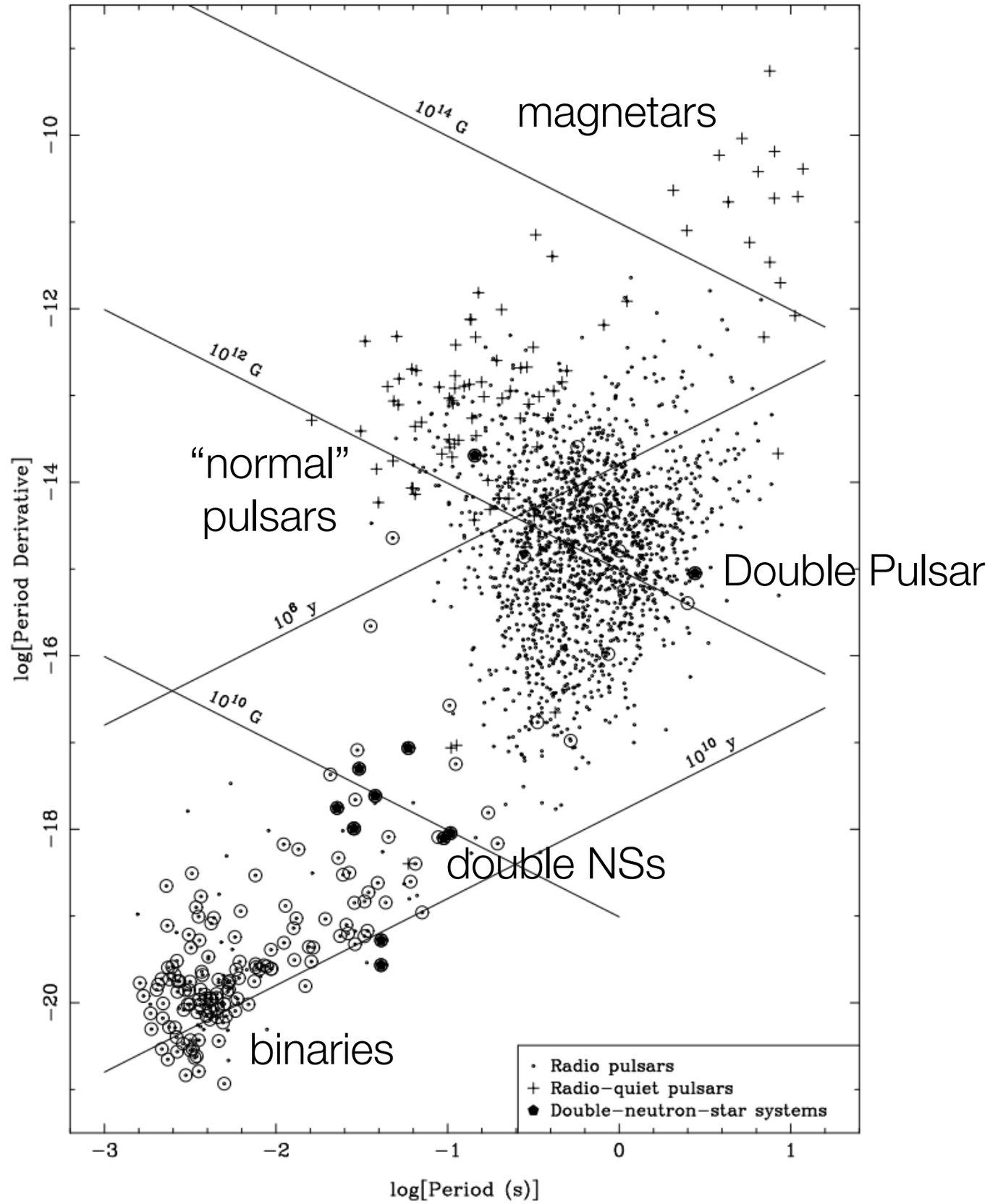
Prospects for testing GR with pulsar-timing gravitational-wave detection

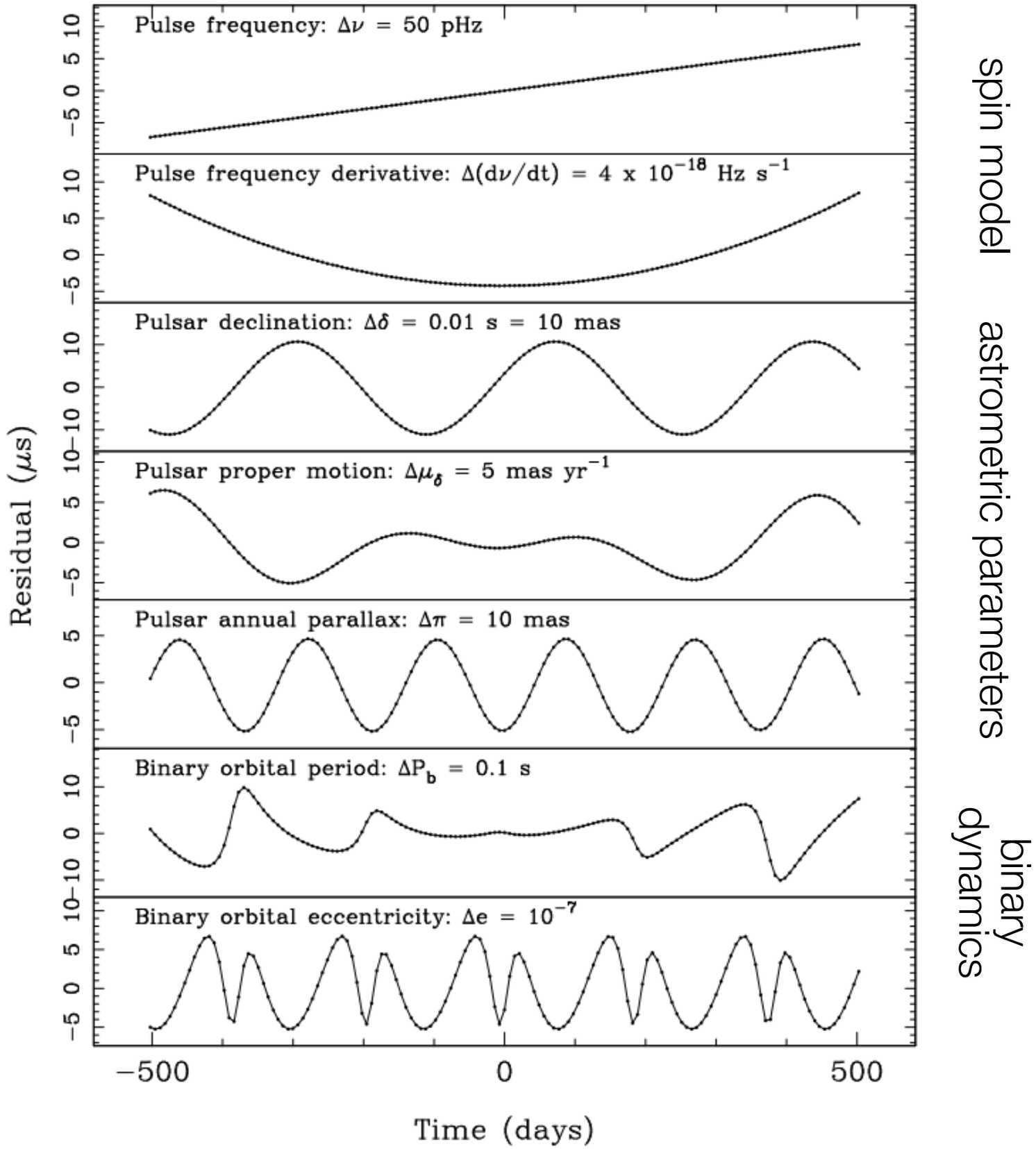
Michele Vallisneri

Jet Propulsion Laboratory
California Institute of Technology



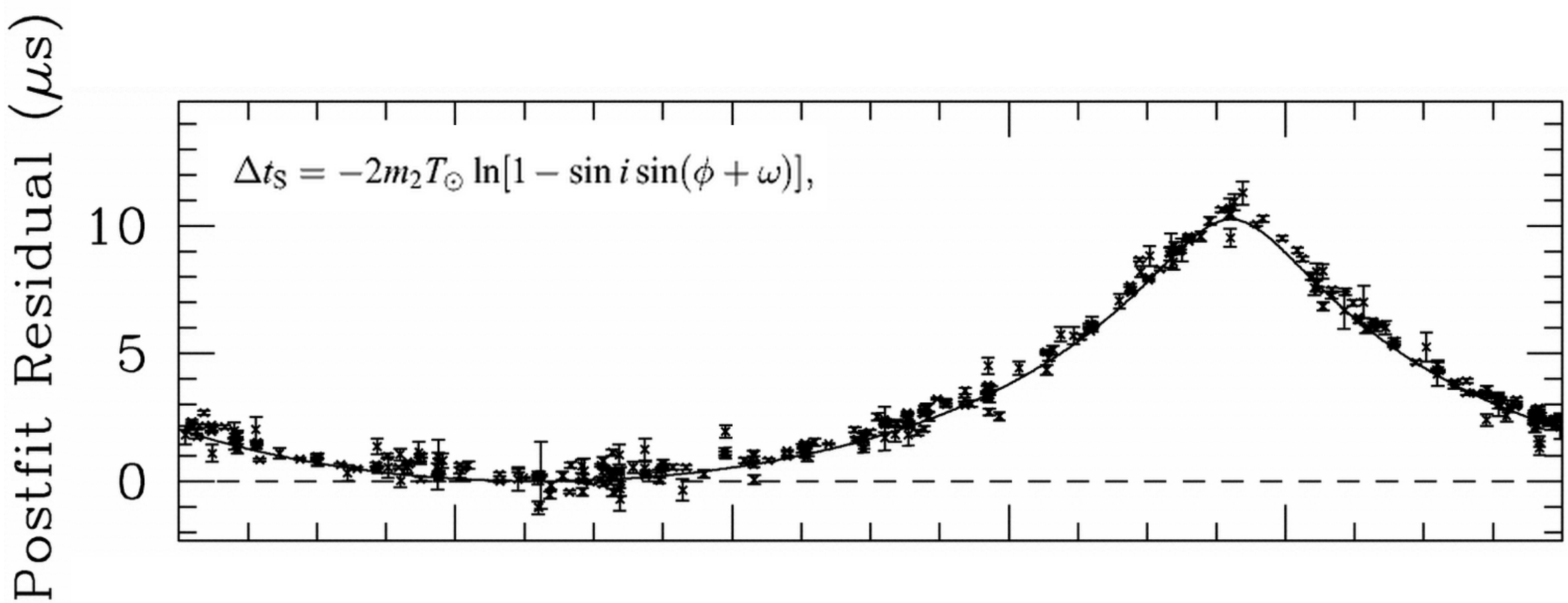
pulsars: Nature's precision clocks

[Manchester 2015]



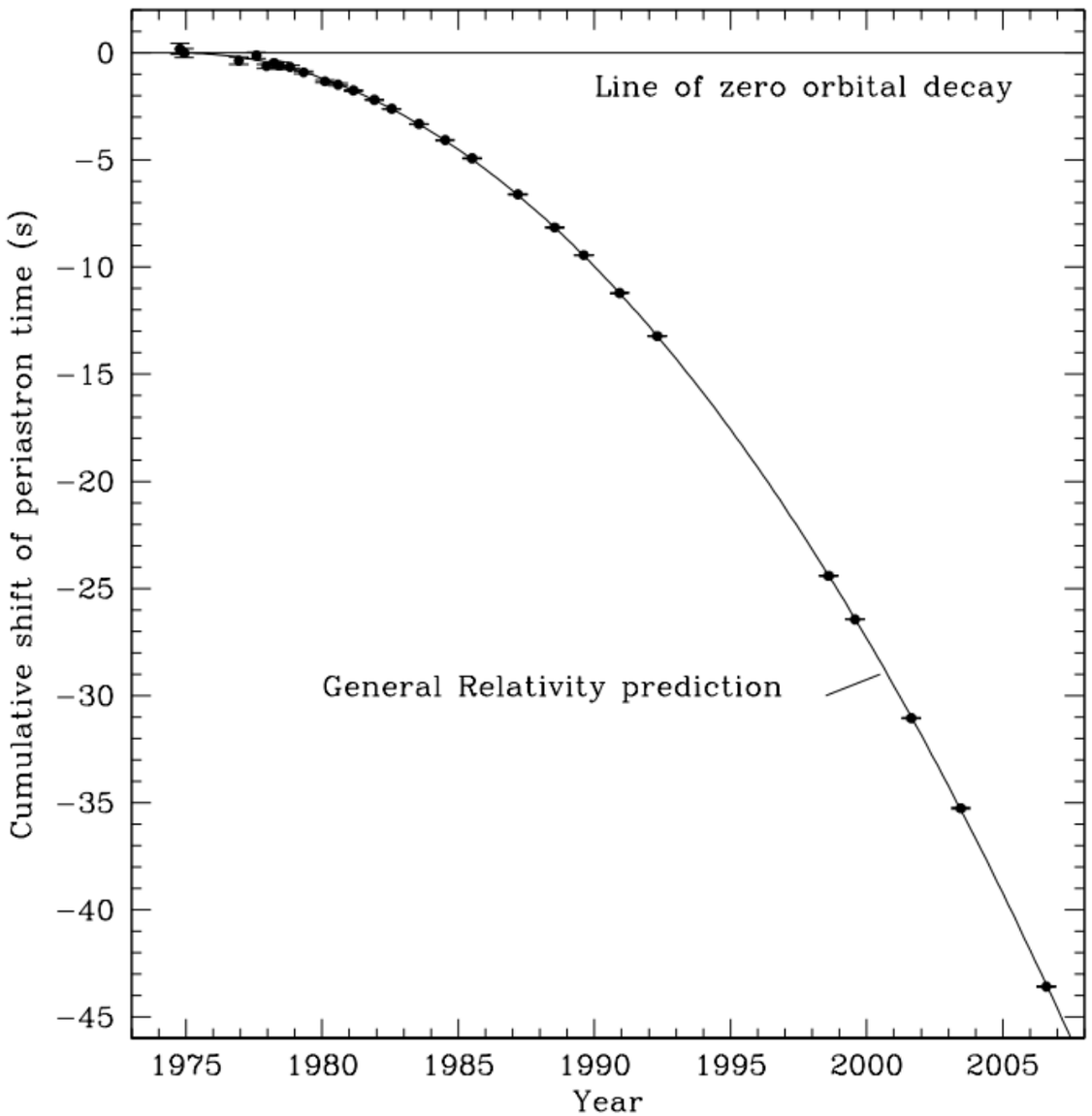
Deterministic effects in timing residuals

f = 300 Hz [Manchester 2015]



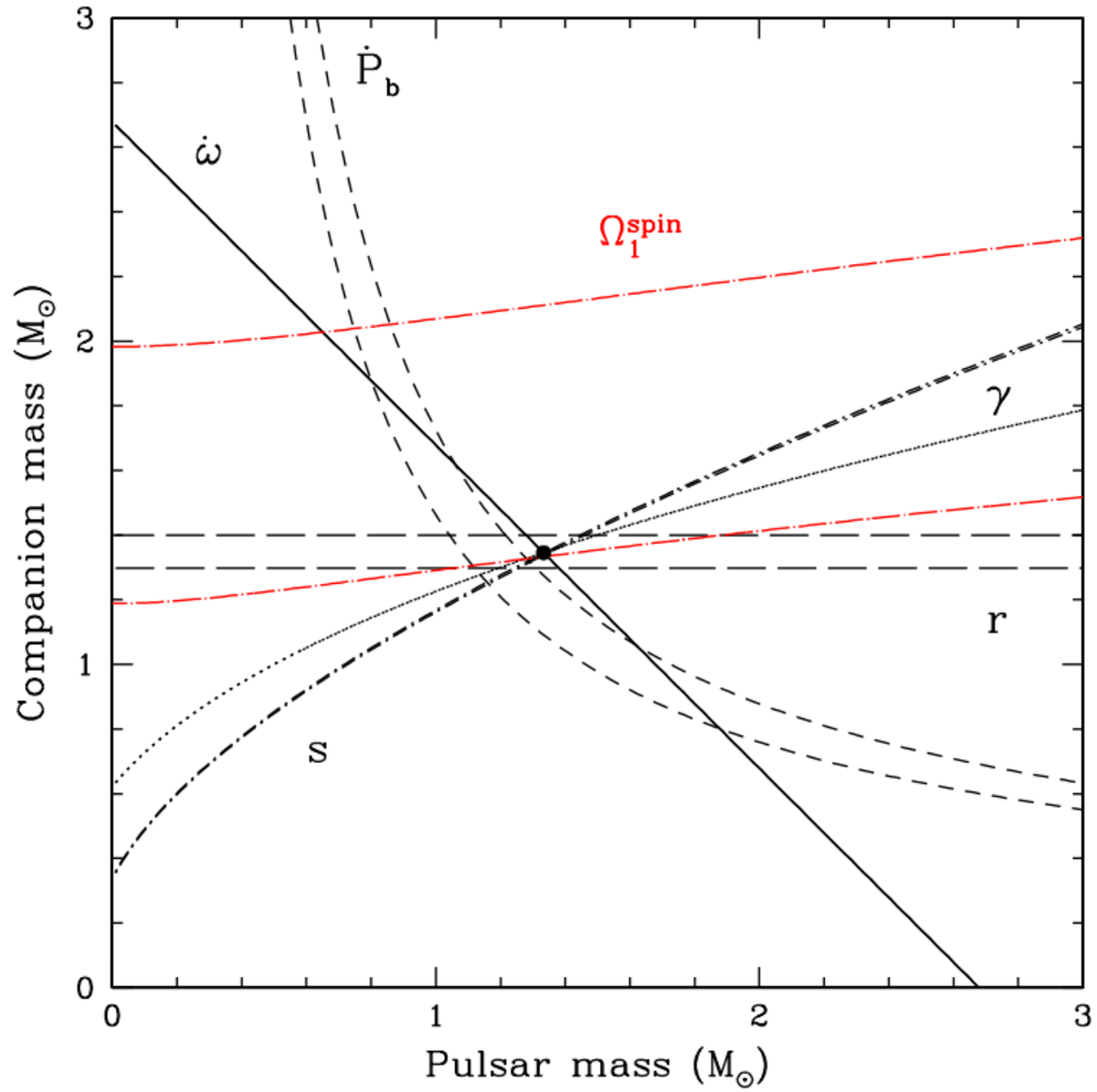
Shapiro delay in PSR J1713

as a function of orbital phase [Splaver et al. 2005]



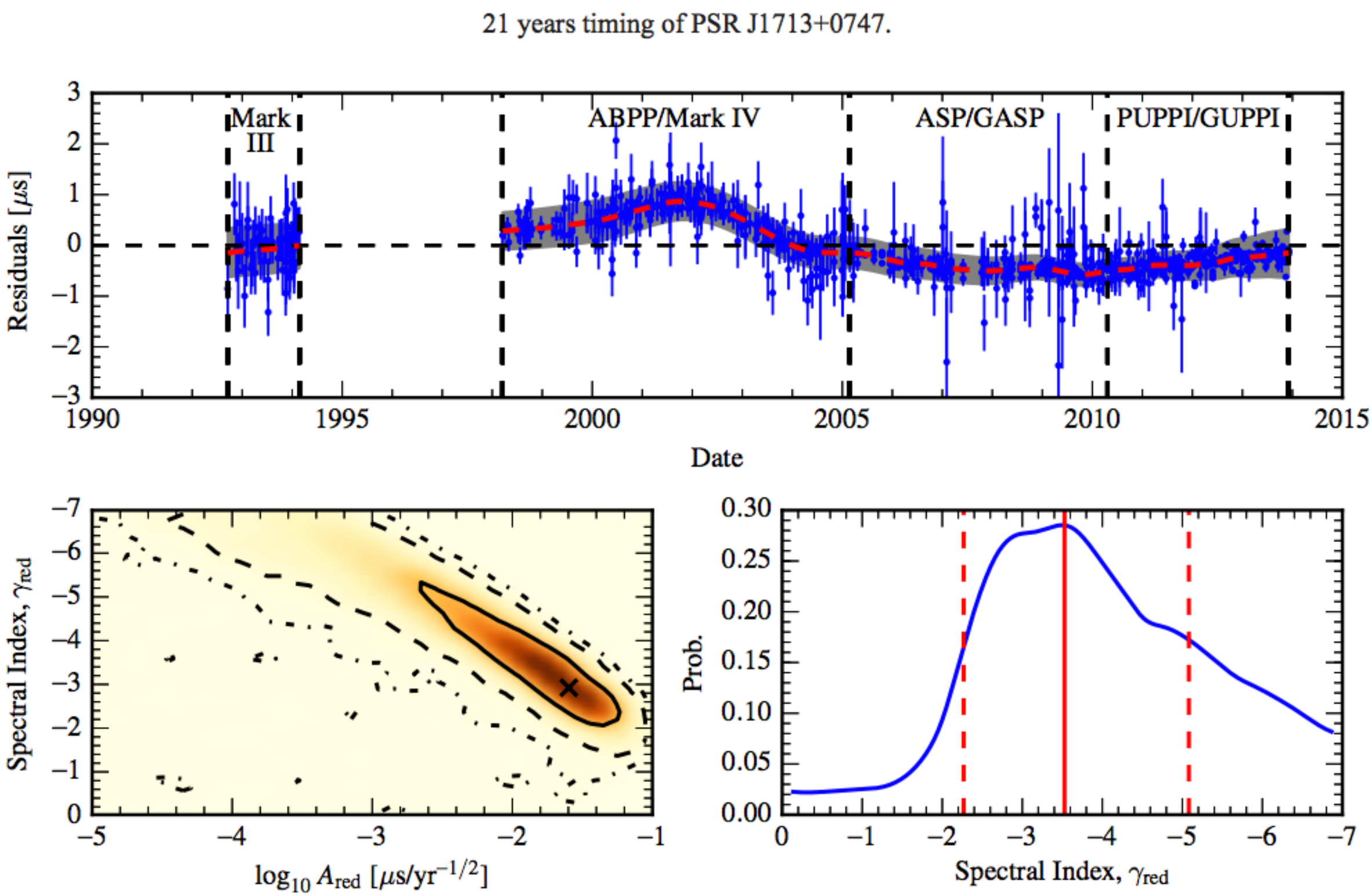
Orbital period variation in B1913+16

[Weisberg et al. 2010]



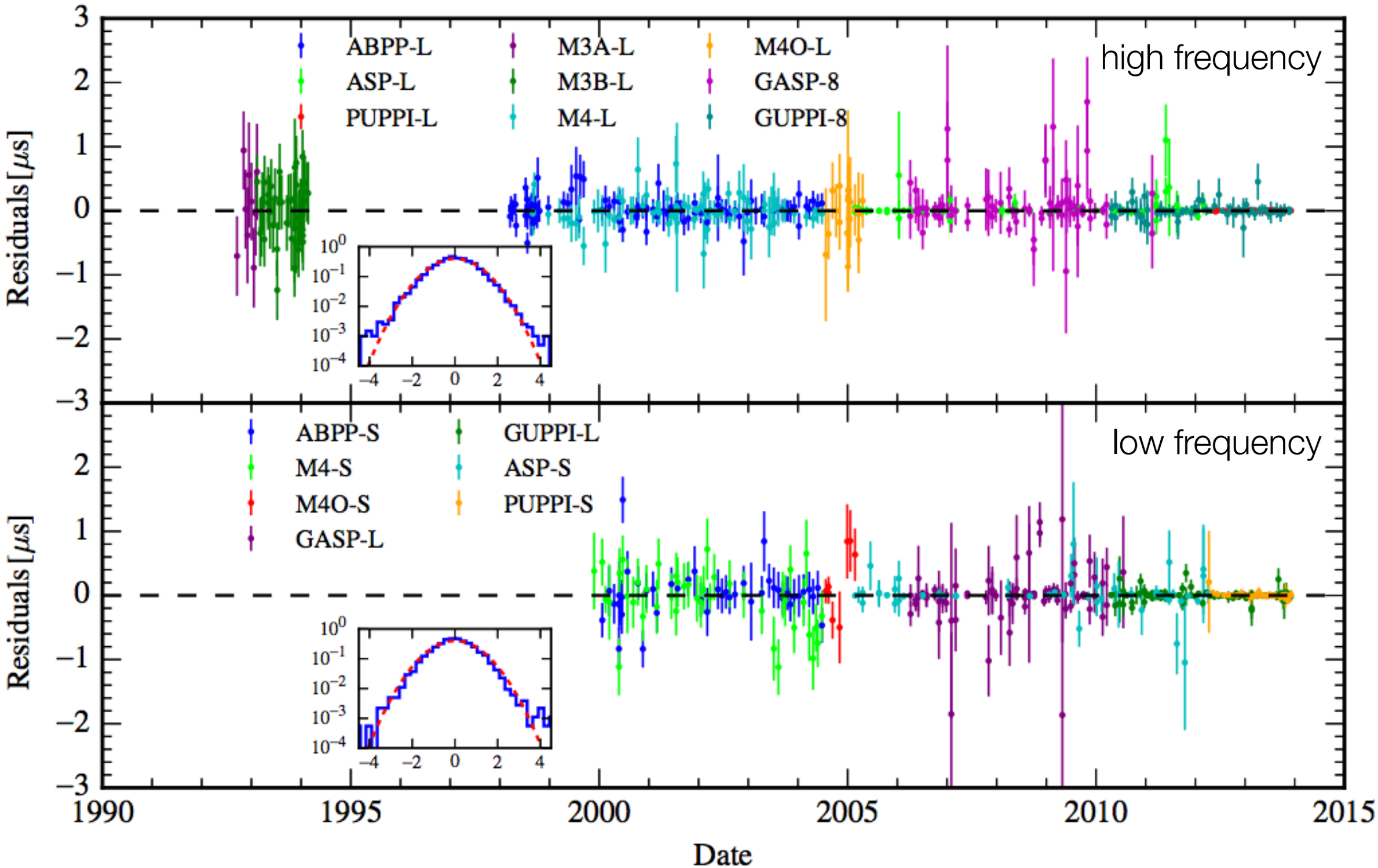
Tests of GR with B1534

[Fonseca et al. 2014]



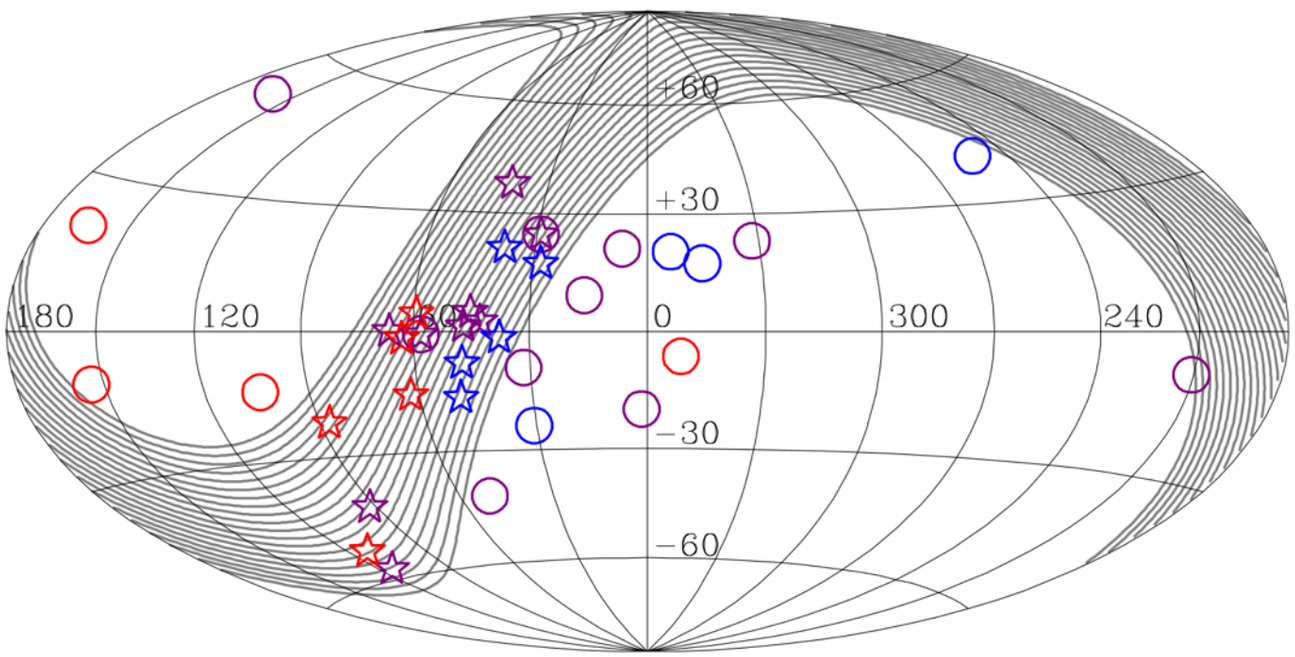
Red noise in PSR J1713

ML red-noise realization [Zhu et al. 2015]



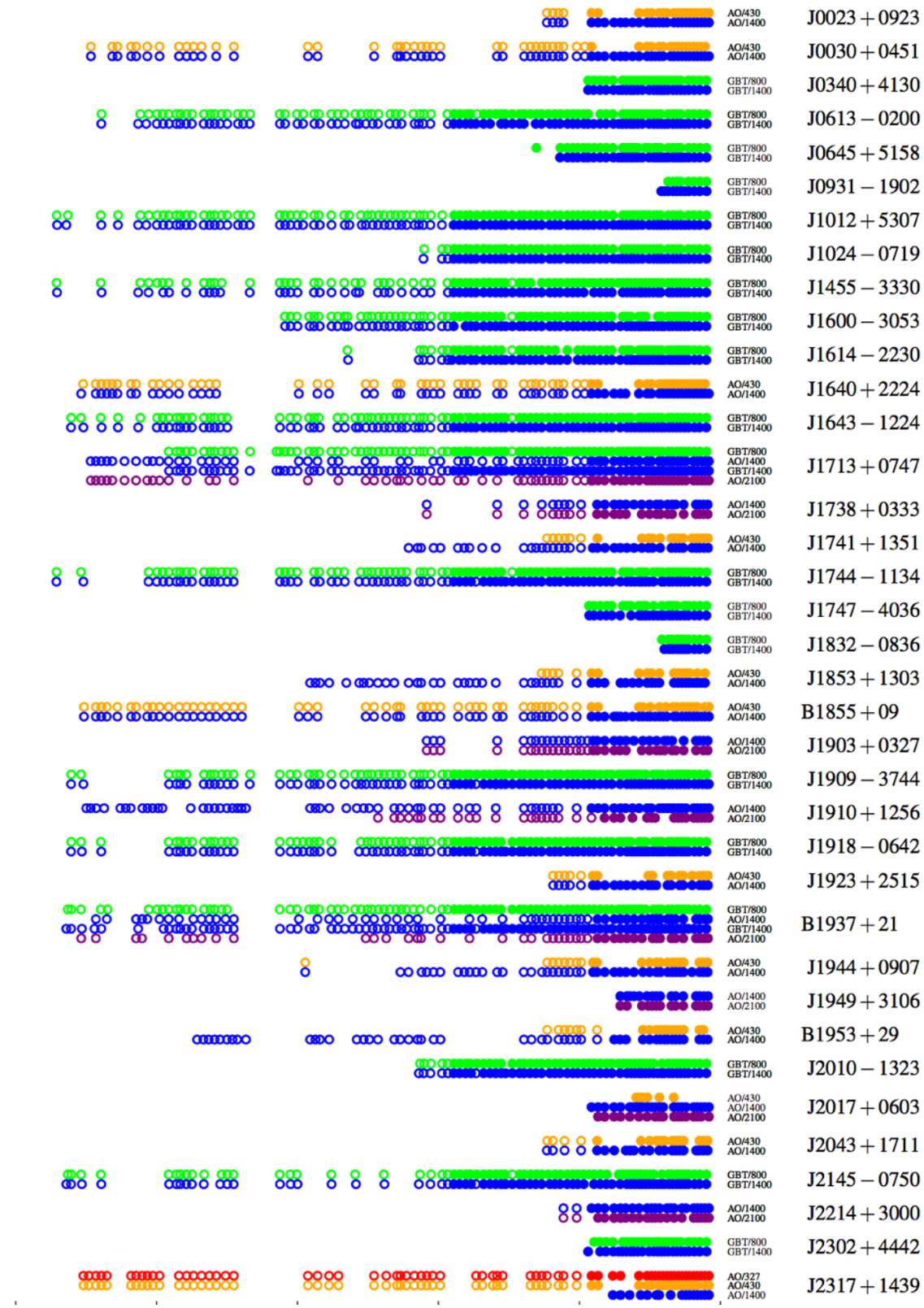
J1713 residuals

daily averages after subtracting ML jitter and red noise [Zhu et al. 2015]



The NANOGrav pulsars

[McLaughlin 2013]



The NANOGrav 9-year dataset

[NANOGrav 2015]

Testing GR: the standard hierarchy of theories of gravitation

WEP Newton's equivalence principle

10^{-13} $m_I = m_G$

EEP Einstein's equivalence principle

= WEP + local Lorentz invariance 10^{-22}

+ local position invariance 10^{-5}

metric theories (what fields?)

SEP EEP, but also for gravitational

10^{-4} experiments

Living Rev. Relativity, **9**, (2006), 3
http://www.livingreviews.org/lrr-2006-3
(Update of lrr-2001-4)

The Confrontation between General Relativity and Experiment

Clifford M. Will
McDonnell Center for the Space Sciences
Department of Physics
Washington University
St. Louis, MO 63130, U.S.A.
email: cmw@wuphys.wustl.edu
http://wugrav.wustl.edu/people/CMW/

Accepted on 27 February 2006
Published on 27 March 2006
Living Reviews in Relativity
Published by the
Max Planck Institute for Gravitational Physics
(Albert Einstein Institute)
Am Mühlenberg 1, 14424 Golm, Germany
ISSN 1433-8351

Abstract

The status of experimental tests of general relativity and of theoretical frameworks for analyzing them is reviewed. Einstein's equivalence principle (EEP) is well supported by experiments such as the Eötvös experiment, tests of special relativity, and the gravitational redshift experiment. Ongoing tests of EEP and of the inverse square law are searching for new interactions arising from unification or quantum gravity. Tests of general relativity at the post-Newtonian level have reached high precision, including the light deflection, the Shapiro time delay, the perihelion advance of Mercury, and the Nordtvedt effect in lunar motion. Gravitational wave damping has been detected in an amount that agrees with general relativity to better than half a percent using the Hulse–Taylor binary pulsar, and other binary pulsar systems have yielded other tests, especially of strong-field effects. When direct observation of gravitational radiation from astrophysical sources begins, new tests of general relativity will be possible.

© Max Planck Society and the authors.
Further information on copyright is given at
http://relativity.livingreviews.org/About/copyright.html
For permission to reproduce the article please contact livrev@aei.mpg.de.

1981 → 2006

Dicke: test of EPs + PPN tests of metric theories

the PPN formalism: metric and potentials

$$
\begin{aligned}
g_{00} = & -1 + 2U - 2\beta U^2 - 2\xi\Phi_W + (2\gamma + 2 + \alpha_3 + \zeta_1 - 2\xi)\Phi_1 + 2(3\gamma - 2\beta + 1 + \zeta_2 + \xi)\Phi_2 \\
& + 2(1 + \zeta_3)\Phi_3 + 2(3\gamma + 3\zeta_4 - 2\xi)\Phi_4 - (\zeta_1 - 2\xi)\mathcal{A} - (\alpha_1 - \alpha_2 - \alpha_3)w^2 U - \alpha_2 w^i w^j U_{ij} \\
& + (2\alpha_3 - \alpha_1)w^i V_i + \mathcal{O}(\epsilon^3), \\
g_{0i} = & -\frac{1}{2}(4\gamma + 3 + \alpha_1 - \alpha_2 + \zeta_1 - 2\xi)V_i - \frac{1}{2}(1 + \alpha_2 - \zeta_1 + 2\xi)W_i - \frac{1}{2}(\alpha_1 - 2\alpha_2)w^i U \\
& - \alpha_2 w^j U_{ij} + \mathcal{O}(\epsilon^{5/2}), \\
g_{ij} = & (1 + 2\gamma U)\delta_{ij} + \mathcal{O}(\epsilon^2).
\end{aligned}
$$

$$
U = \int \frac{\rho'}{|\mathbf{x} - \mathbf{x}'|}\, d^3x',
$$

$$
U_{ij} = \int \frac{\rho'(x - x')_i (x - x')_j}{|\mathbf{x} - \mathbf{x}'|^3}\, d^3x',
$$

$$
\Phi_W = \int \frac{\rho'\rho''(\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3} \cdot \left(\frac{\mathbf{x}' - \mathbf{x}''}{|\mathbf{x} - \mathbf{x}''|} - \frac{\mathbf{x} - \mathbf{x}''}{|\mathbf{x}' - \mathbf{x}''|} \right) d^3x'\, d^3x'',
$$

$$
\mathcal{A} = \int \frac{\rho'[\mathbf{v}' \cdot (\mathbf{x} - \mathbf{x}')]^2}{|\mathbf{x} - \mathbf{x}'|^3}\, d^3x',
$$

$$
\Phi_1 = \int \frac{\rho' v'^2}{|\mathbf{x} - \mathbf{x}'|}\, d^3x',
$$

$$
\Phi_2 = \int \frac{\rho' U'}{|\mathbf{x} - \mathbf{x}'|}\, d^3x',
$$

$$
\Phi_3 = \int \frac{\rho' \Pi'}{|\mathbf{x} - \mathbf{x}'|}\, d^3x',
$$

$$
\Phi_4 = \int \frac{p'}{|\mathbf{x} - \mathbf{x}'|}\, d^3x',
$$

$$
V_i = \int \frac{\rho' v'_i}{|\mathbf{x} - \mathbf{x}'|}\, d^3x',
$$

$$
W_i = \int \frac{\rho'[\mathbf{v}' \cdot (\mathbf{x} - \mathbf{x}')](x - x')_i}{|\mathbf{x} - \mathbf{x}'|^3}\, d^3x'.
$$

the PPN formalism: parameters

Parameter	What it measures relative to GR	Value in GR	Value in semi-conservative theories	Value in fully conservative theories	
γ	How much space-curvature produced by unit rest mass?	1	γ	γ	10^{-5}
β	How much "nonlinearity" in the superposition law for gravity?	1	β	β	10^{-4}
ξ	Preferred-location effects?	0	ξ	ξ	
α_1	Preferred-frame effects?	0	α_1	0	
α_2		0	α_2	0	
α_3		0	0	0	
α_3	Violation of conservation of total momentum?	0	0	0	
ζ_1		0	0	0	
ζ_2		0	0	0	
ζ_3		0	0	0	
ζ_4		0	0	0	

Gravitational radiation...

...is predicted in virtually any metric theory of gravity that embodies Lorentz invariance, but it may differ from GR in:

polarizations

speed of waves

radiation reaction

(tested at low v
with binary pulsars)

Unfortunately, no simple, principled framework like PPN exists for describing radiative systems or systems containing strong internal fields.

So we must consider individual alternative theories, or perform null tests of consistency.

Naïve and sentimental tests of GR consistency

In order of difficulty and un-likelihood:

- If we divide the waveform in segments, do individual SNRs pass a χ^2 test?
- Is there a coherent residual?
- What about the source parameters determined from each segment—are they consistent (within estimated errors) with the parameters determined from the entire waveform?
- Is the shape of the likelihood surface consistent with what's expected for this waveform family?

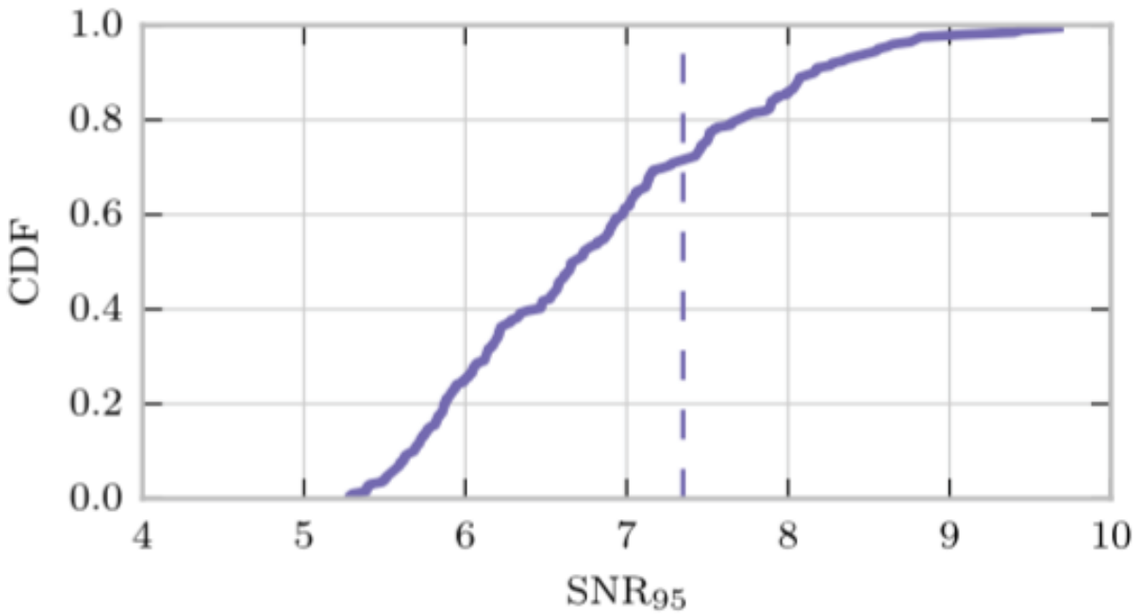
- But before we suspect general relativity: instrument systematics, modeling, data analysis, physical environments…

$$\left\langle \frac{\partial h}{\partial \lambda_{\text{physical}}} \middle| \frac{\partial h}{\partial \lambda_{\text{non-GR}}} \right\rangle \simeq 0$$

Selected for a Viewpoint in *Physics*
PRL **116,** 221101 (2016) PHYSICAL REVIEW LETTERS week ending 3 JUNE 2016

Tests of General Relativity with GW150914

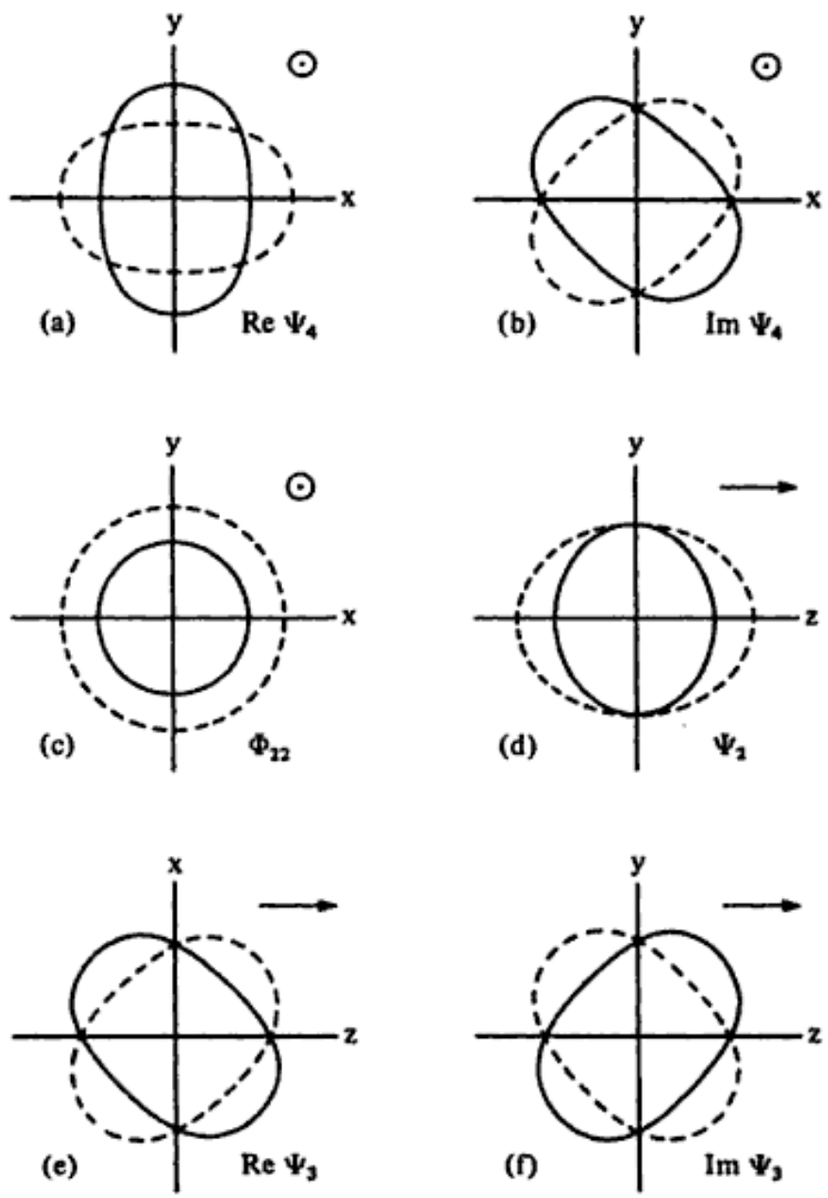
B. P. Abbott *et al.*[*]
(LIGO Scientific and Virgo Collaborations)



SNR in coherent burst analysis of data residual after subtracting best-fit GW150914 waveform

If we assume that $\mathrm{SNR}_{\mathrm{res}}$ is entirely due to the mismatch between the MAP waveform and the underlying true signal, and that the putative violation of GR cannot be reabsorbed in the waveform model by biasing the estimates of the physical parameters [54,55], we can constrain the minimum *fitting factor* (FF) [56] between the MAP model and GW150914. An imperfect fit to the data leaves $\mathrm{SNR}_{\mathrm{res}}^2 = (1 - \mathrm{FF}^2)\mathrm{FF}^{-2}\mathrm{SNR}_{\mathrm{det}}^2$ [57,58], where $\mathrm{SNR}_{\mathrm{det}} = 25.3^{+0.1}_{-0.2}$ is the network SNR inferred by LALINFERENCE [3]. $\mathrm{SNR}_{\mathrm{res}} \leq 7.3$ then implies $\mathrm{FF} \geq 0.96$. Considering that, for parameters similar to those inferred for GW150914, our waveform models have much higher FFs against numerical GR waveforms, we conclude that the noise-weighted correlation between the observed strain signal and the true GR waveform is $\geq$96%. This statement can be read as implying that the GR prediction for GW150914 is verified to be better than 4%, in a precise sense related to noise-weighted signal correlation, and, conversely, that effects due to GR violations in GW150914 are limited to less than 4% (for effects that cannot be reabsorbed in a redefinition of physical parameters).

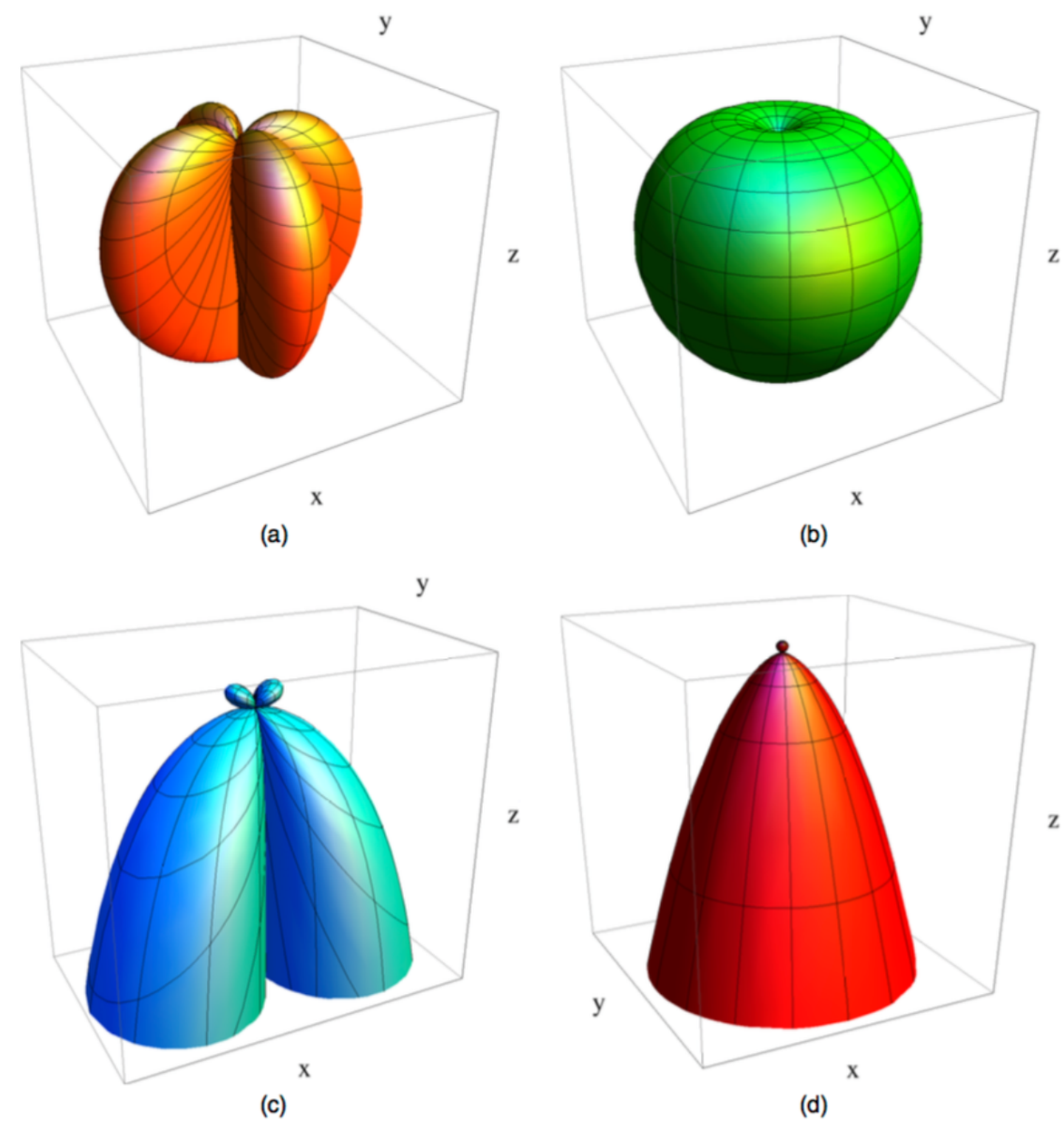
GW polarization and pulsar-timing array antenna patterns



Tensor

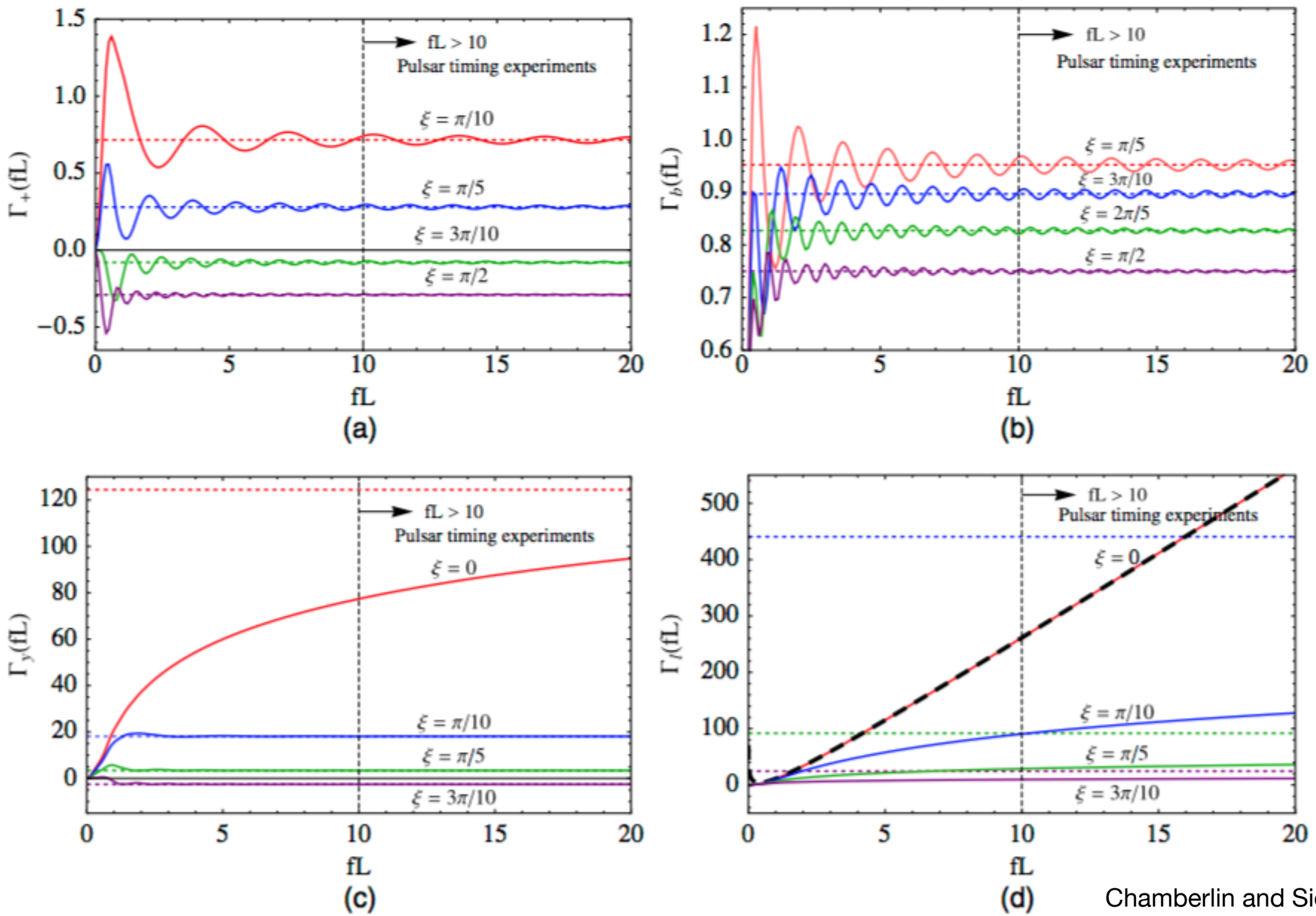
Scalar

Vector



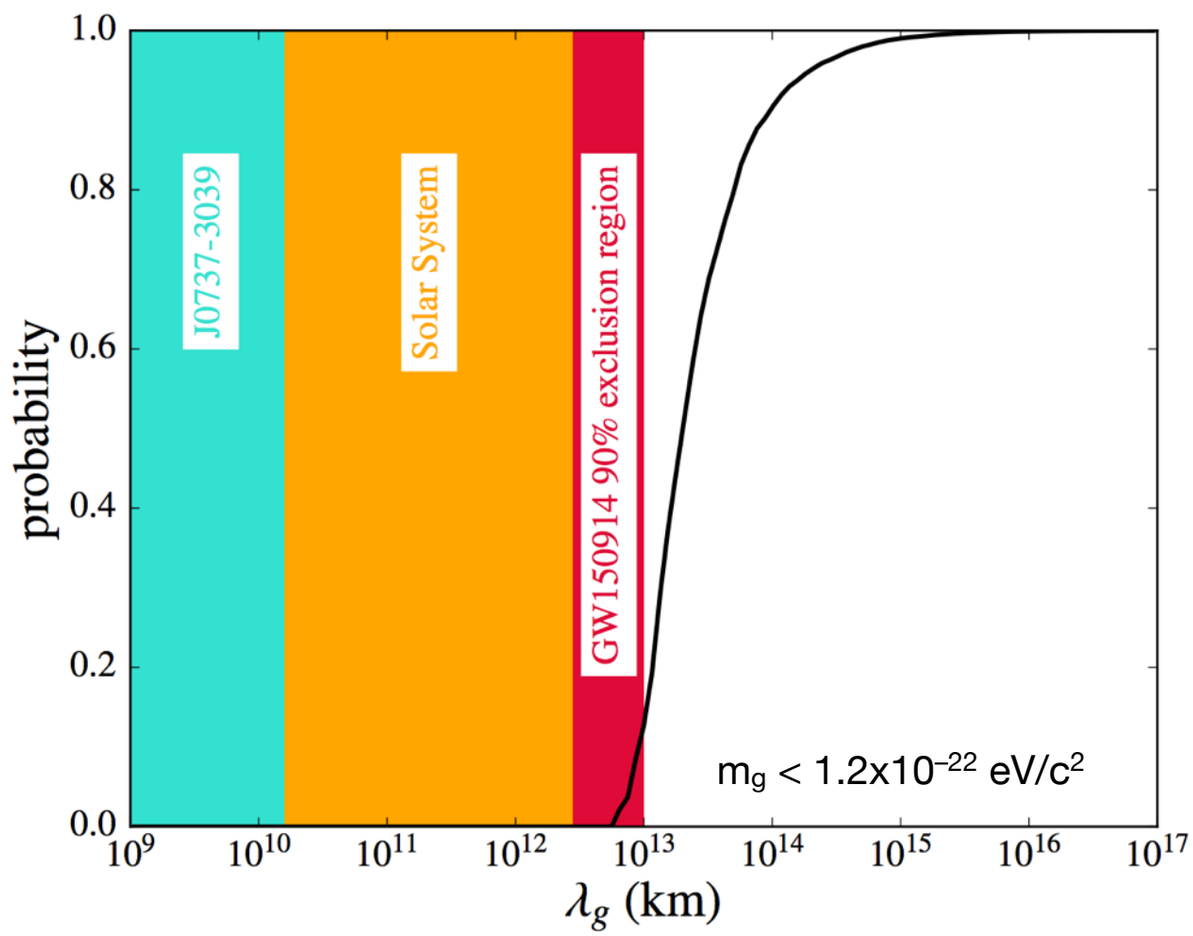
Chamberlin and Siemens 2012

Pulsar-timing-array overlap reduction functions



Chamberlin and Siemens 2012

Speed of waves by dephasing in GW150914
(in future systems with counterparts: compare with EM!)



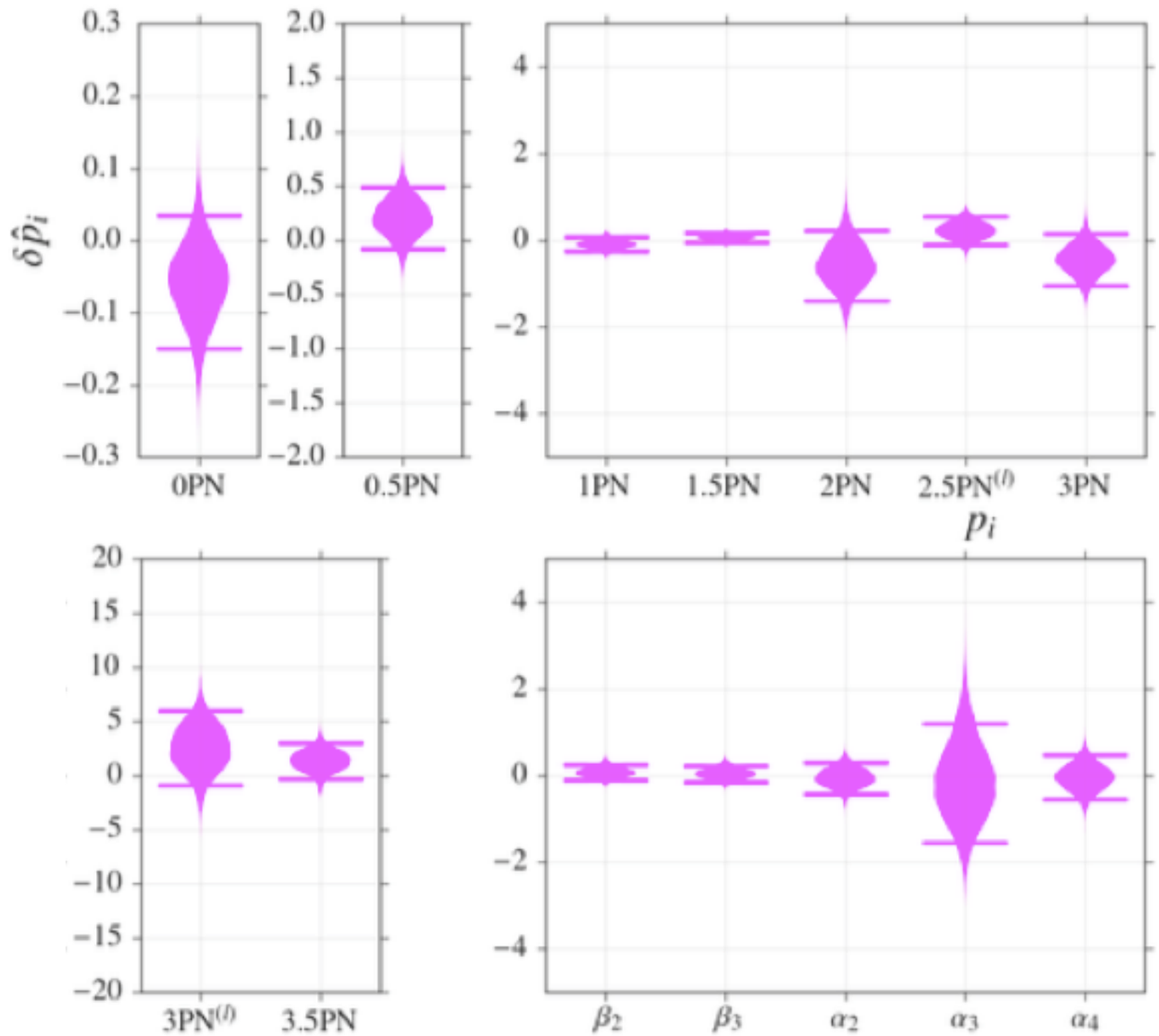
$$h(f) = \frac{1}{D}\frac{\mathcal{A}}{\sqrt{\dot{F}}} f^{2/3} e^{i\Psi(f)}$$

$$\Psi(f) = \sum_i \left[\psi_i + \psi_{il}\log f\right] f^{(i-5)/3} + \Phi^{\mathrm{MR}}[\beta_i, \alpha_i]$$

$$\frac{v_{\mathrm{g}}^2}{c^2} = 1 - \frac{m_{\mathrm{g}}^2 c^4}{E^2}$$

$$\delta\Psi(f) = \frac{\pi D c}{\lambda_g^2 (1+z) f}$$

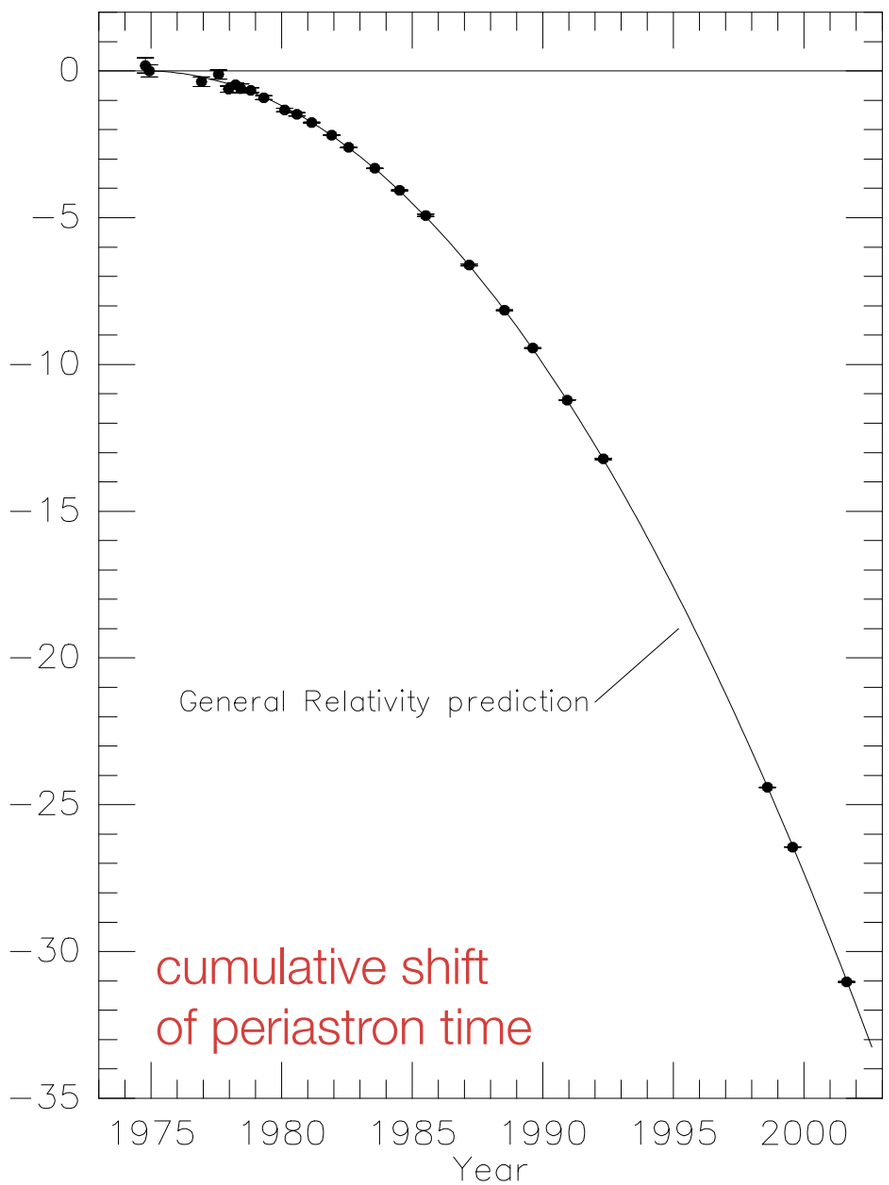
Radiation reaction by waveform coefficients in GW150914 and GW151226 (in NS binaries: dipolar radiation)



$$h(f) = \frac{1}{D}\frac{\mathcal{A}}{\sqrt{\dot{F}}} f^{2/3} e^{i\Psi(f)} \qquad \Psi(f) = \sum_i \left[\psi_i + \psi_{il}\log f\right] f^{(i-5)/3} + \Phi^{\mathrm{MR}}[\beta_i, \alpha_i]$$

For comparison: the timing of NS–NS pulsars allows accurate tests of GR in terms of easily interpreted parameters

PSR B1913+16
[Weisberg 2003]



$$\dot{\omega} = 3\left(\frac{P_\mathrm{b}}{2\pi}\right)^{-5/3}(T_\odot M)^{2/3}\,(1-e^2)^{-1},$$ periastron advance

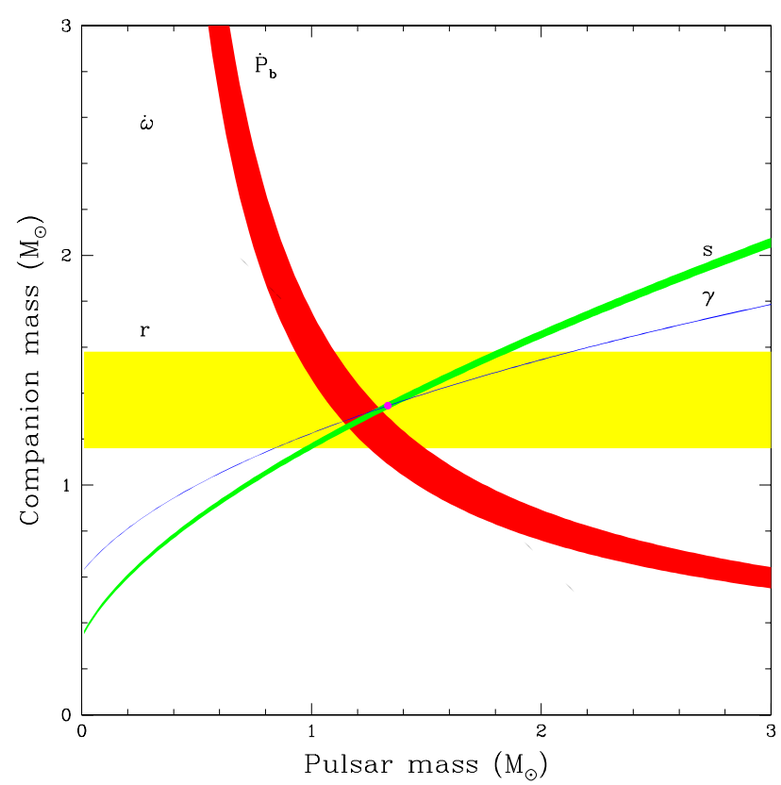
$$\gamma = e\left(\frac{P_\mathrm{b}}{2\pi}\right)^{1/3} T_\odot^{2/3} M^{-4/3}\, m_2(m_1+2m_2),$$ GR redshift

orb. period derivative

$$\dot{P}_\mathrm{b} = -\frac{192\pi}{5}\left(\frac{P_\mathrm{b}}{2\pi}\right)^{-5/3}\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right)(1-e^2)^{-7/2}\,T_\odot^{5/3}\,m_1 m_2\,M^{-1/3}$$

$$r = T_\odot\, m_2,$$ range of Shapiro delay

$$s = x\left(\frac{P_\mathrm{b}}{2\pi}\right)^{-2/3} T_\odot^{-1/3}\,M^{2/3}\,m_2^{-1}.$$ shape of Shapiro delay



PSR B1534+12
[Stairs et al. 2002]

Prospects for testing GR with pulsar-timing gravitational-wave detection

Michele Vallisneri

Jet Propulsion Laboratory
California Institute of Technology