Jet Propulsion Laboratory
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Integrated Wavefront Correction and Exoplanet Detection

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Outline

1. Background
   a) Purpose of wavefront correction
   b) Types of wavefront correction
   c) Detection methods
2. Combining wavefront correction and signal detection

Purpose of WFSC

Goals of Wavefront Sensing & Control (WFSC):

1. Clean up PSF
   - i.e., recover diffraction-limited seeing

2. Reduce wings of stellar PSF back toward ideal level.
   - Easier to see companions directly
   - Reduces photon shot noise from star
   - (A coronagraph can further suppress the stellar PSF wings)

Deformable Mirrors

- Use Deformable Mirrors (DMs) to correct aberrations
- DMs change the path length (phase) of light
- Array of posts behind a continuous facesheet

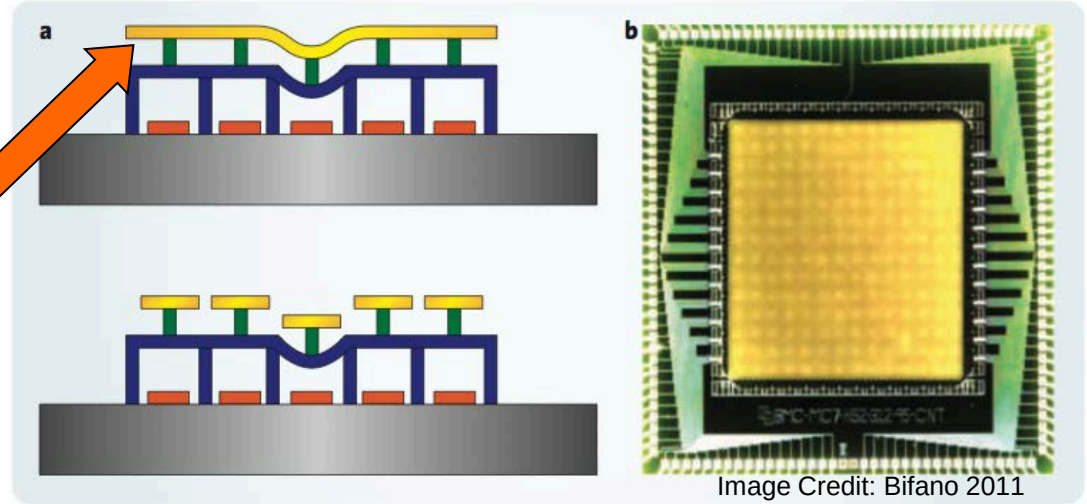
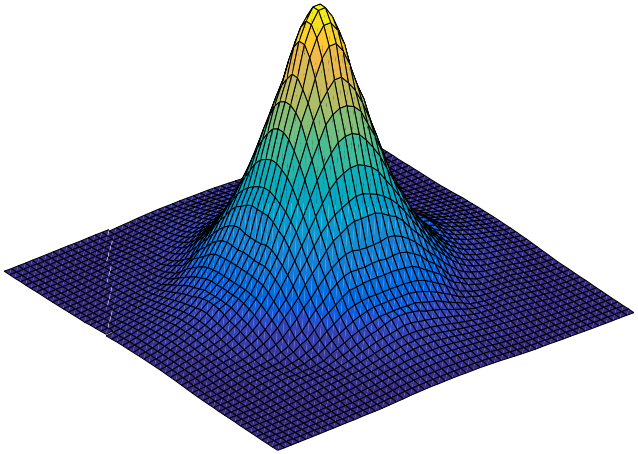


Image Credit: Bifano 2011


Influence Function

$f(x, y)$

- Phase at DM well-approximated as linear superposition of actuator **influence functions**:

$$\phi(x, y) = \frac{2}{\lambda} \sum_{q}^{N_{act}} u_q f(x - x_q, y - y_q)$$

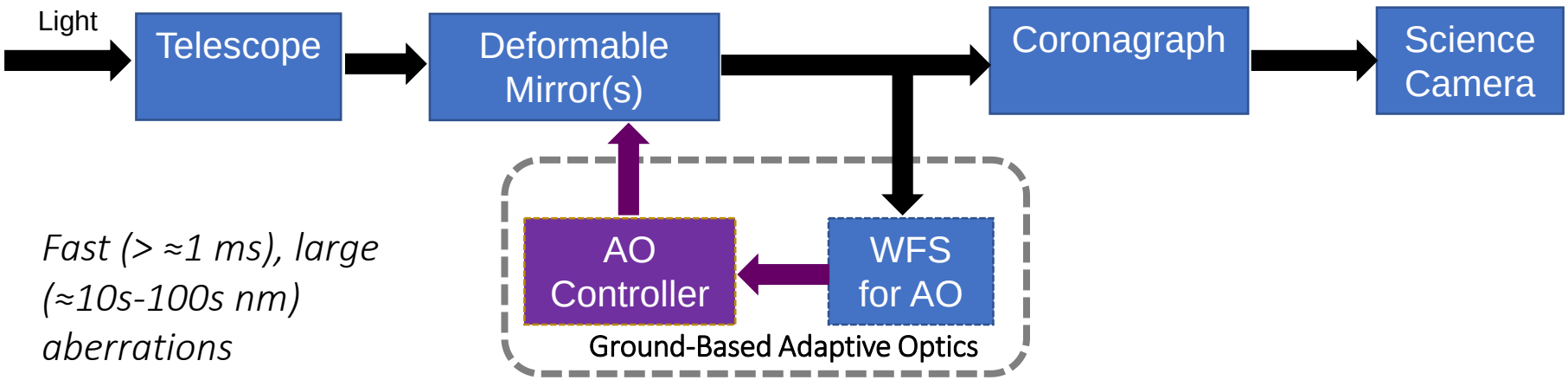
$\lambda =$ Wavelength of light

$u =$ DM actuator displacement commands

$\phi =$ Phase change from DM

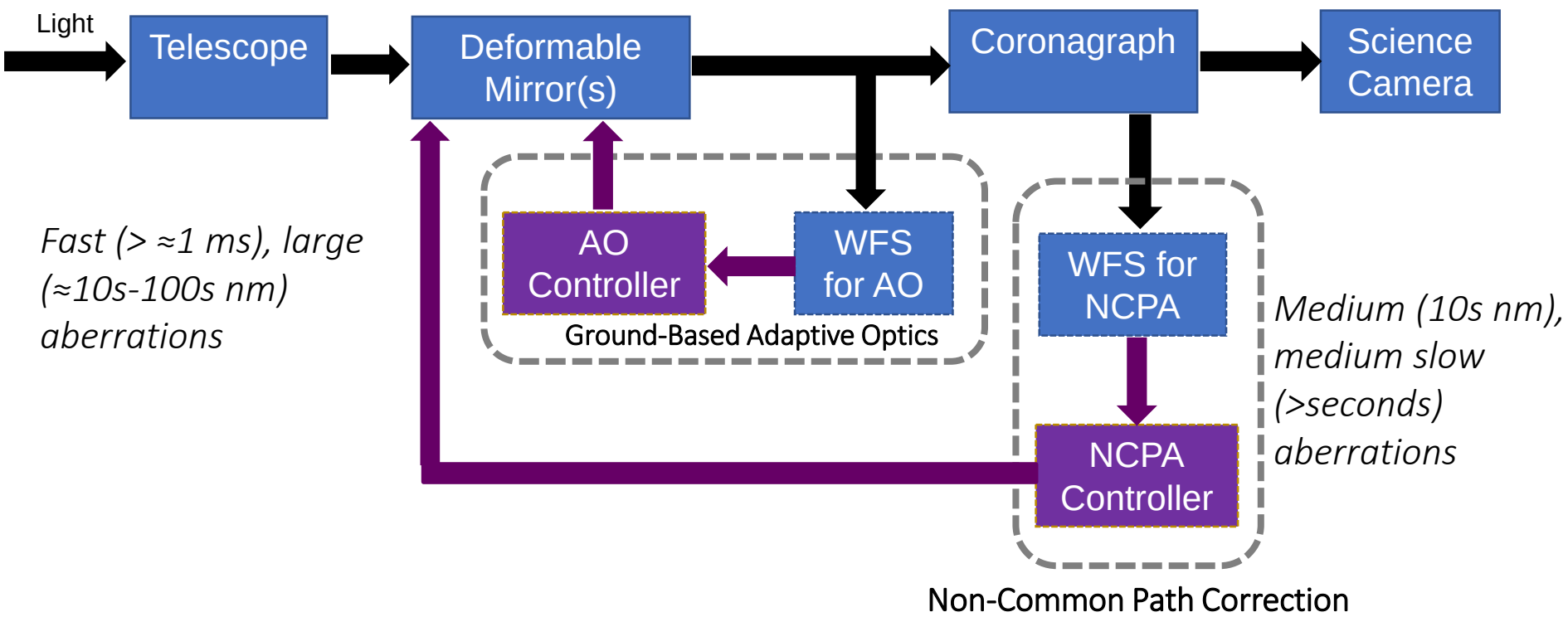
Types of Wavefront Correction

Types of Wavefront Correction

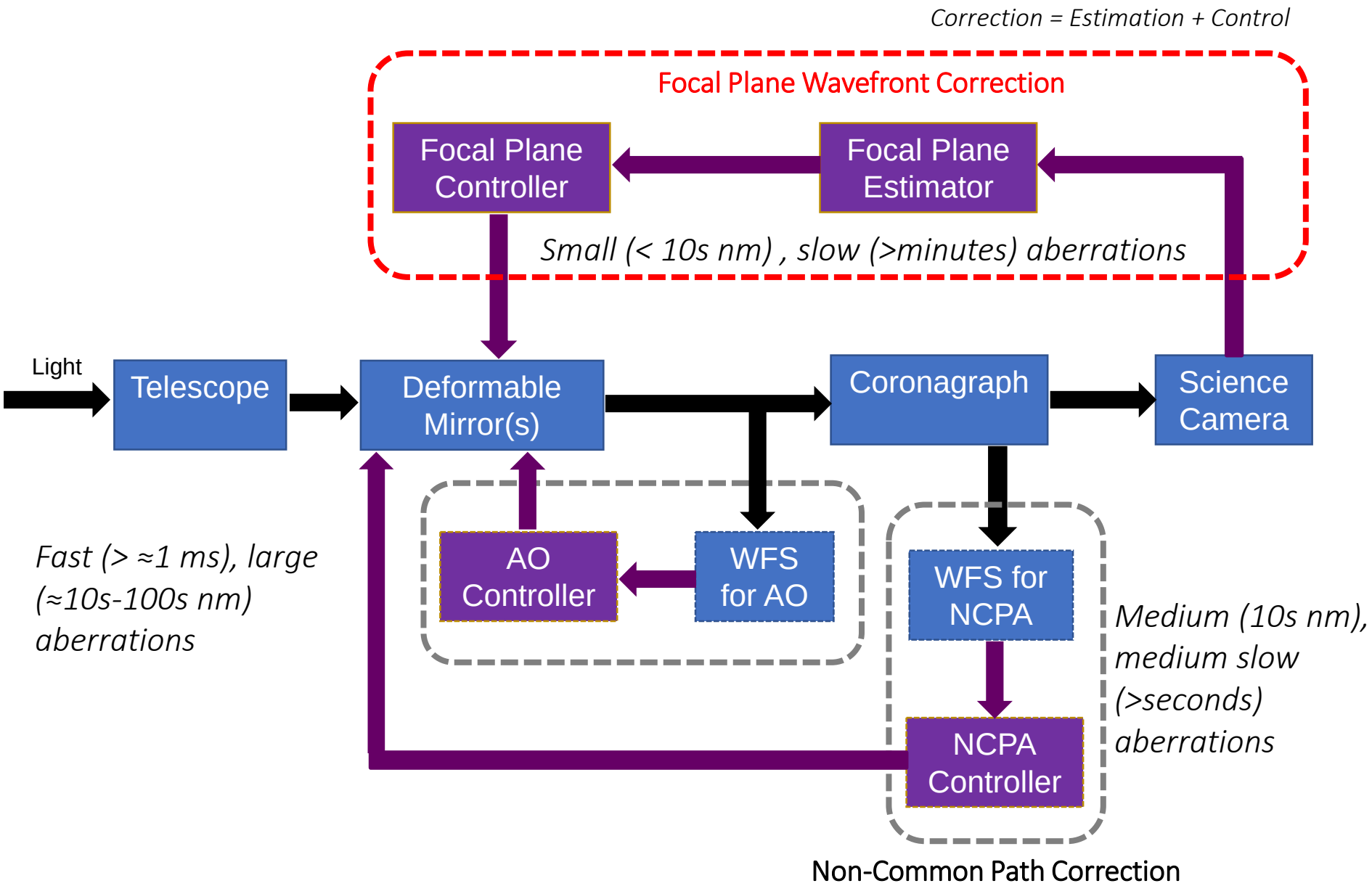


Fast (> ≈1 ms), large (≈10s-100s nm) aberrations

Types of Wavefront Correction



Types of Wavefront Correction



Adaptive Optics for the Ground

Correct phase aberrations from atmospheric turbulence and imperfect optical surfaces

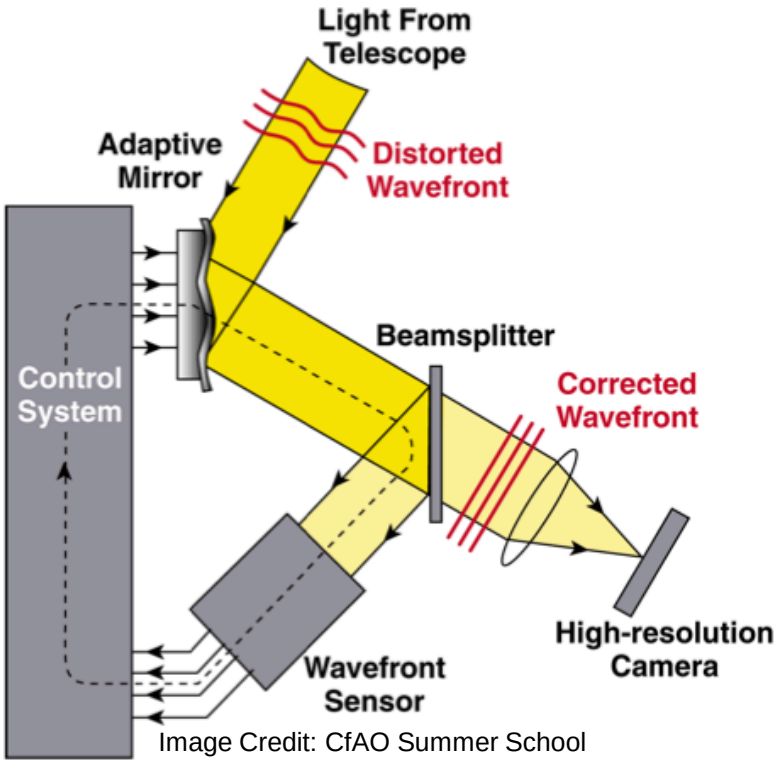


Image Credit: CfAO Summer School

Adaptive Optics (AO):

1. Measure phase errors with wavefront sensor (WFS)
2. Apply opposite phase on DM

Main issues for high-contrast imaging:

- Aberrations after WFS not sensed and corrected
- AO corrects only phase errors
- ➢ Can reach only $\approx 10^{-5}$ contrast

NCPA Correction

Correct non-common path aberrations (NCPAs) upstream of coronagraph

Examples: CAL in P1640, ZELDA in SPHERE, CAL in GPI

Main issues for high-contrast imaging:

- Still doesn't sense aberrations downstream of coronagraph
- ➢ Achievable contrast floor of $\approx 10^{-6} - 10^{-7}$

Focal Plane Wavefront Correction

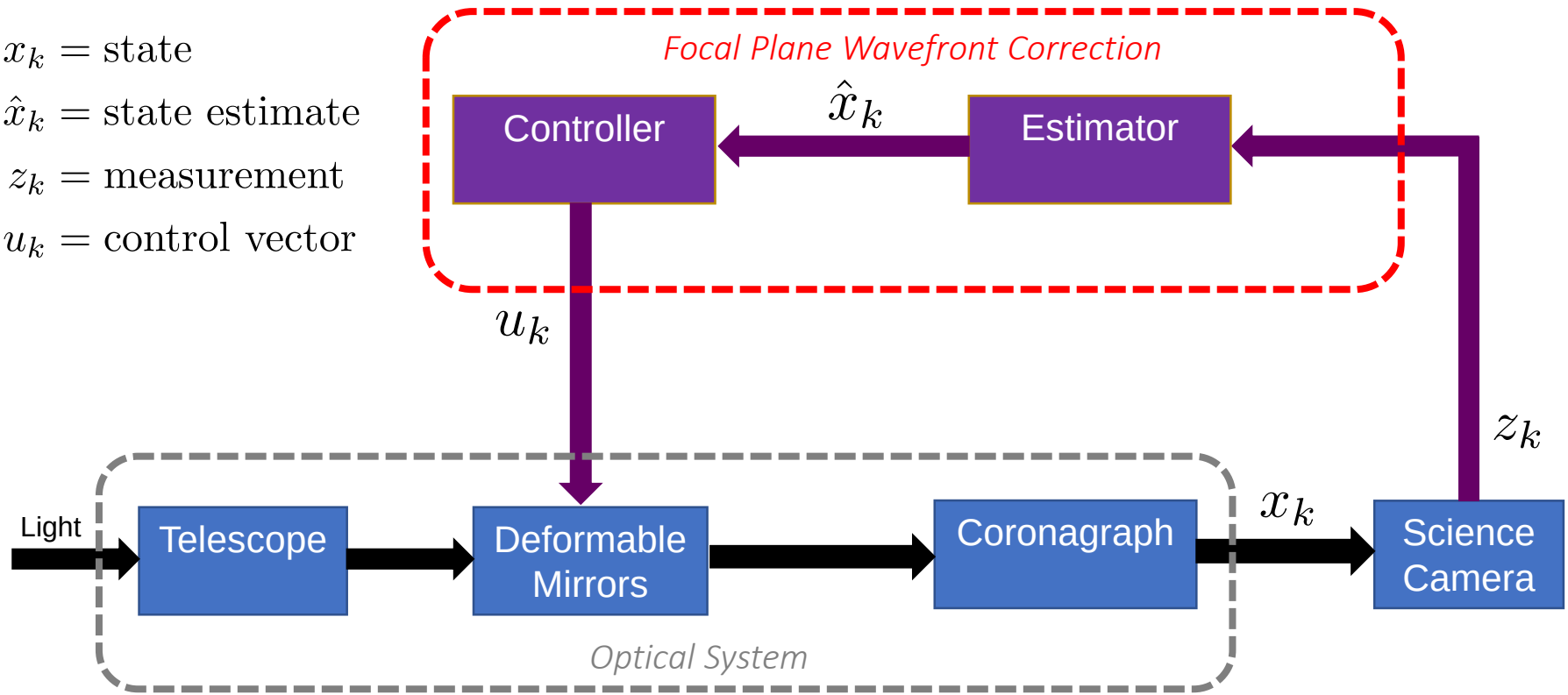
- To correct *all* quasi-static speckles:
  - Estimate and control starlight directly in final **focal plane.**
  - Use **science camera as WFS** to estimate all aberrations.
- **Estimation + Control** (= Correction) is **iterative:**
  - Model errors, estimation errors, nonlinearities

x_k = state

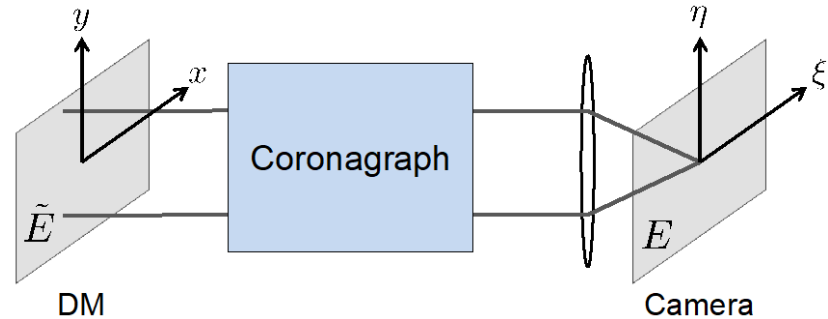
$\hat{x}_k$ = state estimate

z_k = measurement

u_k = control vector



Focal Plane Wavefront Control



Linearized Model of E-field from DM

DM changes phase of incident E-field

$$\tilde{E}_k(x,y) = \tilde{E}_{k-1}(x,y)e^{i\Delta\phi_k(x,y)}$$

DM phase changes are very small → linearize

$$e^{i\Delta\phi(x,y)} \approx 1 + i\Delta\phi(x,y)$$

Fourier optics → linear propagation to camera

$$E_k(\xi,\eta) = \mathcal{C}\{\tilde{E}_{k-1}(x,y)\} + \mathcal{C}\{i\tilde{E}_{k-1}(x,y)\Delta\phi(x,y\}$$
$$= E_{k-1}(\xi,\eta) + G_{k-1}\Delta u_k$$

$\tilde{E}$ = E-field at DM
E = E-field at camera
k = Correction iteration #
$\Delta\phi$ = Change in phase at DM
Δu = Change in DM commands
$i = \sqrt{-1}$
$\mathcal{C}\{\}$ = Linear operator
G = DM response matrix

Optimal Controller: (Stroke Minimization/Electric Field Conjugation) Pueyo 2009, Give'on 2007

- Cost function to minimize:

$$J = E_k^* E_k + \alpha u^T u$$

Minimize energy in dark hole

α = Tuning parameter
$\mathbb{I}$ = Identity matrix
$\mathcal{R}\{\}$ = Real part of complex value

- Optimal command:

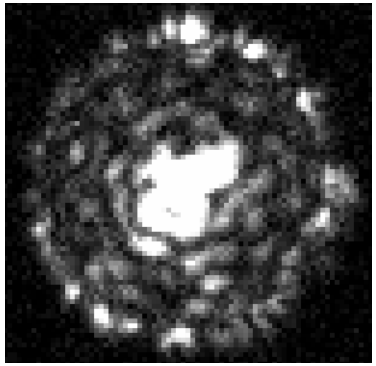
$$u_{opt} = (G_{k-1}^* G_{k-1} + \alpha\mathbb{I})^{-1}\mathcal{R}\{G_{k-1}^* E_{k-1}\}$$

Need an estimate of E_{k-1} to use control…

Focal Plane Wavefront Estimation

What's in an Image?

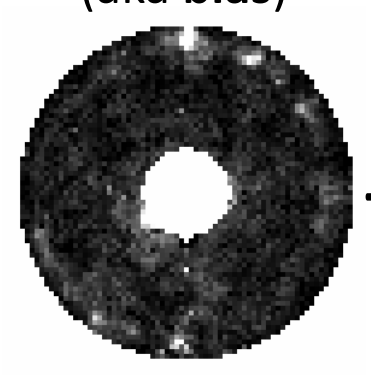
measured intensity



(JPL HCIT lab image)

$=$

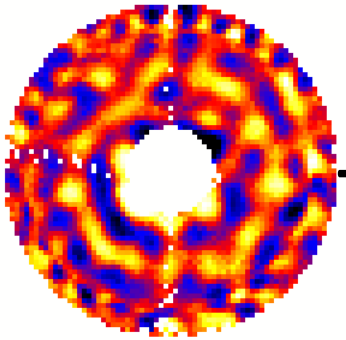
incoherent intensity (aka **bias**)



(exoplanets, disks, stray light, background stars and galaxies)

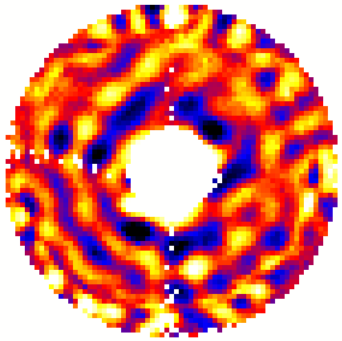
$+$

Starlight: Real{E}



$+i$

Starlight: Imaginary{E}



$\Big|^2$

(starlight only)

Image Credit:
Brian Kern & Eric Cady

- Star and planet are mutually incoherent → add in intensity

- **Estimation Problem**:
  - Controller needs **complex-valued** E-field, but **camera measures intensity only**.
- **Solution**:
  1. Modulate starlight by actuating DMs.
  2. Infer E-field from modulated intensity measurements

Estimation via Pair-Wise DM Probing

k = Correction iteration #
j = Probe #
$\mathbf{p_{k,j}} = \mathbf{G_k u_j}$ = probe field at camera
$I_{k,j}$ = Measured intensity
I_{inco} = Incoherent intensity
$n_{k,j\pm}$ = Measurement noise: shot, readout, dark current

- Estimate light at each pixel separately
- Take images for +/- probe shapes on DM:

$$I_{k,j\pm} = |E_k \pm p_{k,j}|^2 + I_{inco,k} + n_{k,j\pm}$$
$$= |E_k|^2 + |p_{k,j}|^2 \pm 2\mathcal{R}\{E_k^* p_{k,j}\} + I_{inco,k} + n_{k,j\pm}$$

- Subtract +/- probed images to isolate cross term (heterodyne gain)

$$\Delta I_{k,j} = I_{k,j+} - I_{k,j-} = 4\mathcal{R}\{E_k^* p_{k,j}\} + n_{k,j}$$
$$= 4\begin{bmatrix}\mathcal{R}\{p_{k,j}\} & \mathcal{I}\{p_{k,j}\}\end{bmatrix}\begin{bmatrix}\mathcal{R}\{E_k\}\\ \mathcal{I}\{E_k\}\end{bmatrix} + \begin{bmatrix}n_{k,j}\end{bmatrix}$$

At least 2 probes (since 2 unknowns)

$$\underbrace{\begin{bmatrix}\Delta I_{k,1}\\ \vdots \\ \Delta I_{k,N_{pp}}\end{bmatrix}}_{= z_k} = 4\underbrace{\begin{bmatrix}\mathcal{R}\{p_{k,1}\} & \mathcal{I}\{p_{k,1}\}\\ \vdots & \vdots \\ \mathcal{R}\{p_{k,N_{pp}}\} & \mathcal{I}\{p_{k,N_{pp}}\}\end{bmatrix}}_{= H_k}\underbrace{\begin{bmatrix}\mathcal{R}\{E_k\}\\ \mathcal{I}\{E_k\}\end{bmatrix}}_{= x_k} + \begin{bmatrix}n_{k,1}\\ \vdots \\ n_{k,N_{pp}}\end{bmatrix}$$

Measured — Model-based — Unknown

Give'on+ 2007

$$z_k = H_k x_k + n_k$$

Least Squares Estimate:
Batch Process Estimator (BPE)

$$\boxed{\hat{x}_k = (H_k^T H_k)^{-1} H_k^T z_k}$$

Image Post-Processing Methods

RDI: Remove starlight by subtracting a template PSF

Target Star — Template PSF (no companion) — Circumstellar Disk Revealed

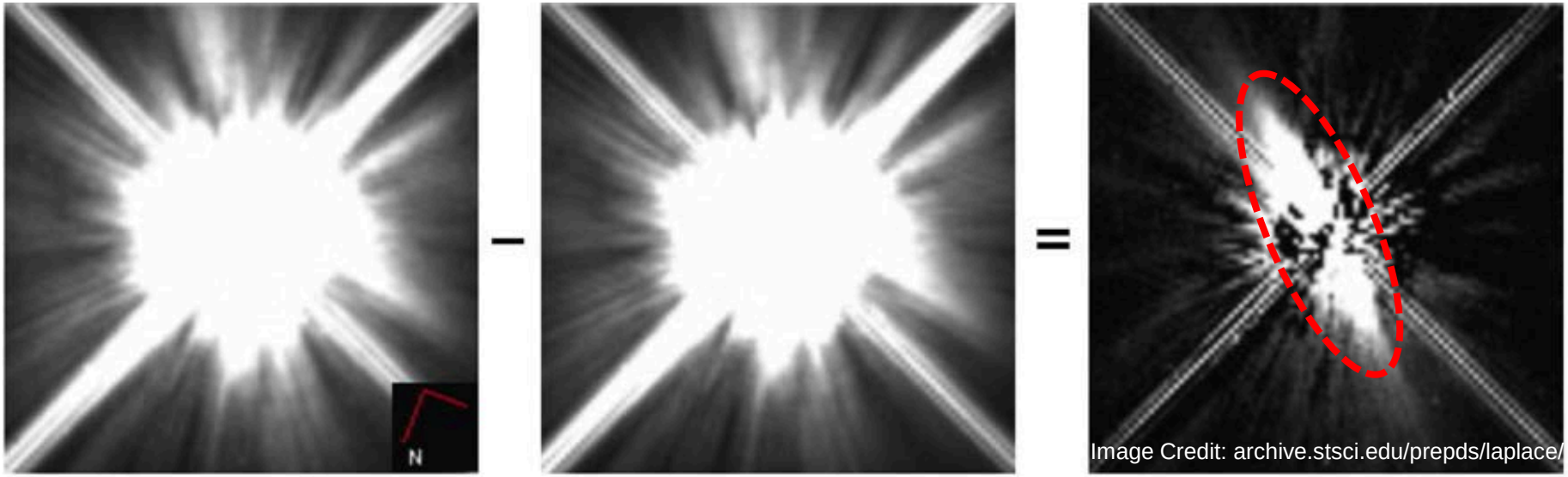


Image Credit: archive.stsci.edu/prepds/laplace/

ADI: Take advantage of planet moving w.r.t. stellar speckles during telescope/sky rotation.

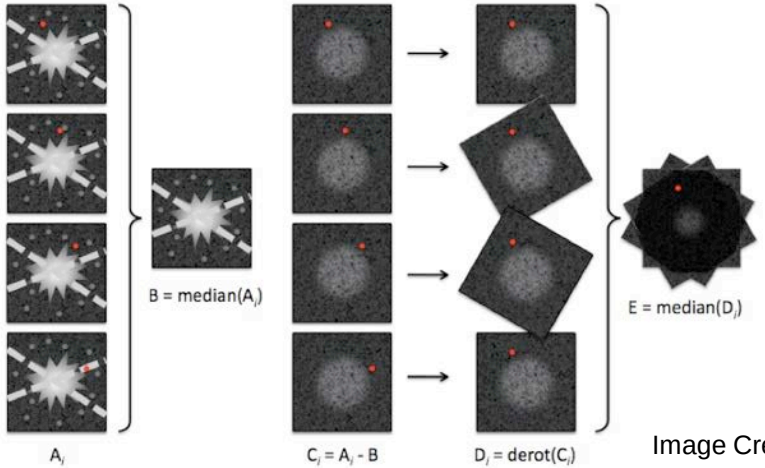


Image Credit: astrobites.org

Coherence Differential Imaging (CDI)

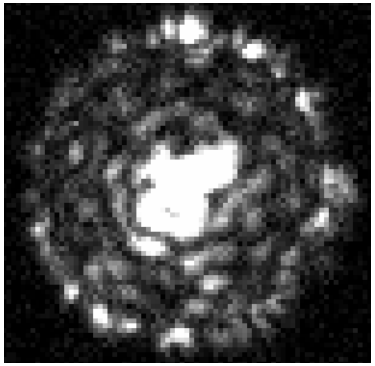
- **Alternative idea**: **Modulate starlight*** without moving telescope
  - Stellar speckles change while planet stays the same
  - Called "**coherence differential imaging**" (**CDI**)
  - Not yet demonstrated on-sky

** See Frazin 2013,2016; Young 2015; Riggs+ 2015*

Recursive Coherence Differential Imaging (RCDI)

- **My PhD work**: Use **wavefront correction** to modulate the starlight
  - ➢ FPWC already separates incoherent light from the starlight **efficiently**

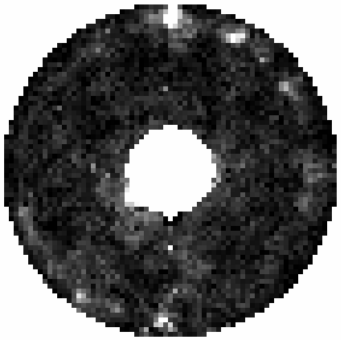
measured intensity



(JPL HCIT lab image)

$=$

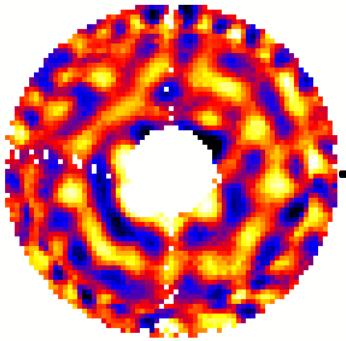
incoherent intensity (aka **bias**)



(exoplanets, disks, stray light, background stars and galaxies)

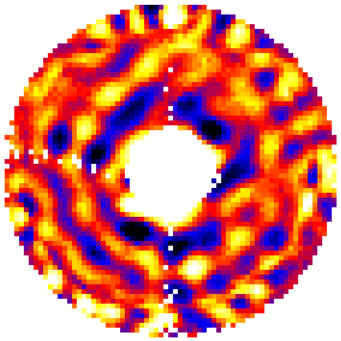
$+$

Starlight: Real{E}



$+i$

Starlight: Imaginary{E}



$\Big|^2$

(starlight only)

Image Credit:
Brian Kern & Eric Cady

1. Wavefront correction time not lost as overhead
2. Use **nonlinear**, near-optimal estimator (iterated extended Kalman filter, **IEKF**) to estimate both incoherent intensity and stellar E-field
3. Combine IEKF with **efficient** pair-wise probing scheme
   - ➢ We call it **Recursive Coherence Differential Imaging (RCDI)**

Comparison of DI Methods

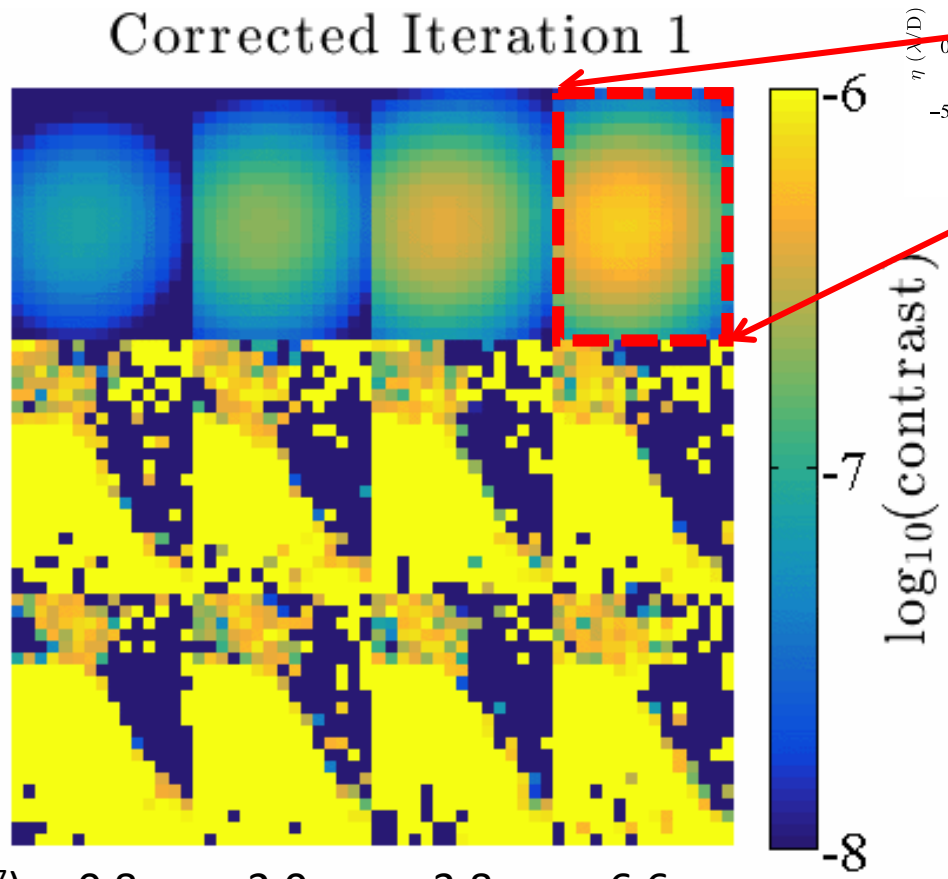
Unique Limitations of Each Method

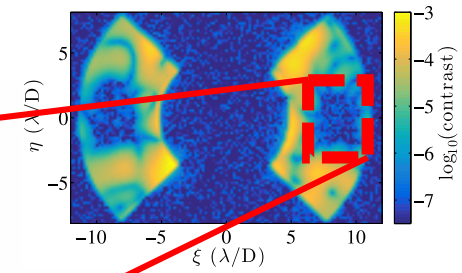
- RDI
  - Wavefront stability during slews
  - Long-term repeatability of dark hole speckles
  - Similarity of stellar size & spectra
  - Number of PSFs in image library
  - Image registration
- ADI
  - Wavefront stability during rolls
  - Image registration
- RCDI
  - Accuracy of the optical model
  - 2x higher stellar shot noise variance from probing

Takeaway: RCDI can be better if stability of stellar wavefront is a concern or slew time is long compared to imaging time.

RCDI Lab Experiment at Princeton

- Planet-like signal injected into the ***testbed***
- 4 trials at different contrasts





Planet is in right-side dark hole

Scaled Template PSF:

KF's Incoherent Estimate (*not* recursive):

IEKF's Incoherent Estimate (recursive):

True Planet Contrast (x 10^{-7}): 0.8 2.0 3.8 6.6

Riggs+, JATIS 2016

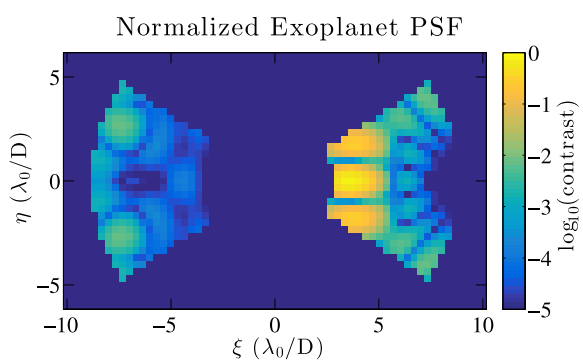
Planet is found using wavefront correction images!

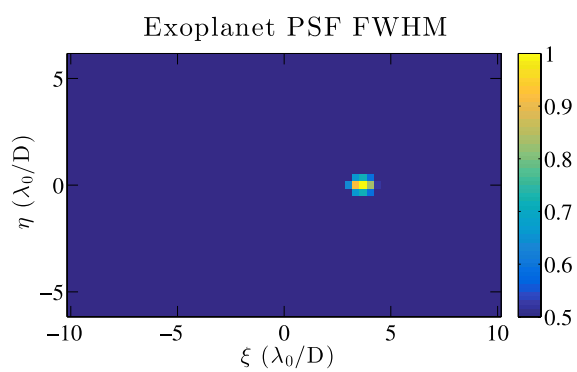
RCDI Monte Carlo Simulations

Key Point: To characterize RCDI detection performance statistically:

- Perform Monte Carlo trials of wavefront correction.
- Include a planet in half the cases.
- Compute the receiver operating characteristic (ROC) curves to compare **probability of detection** vs **false alarm rate**.

- **Monte Carlo** simulations of wavefront correction:
  - Changing optics' phase errors & DM actuator gain errors
  - Photon shot noise only (no dark current or readout noise)
- **100 trials** with/without planet at xi = 3.5 λ_0/D
- Planet at 3×10^{-10} contrast (≈3x below starlight)
- **SPLC** design for WFIRST CGI
- **Static** optical system (no low-order disturbances)
- **Low flux**: 1 photon/image/pixel (at planet peak, on average)



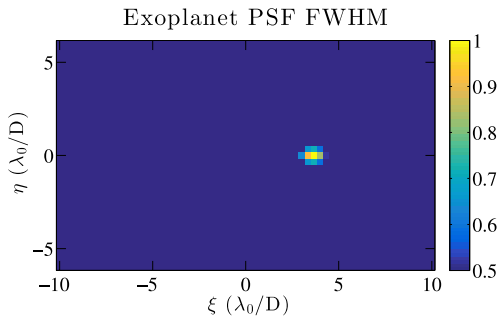


- Linear plot
- 11 pixels within FWHM

Full write-up in Riggs thesis, 2016: http://arks.princeton.edu/ark:/88435/dsp01fb494b824

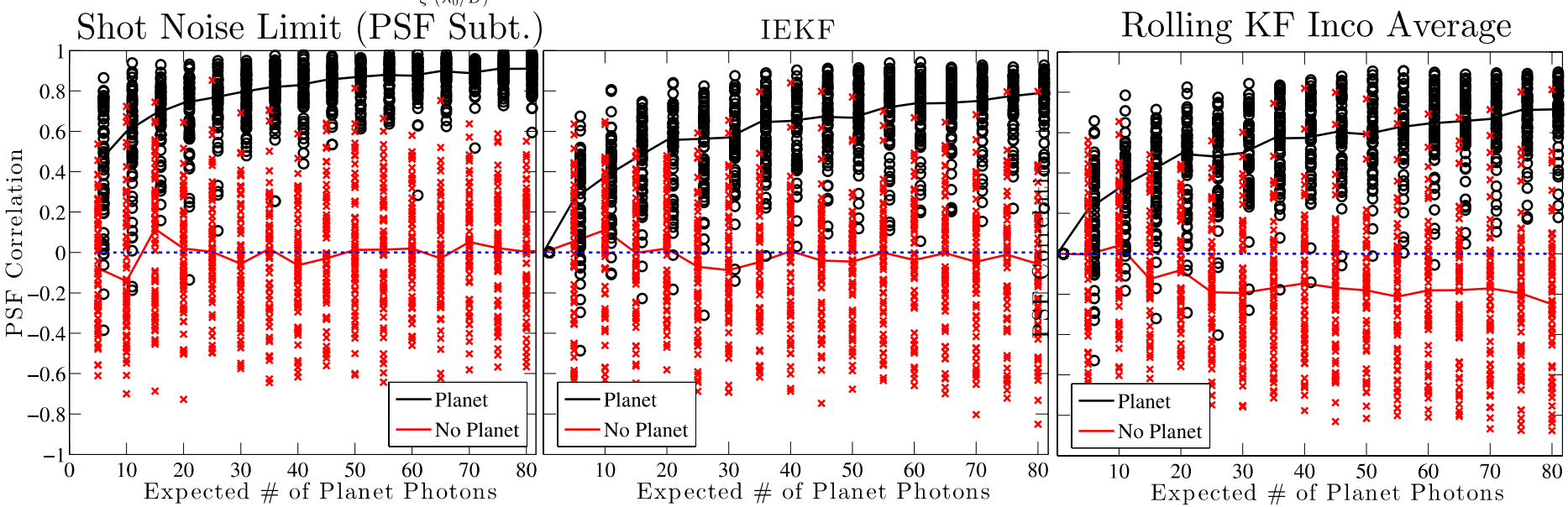
Planet PSF Correlation

Normalized **2-D correlation** between planet's template PSF and incoherent estimate



Model-Based Template PSF

- 11 pixels within FWHM



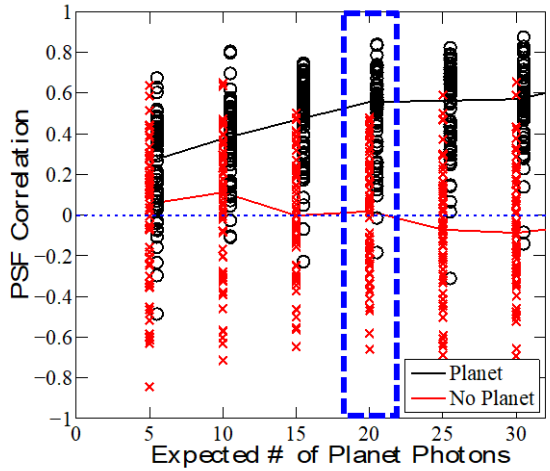
- ➢ PSF correlation increases with exposure time if planet is present
- ➢ Rolling average is biased negative with no planet present

ROC Curve Construction

Receiver Operator Characteristic (ROC) Curve:

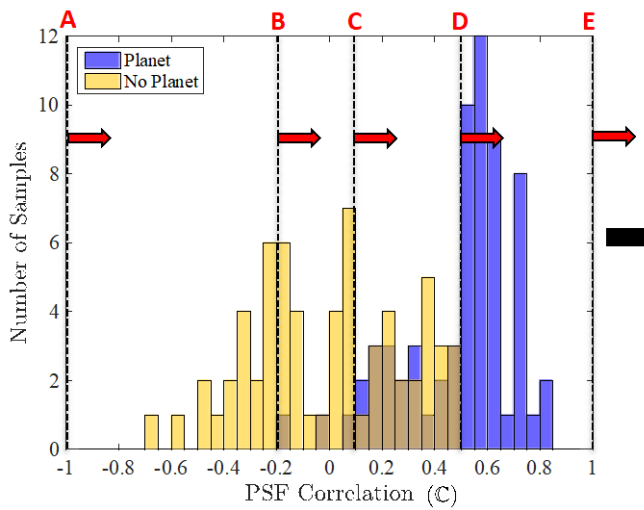
Plots **probability of detection** vs **false alarm rate**

- One ROC curve per time step
- Built by setting minimum PSF correlation value (threshold)

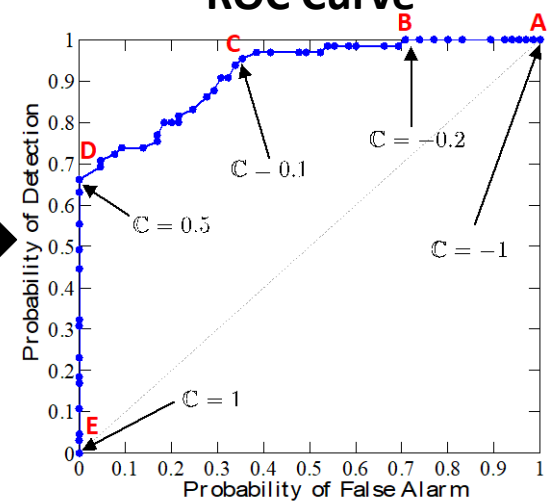


Probability of detection =
- Fraction of all true planets counted

False alarm rate =
- Fraction of spurious signals counted as planets



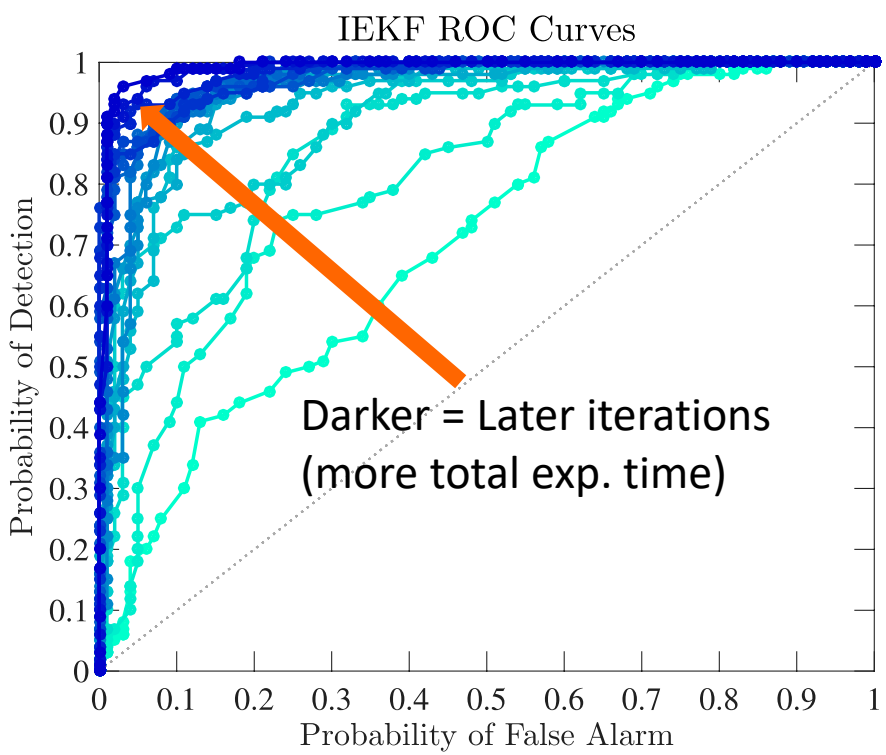
ROC Curve



ROC and AUC Curves

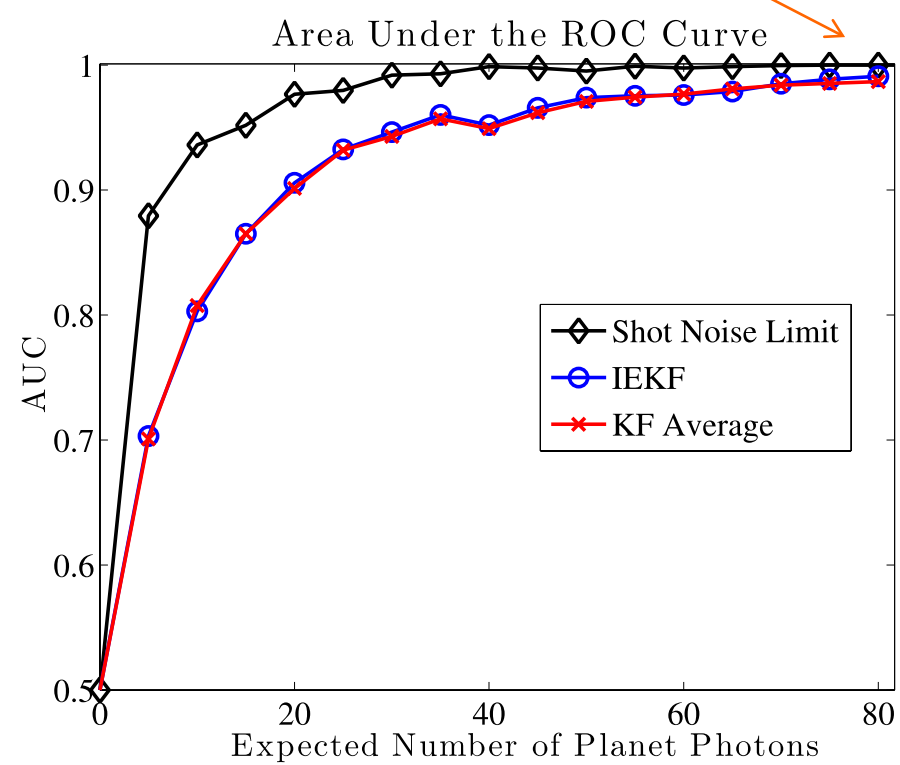
Tradeoffs:

- (Higher) Probability of detection
- (Lower) Probability of false alarm
- (Lower) Total exposure time



- Near-perfect detection after ~6 correction steps

AUC=1 → Perfect classification of signals



- IEKF and rolling average give equal detection performance.
- Perform close to the fundamental noise limit

Summary

- AO corrects atmospheric turbulence
- NCPA correction for some static errors in a telescope
- FPWC required for high contrast (better than 1e-7)
- ***Detection can be done during wavefront correction***
  - Focal plane wavefront estimation already separates incoherent light and the stellar E-field
    - ➢ Exoplanets and disks are in the incoherent signal
  - A recursive estimator (e.g., the IEKF) can build a near-optimal incoherent light estimate from all correction images.
  - Simulations and a lab experiment prove the viability of this recursive CDI (RCDI)

Backup Slides

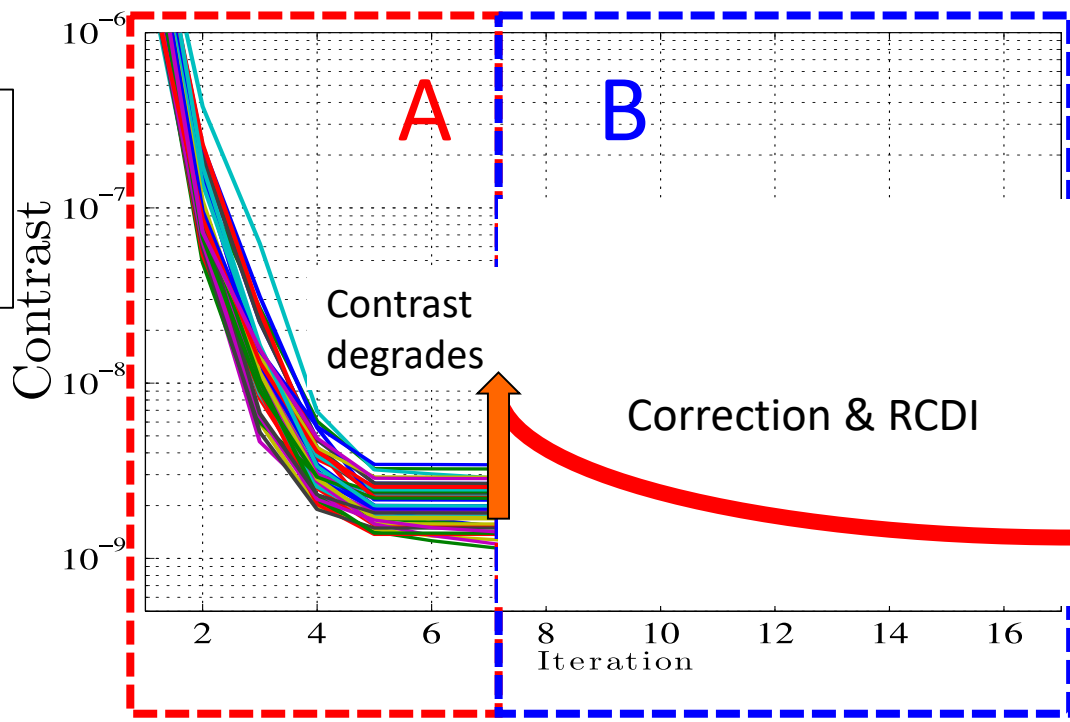
Future Work

Observation Scenarios:

Two phases of correction:

Stage A: "**Bright star**" correction

Stage B: "**Science star**" correction



Add wavefront instability during telescope maneuvers:

- Because of wavefront changes, PSF subtraction isn't as accurate
- RCDI still works

Where is the crossover when RCDI becomes more efficient?

Kalman Filter (KF)

- BPE ignores previous estimates
- KF optimally combines previous data with new measurements
- Enables faster correction and robustness to measurement noise

Kalman Filter Equations (per pixel)

$$\hat{x}_k(-) = \hat{x}(+)_{k-1} + \Gamma u_{k-1}$$
$$P_k(-) = P_{k-1}(+) + Q_{k-1}$$
$$K_k = P_k(-) H_k^T [H_k P_k(-) H_k^T + R_k]^{-1}$$
$$\hat{x}_k(+) = \hat{x}_k(-) + K_k[z_k - H_k \hat{x}_k(-)]$$
$$P_k(+) = [\mathbb{I} - K_k H_k] P_k(-)$$

Model-based updates of state x & state covariance P

Kalman gain: Balances model and measurement error

Measurement-based updates of x & P

Groff & Kasdin 2013

Matrix	Representation	Dimension
Linearized State Response	$\Phi = \mathbb{I}$	2×2
Linear Observation	H_k	$N_{pp} \times 2$
Linearized Complex Response of Probing DM	G	$1 \times N_{act}$
Linearized Response of Probing DM	$\Gamma = \begin{bmatrix} \mathcal{R}\{G[1]\} \cdots \mathcal{R}\{G[N_{act}]\} \\ \mathcal{I}\{G[1]\} \cdots \mathcal{I}\{G[N_{act}]\} \end{bmatrix}$	$2 \times N_{act}$
Disturbance Response	$\Lambda = \Gamma$	$2 \times N_{act}$
State Covariance (Time Update)	$P_k(-) = E[(x_k - \hat{x}_k(-))(x_k - \hat{x}_k(-))^T]$	2×2
State Covariance (Measurement Update)	$P_k(+) = E[(x_k - \hat{x}_k(+))(x_k - \hat{x}_k(+))^T]$	2×2
Process Noise	$Q_k = \Lambda E[w_k w_k^T] \Lambda^T$	2×2
Sensor Noise	$R_k = E[n_k n_k^T]$	$N_{pp} \times N_{pp}$
Kalman Gain	K_k	$2 \times N_{pp}$

Incoherent estimate is not recursive:

$$\hat{I}_{inco,k} = I_k - |\hat{E}_k|^2$$

Unprobed image

Starlight estimate

Exoplanets are in the incoherent signal

Extended Kalman Filter (EKF)

Goal: estimate **both** stellar E-field and incoherent intensity **recursively**

- Because planets are in incoherent signal
- Requires **nonlinear** filter

Riggs et al. 2016

	BPE & KF		**EKF**	
State:	$x_k = \begin{bmatrix} \mathcal{R}\{E_k\} \\ \mathcal{I}\{E_k\} \end{bmatrix}$	→	$x_k = \begin{bmatrix} \mathcal{R}\{E_k\} \\ \mathcal{I}\{E_k\} \\ I_{inco,k} \end{bmatrix}$	• Add incoherent signal to state
Measurement:	$z_k = \begin{bmatrix} I_{k,1+} - I_{k,1-} \\ \vdots \\ I_{k,N_{pp}+} - I_{k,N_{pp}-} \end{bmatrix}$	→	$z_k = \begin{bmatrix} I_k \\ I_{k,1+} \\ I_{k,1-} \\ \vdots \\ I_{k,N_{pp}+} \\ I_{k,N_{pp}-} \end{bmatrix}$	• Stop differencing images • Add unprobed image to state

Differences out incoherent signal

EKF Equations

Measurement Vector:

$$z_k = \begin{bmatrix} I_k & I_{k,1+} & I_{k,1-} & \cdots & I_{k,N_{pp}+} & I_{k,N_{pp}-} \end{bmatrix}^T$$
$$= h(x_k) + n_k$$

Riggs et al. 2016

Quadratic Measurement Function:

$$h(x_k) = \begin{bmatrix} |E_k|^2 + I_{inco,k} \\ |E_{k,1+}|^2 + I_{inco,k} \\ |E_{k,1-}|^2 + I_{inco,k} \\ \vdots \\ |E_{k,N_{pp}+}|^2 + I_{inco,k} \\ |E_{k,N_{pp}-}|^2 + I_{inco,k} \end{bmatrix} \approx \begin{bmatrix} |E_k|^2 + I_{inco,k} \\ |E_k + Gu_1|^2 + I_{inco,k} \\ |E_k - Gu_1|^2 + I_{inco,k} \\ \vdots \\ |E_k + Gu_{N_{pp}}|^2 + I_{inco,k} \\ |E_k - Gu_{N_{pp}}|^2 + I_{inco,k} \end{bmatrix}$$

Linearized Observation Matrix:

$$H_k = \left. \frac{\partial h(\hat{x}_k)}{\partial \hat{x}_k} \right|_{\hat{x}_k = \hat{x}_k(-)}$$

Extended Kalman Filter Equations

$$\hat{x}_k(-) = \hat{x}(+)_{k-1} + \Gamma u_{k-1}$$
$$P_k(-) = P_{k-1}(+) + Q_{k-1}$$
$$K_k = P_k(-) H_k^T [H_k P_k(-) H_k^T + R_k]^{-1}$$
$$\hat{x}_k(+) = \hat{x}_k(-) + K_k [z_k - h(\hat{x}_k(-))]$$
$$P_k(+) = [\mathbb{I} - K_k H_k] P_k(-)$$

- Nearly same form as KF's
- Different matrix definitions because of different x & z

Iterated Extended Kalman Filter (IEKF)

- **Problem**: EKF estimates known to be biased
- **Solution**: Iterating the EKF can reduce the bias error
  1. Run EKF
  2. Relinearize about new estimate
  3. Re-compute H & K.
  4. Re-compute x & P.
  5. Repeat steps 2-4 until estimates converge.

Iterated Extended Kalman Filter (IEKF) Equations

$$H_{k,i} = \left.\frac{\partial h(x)}{\partial x}\right|_{x=\hat{x}_{k,i}(+)}$$

$$K_{k,i} = P_k(-)H_{k,i}^T[H_{k,i}P_k(-)H_{k,i}^T + R_k]^{-1}$$

$$\hat{x}_{k,i+1}(+) = \hat{x}_k(-) + K_{k,i}\big(z_k - h(\hat{x}_{k,i}(+)) - H_{k,i}[\hat{x}_k(-) - \hat{x}_{k,i}(+)]\big)$$

$$P_{k,i+1}(+) = [\mathbb{I} - K_{k,i}H_{k,i}]P_k(-)$$

Estimator Comparison Experiments

Review:

Pair-wise Probing: DM modulates image plane with pairs of probes

1. Batch process estimator (BPE): linear model, no previous data used
2. Kalman filter (KF): linear model, recursive **star** estimation
3. Extended Kalman Filter (EKF): nonlinear model, recursive **star** and **planet** estimation
4. Iterated Extended Kalman Filter (IEKF): nonlinear model, recursive **star** and **planet** estimation, less bias error

Borde & Traub 2006; Give'on+ 2007, 2011;
Groff & Kasdin 2013; Groff et al. 2016; Riggs+ 2016