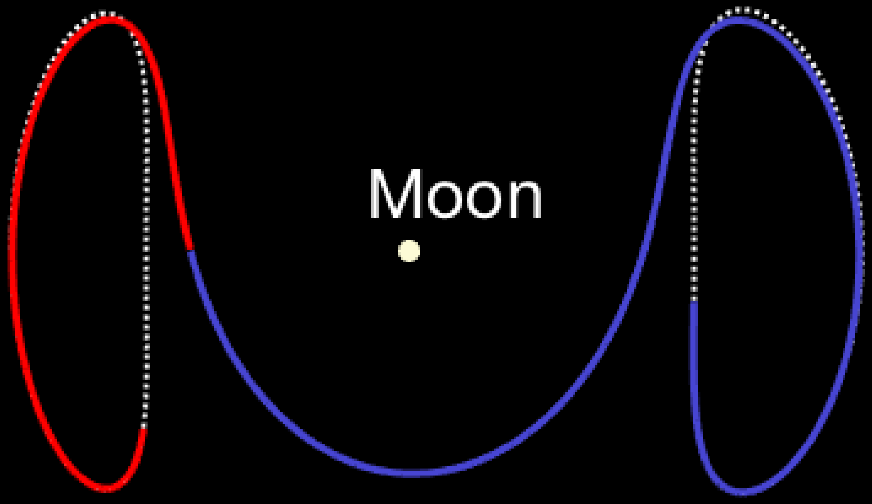
A Monte.Poincare Interactive Tool for Computing Orbit Connections

Nick LaFarge

Mentors: Mar Vaquero
& Juan Senent

Final Presentation
July 19th, 2017



Jet Propulsion Laboratory
California Institute of Technology

Nick LaFarge Bio

- **2014** – B.S. from University of Colorado Boulder in Computational Mathematics and Japanese (minor in Computer Science)
- **2014-2016** – Software Engineer using Java & Python
- **Fall 2016** – Began at Purdue University pursuing a M.S. in Aerospace Engineering
    - Advisor: Dr. Kathleen Howell

Poincare Background

Jet Propulsion Laboratory
California Institute of Technology

Poincare: Trajectory design tool in Monte based on a catalog of three-body science orbits and a differential corrector to compute connecting transfer arcs between orbits in multi-body systems

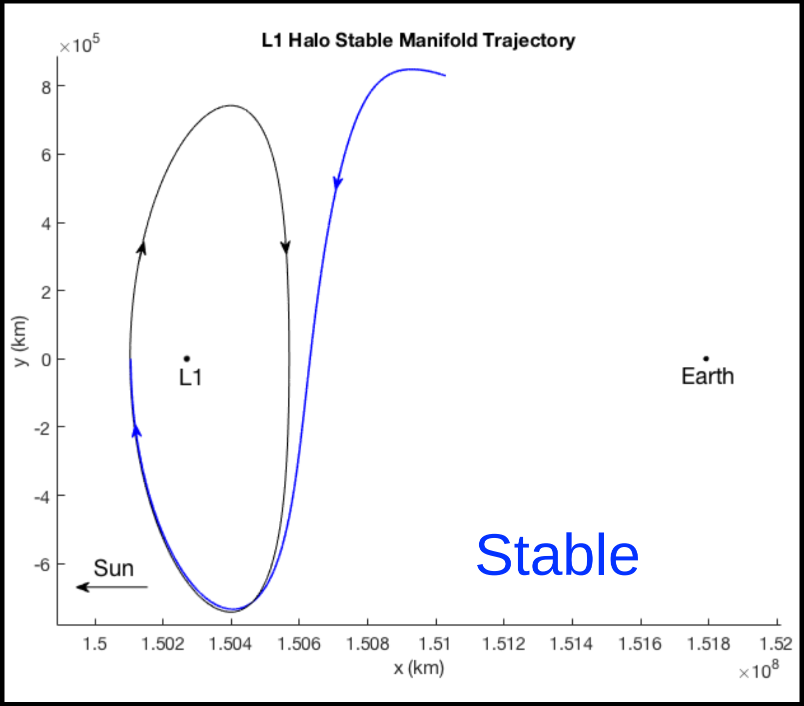
- In Place: Science Orbit Design Tool to facilitate rapid and well-informed decisions regarding the selection of periodic orbits for a particular mission and enable the simultaneous study of various orbit alternatives
- Currently Under Development: Reference Trajectory Design Tool to compute orbit connections via dynamical systems structures
  - My Role: prototype development
- Future Work: Calculate end-to-end reference trajectories in the ephemeris model

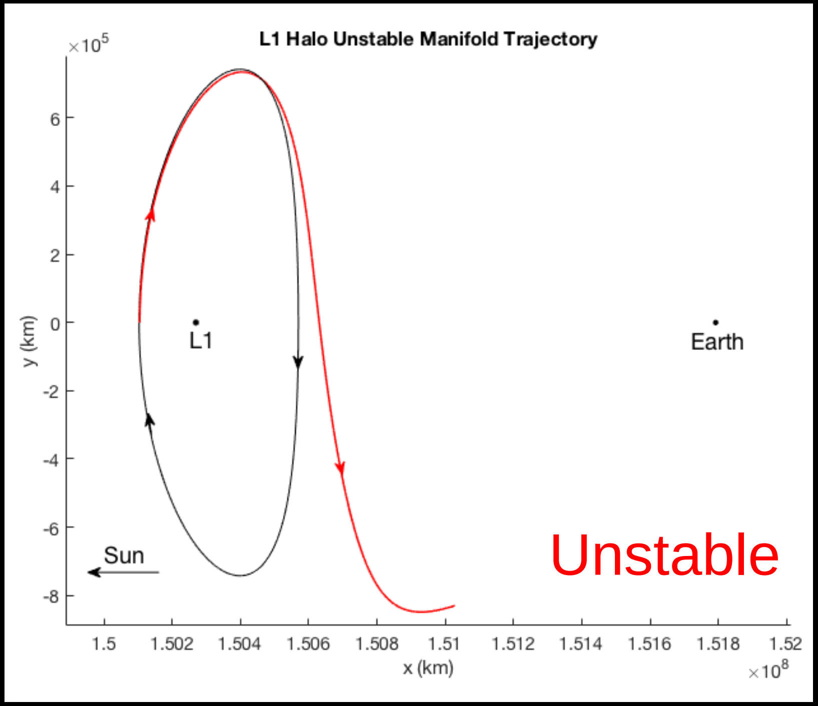
Source: Vaquero/Senent Monte U Seminar Presentation

Invariant Manifolds

Stable Manifold: Consists of trajectories that asymptotically *approach* the orbit

Unstable Manifold: Consists of trajectories that asymptotically *depart* the orbit



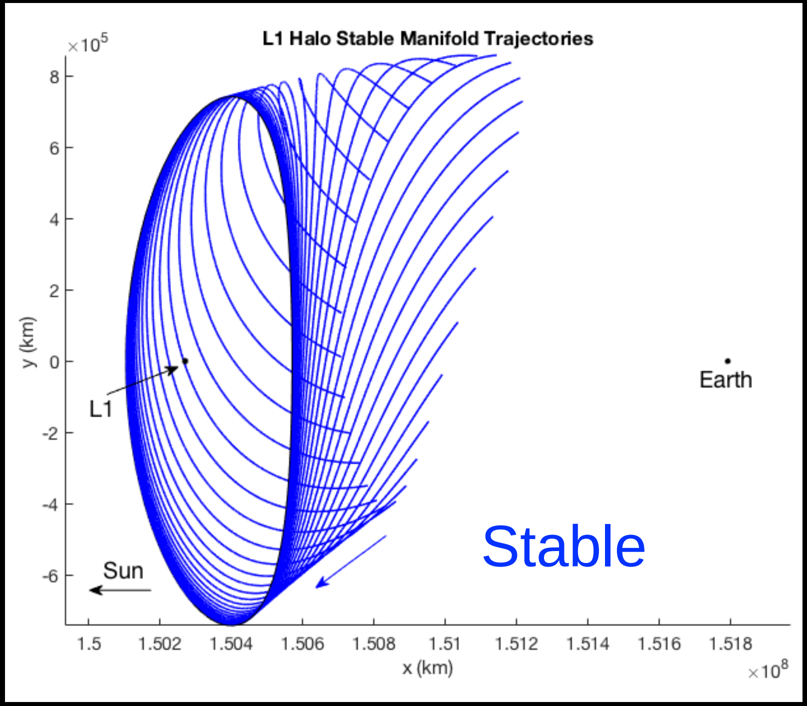


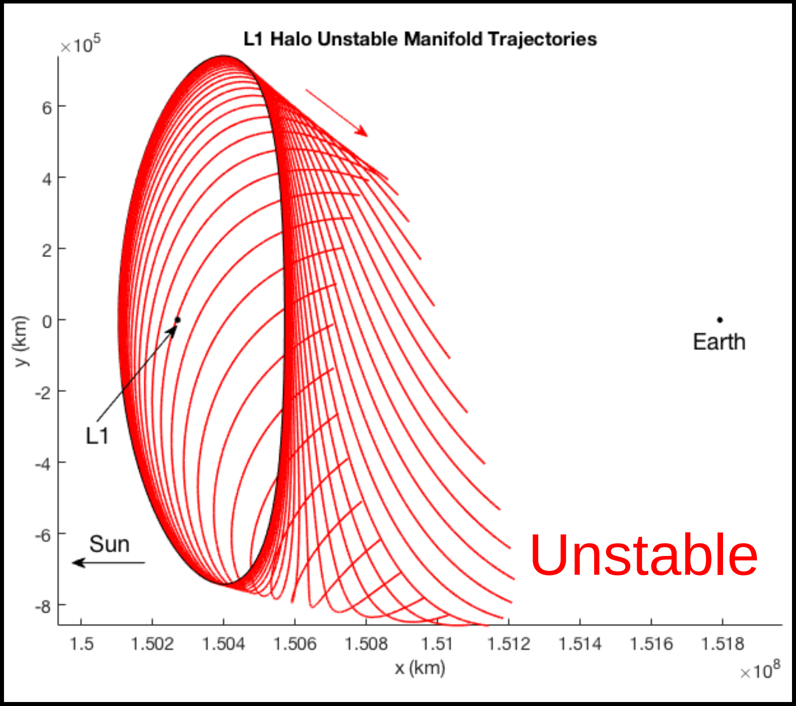
Definition *Source: Vaquero (2013) PhD Thesis Defense*

Invariant Manifolds

Stable Manifold: Consists of trajectories that asymptotically *approach* the orbit

Unstable Manifold: Consists of trajectories that asymptotically *depart* the orbit



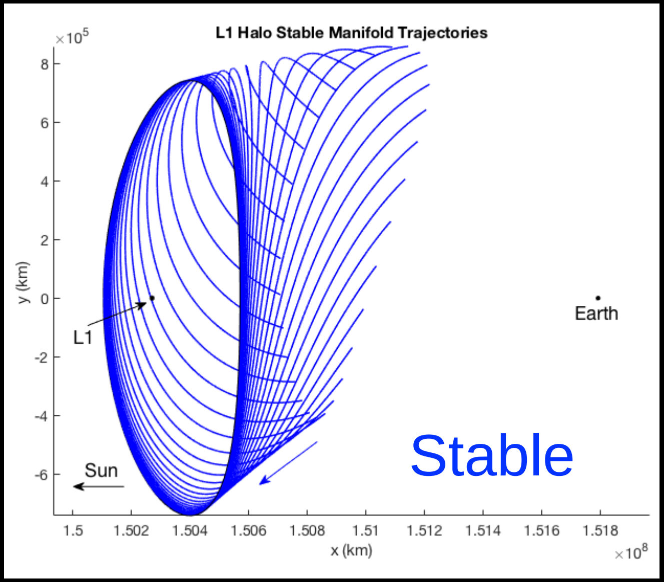


Definition *Source: Vaquero (2013) PhD Thesis Defense*

Finding Invariant Manifolds in *Poincare*

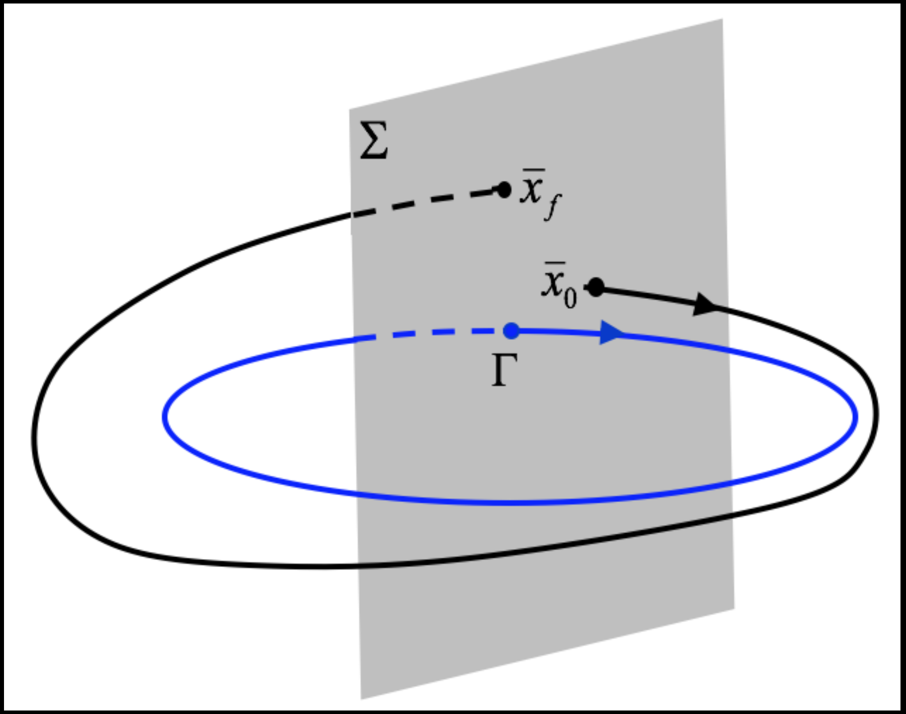
```
# Initialize Manifold Finder
manifold_finder = ManifoldFinder(sun_earth_system, body, SUN_RADIUS, EARTH_RADIUS)

# Propagate 60 stable manifold trajectories for 200 days each
manifold_finder.find_manifolds_for_orbit(initial_state,      # Orbit State From Catalog
                                         orbital_period,     # Orbit Period From Catalog
                                         step_distance=200*km,
                                         integration_period=200*day,
                                         num_manifold_trajectories=60,
                                         manifold_type=ManifoldType.STABLE)
```



Finding Transfers via Poincaré Maps

Generated by defining a hyperplane Σ, and recording every crossing of a trajectory across Σ



Γ = periodic orbit
Σ = surface of section
$\bar{x}_0$ = initial state
$\bar{x}_f$ = first return to the map

Source: Vaquero (2013) PhD Thesis Defense

<u>Motivations</u>

- Reduce dimensionality
- Reveal dynamic structure in system
- Conceptual Clarity

<u>Uses in *Poincare*</u>

- Find intersections between stable and unstable manifolds
- Visualization/Interactive Tool
- Available map types: Hyperplane, Periapsis, Apoapsis, Close Pass

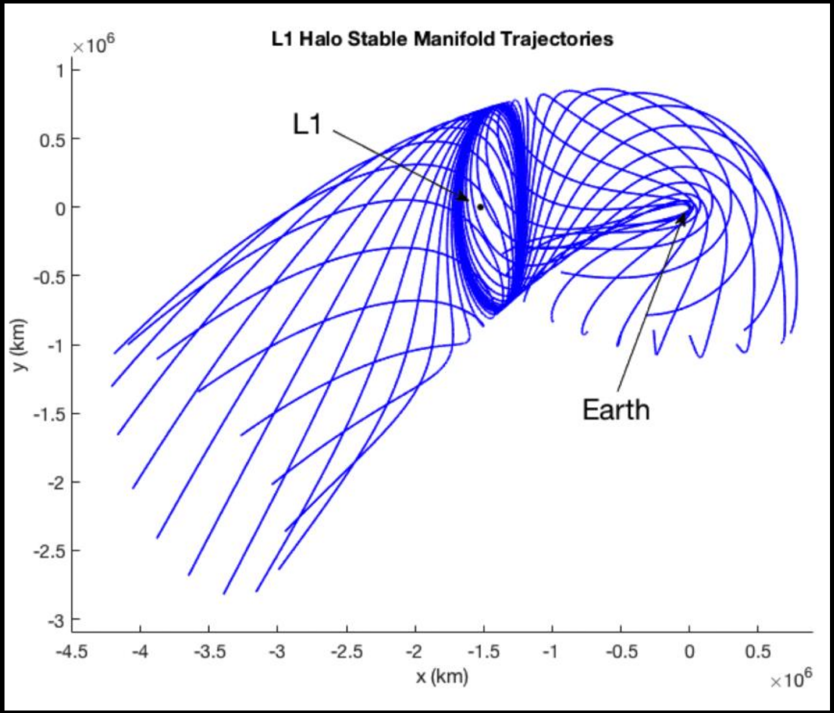
Generating Poincaré Maps in Monte.*Poincare*

```
pm = PoincareMap(
    sun_earth_system,
    maps.map_types['periapsis'],
    initial_state,
    orbital_period,
    primary_radius=SUN_RADIUS,
    secondary_radius=EARTH_RADIUS)

pm.find_stable_manifold_states(
    sun_earth_system.boa,
    sun_earth_system.secondary,
    sun_earth_system.rotatingFrame)

pm.plot_states(
    'x',
    'y',
    ManifoldType.STABLE)
```

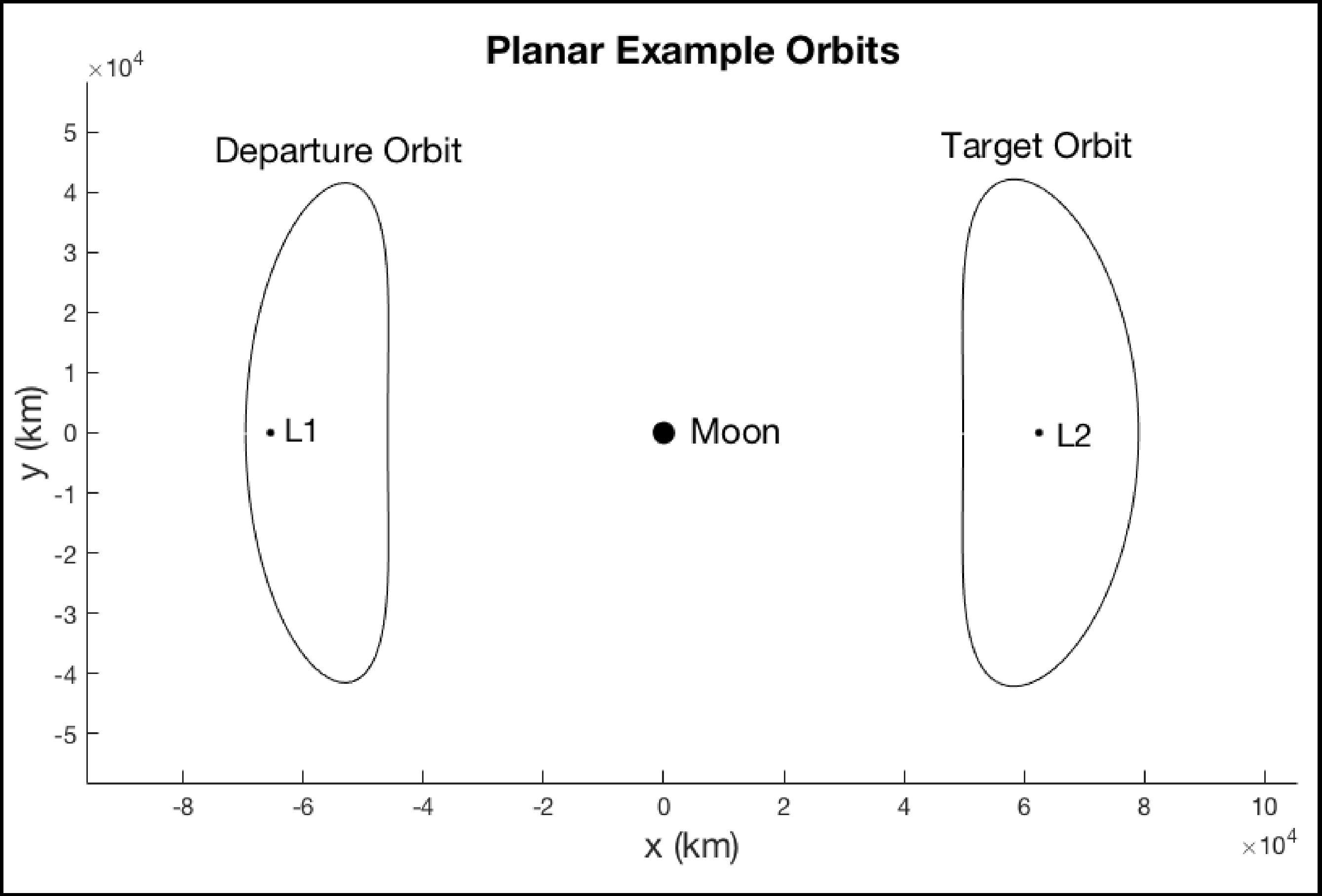
Sun/Earth L1 Halo Stable Manifold Trajectories Propagated for 250 Days



2D Transfer Example

L1 Lyapunov to L2 Lyapunov

$C = 3.14324$

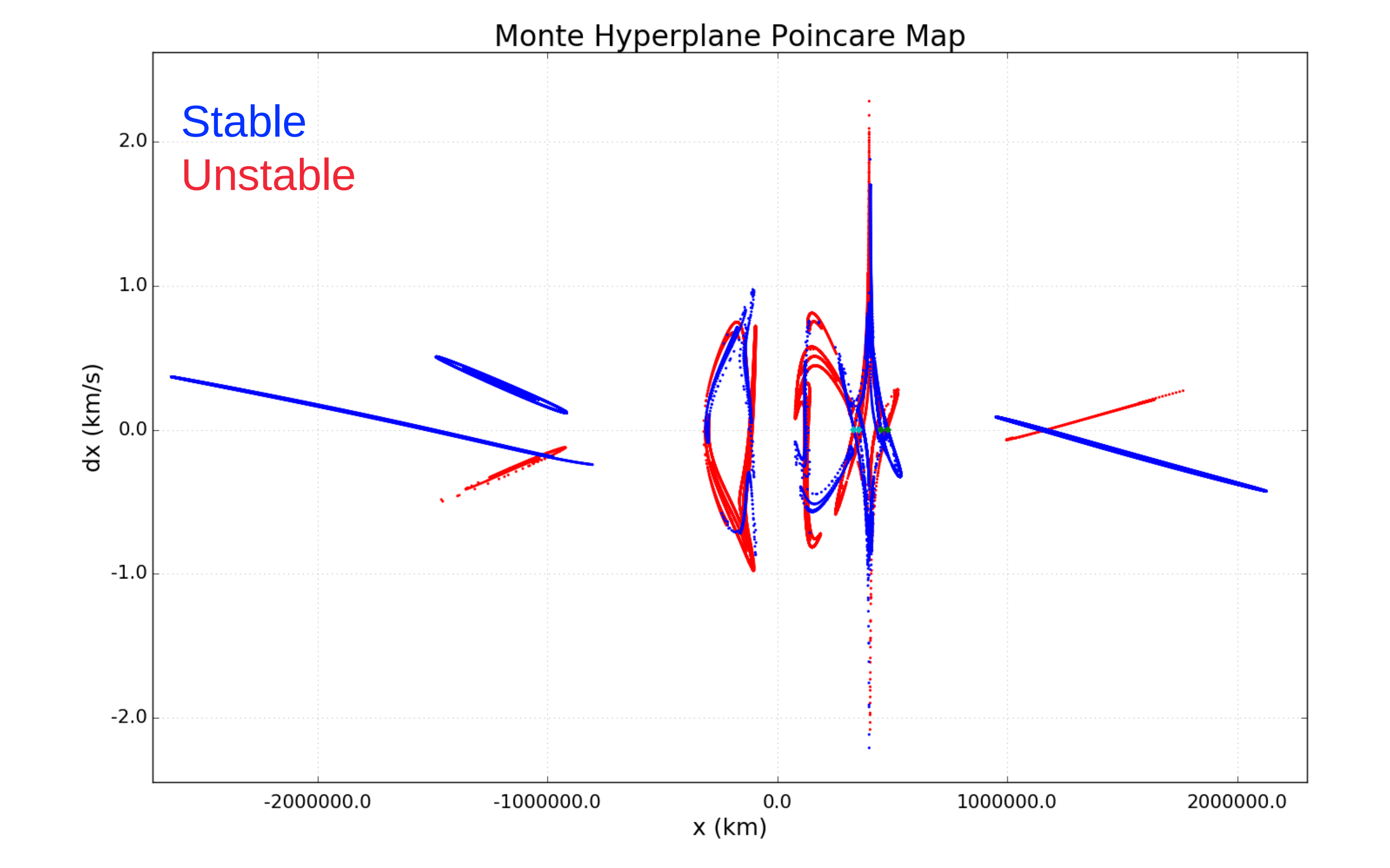


Goal: Find free transfers between planar orbits

2D Transfer Example

$y = 0$ Event

$C = 3.14324$

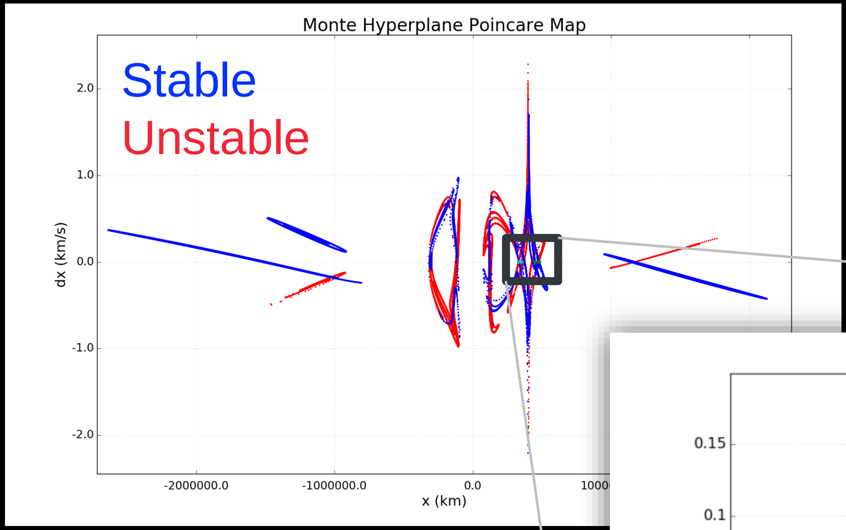


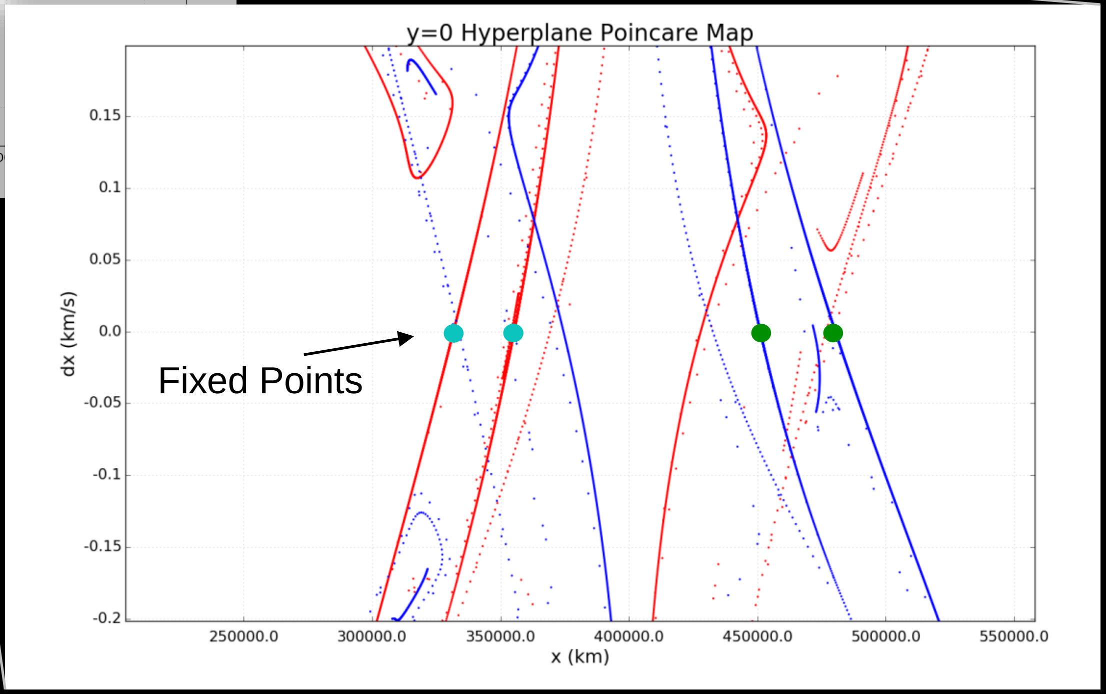
$$y = 0$$
$$z = 0$$
$$dz = 0$$

Goal: Find free transfers between planar orbits

2D Transfer Example

$y = 0$ Event





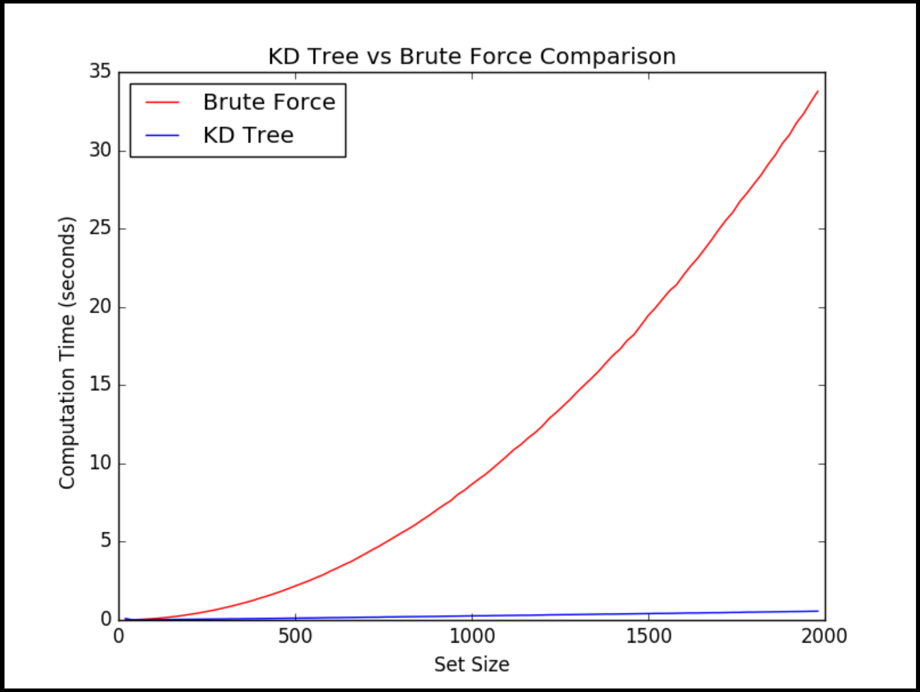
22658 Stable States
23478 Unstable States

Too many intersections
to manually search

Nearest Neighbor Searching

KD Tree ("K-Dimensional Tree")

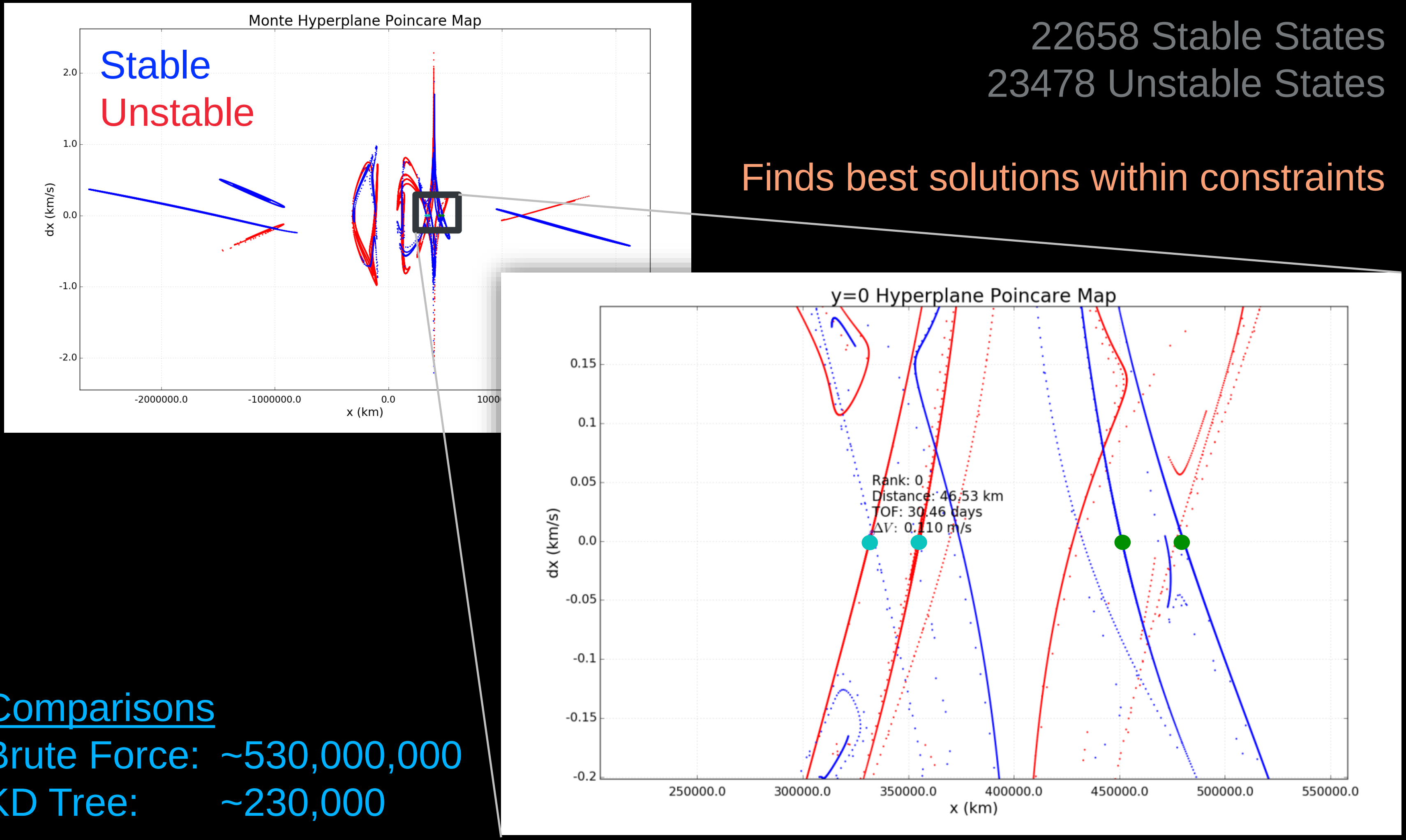
- Similar to a Binary Search Tree
- Implemented in scipy
- Brute Force Solution: $O(\mathrm{n}^2)$
- Extrapolates to higher dimensions



Actual Sets Much Larger

Creation	$O(n \log n)$
Nearest Neighbor	$O(\log n)$
Entire Search	$O(n \log n)$

2D Transfer Example



22658 Stable States
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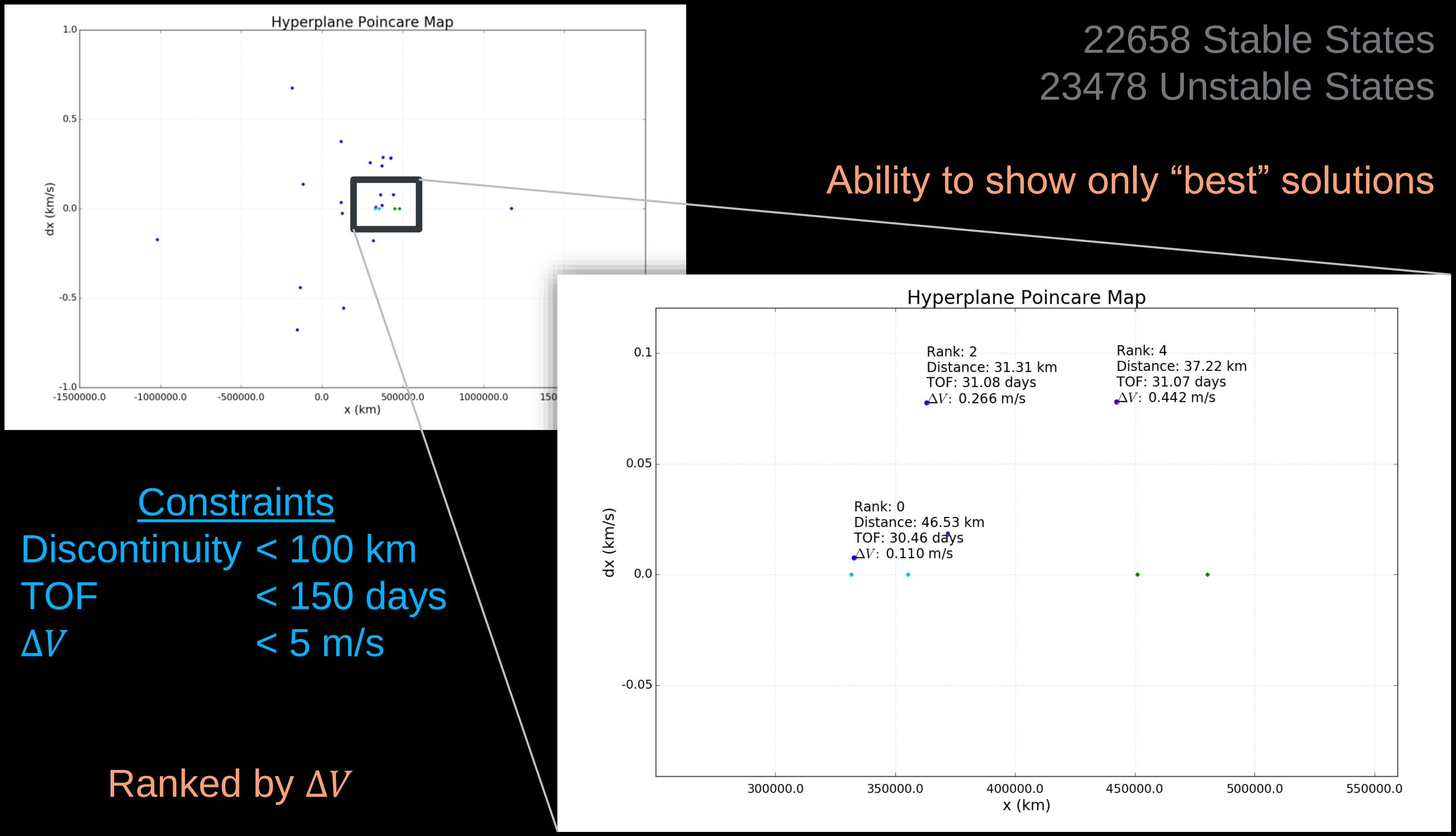
Finds best solutions within constraints

Comparisons
Brute Force: ~530,000,000
KD Tree: ~230,000

2D Transfer Example



22658 Stable States
23478 Unstable States

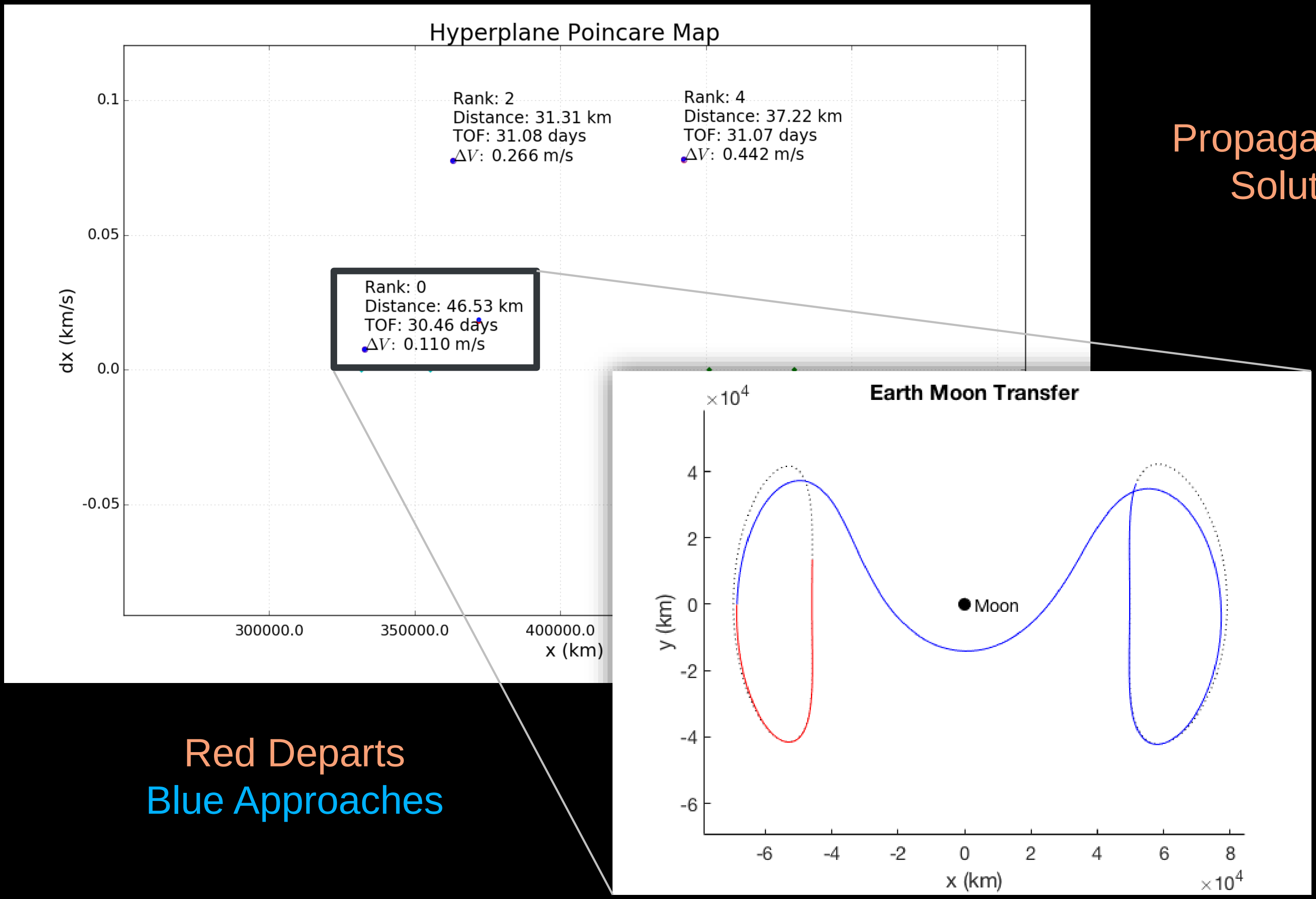
Ability to show only "best" solutions

Constraints

Discontinuity < 100 km
TOF < 150 days
ΔV < 5 m/s

Ranked by ΔV

2D Transfer Example

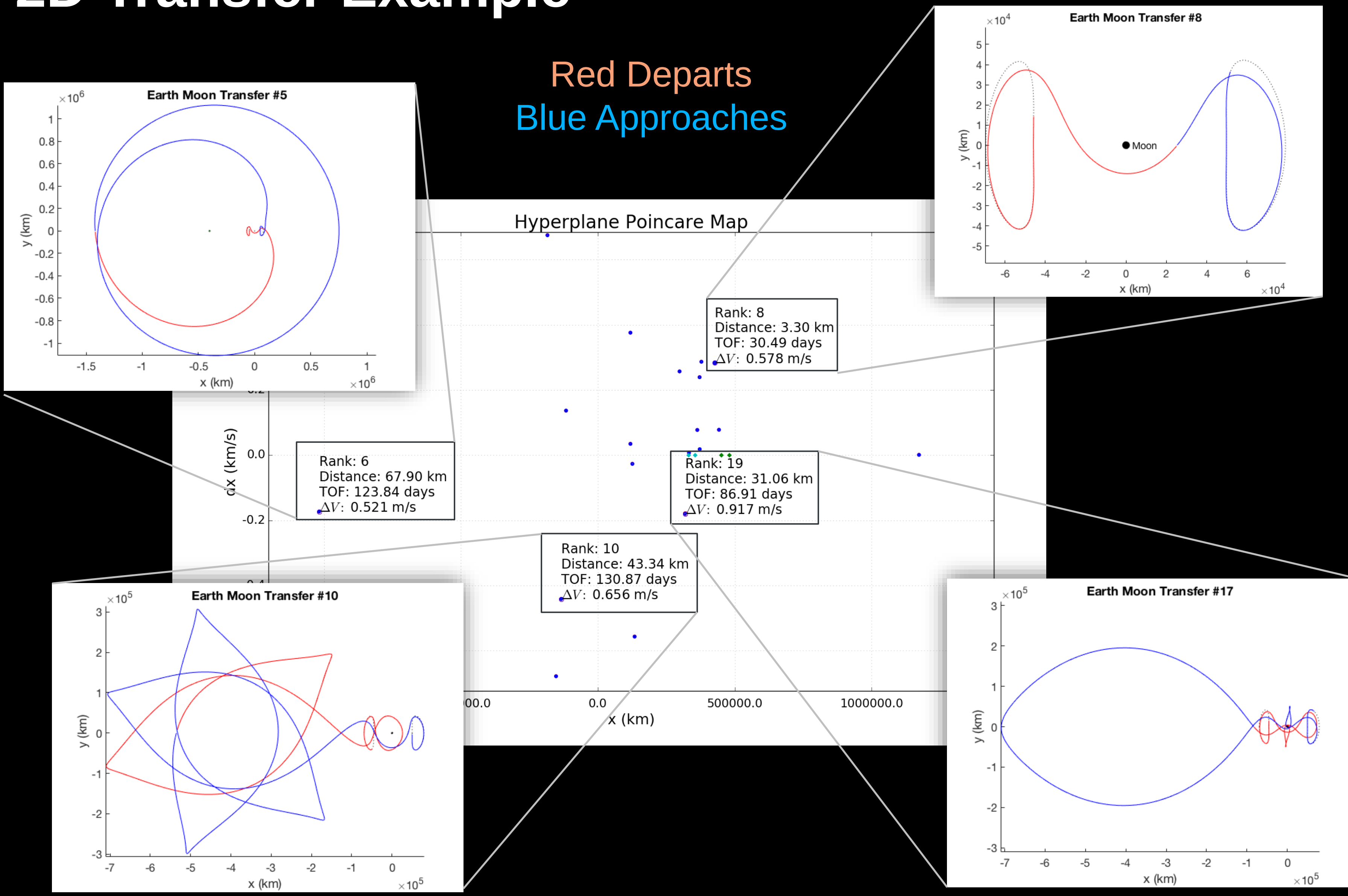


Propagate Any Solution

Red Departs
Blue Approaches

2D Transfer Example

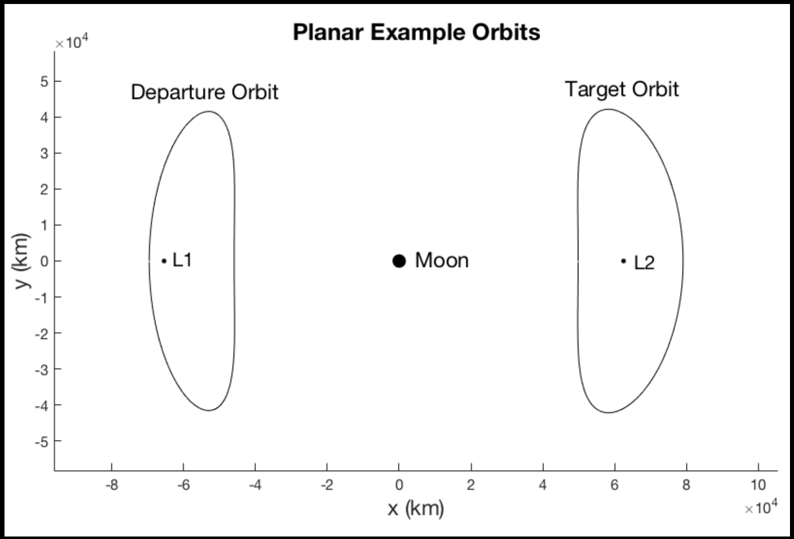
Red Departs
Blue Approaches



2D Transfer Example

Process in *Poincare*

1. Query Database
2. Propagate Stable Manifolds for Target orbit, and Unstable Manifolds for Departure Orbit
3. $y = 0$ Event Search
4. (Visualize Poincaré Map)
5. Find Intersections in Map
6. Select Potential Transfers

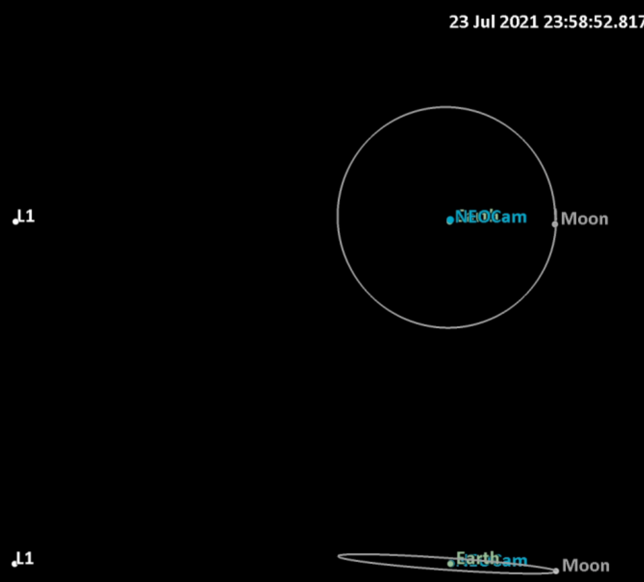


NEOCam

Transfer from Earth to L1 Halo

Mission: Observe near-Earth objects that have the potential to cause major damage if an impact occurs

Orbit: Sun-Earth L1 Halo Orbit to provide a vantage point form which to observe asteroids



Source: Mar Vaquero

NEOCam

Transfer from Earth to L1 Halo

Orbit Size

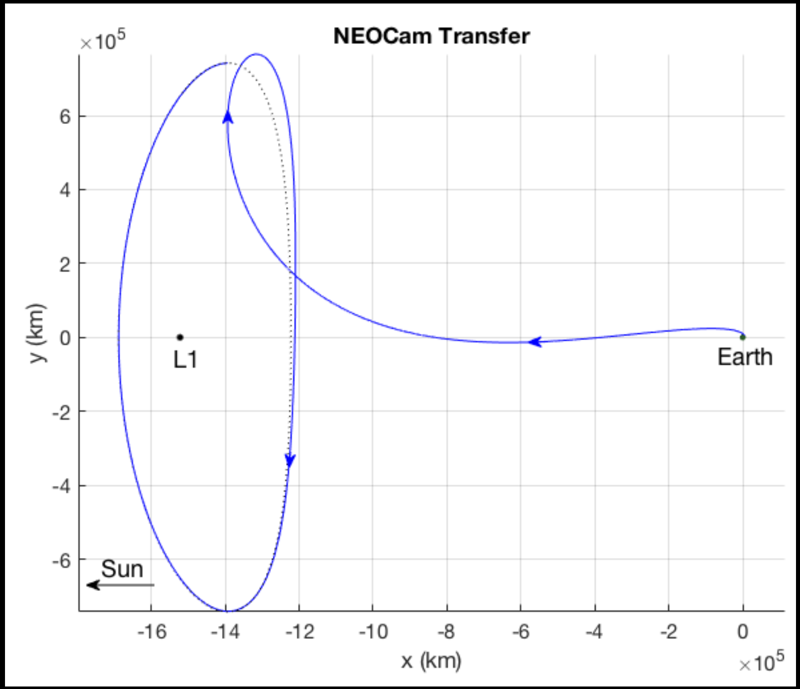
$y = 750{,}000\ km$

$z = 300{,}000\ km$

Process in *Poincare*

1. Query Database
2. Propagate Stable Manifold Trajectories
3. Event Search
4. Global Minimum
5. Save Transfer

Transfer found by *Poincare*

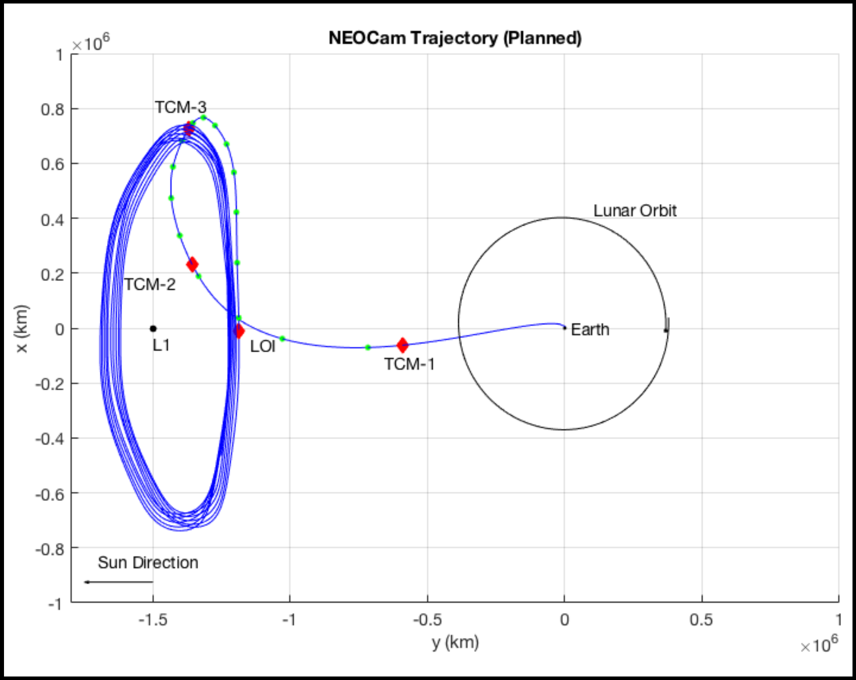


Source: Mar Vaquero

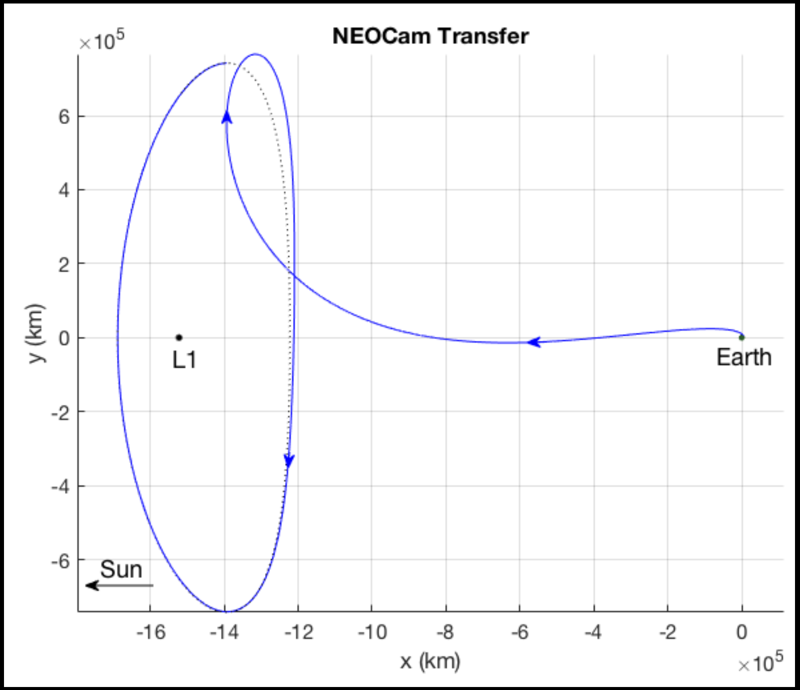
NEOCam

Transfer from Earth to L1 Halo

Actual Planned Transfer



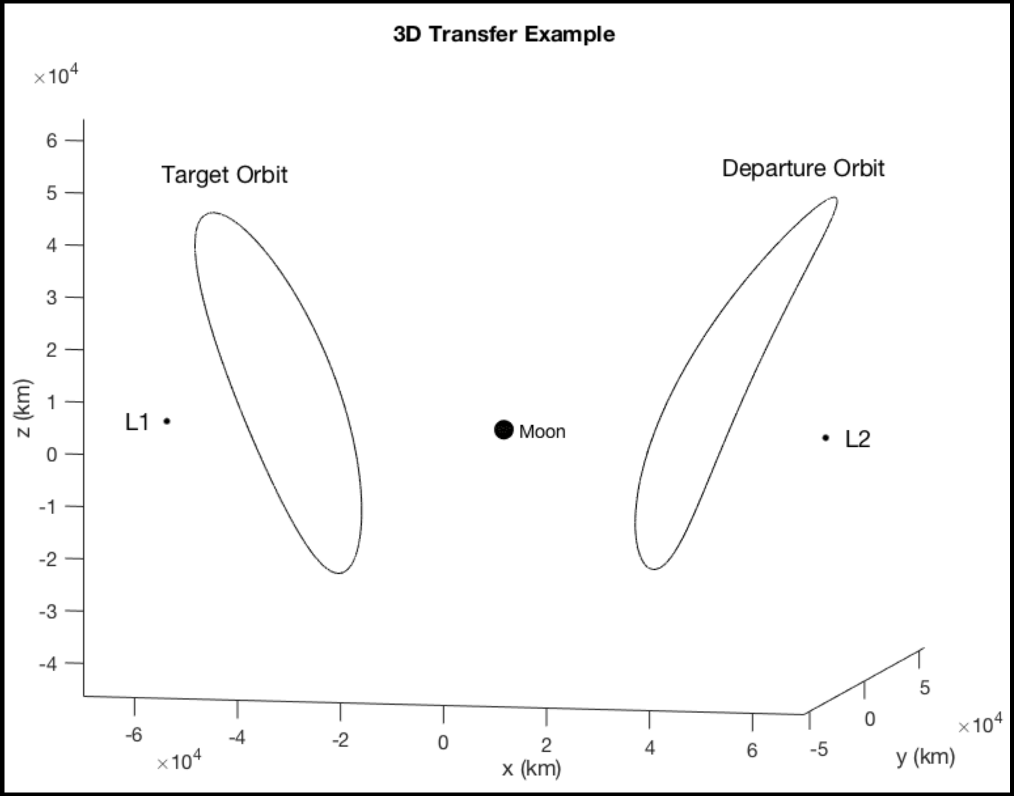
Transfer found by *Poincare*



Source: Mar Vaquero

3D Transfer Example

L2 Halo to L1 Halo

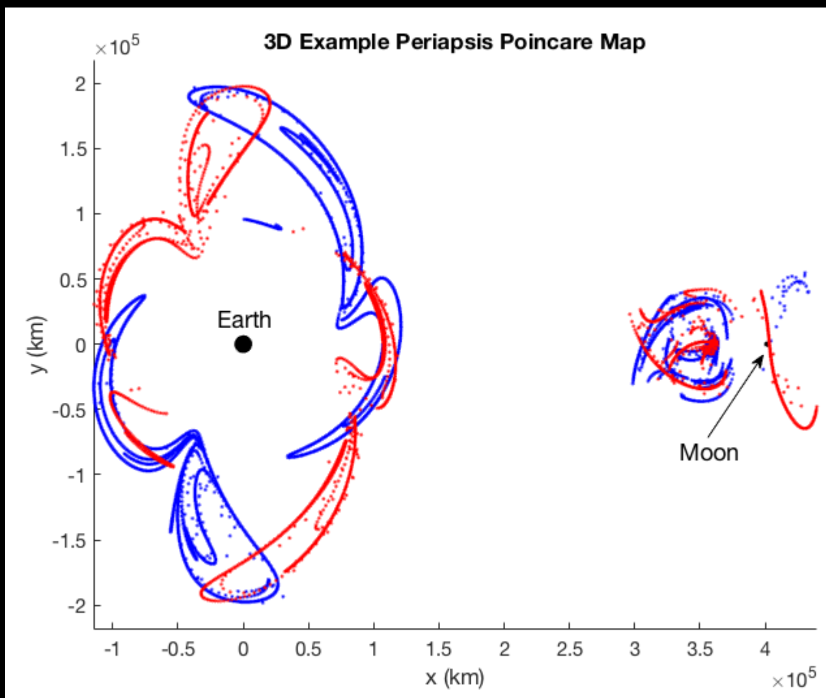


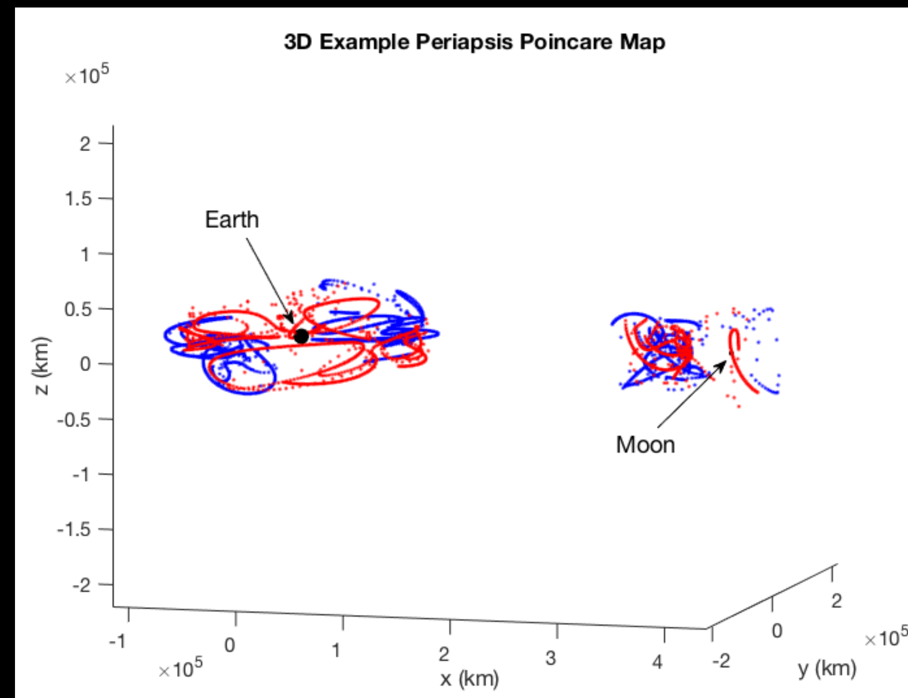
Goal: Find low-cost transfers between 3D orbits

3D Transfer Example

$z \neq 0$
$dz \neq 0$

Tree search still possible!



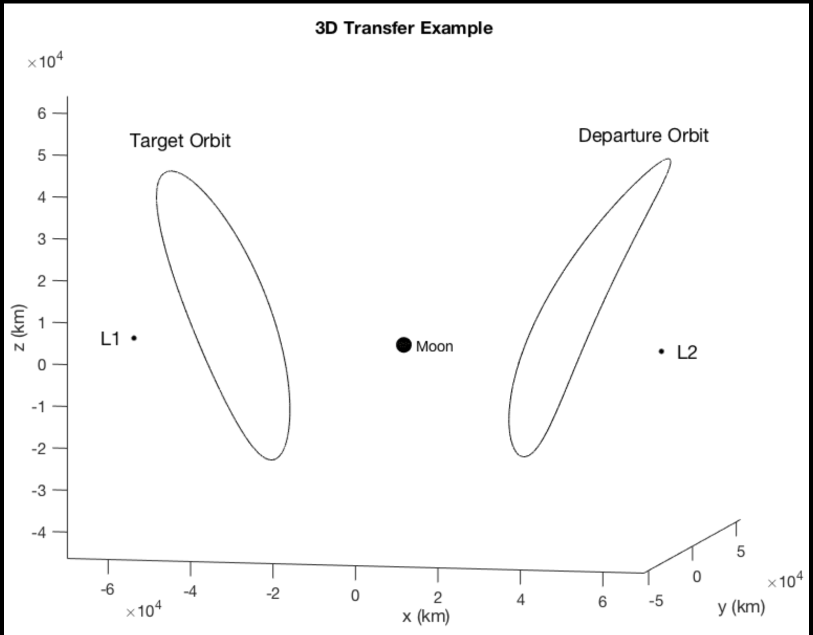


With added dimension, 2D Poincaré map visualization not as useful

3D Transfer Example

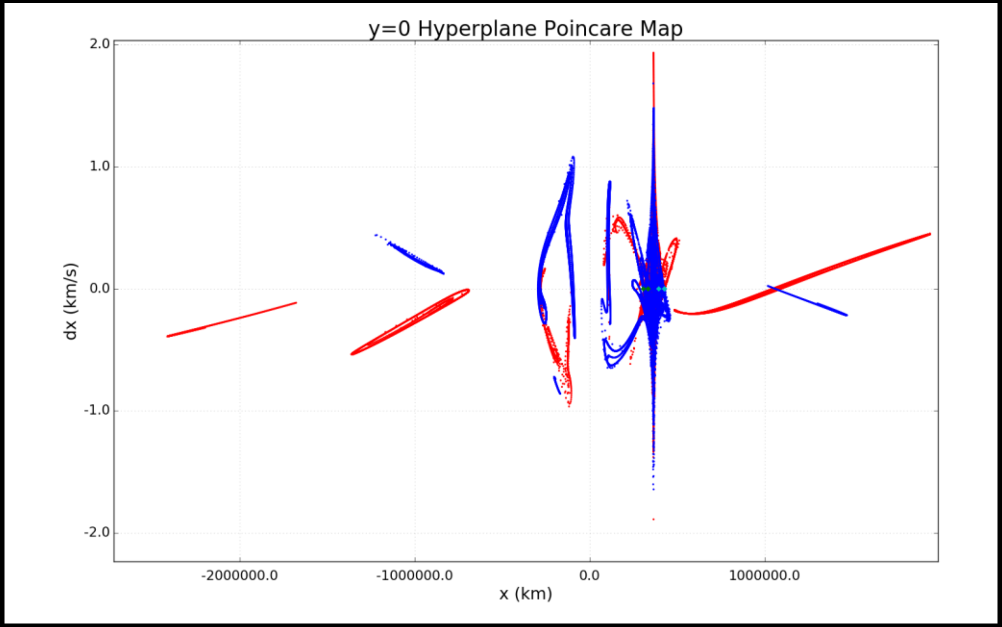
Process in *Poincare*

1. Query Database
2. Propagate Stable Manifolds for
3. Target orbit, and Unstable Manifolds for Departure Orbit
4. $y = 0$ Event Search
5. ~~(Visualize Poincaré Map)~~
6. Find Intersections in Map via KD Tree
7. Select Potential Transfers



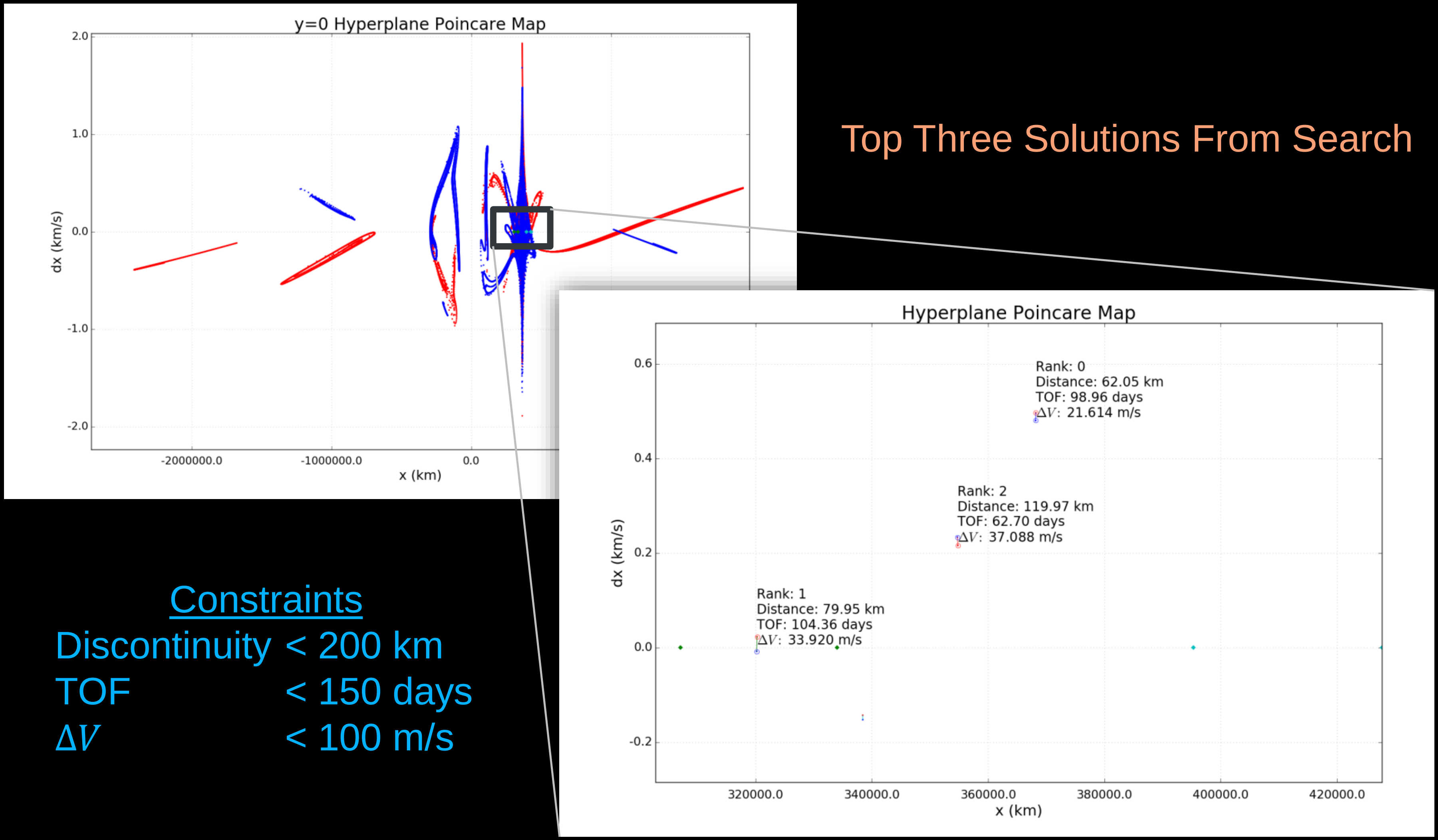
3D Transfer Example

$y = 0$ Hyperplane Event




```
mi.set_map_constraints(
    max_distance=200 * km,
    max_tof=150 * day,
    max_delta_v=100 * m / sec
)
intersections = mi.find_closest_points(
    ['x', 'z', 'dx'],
    num_points=20,
    comparison_count=30)
```

3D Transfer Example

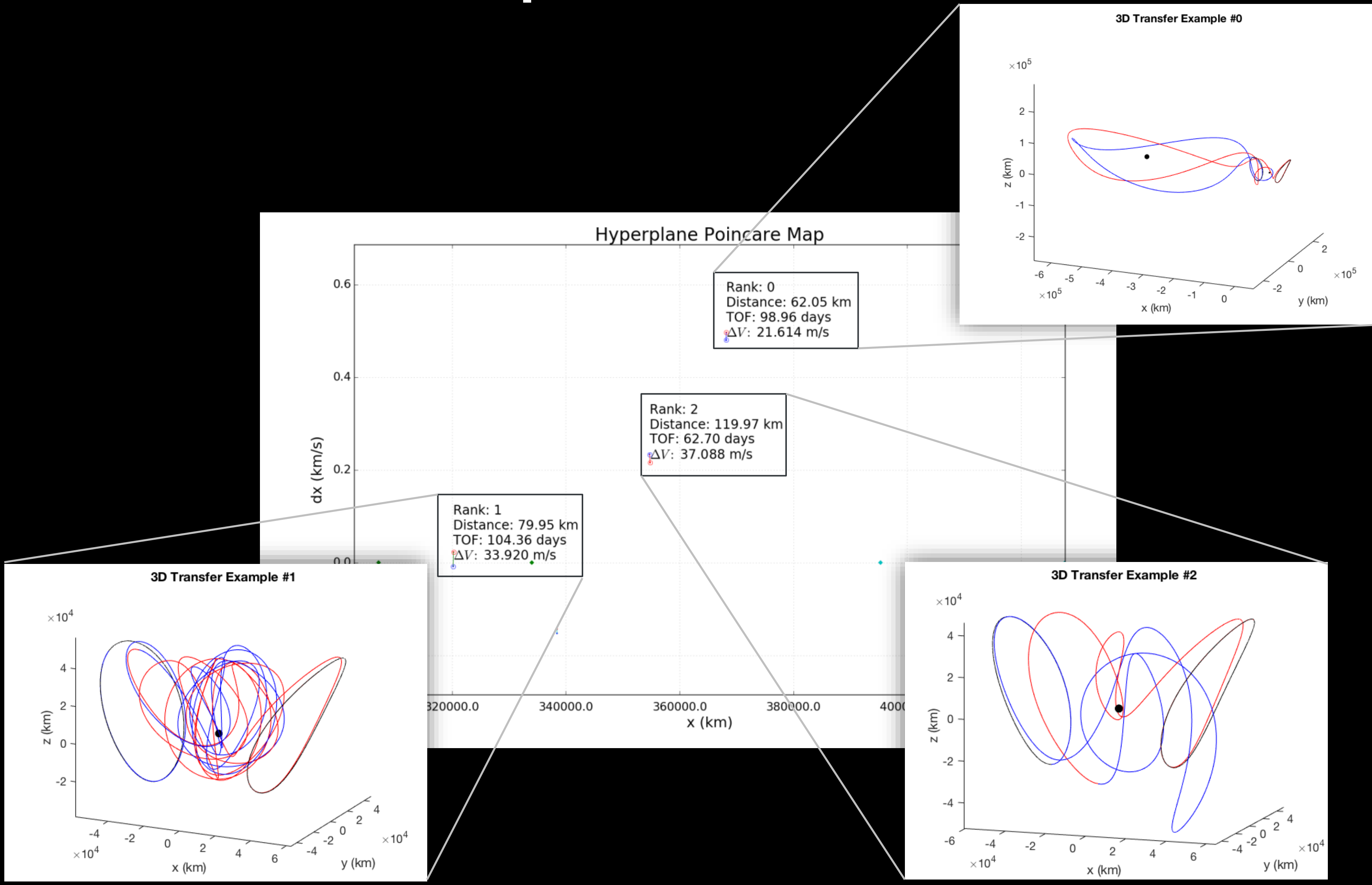


Top Three Solutions From Search

Constraints

Discontinuity	< 200 km
TOF	< 150 days
ΔV	< 100 m/s

3D Transfer Example



3D Transfer Example

Time Complexities

Step	Time Taken
Propagate 8000 Manifold Trajectories	15 Minutes*
Event Search	3 Minutes*
KD Tree Search	4 Minutes*

Start-to-finish runtime of approximately 20-25 minutes*

* Will be sped up in the future by running in parallel

Conclusion

My Contributions

- Prototype built in Monte using *Poincare*
- Real-time initial guess computations of orbit connections
- Generate and interact with Poincaré Maps
- Efficient nearest neighbor search (KD Tree)

Future Work

- Tier-2 Implementation
- Differential Corrector
- End-to-end higher fidelity solutions

Acknowledgments

Mar Vaquero and Juan Senent

For giving me the opportunity to be a part of JPL, mentoring me, and continuously supporting throughout the summer.

Dr. Kathleen Howell

For initially putting me in touch with Mar and Juan, and for helping me prepare for this internship.

Jet Propulsion Laboratory
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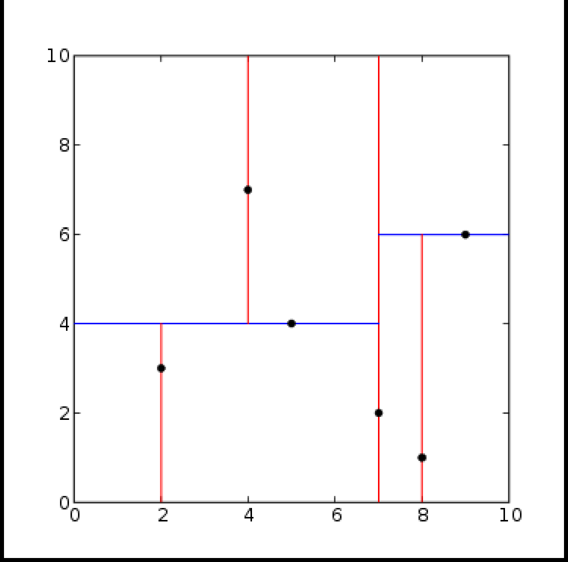
Backup Slides

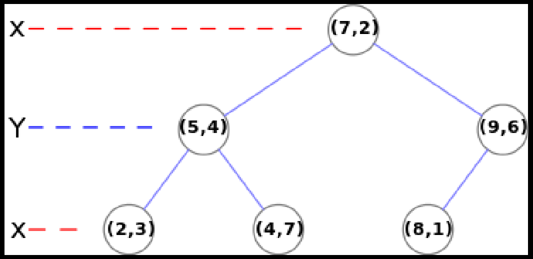
KD Tree Search

Creation

- Split set of points alternating between x/y
  - Even number of points with respect to x on each side of a tangent hyperplane.
  - Next, subdivide each section evenly with respect to y on each side of a tangent hyperplane.



(2,3), (5,4), (9,6), (4,7), (8,1), (7,2)

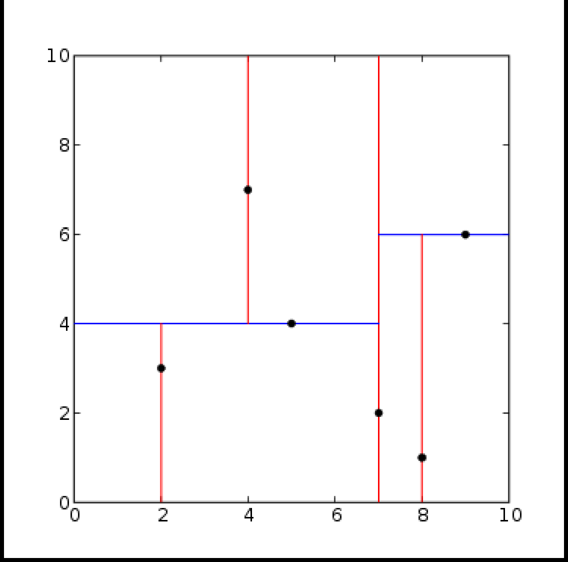


Source: Wikipedia

KD Tree Search

<u>Complexity</u>

- Create KD Tree for all states associated with the unstable manifold $O(n \log n)$
- Find nearest neighbor in the KD Tree for each stable manifold state $O(n \log n)$



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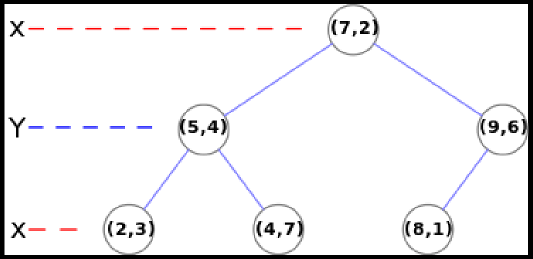
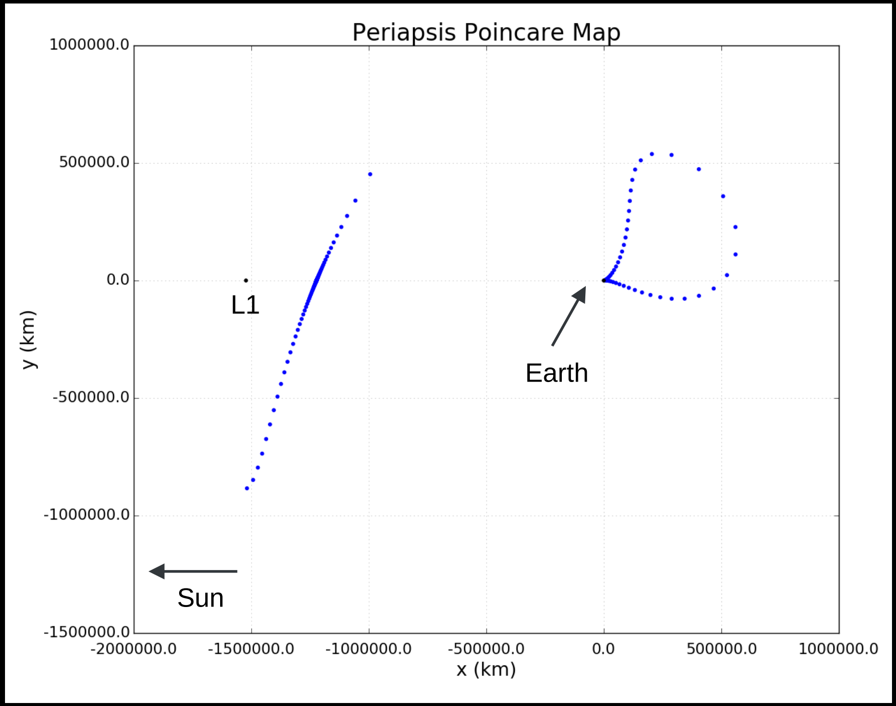
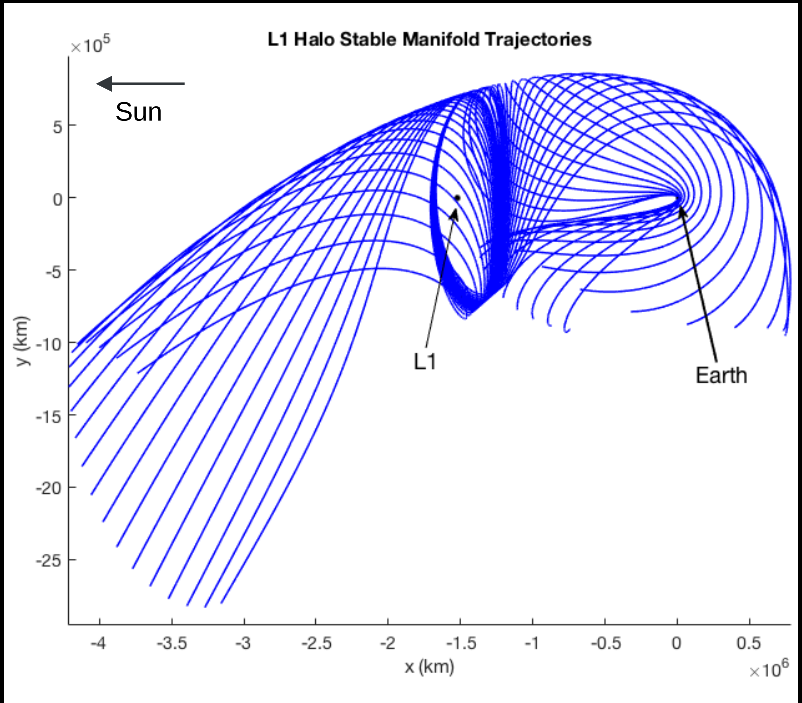


Source: Wikipedia

Generating Poincaré Maps in Monte.*Poincare*

Sun/Earth L1 Halo Stable Manifold Trajectories Propagated for 250 Days

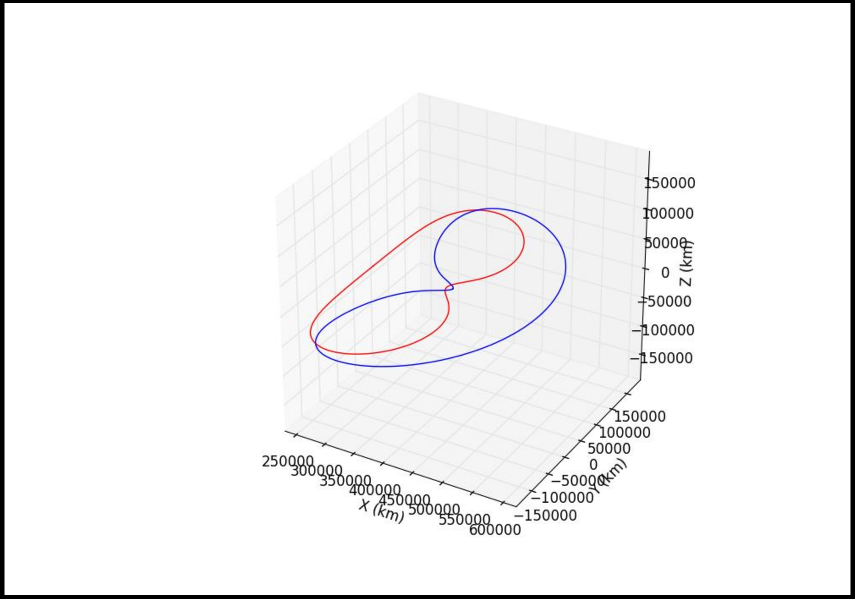




Poincare Map Example — All Types

Earth/Moon Transfer Example

L1 Lyapunov to L2 Lyapunov

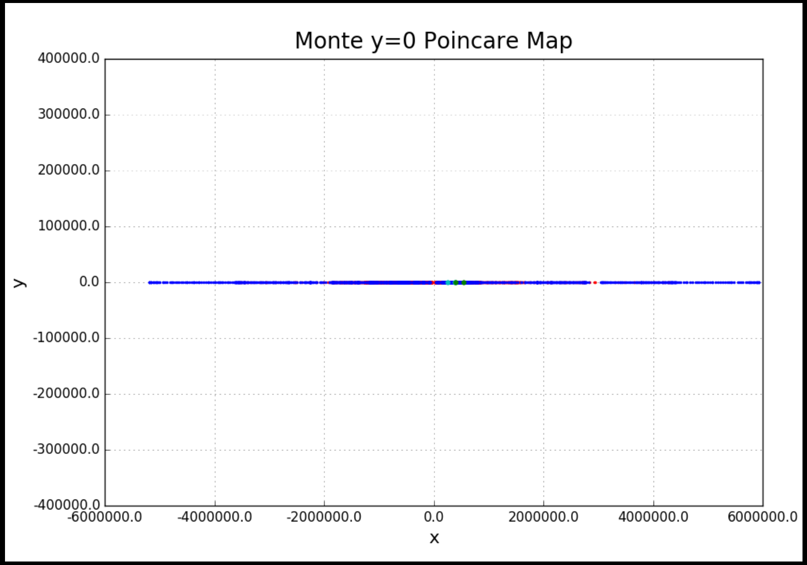


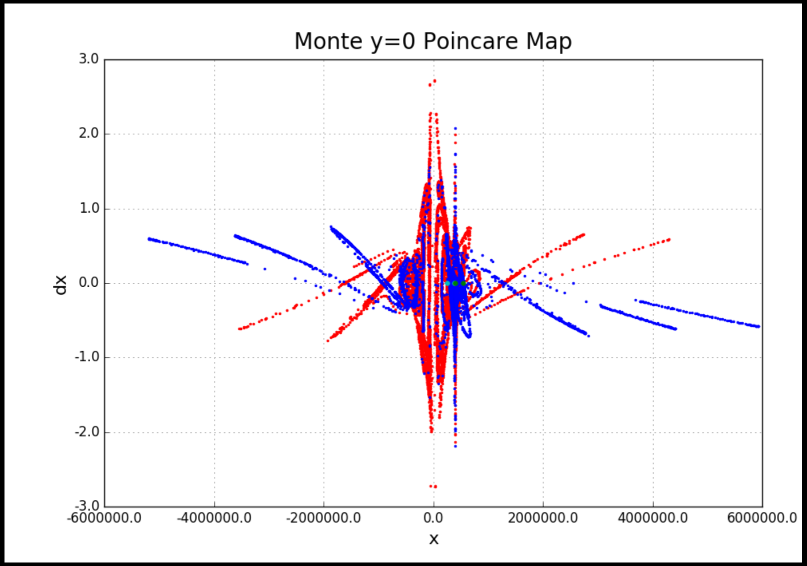
Jacobi Constant: $C = 2.9136$

Jacobi Error: 2.899e-7

Earth/Moon Transfer Example

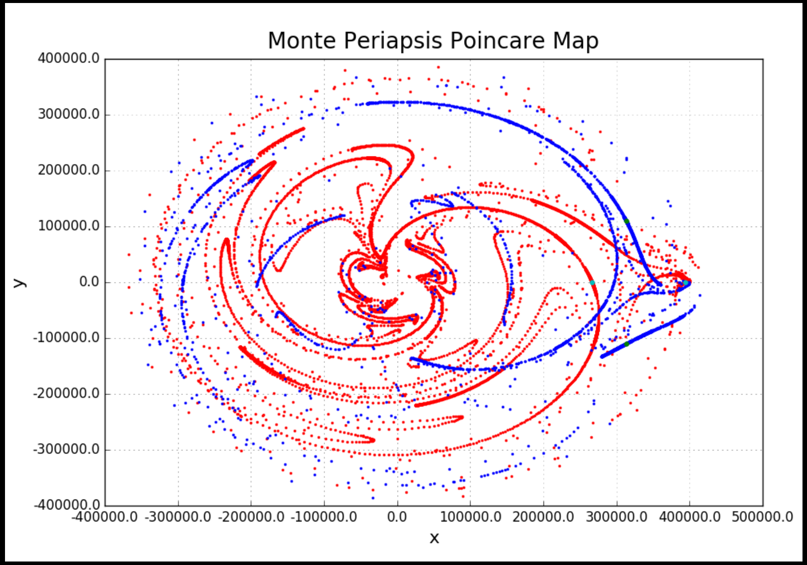
Map Generation – y=0

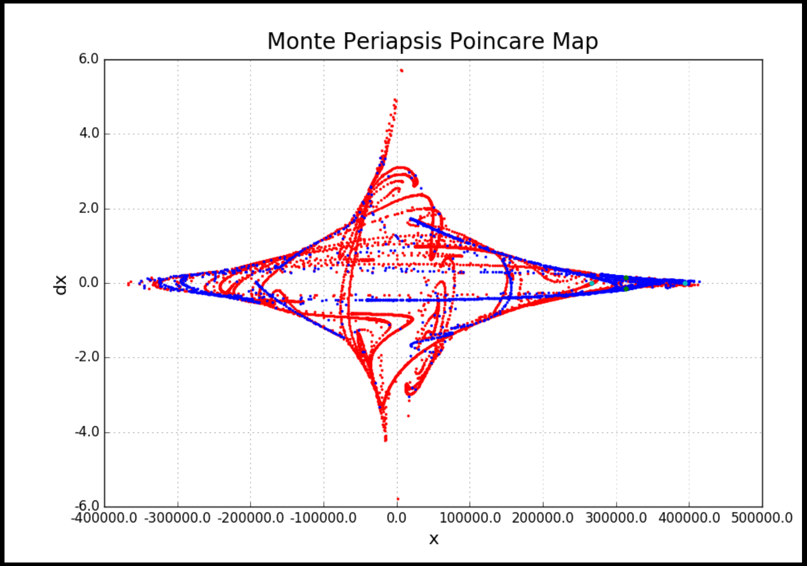




Earth/Moon Transfer Example

Map Generation – Periapsis





Earth/Moon Transfer Example

Map Generation – Apoapsis