Electron transport by the EXB-driven ion acoustic instability in a Hall thruster based on r-z multi-fluid simulations

A. Lopez Ortega, I. Katz, V. Chaplin, I. Mikellides
Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA

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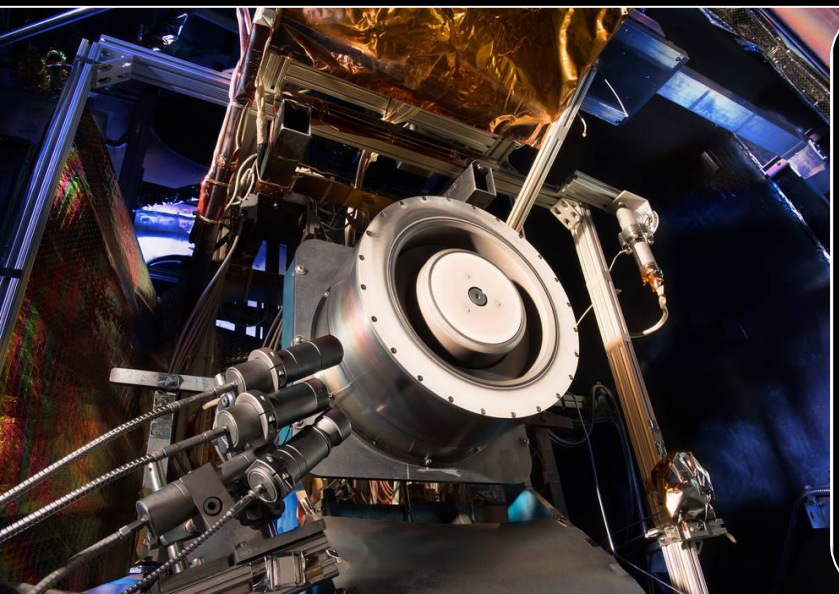
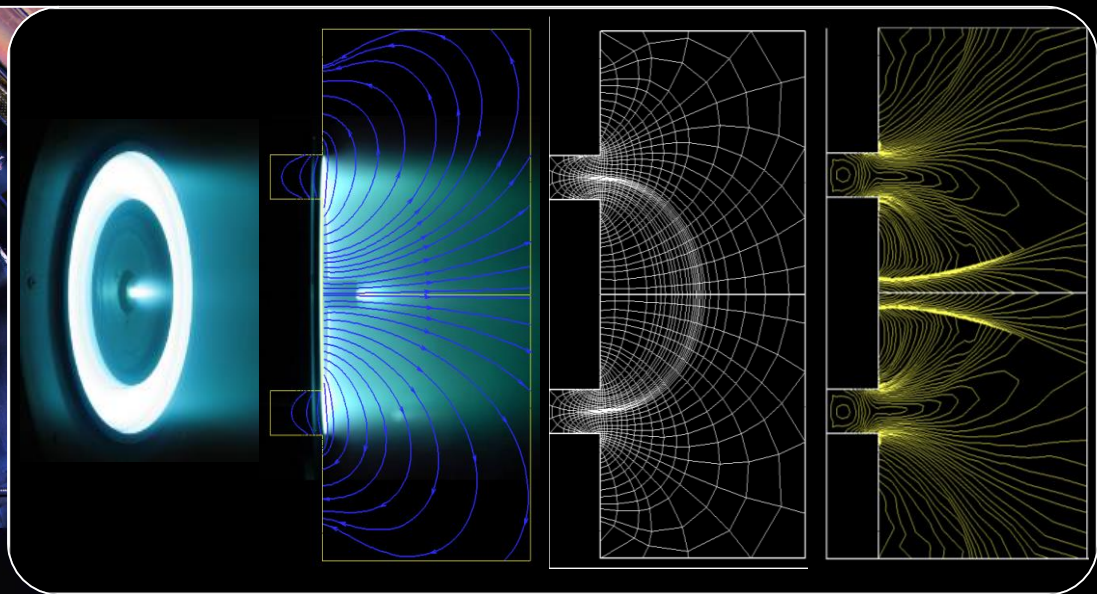
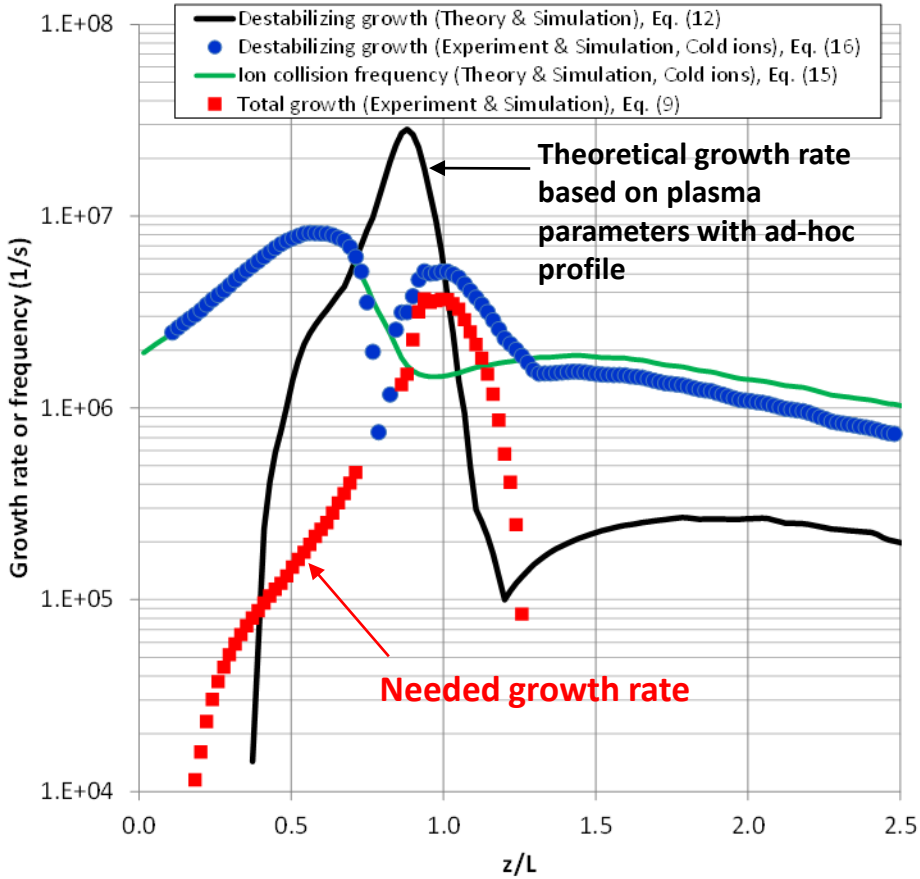
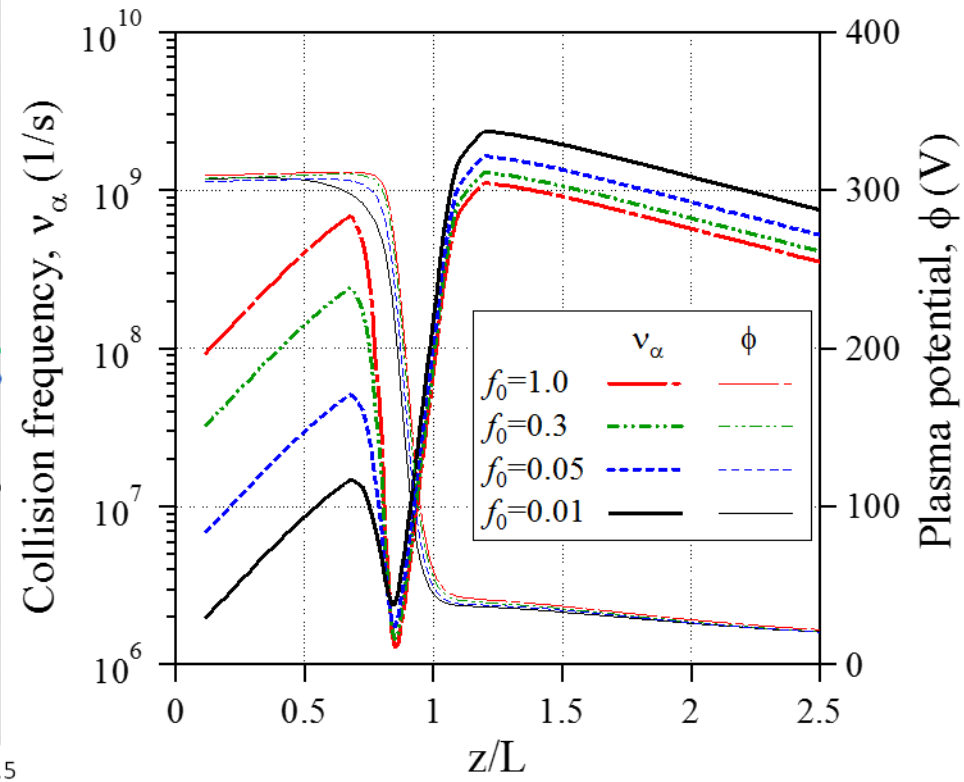




Theoretical growth of instability in acceleration region is higher than the "needed" growth





- The anomalous collision frequency is found to have a minimum at the acceleration region. However, the theoretical growth rate predicted by such plasma potential profile is conductive to large growth rates of the instability, which in turn should result in large values of the wave energy density and the anomalous collision frequency.

- Our efforts have been focused on eliminating the growth in the acceleration region by means of, for instance, Landau damping. All our efforts have been unsuccessful as even significant Landau damping cannot completely counteract the maximum growth rate.

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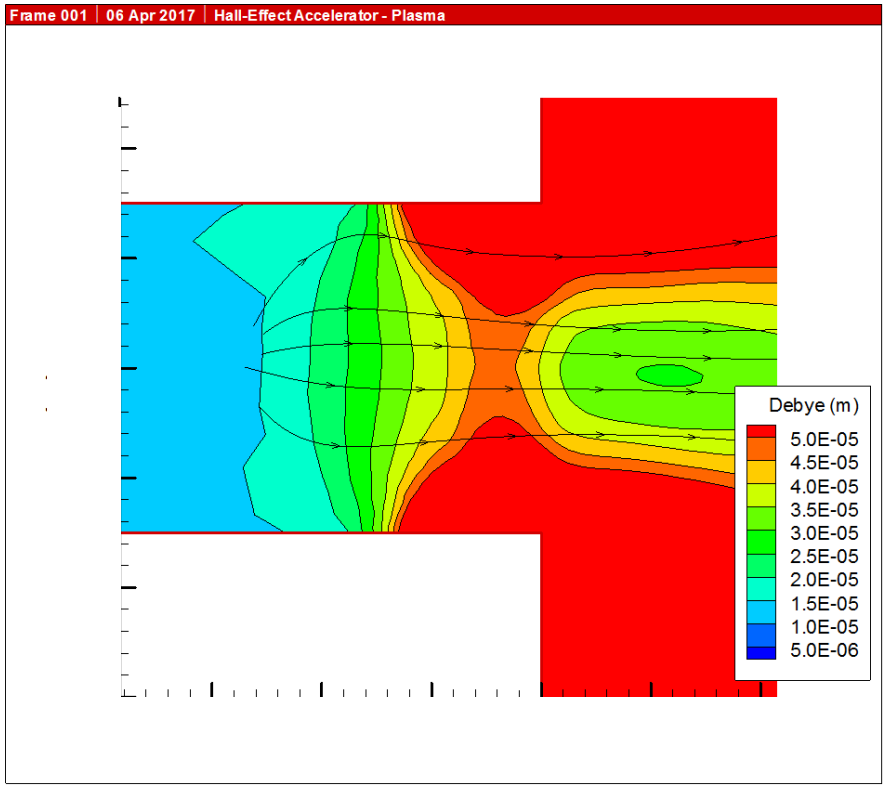
Can accounting for multiple wave-lengths improve our self-consistent model?



- For the H6, the Debye length increases by a factor of 5, following ion streamlines.
- Neglecting Landau damping, the maximum growth occurs for $k = 1/\sqrt{2\lambda_{De}^2}$. Waves of wavelength that have maximum growth inside the channel may be damped as the Debye length increases, while other waves with lower k may start growing further downstream.
- Addition of Landau damping may change the wave-length of maximum growth at a given location.
- Proposed algorithm: solve a discrete number of equations for the wave actions associated with multiple wavelengths. The total anomalous collision frequency can be obtained as the sum of all contributions.

Summary of equations for discrete wave-length model for wave action

$$\frac{\partial N_k}{\partial t} + \nabla \cdot \left(\left(\nabla_k \omega_{r,k} \right) N_k \right) - \nabla_k \cdot \left(\left(\nabla \omega_{r,k} \right) N_k \right) = 2\omega_{i,k} N_k$$

$$\omega_{r,k} = \mathbf{u}_i \cdot \mathbf{k} + \sqrt{qT_e/m_i} \frac{k}{\left(1 + k^2 \lambda_{De}^2\right)^{1/2}}$$

$$\omega_{i,k} = \left(\frac{\pi}{8}\right)^{1/2} \frac{\sqrt{qT_e/m_i}\,k}{\left(1 + k^2 \lambda_{De}^2\right)^{3/2}} \left(\frac{\mathbf{k} \cdot \mathbf{u}_{ei} - k\sqrt{qT_e/m_i} \Big/ \left(1 + k^2 \lambda_{De}^2\right)^{1/2}}{k\sqrt{qT_e/m_e}} \exp\left(-\frac{1}{2} \left(\frac{\mathbf{k} \cdot \mathbf{u}_{ei} - k\sqrt{qT_e/m_i} \Big/ \left(1 + k^2 \lambda_{De}^2\right)^{1/2}}{k\sqrt{qT_e/m_e}} \right)^2 \right) - \sum_i \frac{n_i}{n_e} \frac{T_e^{3/2}}{T_i^{3/2} \left(1 + k^2 \lambda_{De}^2\right)^{1/2}} \exp\left(-\frac{T_e}{2T_i \left(1 + k^2 \lambda_{De}^2\right)} \right) \right) - \frac{1}{2} \nu_i$$

$$\nu_{an} = \left(\frac{\pi}{2}\right)^{1/2} \sum_k \frac{qk^2 N_k}{n_e \sqrt{m_e m_i} \left(1 + k^2 \lambda_{De}^2\right)^{3/2}} \exp\left(-\left(\frac{\hat{k} \cdot \mathbf{u}_{ei} - c_s \Big/ \left(1 + k^2 \lambda_{De}^2\right)^{1/2}}{v_{te}} \right)^2 \right)$$

We also accounted in our model for anomalous heating of ions:

$$\frac{3}{2} n_i \frac{\partial T_i}{\partial t} + n_i \nabla \cdot \left(\mathbf{u}_i T_i \right) + \frac{n_i \mathbf{u}_i}{2} \cdot \nabla T_i = Q_i + \dot{n}_i \left(\varepsilon_i - \frac{3}{2} T_i \right) + Q_{an(i)}$$

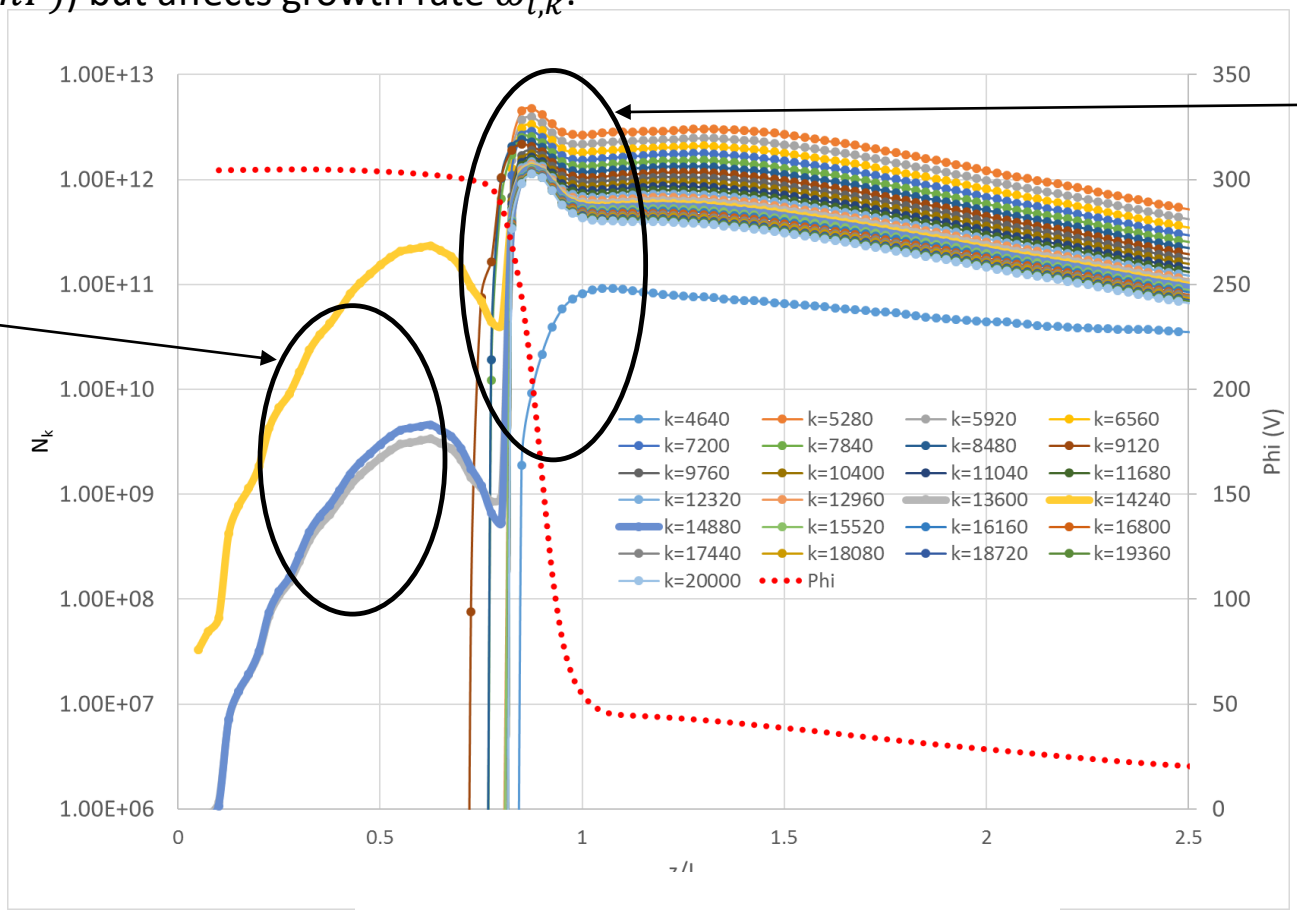
$$Q_{an(i)} = \frac{qn_i}{n_e} \sum_k N_k \sqrt{qT_e/m_i}\,k \left(\frac{\sqrt{\pi/2}\,\omega_{r,k}}{\left(1 + k^2 \lambda_{De}^2\right)^{3/2}} \left(\frac{\mathbf{k} \cdot \mathbf{u}_{ei} - k\sqrt{qT_e/m_i} \Big/ \left(1 + k^2 \lambda_{De}^2\right)^{1/2}}{k\sqrt{qT_e/m_e}} \exp\left(-\frac{1}{2} \left(\frac{\mathbf{k} \cdot \mathbf{u}_{ei} - k\sqrt{qT_e/m_i} \Big/ \left(1 + k^2 \lambda_{De}^2\right)^{1/2}}{k\sqrt{qT_e/m_e}} \right)^2 \right) \right) - \frac{\omega_{i,k}}{\left(1 + k^2 \lambda_{De}^2\right)^{1/2}} \right)$$

Wave action distribution along centerline based on ad-hoc solution background plasma

- First check for self-consistent model: **resistivity computed with ad-hoc solution** as background for computing self-consistent anomalous collision frequency. **Then compare ad-hoc and self-consistent anomalous collision frequencies.**
- **Ion temperature is computed including anomalous heating**. This has negligible effect of momentum ($nE \gg \nabla(nT)$) but affects growth rate $\omega_{i,k}$.

Growth of wave action in region of large electric fields for all wavelengths

Some wavelengths exhibit growth inside channel



Multiple wave-length model do not produce results that agree with measurements

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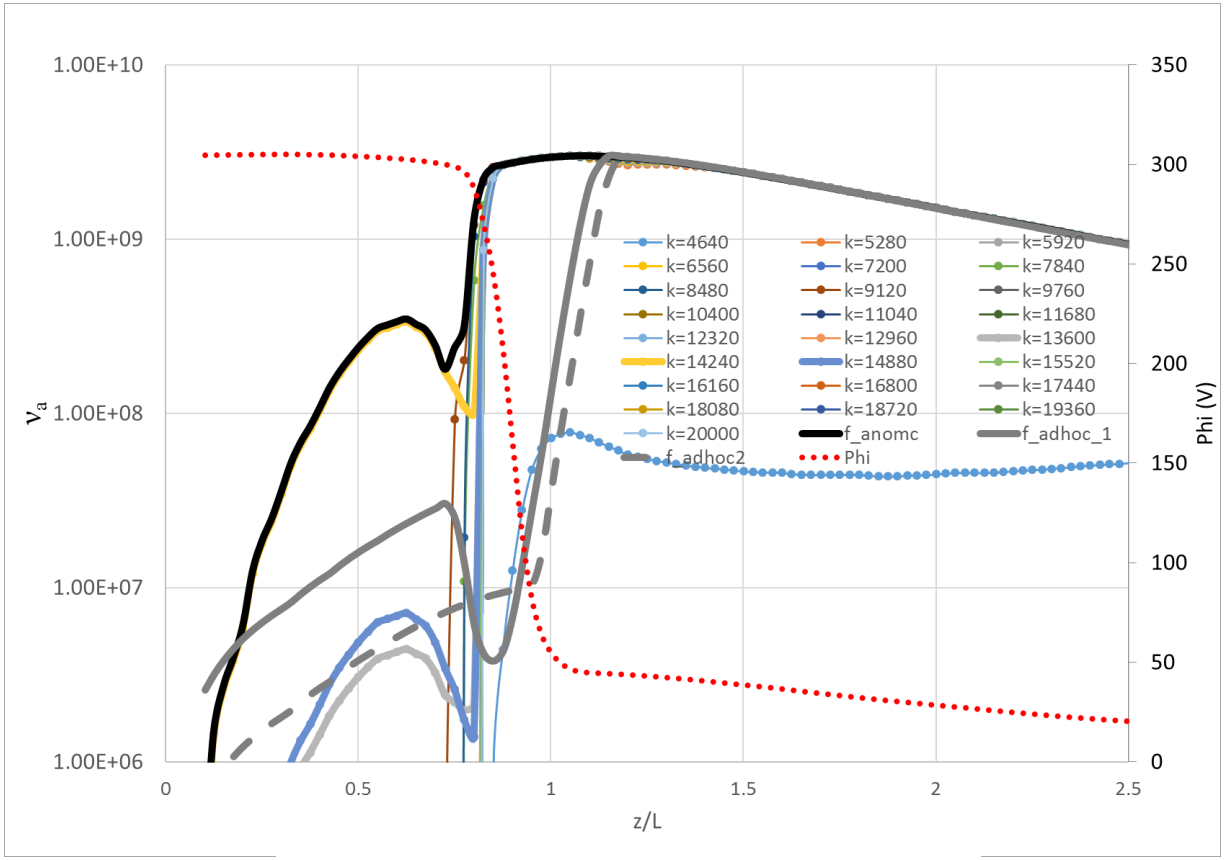
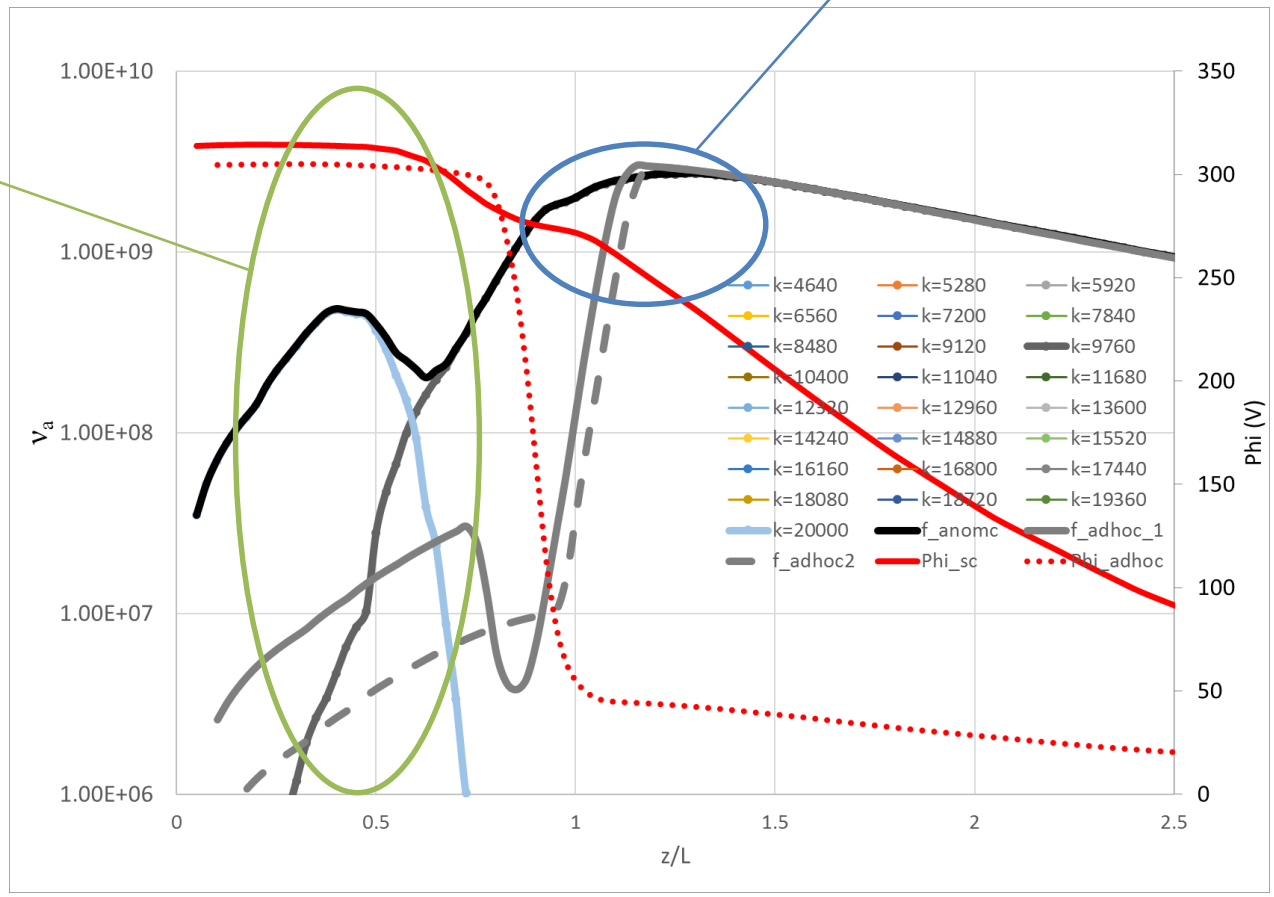


Multiple wave-length model do not produce results that agree with measurements

- Results of the first test suggest that a **fully self-consistent simulation** will produce results that are very different to those of the ad-hoc solution.

Very flat plasma potential

Due to small electric field, only two wavelengths exhibit growth



Total resistivity in the plasma is low, discharge current 57 A vs 20 A of ad-hoc solution

Accepting that there is going to be growth of the instability in the acceleration region

- So far, we relied on trying to decrease the growth rate of the instability in the acceleration region by means of a damping mechanism, such as Landau damping.

- Follow a new approach: what if growth of the instability still occurs, but the electrons "do not feel" the effect of the instability?

- **One basic idea is to compare the energy associated with the ion drift with the energy of the wave perturbations**

$$\phi_{drift} \approx \frac{m_e |\mathbf{E}|^2}{2q|\mathbf{B}|^2} \qquad \phi_k = \sqrt{\frac{kT_e^2 N_k}{n_e m_i \left(1+k^2\lambda_{De}^2\right)^{3/2}}}$$

- We compute an auxiliary variable ξ by means of a wave equation

$$\frac{\partial \xi_k}{\partial t} - K_1 \nabla^2 \xi_k = \frac{\phi_{drift}}{\phi_k}$$

- And assume that when ξ is large, the anomalous collision frequency is not affected by the wave action

$$\nu_{an} = \left(\frac{\pi}{2}\right)^{1/2} \sum_k \boxed{f_k} \frac{qk^2 N_k}{n_e \sqrt{m_e m_i}\left(1+k^2\lambda_{De}^2\right)^{3/2}} \exp\left(-\left(\frac{\hat{k}\cdot\mathbf{u}_{ei} - c_s \big/ \left(1+k^2\lambda_{De}^2\right)^{1/2}}{v_{te}}\right)^2\right)$$

$$f_k = \exp\left(-K_2 \xi_k\right)$$

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Setting a limit to the drift velocity

- Simulations showed that the anomalous collision frequency can become almost zero with this model as a consequence of the plasma potential gradient becoming very steep.

- To avoid this circumstance, we also included a simple model for computing the floor value of the anomalous collision frequency.

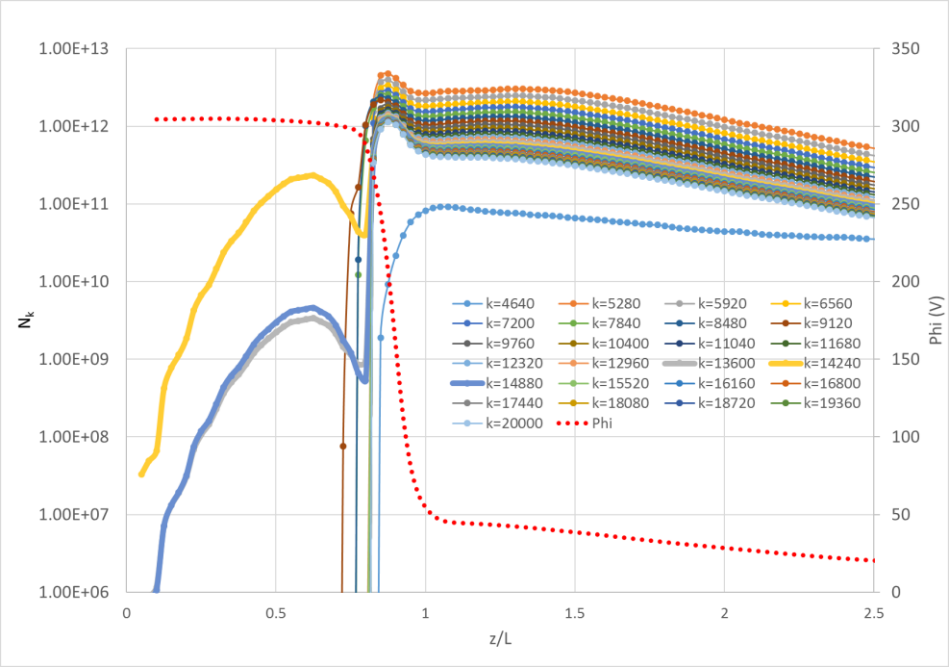
- The model relies on the fact that the Mach number for electrons cannot exceed 1, as other instabilities characterized by shorter time-scales (i.e., two-stream instability for electrons) may occur.

$$\sqrt{\frac{qT_e}{m_e}} \geq \frac{j_{e\wedge}}{qn_e} = \Omega_e \frac{j_{e\perp}}{qn_e} = \frac{qB}{m_e\left(\nu_{ei} + \nu_{en} + \nu_a\right)} \frac{j_{e\perp}}{qn_e}$$

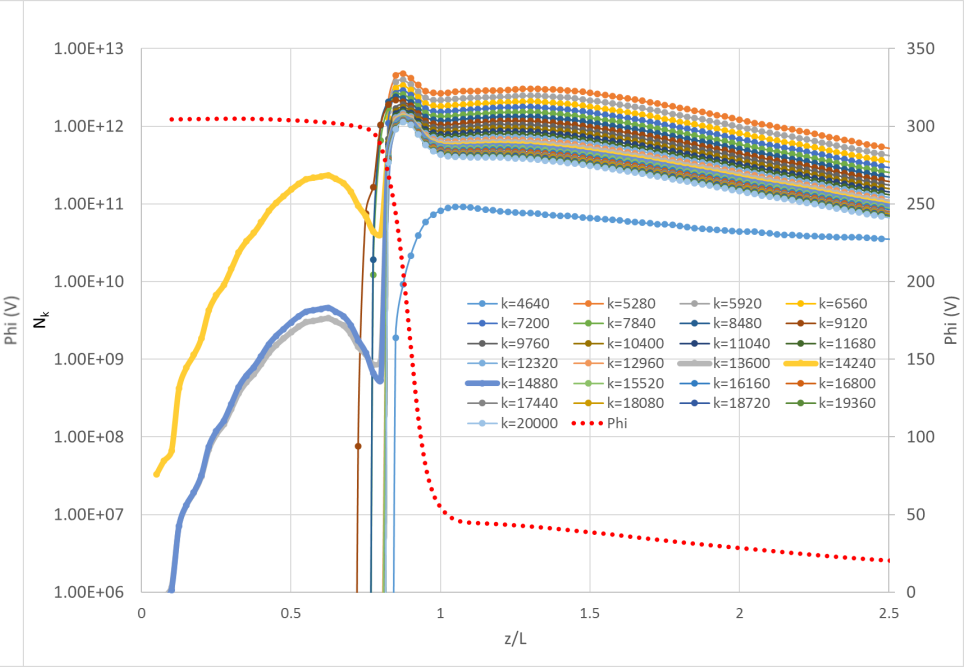
$$\nu_{a,\,floor} = \frac{B}{m_e n_e} \frac{j_{e\perp}}{\sqrt{qT_e / m_e}} - \nu_{ei} - \nu_{en}$$

Wave action distribution along centerline with new model is unmodified (as expected)

- First check for self-consistent model: **resistivity computed with ad-hoc solution** as background for computing self-consistent anomalous collision frequency. **Then compare ad-hoc and self-consistent anomalous collision frequencies.**



Model with correction factor in anomalous collision frequency

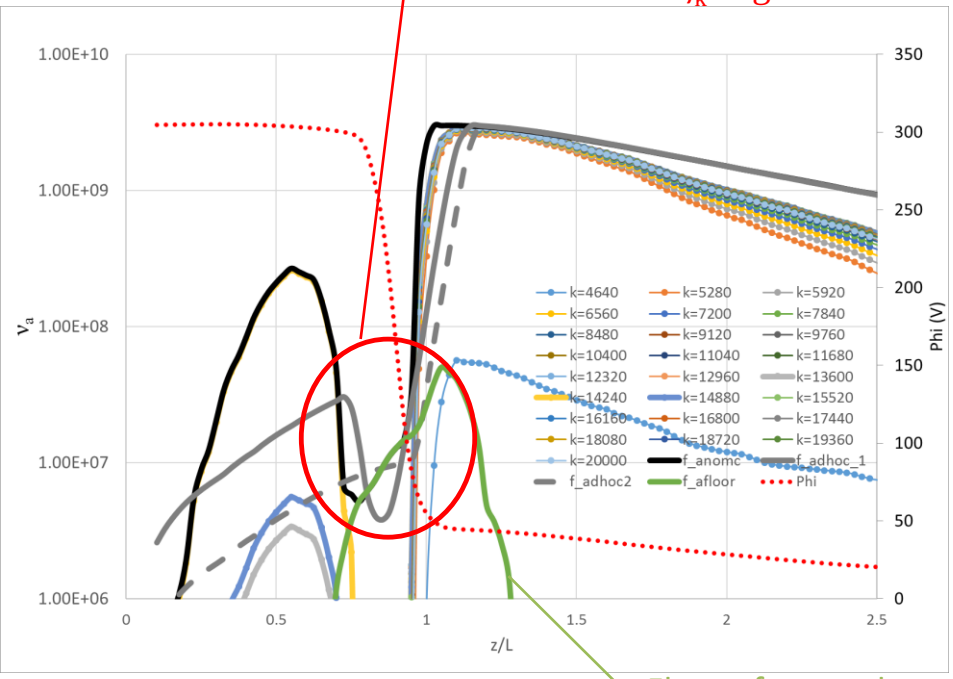


Model with no correction factor in anomalous collision frequency

The effect of the correction factor is noticeable when computing the anomalous collision frequency

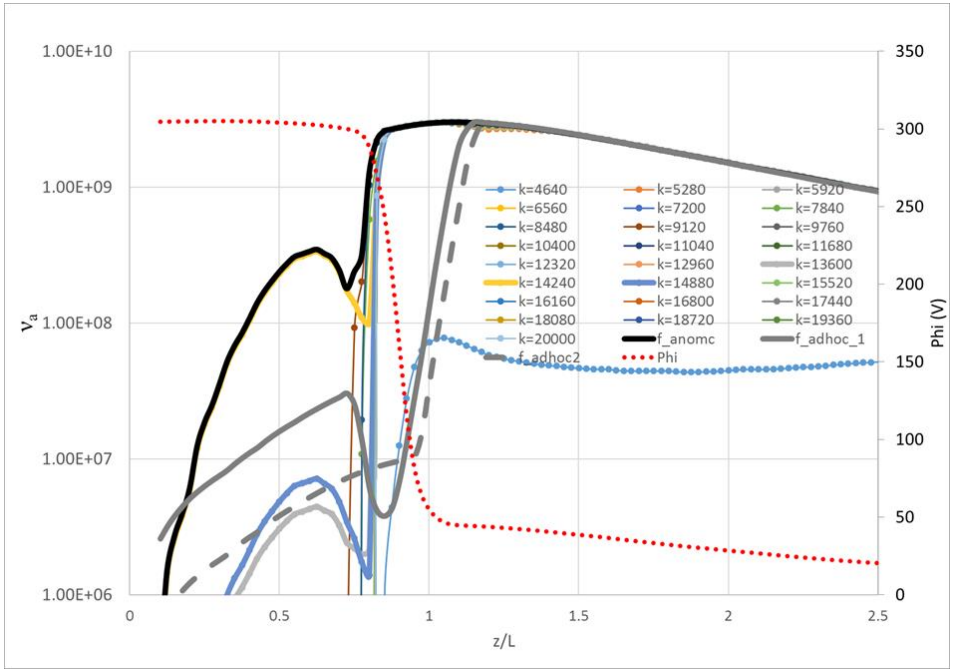
- First check for self-consistent model: **resistivity computed with ad-hoc solution** as background for computing self-consistent anomalous collision frequency. **Then compare ad-hoc and self-consistent anomalous collision frequencies.**

Anomalous collision frequency decreases when ξ_k large



Floor of anomalous collision frequency

Model with correction factor in anomalous collision frequency



Model with no correction factor in anomalous collision frequency

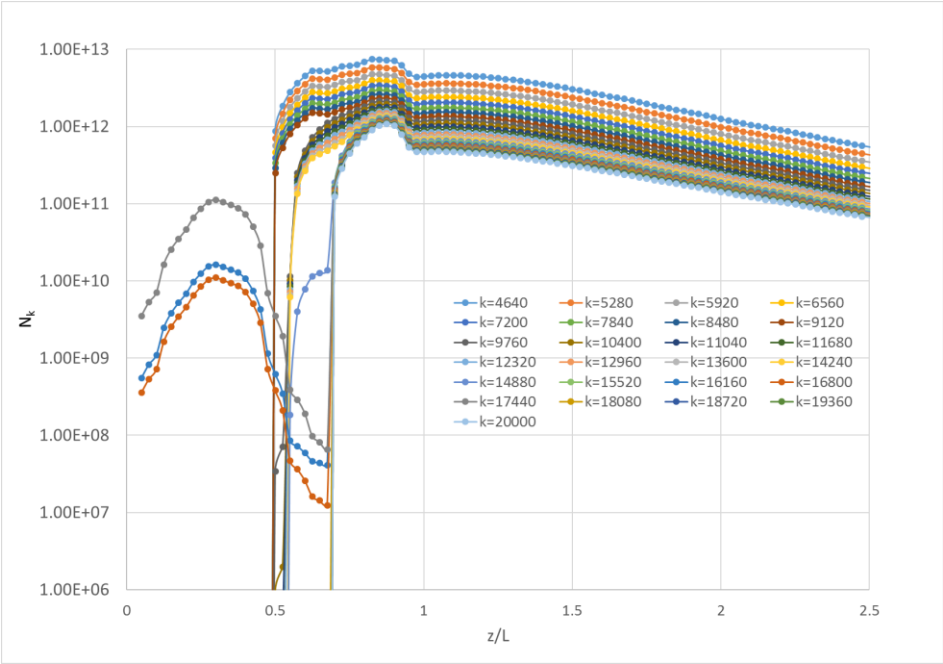
Full self-consistent simulation results exhibit excellent agreement with ad-hoc solution

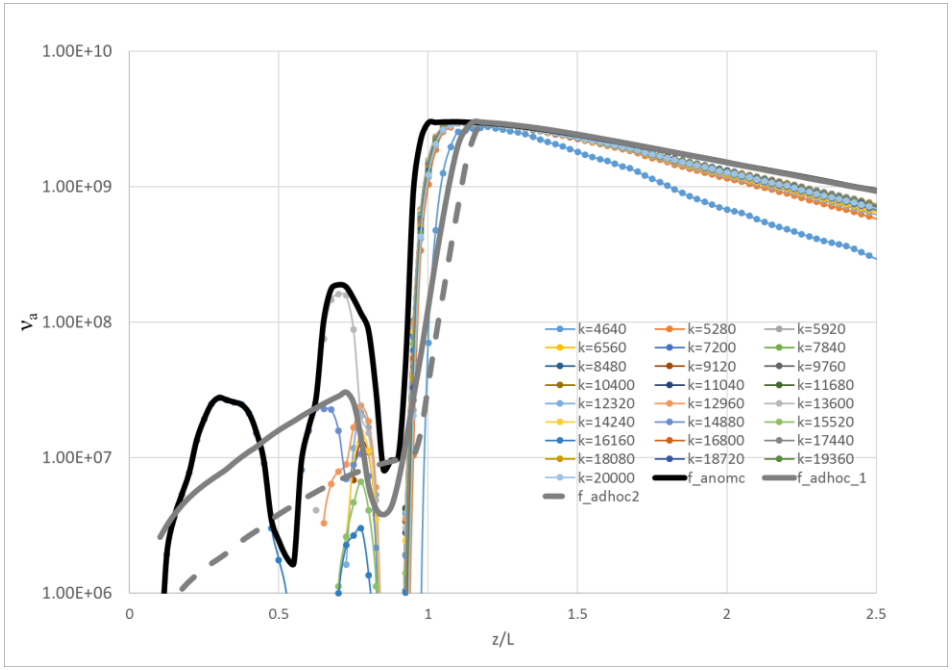
- As the comparison between the self-consistent and the ad-hoc profiles of the anomalous collision frequency (when using the ad-hoc solution as background) has been greatly improved, we proceed to run a full self-consistent simulation.

$$\frac{\partial \xi_k}{\partial t} - K_1 \nabla^2 \xi_k = \frac{\phi_{drift}}{\phi_k} \qquad \nu_{an} = \left(\frac{\pi}{2}\right)^{1/2} \sum_k f_k \frac{q k^2 N_k}{n_e \sqrt{m_e m_i}\left(1 + k^2 \lambda_{De}^2\right)^{3/2}} \exp\left(-\left(\frac{\hat{k} \cdot \mathbf{u}_{ei} - c_s \big/ \left(1 + k^2 \lambda_{De}^2\right)^{1/2}}{v_{te}}\right)^2\right) \qquad f_k = \exp\left(-K_2 \xi_k\right)$$

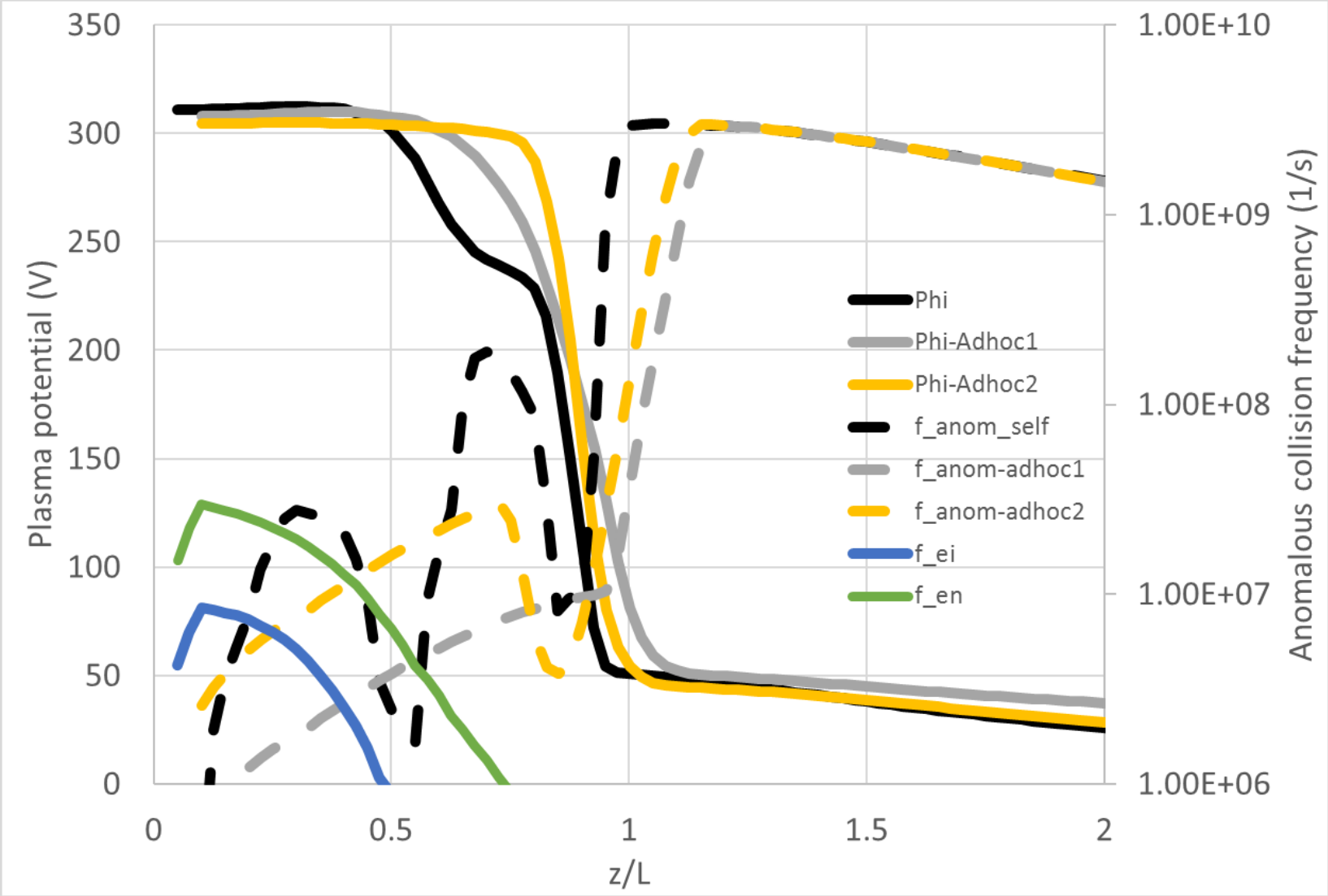
K_1 = fixed

K_2 = iterative procedure to achieve discharge current of 20 A

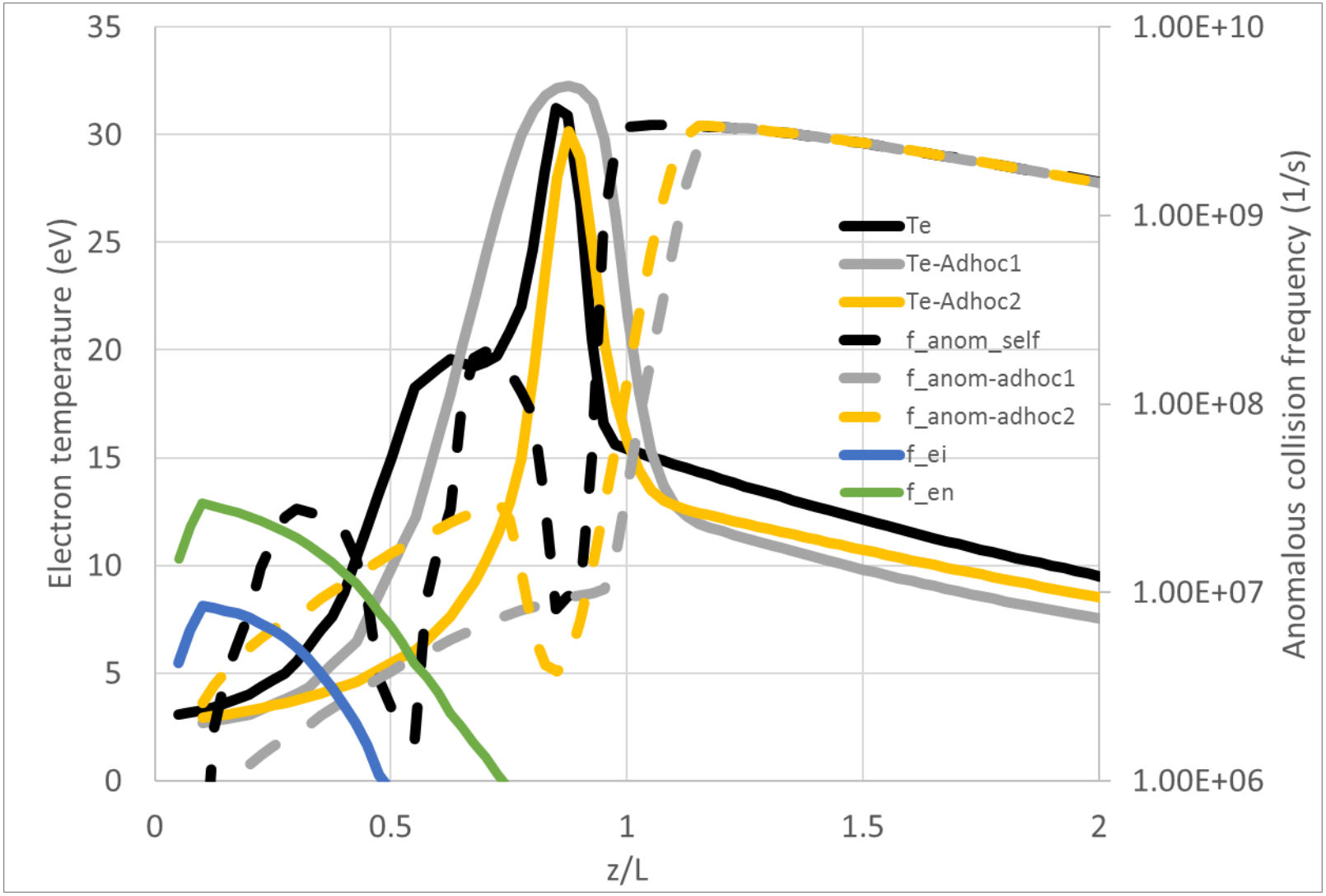




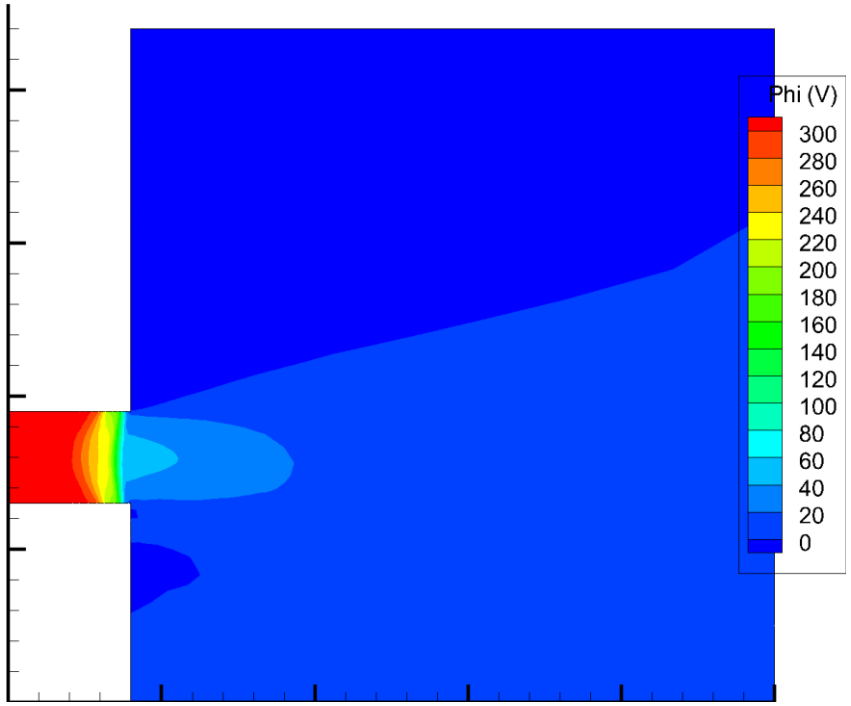
Electron temperature and anomalous collision frequency comparison



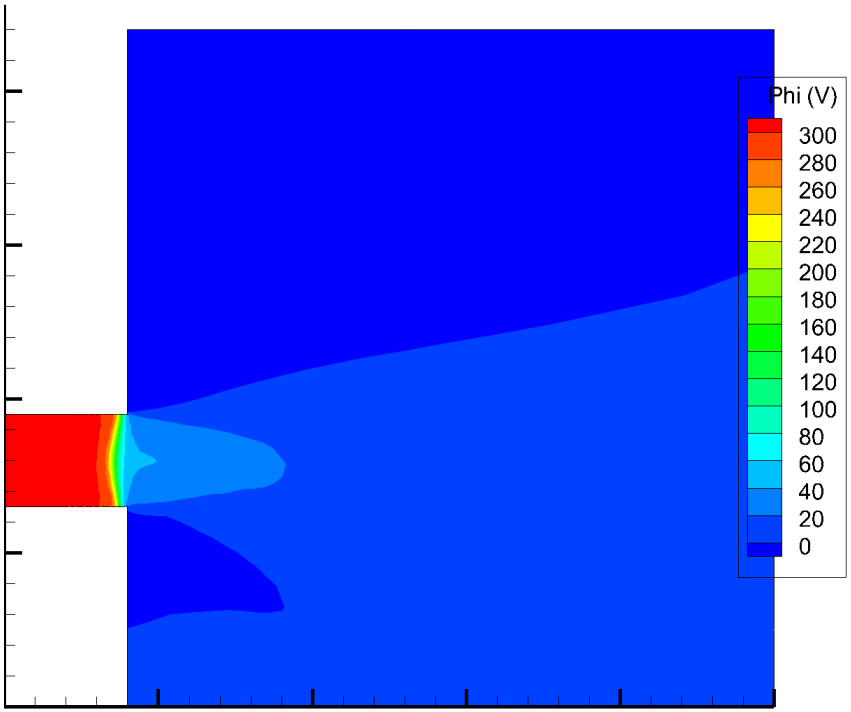
Plasma potential and anomalous collision frequency comparison



2-D Plasma potential comparison

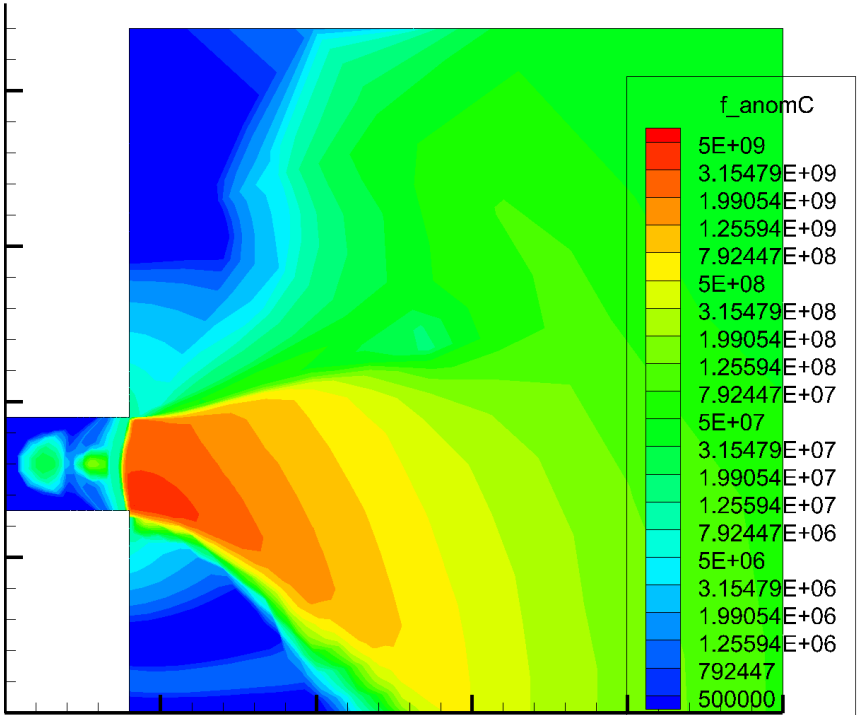


Self-consistent model

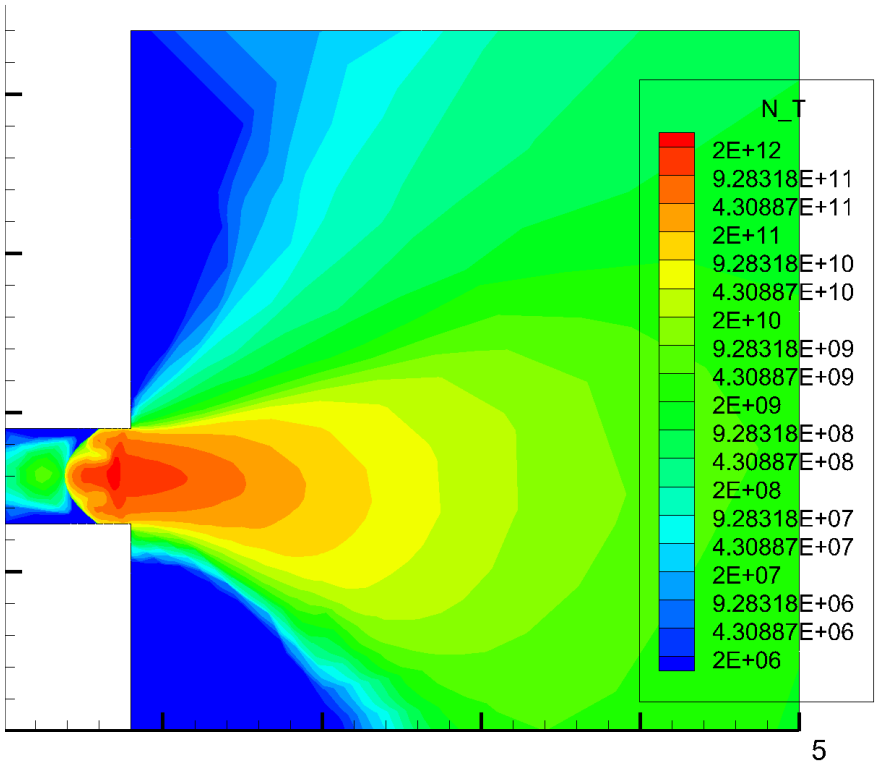


Ad-hoc model

2-D wave energy density and anomalous collision frequency



Anomalous collision frequency



Total wave action

Concluding remarks

- By examination of the ad-hoc solution, we came to the conclusion that its associated anomalous collision frequency profile was not achievable when employing a wave energy equation with growth terms dictated by liner theory of ion acoustic waves.

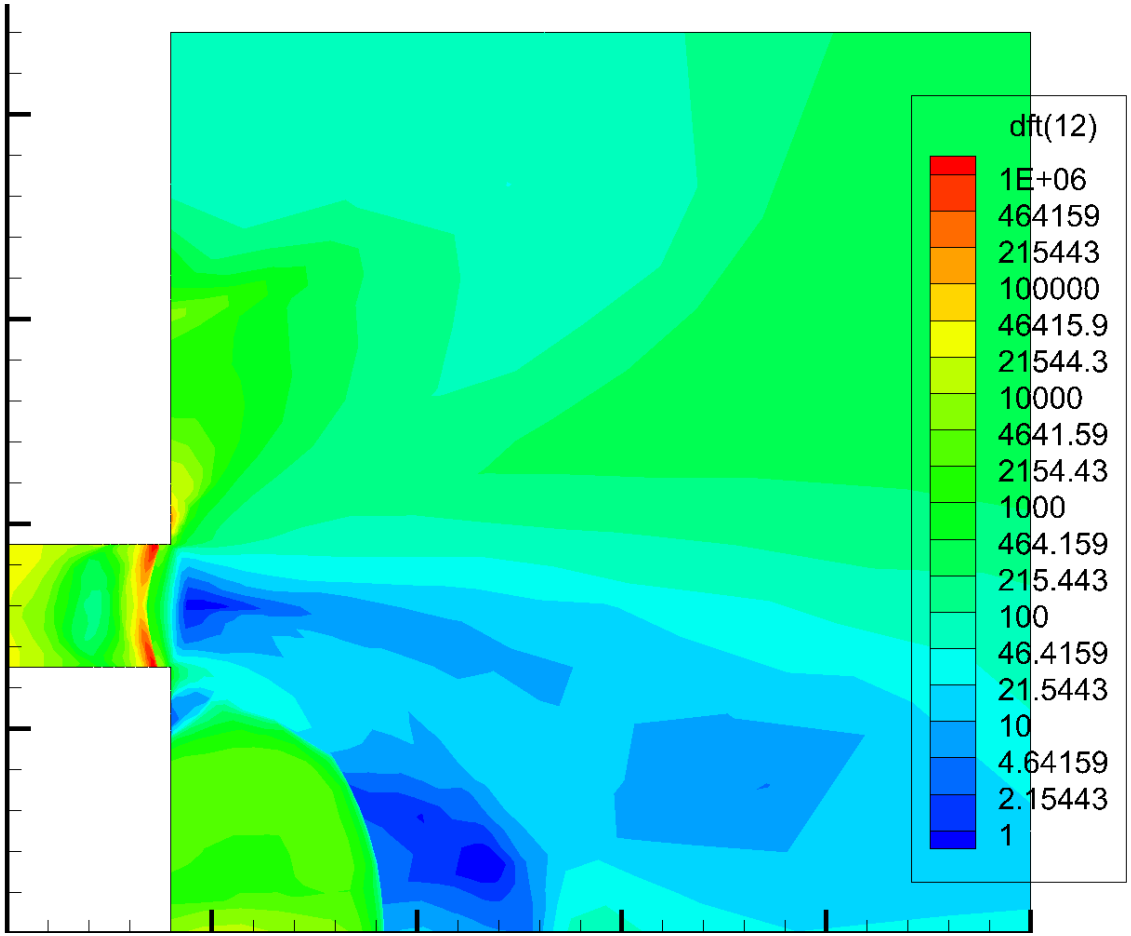
- The major difficulty was to reconcile the fact that the anomalous collision frequency value is minimum when the electron drift is maximum (and thus the growth rate of the instability is maximum).

- We attempted to decrease the growth rate in the acceleration region by separating the contributions of multiple wave-lengths to the wave action and also by including the additional anomalous ion heating. This approach was proven unsuccessful as the growth rate in the acceleration region is typically much larger than Landau damping.

- We finally focused on the hypothesis that the electron transport may not be affected by ion-acoustic waves in the acceleration region. We proposed to use a simple comparison between the drift energy of the electrons and the energy of the waves to quantify this phenomenon. We also limited the floor value of the anomalous collision frequency so the Mach number for electrons never exceeds 1.

- The anomalous collision frequency profile obtained with this model exhibited good agreement with the ad-hoc profile when using the ad-hoc plasma solution as background. Full self-consistent simulations also predicted correctly the location of the acceleration region.

- Based on the promising results of this investigation, we believe that the path forward involves a detailed study of the physics of the acceleration region and whether the response of the electrons to the presence of waves can be quantified with the anomalous collision frequency expression that was derived from quasi-linear theory.

BACK-UP CHARTS

Example of ξ_k (for k=11680 m^{-1})