a **shared-world** approach, where agents exchange their state and observations so that every robot has full knowledge of the entire system's state. However, shared-world algorithms have very onerous communication requirements (due to large messages and all-to-all communication) and high computation complexity, since each agent must solve the full centralized optimization problem.

3. CONCLUSION

The proposed taxonomy and the properties shown in Table 1 highlight some surprising characteristics of collective behavior algorithms. The majority of existing mathematical techniques is tailored to either low-level spatially organizing tasks (e.g., bio-inspired algorithms and density-based control) or high-level coordination applications (e.g., state machines and optimization-based algorithms). Only a small number of mathematical techniques (in particular, Artificial Potential Functions) can be adapted to a wide variety of tasks that include both low-level and high-level application. This prompts further research into non-APF algorithms for multi-agent systems that share APF's key properties of simplicity, scalability, and high expressivity.

Very few algorithms are mature and field-tested. Such algorithms exchange very simple information (e.g. the agents' locations) or rely on centralized implementations: this may be justified by the difficulty of characterizing and certifying the behavior of an entire multi-agent system when distributed algorithms are used. To overcome this, (i) research in formal methods and adoption of tools from the distributed algorithms literature to provide stronger guarantees for distributed systems and (ii) creation of standardized software and hardware test-beds to characterize the end-to-end behavior of such systems are needed.

Several avenues for future research are of interest. In particular, we hope to evaluate the performance of collective behavior algorithms according to additional metrics including 1) bandwidth use in broadcast and in point-to-point networks, 2) computational complexity, 3) availability of formal guarantees, 4) resilience to disruptions in communication network and to *adversarial* failures, and 5) availability of a reference implementation. We also wish to explore other possible taxonomies for coordination algorithms based, e.g., on the content of messages exchanged by the agent (which vary from simple "beacon" messages reporting the agent's location to complex messages carrying intentions and bids), and the communication topology induced by the algorithm (single-hop vs. multi-hop) Finally, we plan to further explore high-level multi-agent tasks, including adversarial "swarm vs. swarm" problems, and to assess the applicability and performance of collective behavior algorithms with respect to such tasks.

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