Jet Propulsion Laboratory
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Forward Modeling of Ceres' Gravity Field for Planetary Protection

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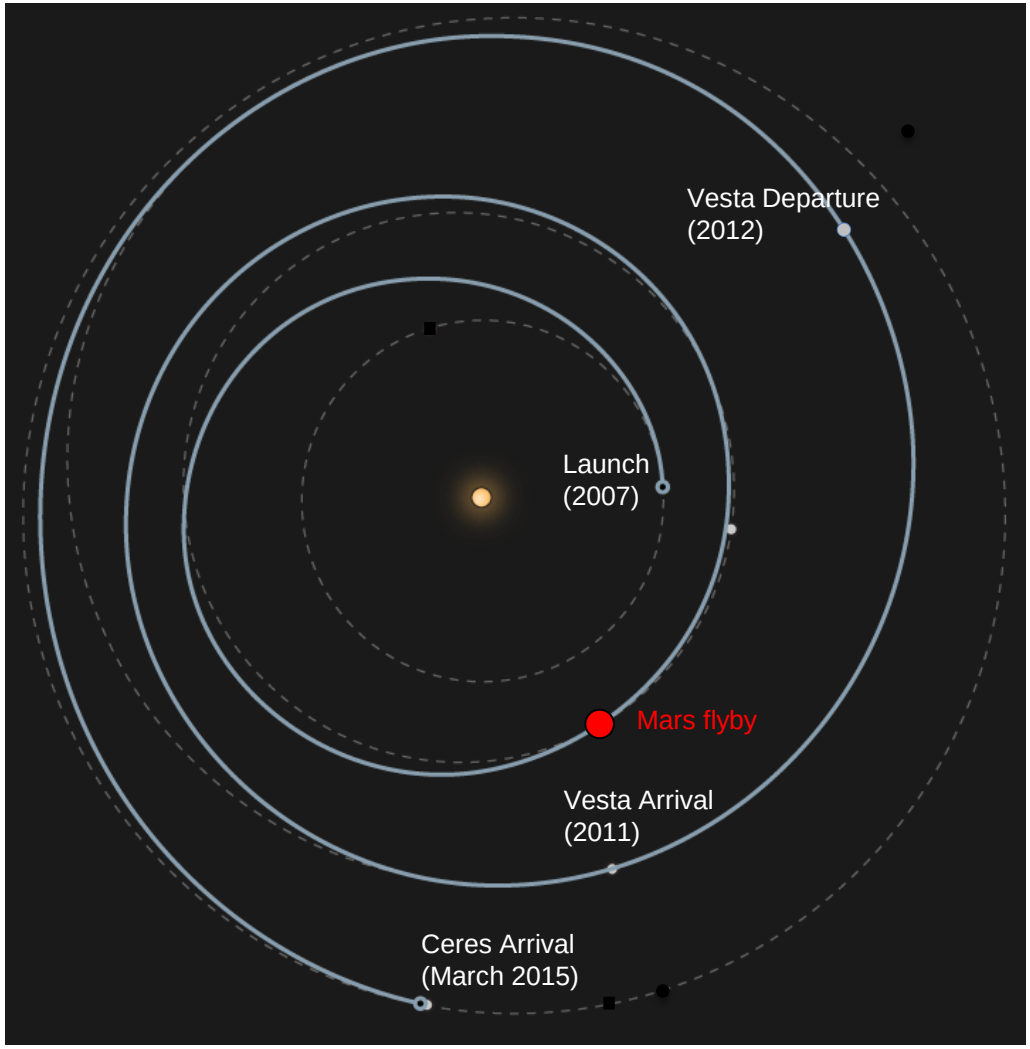
AIAA Space 2016

The Dawn mission

Dawn's Journey

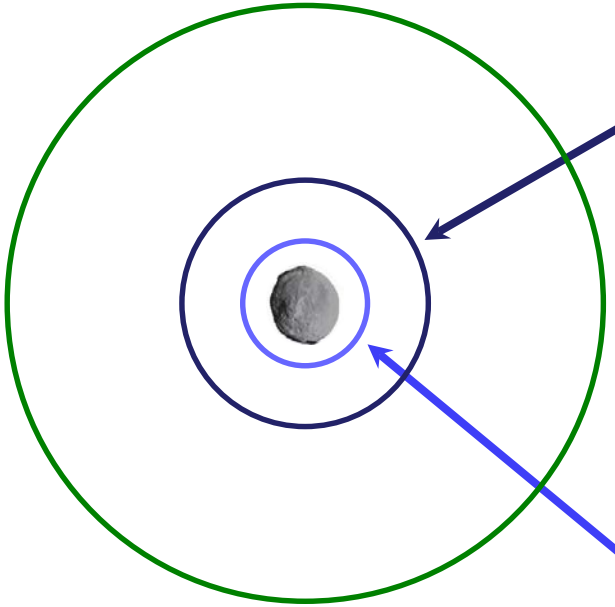


Science Orbits

Ceres
470 km mean radius,
Rotation period 9.1 hrs

Rotation Characterization 3 (RC3)
- 14,000 km radius, 15 days, 1 orbit
- Nadir rotation movies, high phase observations

HAMO
- 1950 km radius, 19 hrs, 6x14 orbits (56 days)
- Nadir and fixed off-nadir mapping



Survey
- 4900 km radius, 75 hrs, 7 orbits (22 days)
- Nadir mapping, limb observations

LAMO
- 850km radius, 5.4 hrs, 404 orbits (92 days) *plan
- Mapping gravity field & elemental composition (GRaND)

Planetary Protection

- Category runs from I through V
  - Higher the number, the more stringent the requirements
- Vesta and Ceres are category II target bodies
  - Mission type: any
  - Of significant interest relative to the process of chemical evolution
  - A remote chance that contamination would compromise future investigations
- Mars flyby component of the mission makes the Dawn a category III mission
  - Mission type: flyby or orbiter
  - Of significant interest relative to the process of chemical evolution and/or the origin of life
  - A significant chance that contamination would compromise future investigations and scientific opinion

Goal

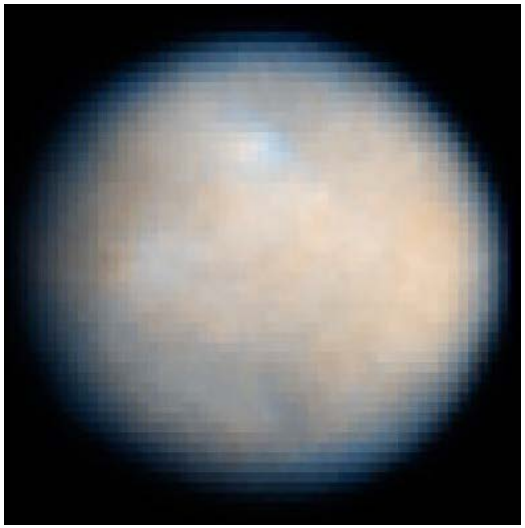
- Make sure that Dawn does not impact Ceres for at least ***20*** years after the end of mission, based on the ***worst-case credible gravity field***
  - The analysis is mostly done pre-Dawn arrival
  - The Hubble images were the best observations we had
  - What is the worst-case credible gravity field?

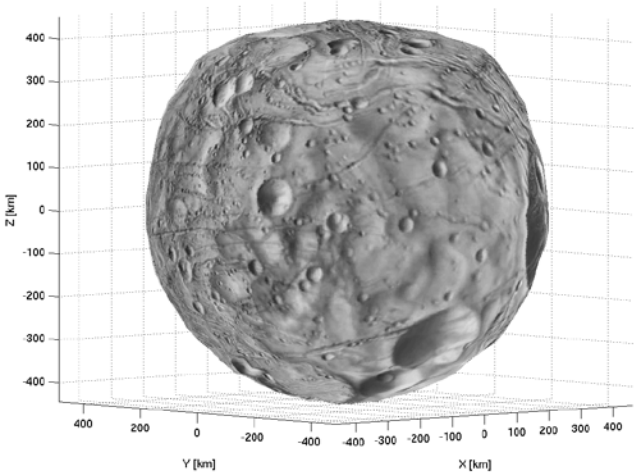
- If Ceres is found biologically interesting
  - The 20-year requirement becomes a minimum
  - The lifetime requirement becomes 50 years or more

- The project decided to pursue the 50-year requirement for the planetary protection analysis

Which one is the real Ceres?

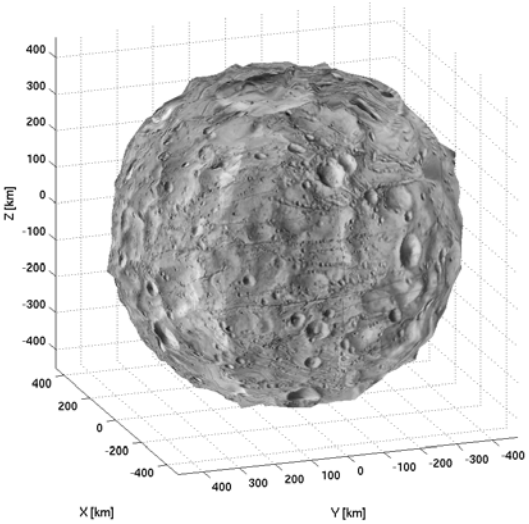
HST image

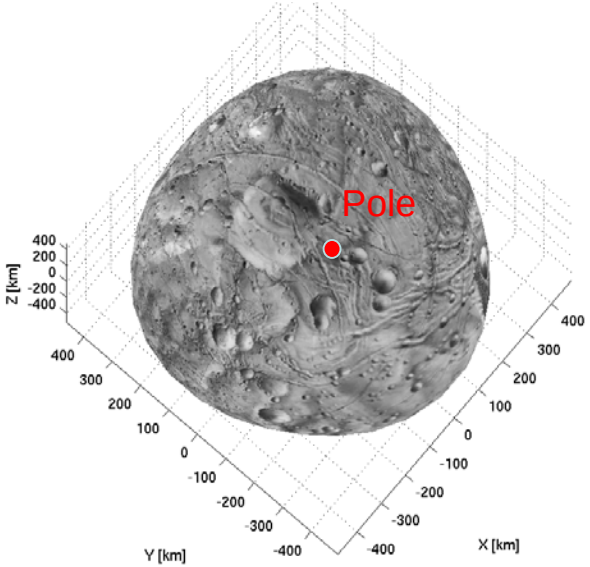




Missing Pole (Vesta Like)

Pimples?





Tri Lobe

Pre-arrival HST model

- Pre-Dawn arrival analysis with HST data
  - Axi-symmetric shape
  - Two-layer (core - surface) model
  - Hydrostatic equilibrium inside Ceres

Table 1. Ceres dimensions.

References	$a = b$ (km)	c (km)	Δ (km)	$a - c$ (km)
Millis et al. (1987)	479.6	453.4	4.5	26.2
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Drummond & Christou (2008)	486.5	454	3.5	32.5
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Drummond et al. (2014)	483.5	446	5	37.5

Notes. Columns: equatorial radius ($a = b$), polar radius (c), uncertainties (Δ), and radii difference.

Source: Rambaux et al., A&A, Volume 584, 2015, Third-order development of shape, gravity, and moment of inertia for highly flattened celestial bodies. Application to Ceres.

Sensitivity Study

Sensitivity Study

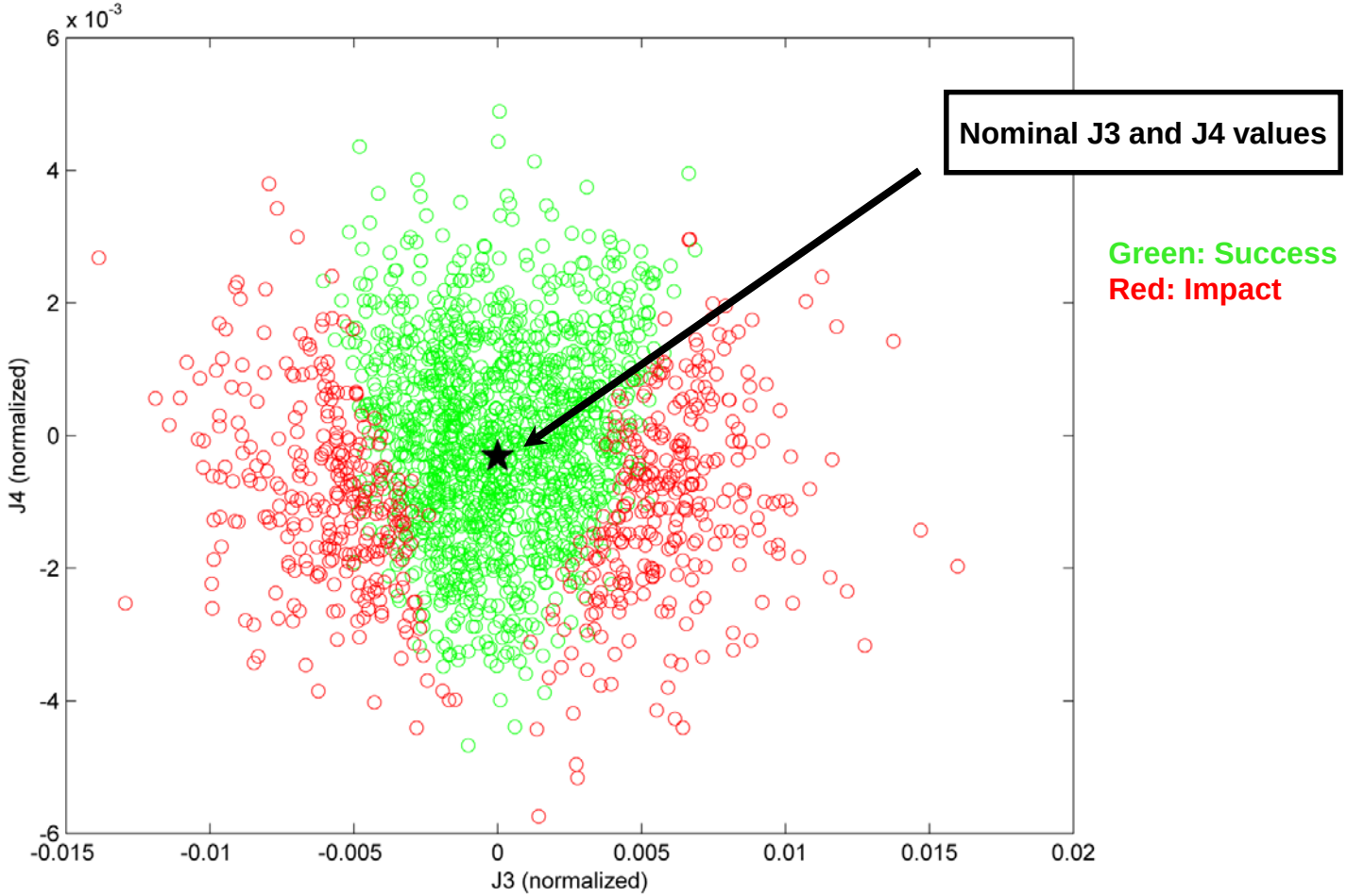
- We perturb harmonic coefficients to understand their sensitivity to the long-term orbit propagation in LAMO
  - GM is from Baer et. al (2011)
  - The nominal harmonic values are supplied by the project science team

$$U = \frac{GM^*}{R^*} \sum_{n=0}^{\infty} \sum_{m=0}^{n} \left(\frac{R^*}{r}\right)^{n+1} P_{nm}(\sin\phi) \cdot \begin{bmatrix} \cos(m\lambda) \\ \sin(m\lambda) \end{bmatrix} \cdot \underline{\begin{bmatrix} C_{nm} \\ S_{nm} \end{bmatrix}}$$

Spherical harmonics

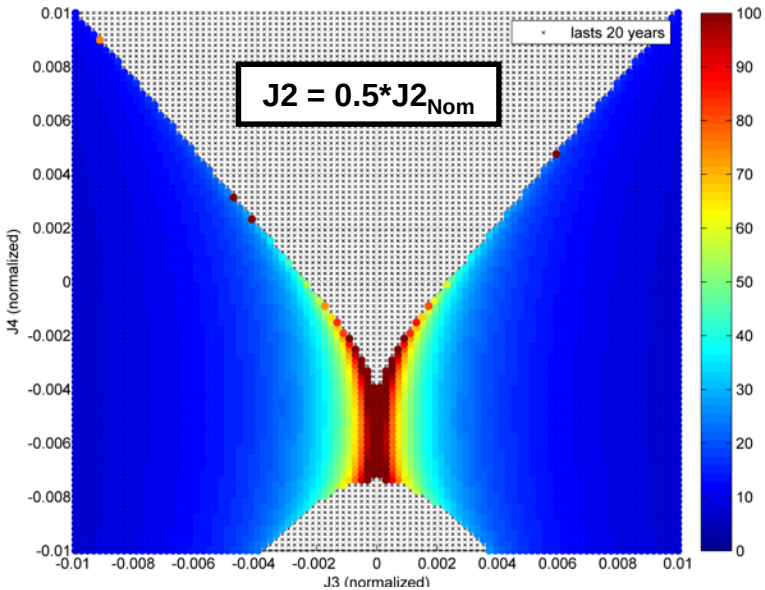
  - The 1-sigma uncertainties are provided by the injection covariance into RC3 (i.e., largest science orbit)
  - We initially tested 1000 samples
  - The initial conditions are circular orbit and the same for all samples
  - This analysis is a numerical, non-linear orbital propagation

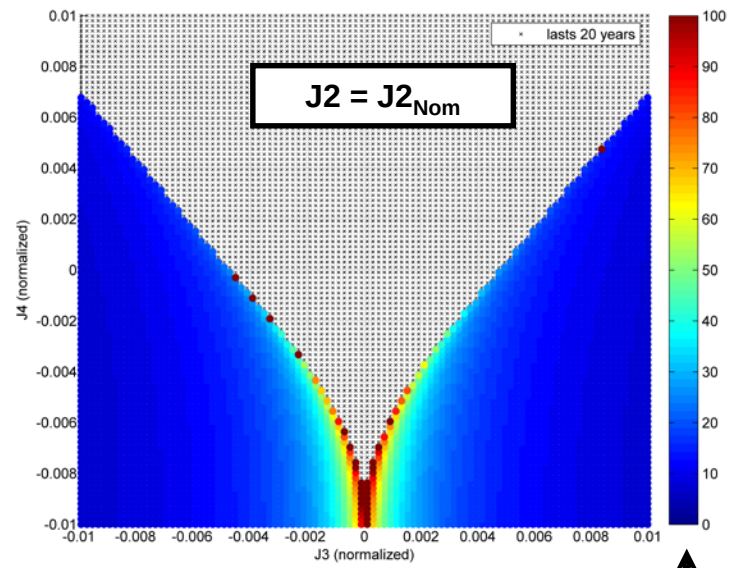
Sensitivity Study

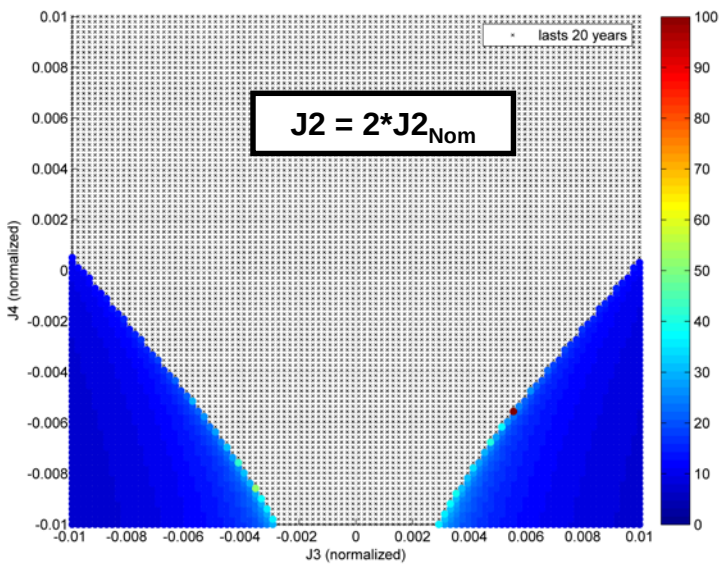


- Clear correlation between J3 and J4
- J2 also has sensitivity
- Impacting trajectories have high eccentricity

Sensitivity Study







Days to impact

- Larger J2 has a stabilizing effect
- Most of the impacting cases impact within tens of days and less than 100 days at most

Sensitivity Study

- We can reproduce the same results analytically
  - To first order, the argument of periapse and eccentricity follow the equations below:

$$\frac{d\omega}{dt} = \frac{3nJ_2R^2}{a^2(1-e^2)^2}\left(1-\frac{5}{4}\sin^2 i\right)\left[1+\frac{J_3R}{2J_2a(1-e^2)}\left(\frac{\sin^2 i - e\cos^2 i}{\sin i}\right)\frac{\sin\omega}{e}\right]$$

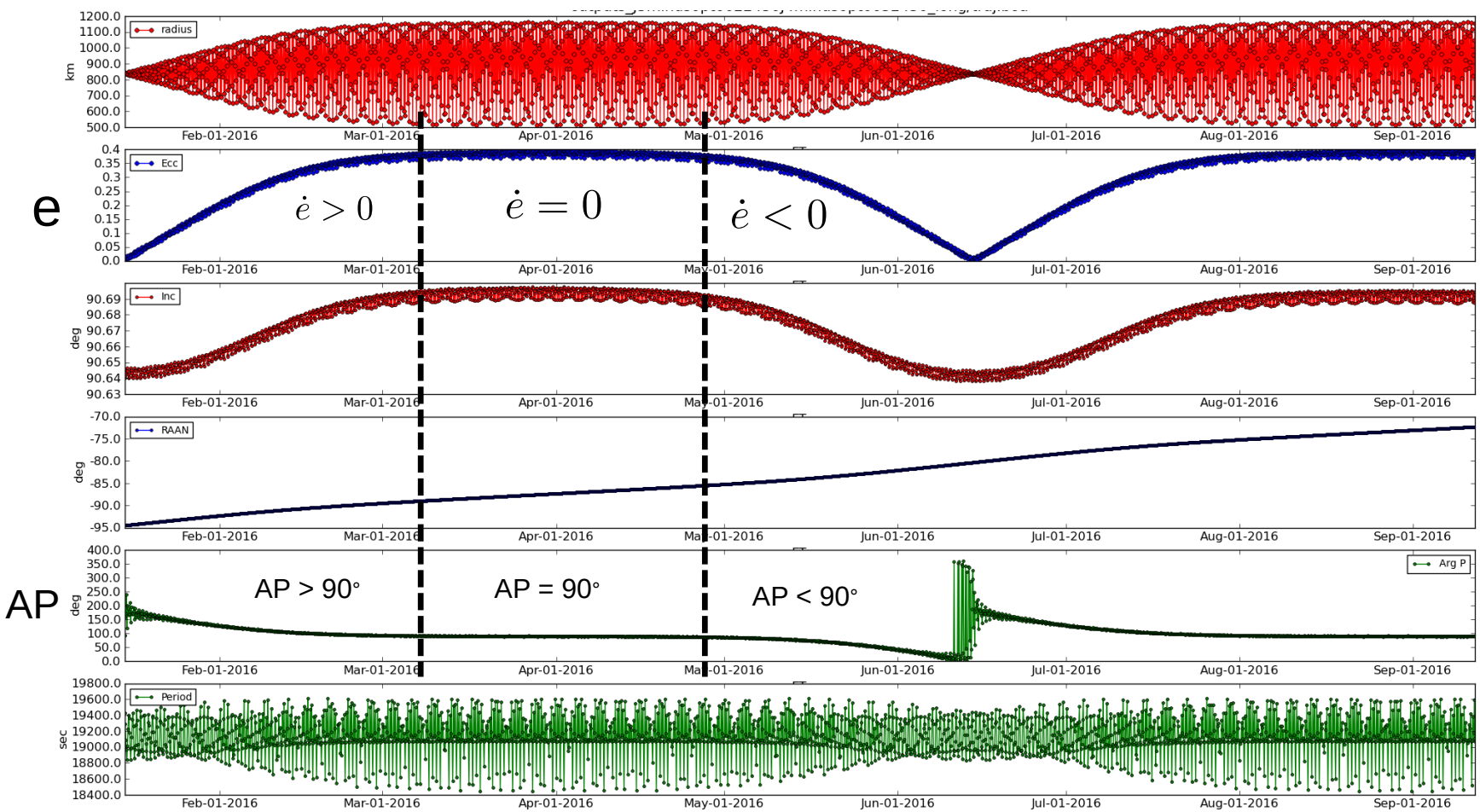
$$\frac{de}{dt} = -\frac{3nR^3J_3\sin i}{2a^3(1-e^2)^2}\left(1-\frac{5}{4}\sin^2 i\right)\cos\omega$$

$$h = e\sin\omega$$

$$k = e\cos\omega$$

- Eccentricity is small
- Use semi-equinoctial elements
- dh/dt and dk/dt become functions of inclination functions
- The previous slides can be reproduced for different J_n values

Non-impacting trajectory

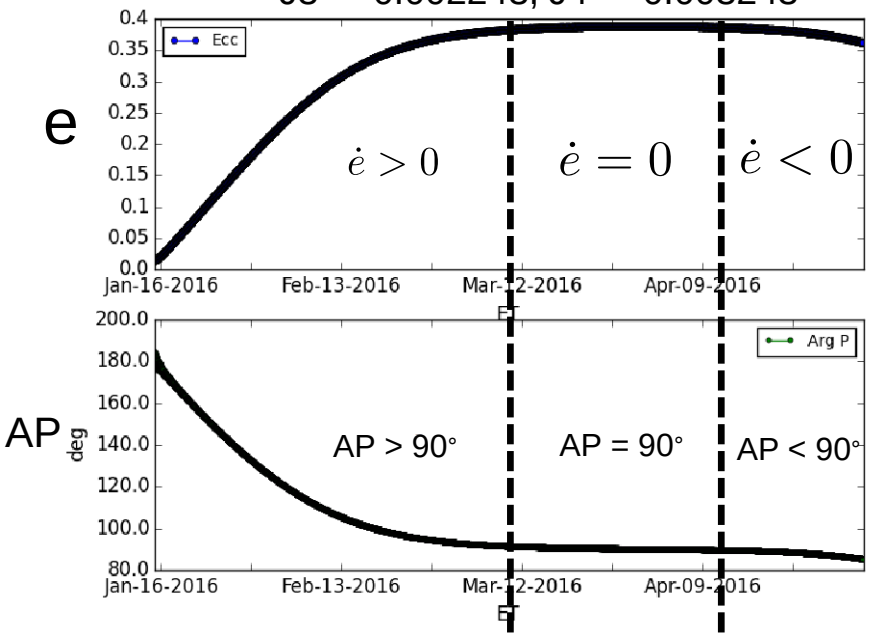


- AP: 180° → 90° → 0° → 180° → ...
- Eccentricity: 0 → 0.4 → 0 → 0.4 → ...

Sensitivity Study

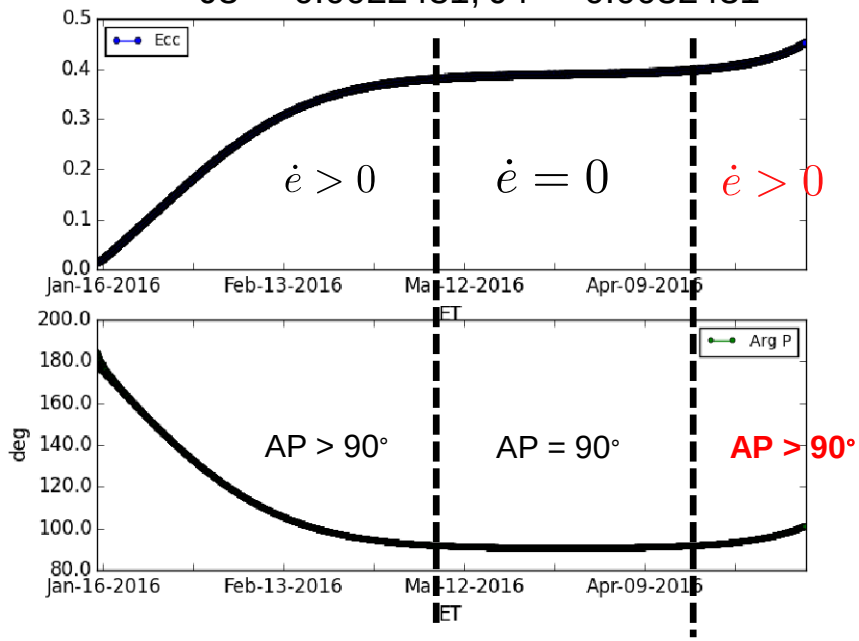
Survives

J3 = -0.002243, J4 = -0.003243



Impacts

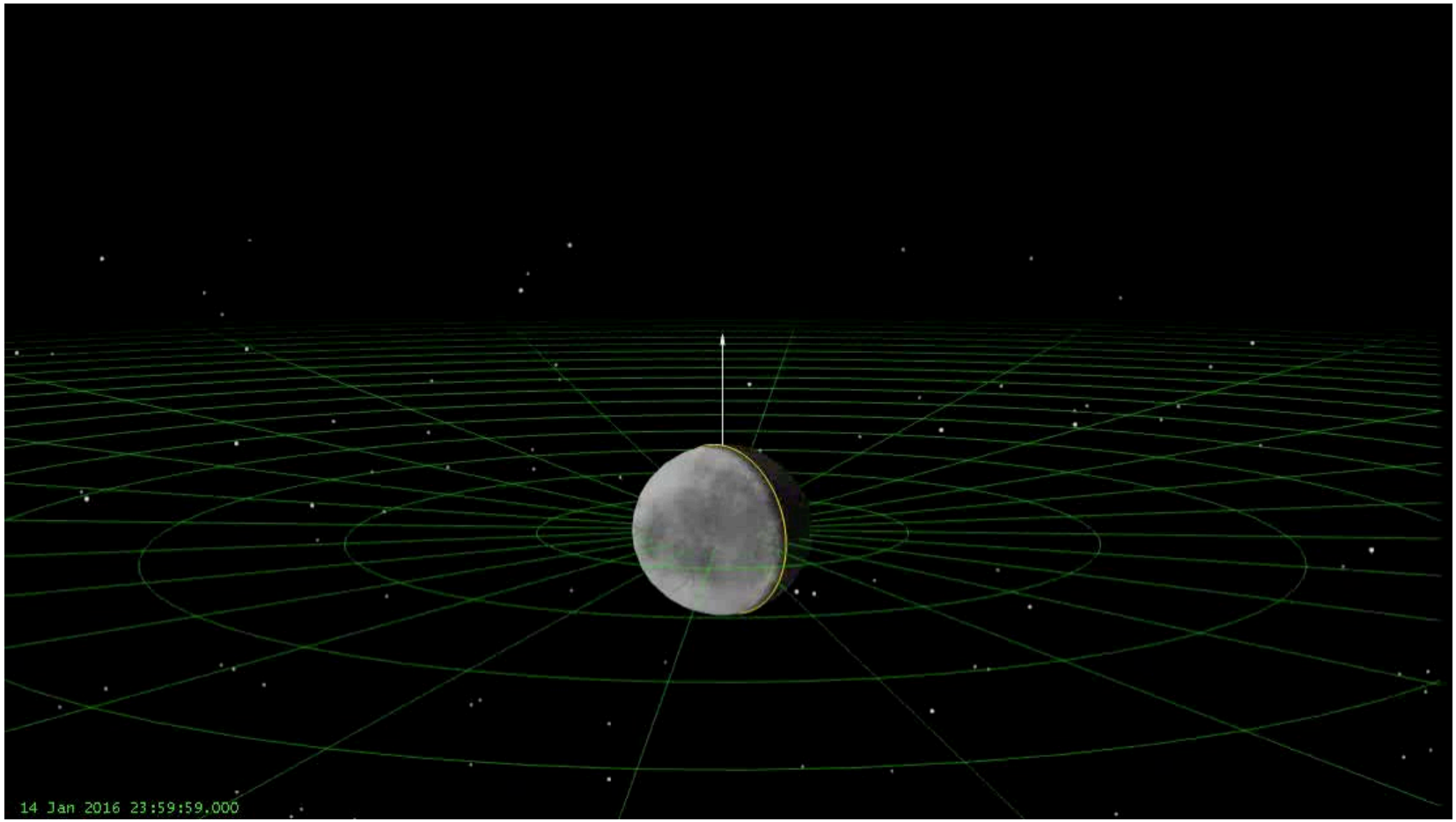
J3 = -0.0022431, J4 = -0.0032431



- J3 and J4 values are different only by 1e-7
- Altitude drops to 20 km when e ~ 0.4
- Impacting cases remain > 90° for AP and eccentricity keeps growing

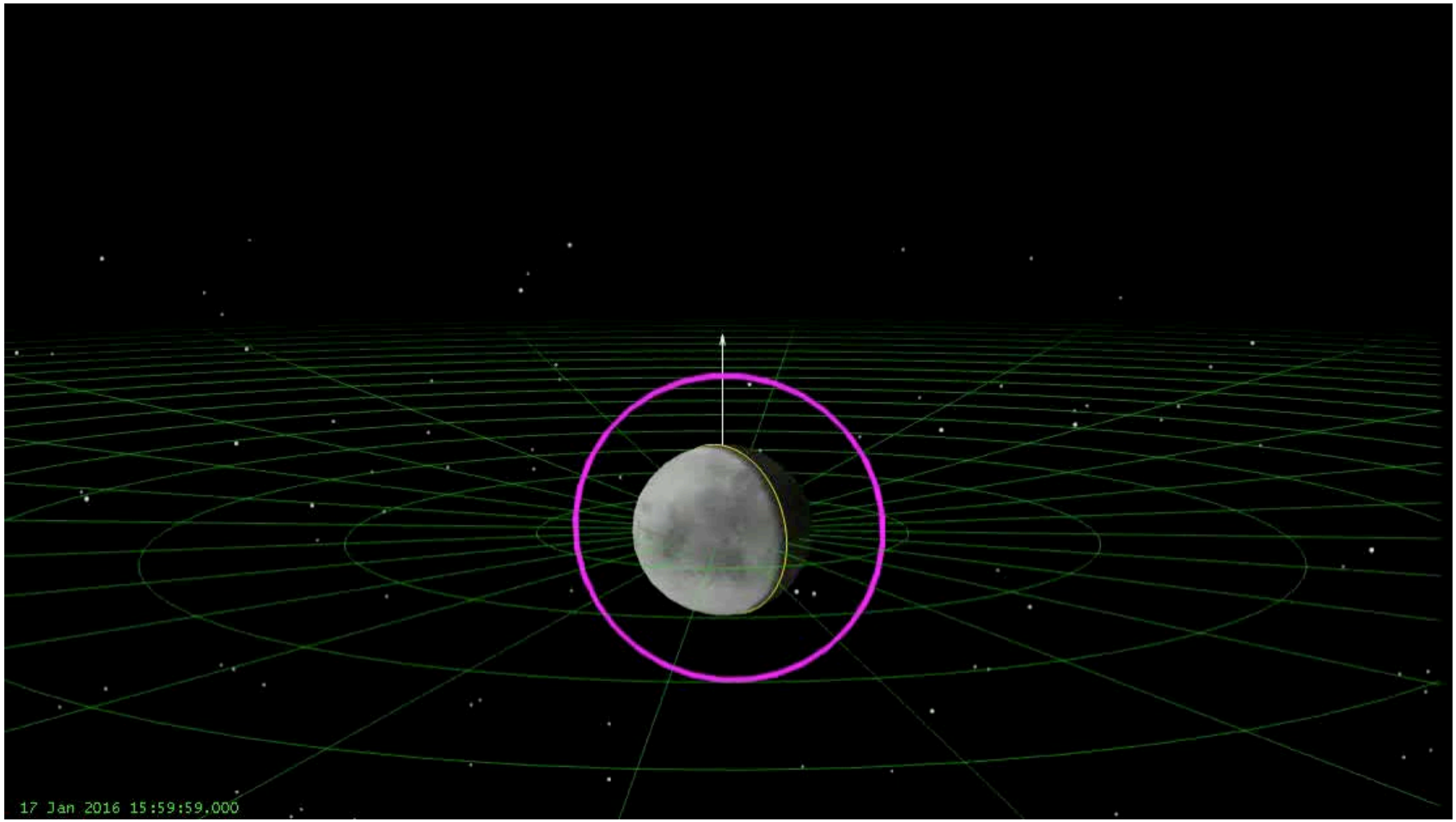
Impacting case

- Orbit propagation video of STK (impact)



Non-impacting case

- Orbit propagation video of STK (surviving case)



Gravity from Shape Model

Gravity from Shape Model

- We now understand that J2, J3, and J4 are the key parameters for long-term orbit propagation
- What kind of Ceres might possess a gravity that results in impact?
  - We can generate a gravity field from a shape model and assumed density distribution
  - We can generate many samples to get statistics
- We look at
  - Direct expansion model
  - Gravity Monte Carlo

Direct Expansion Model

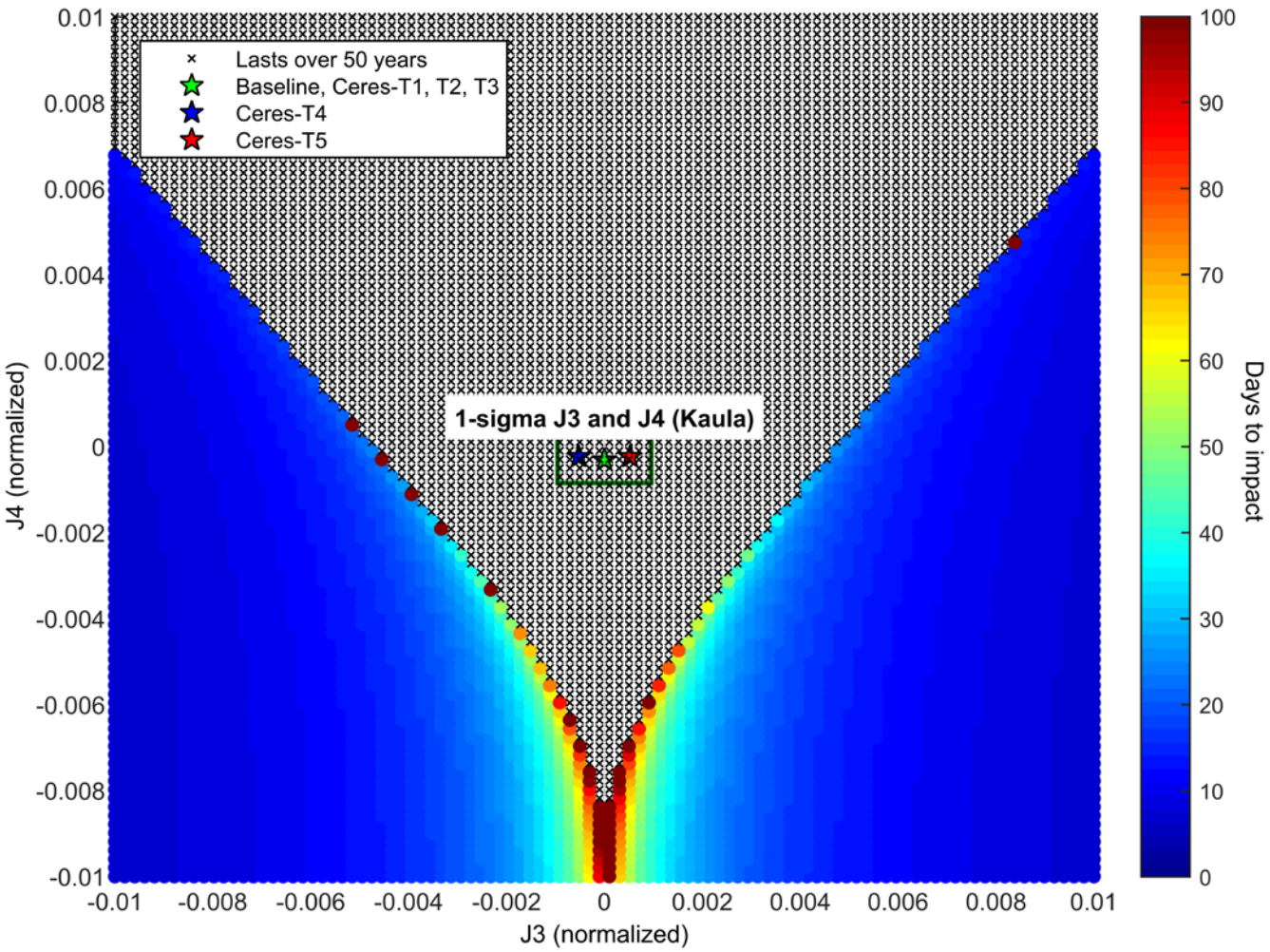
Direct Expansion Model

- Gravity field models with science inputs
- Direct expansion model seeks a stressed case where the shape is strongly perturbed
  - Constraints on the flattening based on Hubble
  - Constraints on the overall shape based on HST data and hydrostatic equilibrium

$$r = r_{mean} \sum_{n=0}^{\infty} \sum_{m=0}^{n} P_{nm}(\sin\phi) \begin{bmatrix} \cos(m\lambda) \\ \sin(m\lambda) \end{bmatrix} \cdot \begin{bmatrix} T_{nm} \\ U_{nm} \end{bmatrix}$$

- The density distribution model is a radially stratified two-layer model
  - Rocky core: ρ_{core} = 2700 kg/m^3, $r_{mean,\ core}$ = 409.1 km
  - Icy layer: ρ_{ice} = 1000 kg/m^3, $r_{mean,\ ice}$ = 476.2 km

Direct Expansion Model



- Direct expansion models do not threaten planetary protection

Gravity Monte Carlo

Gravity Monte Carlo

- Nav internal gravity field
  - Constraints not imposed from the Hubble observations
- Statistics of the gravity fields are sought after
  - Kaula rule

$$\kappa_n = \sqrt{\frac{\sum_{m=0}^{n} C_{nm}^2 + S_{nm}^2}{2n+1}} \approx \frac{K_{Ceres}}{n^2}$$

$$K_{Ceres} = K_{Earth} \left(\frac{M_{Earth}}{M_{Ceres}}\right)^2 \left(\frac{R_{Ceres}}{R_{Earth}}\right)^4 = 0.0086$$

  - Shape deformation and density variation
    - Core – icy layer model
    - The surface vertex is modulated with 12-km amplitude perturbation (2-sigma of Hubble observations)
    - Frequency = 50 km ~ 100 km
    - 75000 samples are generated

Gravity Monte Carlo

- Kaula uncertainty > Gravity Monte Carlo uncertainty
  - It is very challenging to perturb the shape and density distribution enough to see substantial differences in the gravity field
  - We know that Kaula rule is very conservative (i.e., large) for lower-degree harmonics
- Orbital sensitivity study used larger uncertainty for all harmonics but J2
  - Orbital sensitivity analysis is conservative
  - The impacting cases are likely to have resulted from aphysical gravity field

Hand-picked pre-arrival best Ceres gravity model

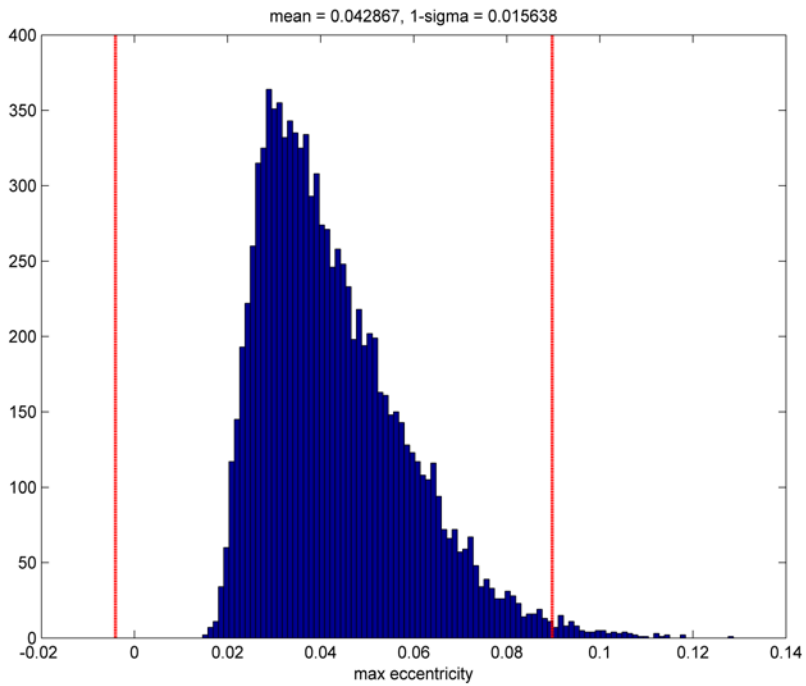
CBE Ceres Gravity

- Members of Nav and science teams gather to choose the mean and 1-sigma uncertainty for each degree and order harmonics
  - 6x6 current best estimate (CBE) Ceres gravity model
  - No correlations are assumed, which results in a more conservative gravity field
- Orbital propagation is rerun for 50 years
  - 10000 trajectory samples are generated

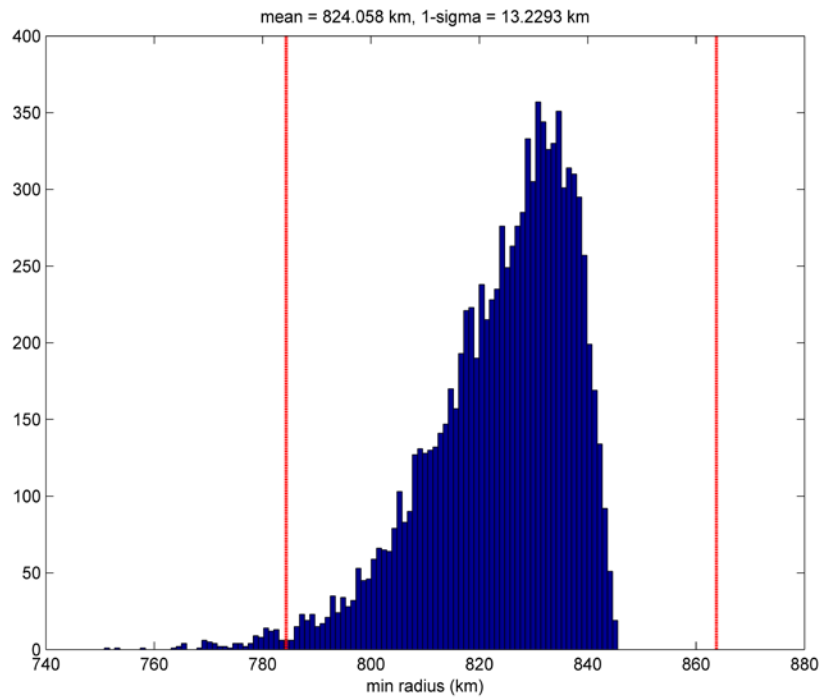
CBE Ceres Gravity

- **None of the Monte Carlo samples impacted Ceres within 50 years.**

Eccentricity



Radius



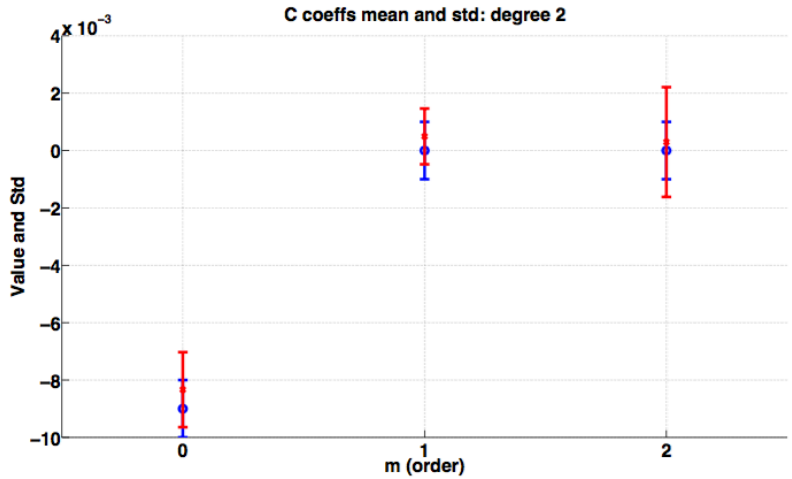
- Mean radius of Ceres is 470 km
- Impact occurs at e = 0.4
- The red lines are the 3-sigma lines

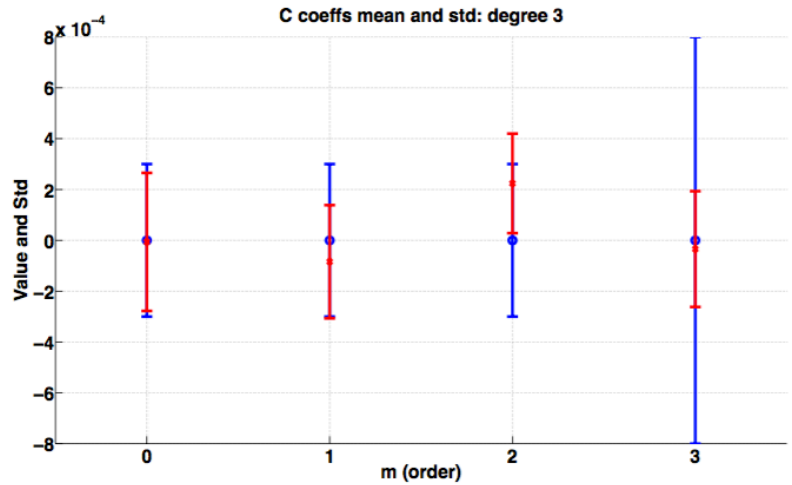
Gravity field with updated shape model

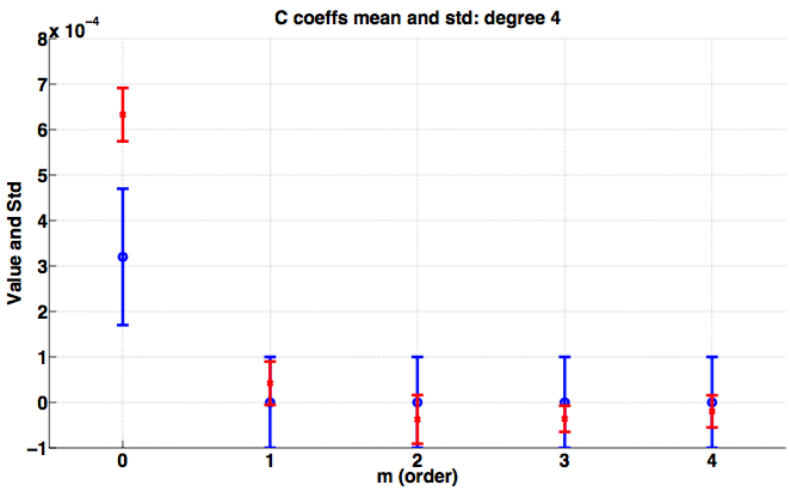
Gravity with updated shape model

- Dawn's images exceeded Hubble images by late January, 2015
- Opnav team constructed a new shape model
- In March 2015, the gravity Monte Carlo was rerun with the updated shape model
- At this time Dawn was in Survey (~5000 km radius)
  - J2 = 7.6e-3 ± 9e-4
  - J3 and J4 were unobservable

Gravity with updated shape model







Blue: CBE (pre-arrival)
Red: Post-arrival

- Pre-arrival statistics encompass post-arrival statistics at 2-sigma level

CBE vs HAMO gravity field

CBE vs HAMO gravity field

- Prior to descending into LAMO, the estimated values of the spherical harmonics in HAMO and their uncertainties are compared against CBE gravity field
  - They agree within 3-sigma
  - They are well within the no-impact region of the butterfly plot

Conclusion

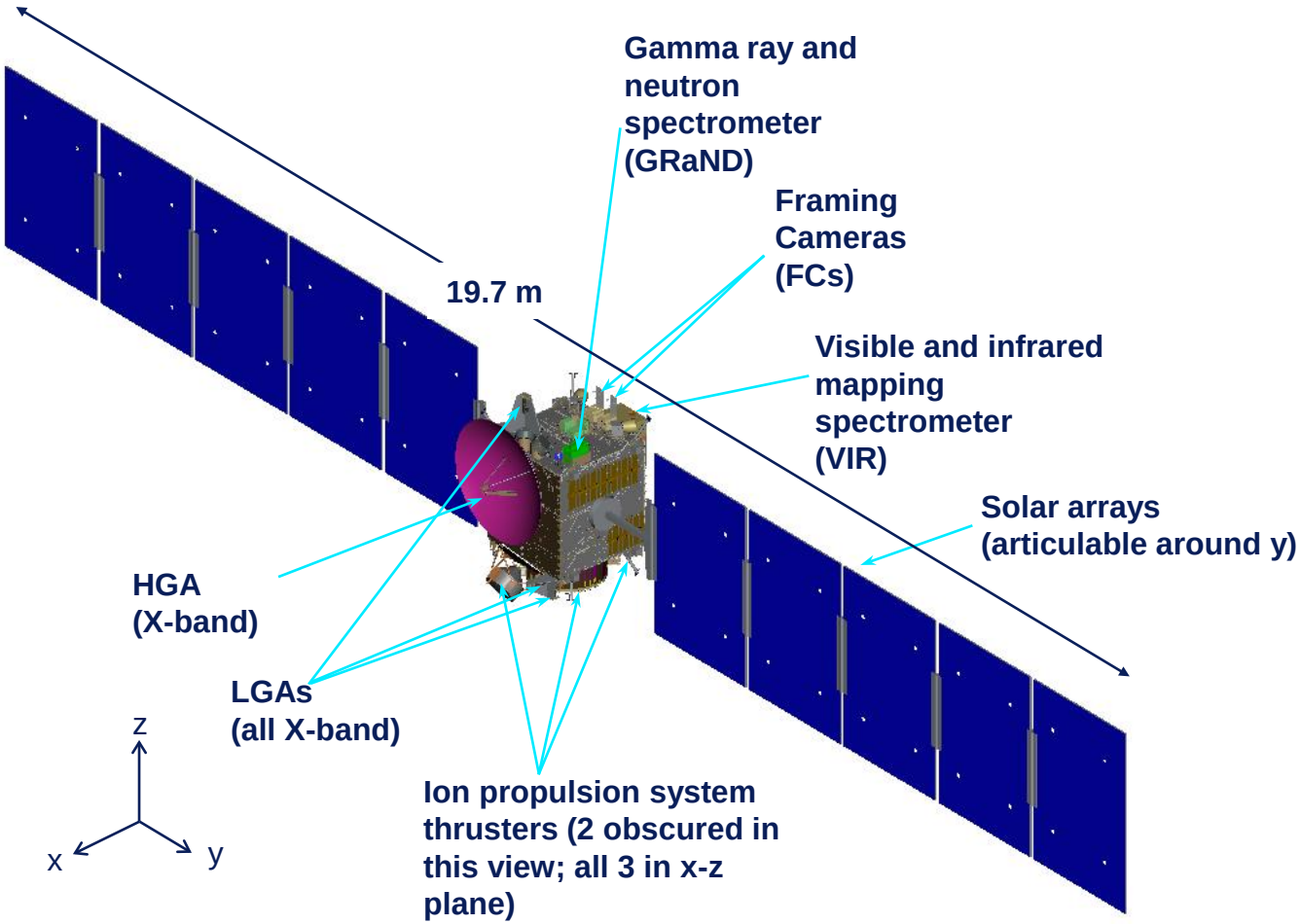
Conclusion

- We have identified sensitivity of long-term orbit propagation to J2, J3, and J4
- If the argument of perigee does not drop below 90°, the eccentricity keeps growing
- All impacts occur within < 100 days
- We have shown that the LAMO orbit does not threaten the planetary protection requirement with
  - Pre-arrival Hubble data
  - Post-arrival Dawn data

Questions?

Backup

Dawn Flight System Configuration



Dawn Navigation Team

Navigation

Optical Navigation

- Process images
- Generate surface "landmarks"
- Estimate pole orientation and spin rate

Orbit Determination

- Process optical, radiometric, and DDOR data
- Estimate bias, stochastic, and state parameters
- Map uncertainty forward in time

Maneuver Design

- Use state and parameter estimations from OD
- Produce low-thrust vectors for maneuvers

Gravity Monte Carlo

- Shell: a,b = 480 – 492km, c = 450 – 462 km (base: a = b = 480km, c = 450km)
- Core: a,b,c = 380 – 420 km
- Density:
  - Shell = 1.0 g/cm^3
  - Core = 2.7 – 2.95 g/cm^3
- Topology perturbation
  - Shell flattening: ab: < 0.05, ac, bc: 0.05 – 0.08
  - Core flattening: ab: < 0.01, ac, bc: 0.025 – 0.08
  - Amplitude = ~12 km (combination of low frequency + high frequency, one sigma)
  - Frequency = 50 km ~ 100 km

Pre-arrival HST model

- Pre-Dawn arrival analysis with HST data
  - Axi-symmetric shape
  - Two-layer (core - surface) model
  - Hydrostatic equilibrium inside Ceres

Table 1. Ceres dimensions.

References	$a = b$ (km)	c (km)	Δ (km)	$a - c$ (km)
Millis et al. (1987)	479.6	453.4	4.5	26.2
Thomas et al. (2005)	487.3	454.7	1.8	32.6
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Drummond et al. (2014)	483.5	446	5	37.5

Notes. Columns: equatorial radius ($a = b$), polar radius (c), uncertainties (Δ), and radii difference.

Table 2. Geophysical parameters of Ceres.

Ceres parameters	From Thomas et al. (2005)	From Drummond et al. (2014)
Mass M	$9.395 \pm 0.125 \times 10^{20}$ kg[a]	9.4062×10^{20} kg[b]
Mean spherical radius R	476.200 km[a]	470.500 km[c]
Rotational period	9.076 h[a]	9.074170 ± 0.000001 h[d]
Deduced parameters		
Geodetic parameter m	0.06368	0.06135
Mean density $\bar{\rho}$	2077.02 kg m^{-3}	2156.00 kg m^{-3}

Notes. [(a)] Thomas et al. (2005). [(b)] deduced from the GM of Kuchynka & Folkner (2013). [(c)] Drummond et al. (2014). [(d)] Chamberlain et al. (2007).

Source: Rambaux et al., A&A, Volume 584, 2015, Third-order development of shape, gravity, and moment of inertia for highly flattened celestial bodies. Application to Ceres.

Pre-arrival HST model

Baseline model

Parameter	Value
a	487.218 km
b	487.218 km
c	454.164 km
a_{core}	418.585 km
b_{core}	418.585 km
c_{core}	390.187 km
$R_{ice,\ mean}$	476.2 km
$R_{core,\ mean}$	409.1 km
ρ_{ice}	1000 kg/m^3
ρ_{core}	2700 kg/m^3
ρ_{mean}	2078.3 kg/m^3

a, b, c = dimensions of an ellipsoid
R = radius
ρ = density

Sensitivity Study

- Gravity field sample statistics

	Nominal	1-sigma
GM (km^3/s^2)	63.13	0.0135
J_2	0.00940	0.0016
J_3	0	0.0209
J_4	-0.000316	0.0075
J_5	0	0.0005
J_6	0.0000305	0.00015

Direct Expansion Model

	CERES-T1	CERES-T2	CERES-T3	CERES-T4	CERES-T5
T_{20} r_{mean} (km)	-21.53	-21.53	-21.53	-21.53	-21.53
$T_{20}c$ r_{mean} (km)	-18.5	-18.5	-18.5	-18.5	18.5
T_{30} r_{mean} (km)				3.4	-3.4
$T_{33}c$ r_{mean} (km)		0.3	0.8		
J_2	1.01 x 10-2	1.02 x 10-2	1.02 x 10-2	1.01 x 10-2	1.01 x 10-2
J_3				-5.23 x 10-4	5.23 x 10-4
C_{33}		-7.71 x 10-4	-2.32 x 10-3		
J_s	-2.39 x 10-4	-2.41 x 10-4	-2.60 x 10-4	-2.41 x 10-4	-2.41 x 10-4

Gravity Monte Carlo

- Kaula uncertainty > Gravity Monte Carlo uncertainty
  - It is very challenging to perturb the shape and density distribution enough to see substantial differences in the gravity field
  - We know that Kaula rule is very conservative (i.e., large) for lower-degree harmonics
- Orbital sensitivity study used larger uncertainty for all harmonics but J2
  - Orbital sensitivity analysis is conservative
  - The impacting cases are likely to have resulted from aphysical gravity field

Degree	Kaula rule for each degree (1-sigma)	Maximum coefficient uncertainty for each degree (1-sigma)
2	2e-3	1e-3
3	1e-3	8e-4
4	5e-4	1.5e-4
5	3e-4	1e-4
6	2e-4	1e-4

CBE Ceres Gravity

Coefficient	Mean (1-sigma)	Uncertainty (1-sigma)
J2	9e-3	1e-3
Other 2nd-degree terms	0	1e-3
J3	0	3e-4
Other 3rd-degree terms	0	3e-4, except σ_{C33} = 8e-4
J4	-3.2e-4	1.5e-4
Other 4th-degree terms	0	1e-4
J5	3e-5	1e-4
Other 5th-degree terms	0	1e-4

CBE vs HAMO gravity field

- Prior to descending into LAMO, the estimated values of the spherical harmonics in HAMO and their uncertainties are compared against CBE gravity field
  - They agree within 3-sigma
  - They are well within the no-impact region of the butterfly plot

	CBE Value	CBE 1-sigma uncertainty	HAMO Value	HAMO 1-sigma uncertainty
J_2	9.4e-3	1e-3	1.1506e-2	1e-6
J_3	0.0	3e-4	-4.96e-5	1.4e-6
J_4	-3.16e-4	1.5e-4	-5.25e-4	3e-6