Optical Error Budgeting Using Linearized Ray-Trace Models

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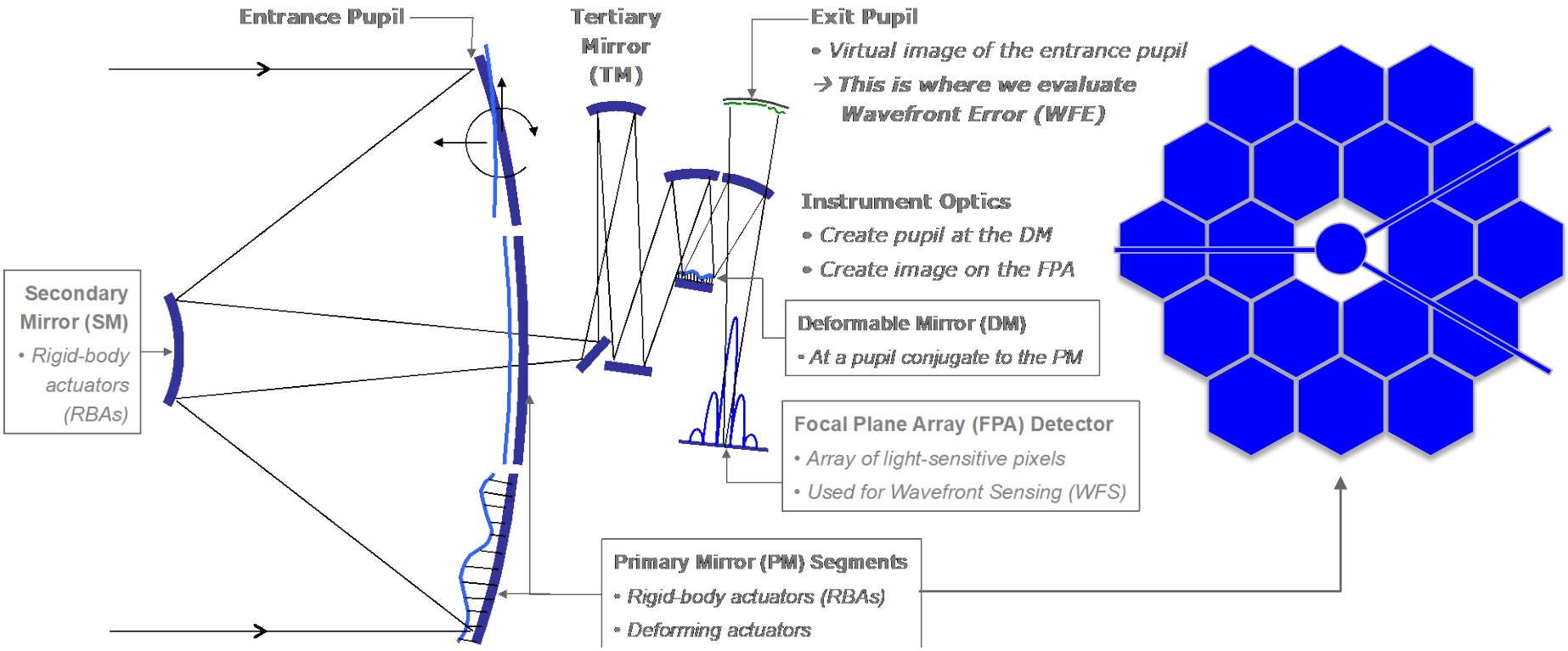
Error Models for Performance Budgeting

- **Classic system engineering practice utilizes "RSS" Wavefront Error (WFE) budgets, wherein:**
  - $WFE = \mathrm{sqrt}((coef_1 * \mathrm{std}(error_1))^2 + (coef_2 * \mathrm{std}(error_2))^2 + \ldots)$
  - Here the errors of interest include misalignments, figure errors, and sensing and control errors
  - For both static and drift conditions

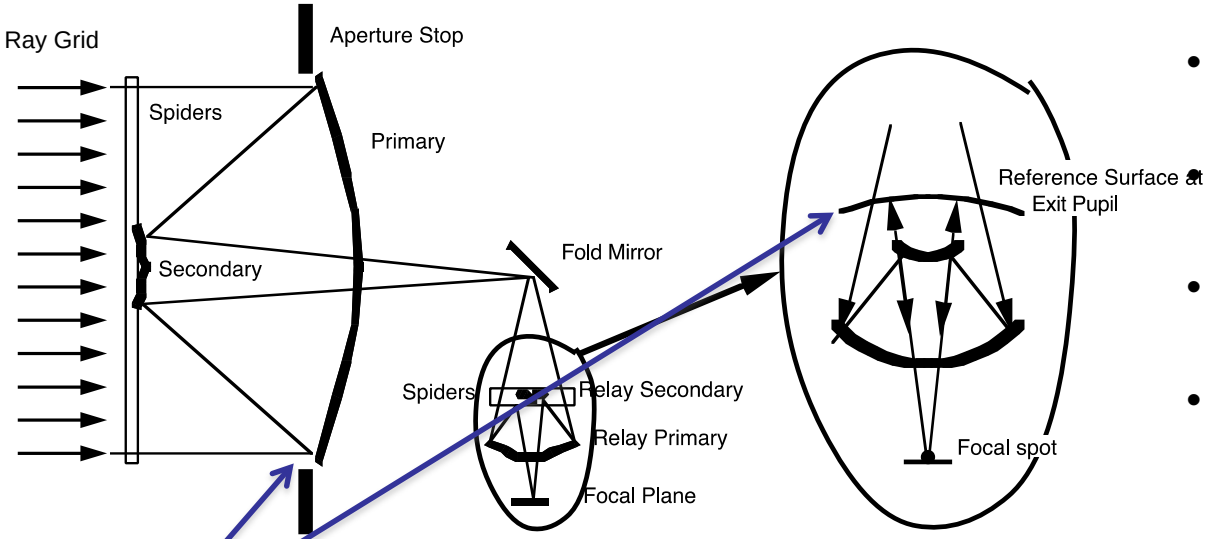
- **This paper:**
  1. Derives WFE coefficients for component errors, based on detailed linearized ray-trace models
  2. Derives models for end-to-end performance, including active optical controls, such as rigid-body alignment actuators, or deformable mirrors
  3. Shows that RSS error budgets are covariance analyses
  4. Illustrates the error modeling process for a notional "LUVOIR"-like large active space telescope
  5. Briefly discusses extensions to other metrics (Strehl, MTF, ...) and to other conditions (correlated errors, multiple fields of view, ...)

Example Telescope

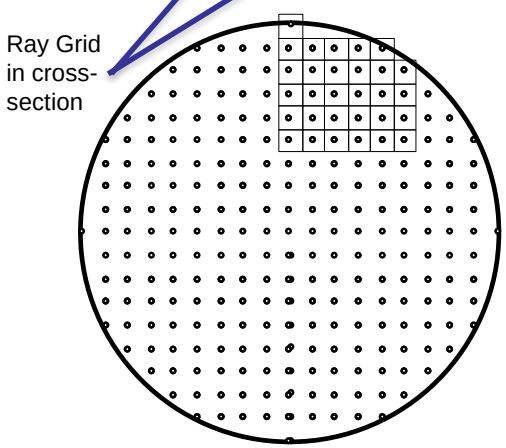


- Three-Mirror Anastigmat (TMA) wide field notional design, from the 2008 “ATLAST” study (*see references*)
  - Segmented, 8-meter class aperture
  - Rigid-Body Actuators (RBAs) on the PM segments and SM
  - Deformable Segments, and a Deformable Mirror at a reimaged pupil

Modeling Image Formation – in a Nutshell



- Source at infinity: modeled as a grid of parallel rays
- Each ray samples a small patch of the beam
- Rays trace through the entire optical train to the FPA
- Then back to the exit pupil, where the grid is restored



- The WF phase is computed as the Optical Path Difference (OPD) of the ray grid at the exit pupil → **WFE metric**
- The ray grid is mapped to the elements of a complex amplitude (CA) matrix, determining CA phase (and support)
- Fourier physical optics propagates the CA to the image plane, capturing diffraction effects, via an FFT + phase shift
- The modulus squared of the field at the FPA is resampled to match FPA pixel size – voila, an image! → **Strehl, EE metrics**

Linearized Optical Models: Wavefront

- **In quantifying optical performance, it is convenient to vectorize the various performance variables...**
- OPD **matrix is a 2D map of the WF, computed by an optical analysis code, or measured by a WF sensor, interferometer, etc.**

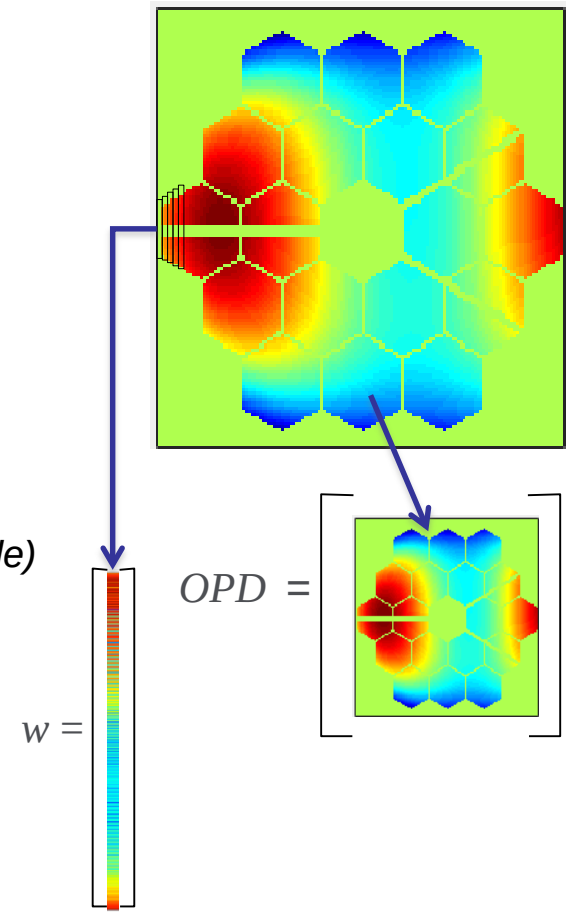
$$OPD = \text{MACOS}(\text{prescription data, various commands})$$ *(or other code)*

- w **is the vector form of the OPD, formed by stacking the columns of OPD**

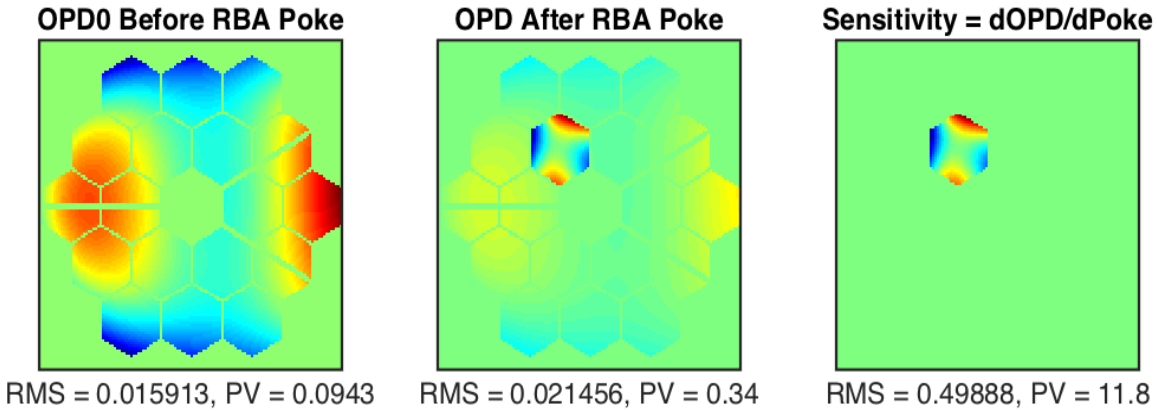
$$w = \text{vector}(OPD) \qquad OPD = \text{matrix}(w)$$

- WFE **is the RMS of the OPD**

$$WFE = \text{RMS}(w) = \text{RMS}(\text{nonzeros}(OPD))$$



Linearized Optical Models: Sensitivity Matrices



- This shows the dOPD due to a particular RBA action: a twist motion of Segment 2
- The sensitivity of the WF w to that poke is the matrix dw/dPoke:

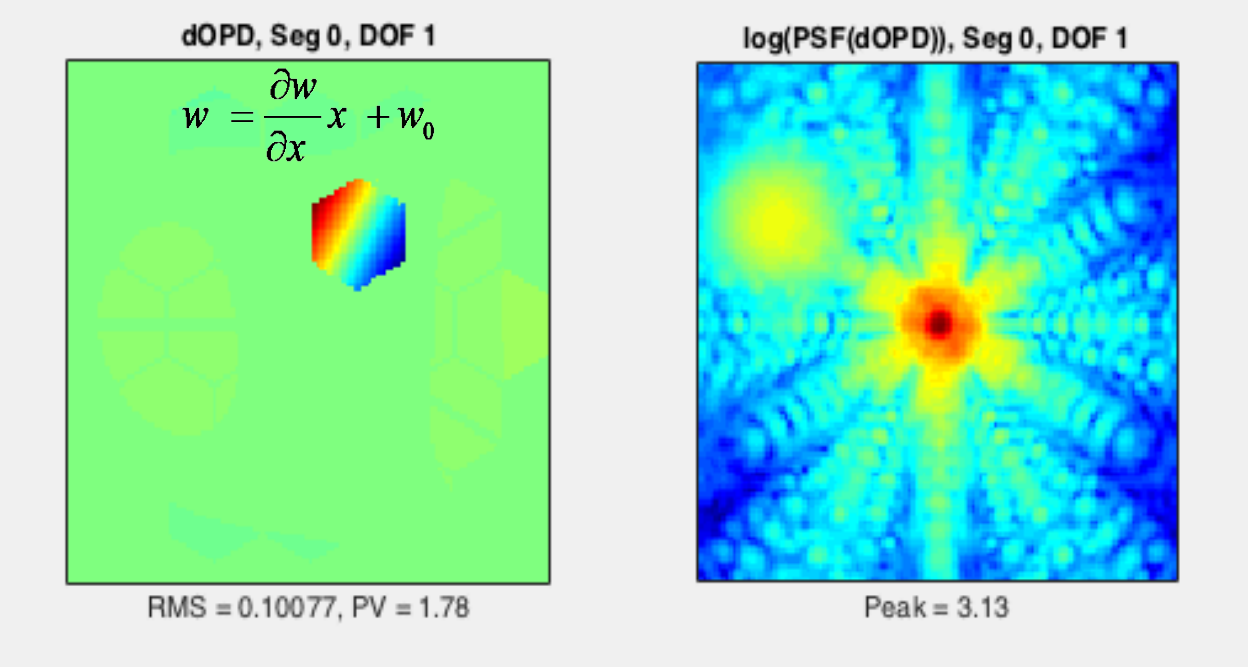
$$w_{Poke} = \frac{\partial w}{\partial x_{Poke}} x_{Poke}$$

- Here $\frac{\partial w}{\partial Poke} = \text{vector}(OPD_0) - \text{vector}(OPD_{Poke})$
- The total WF w is a linear function of the optical state x, figure errors z, RBA controls u_{RB}, DM controls u_{DM}, segment figure controls u_{FCA}, design WF w_{nom}:, and any other terms of interest

$$w = \frac{\partial w}{\partial x} x + \frac{\partial w}{\partial z} z + \frac{\partial w}{\partial u_{RB}} u_{RB} + \frac{\partial w}{\partial u_{DM}} u_{DM} + \frac{\partial w}{\partial u_{FCA}} u_{FCA} + w_{nom}$$

- The next few slides illustrate each of these terms

The Effect of Segment and SM Motions

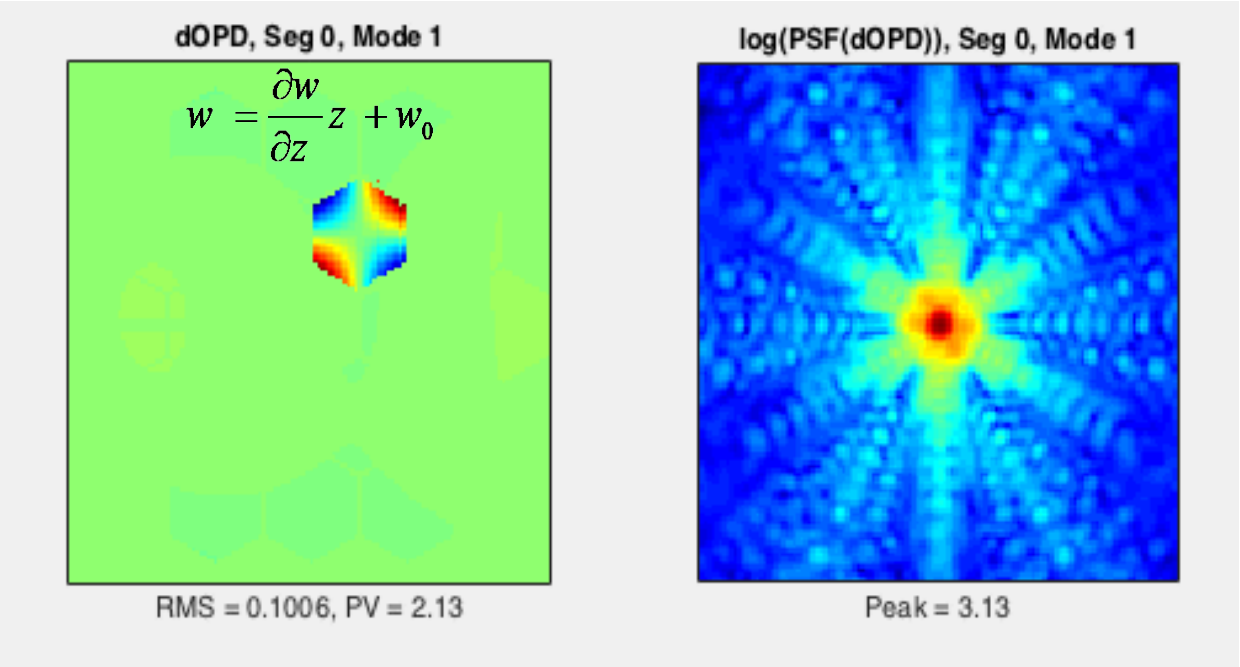


- OPD and Point Spread Function (PSF) image after "poking" individual RB Degrees of Freedom (DOF)
- RB perturbations are due to:
  - Misalignments
  - Controls u_{RB}

- The optical state vector x captures the RB perturbations of the optics, the rotational (θ) and translational (δ) pose changes of each optic:
  - For the ith optic: $$x_i = \begin{bmatrix} \theta \\ \delta \end{bmatrix}$$
  - For all optics: $$x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$$
  - RBA controls u_{RB} couple to x via the actuator kinematics (hexapod, e.g.):

$$x_u = \frac{\partial x}{\partial u} u_{RB}$$

The Effect of Surface Figure Errors



- OPD and Point Spread Function (PSF) image after "poking" individual SFE terms
- SFE perturbations are due to:
  - Fabrication errors
  - Thermal and other deformation effects

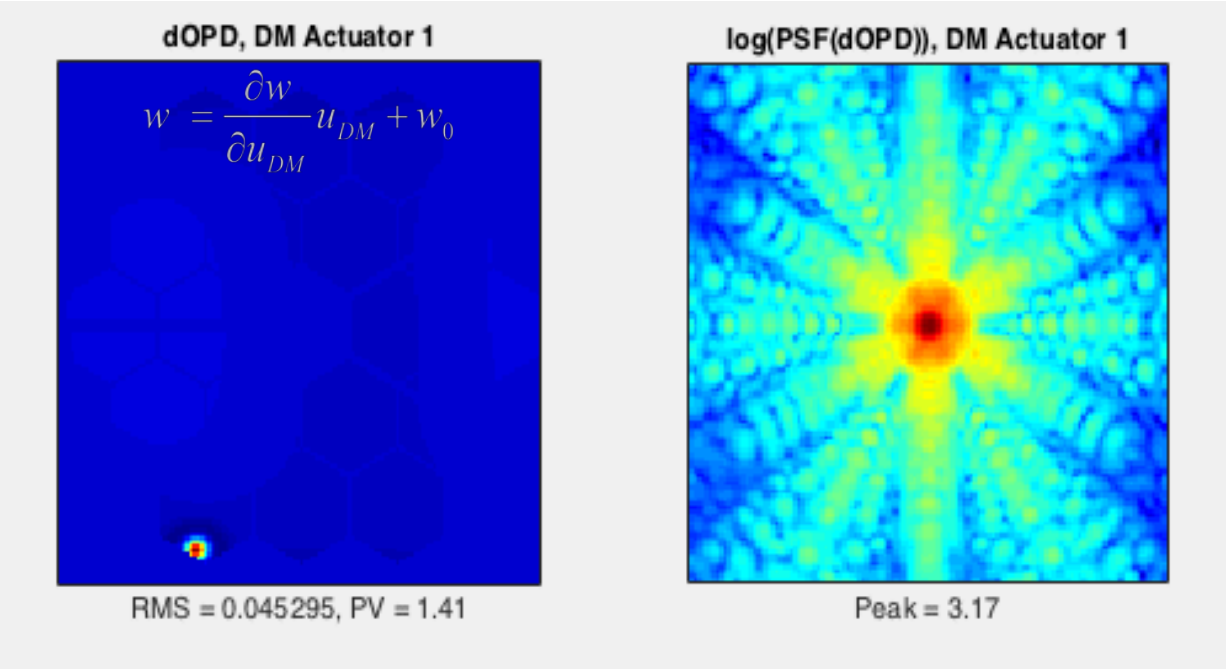
- The Surface Figure Error (SFE) vector z represents the low-order Zernike aberrations of each optic:
  - For the ith optic:

$$z_i = \begin{bmatrix} Z_4 \\ \vdots \\ Z_{13} \end{bmatrix}$$

For all optics:

$$z = \begin{bmatrix} z_1 \\ \vdots \\ z_n \end{bmatrix}$$

The Effect of DM Actuators

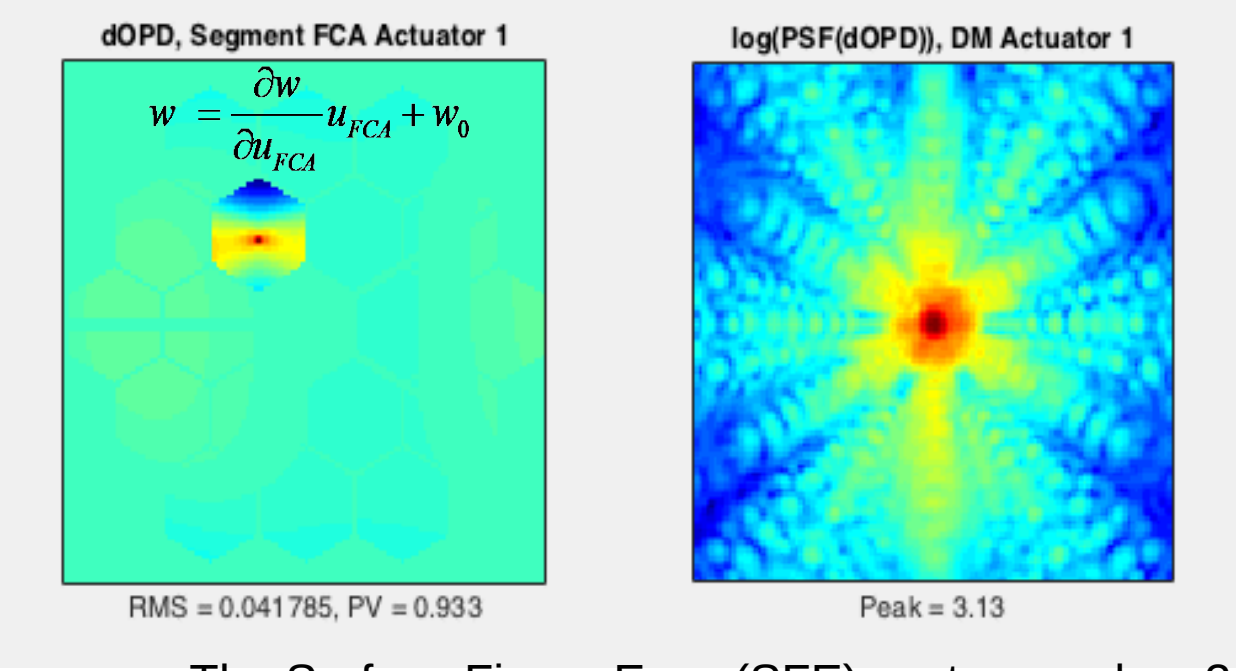


- OPD and Point Spread Function (PSF) image after "poking" individual DM actuators
- DM is of the "bed-of-nails" type, with actuators pushing normal to a thin facesheet

– The DM control vector u_{DM} has 690 actuators (of 1313) in the telescope on-axis clear aperture

- Simulated DM influence function is based on Timoshenko membrane model
  – Coupled response over ~ 5x5 actuator spacings

The Effect of Segment Figure Control Actuators



- OPD and Point Spread Function (PSF) image after "poking" individual PM segment Figure Control Actuators (FCAs)
- Actuators are of the "surface parallel" type, based on AHM design
  - FEM simulation

- The Surface Figure Error (SFE) vector u_{FCA} has 342 actuator voltages for each of 18 segments, for a total of 6156 actuators in the clear aperture:
  - For the ith optic:

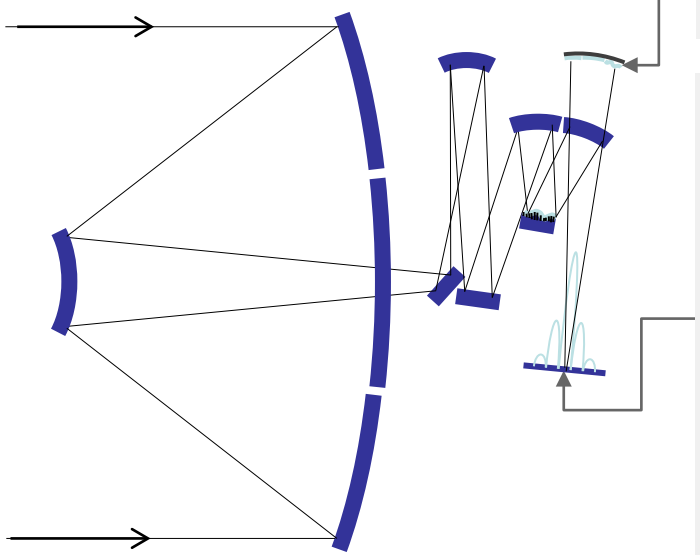
$$u_{FCAi} = \begin{bmatrix} a_1 \\ \vdots \\ a_{342} \end{bmatrix}$$

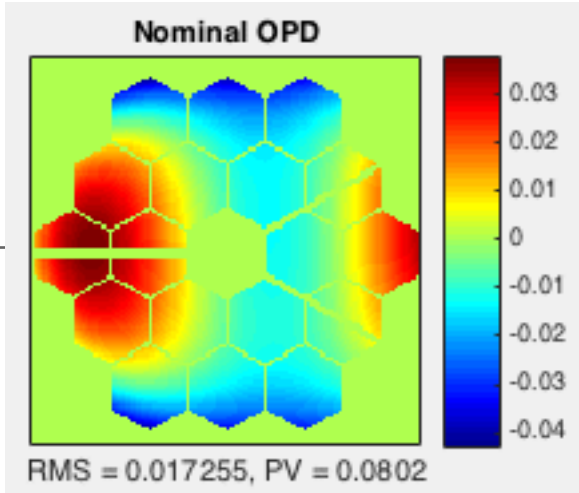
For all optics:

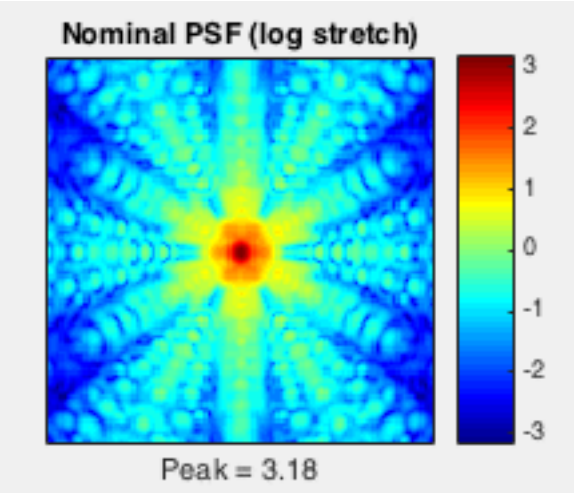
$$u_{FCA} = \begin{bmatrix} u_{FCA1} \\ \vdots \\ u_{FCA18} \end{bmatrix}$$

Wavefront Control

- **Wavefront control (WFC) objective:**
  - Maximize image quality, by minimizing WF error
    - Across the full science Field of View







- **Do this by measuring the WF across the FOV**
  - Using WF sensing, looking at stars
- **Then control the WF to match the nominal OPD**
  - Using PM segment and SM Rigid Body Actuators, Segment Figure Control Actuators, and the DM actuators
- **Can actively maintain OPD using metrology feedback**

Wavefront Control Law

- Wavefront Control (WFC) begins with Wavefront Sensing (WFS) – some means of estimating the system WF, as w_{est}
- Then the control problem is to choose values for u that minimize the cost function: $J = 0.5\, w^T w$,

  subject to: $w_c = \frac{\partial w}{\partial u} du + w_{est}$ and: $u_{LB} \le u \le u_{UB}$, where $u = u_0 + du$

- Substituting w_c for w in the cost function:

$$2J = u^T \left(\frac{\partial w}{\partial u}\right)^T \frac{\partial w}{\partial u} u + w_{est} \left(\frac{\partial w}{\partial u}\right)^T \frac{\partial w}{\partial u} w_{est}$$

- The control solution comes at the stationary point $dJ = 0$:

$$dJ = 0 = du \left[\left(\frac{\partial w}{\partial u}\right)^T \frac{\partial w}{\partial u} u + \left(\frac{\partial w}{\partial u}\right)^T w_{est} \right]$$

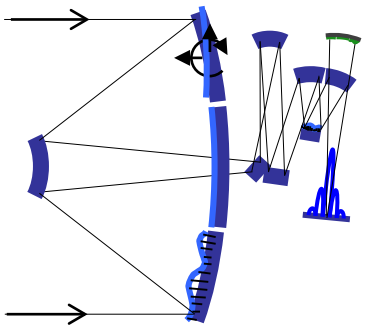
- The classic "pseudo-inverse" control law is thus (don't forget the bounds!):

$$u = -\left[\left(\frac{\partial w}{\partial u}\right)^T \frac{\partial w}{\partial u} \right]^{-1} \left(\frac{\partial w}{\partial u}\right)^T \left(w_{est} - w_{nom}\right) \equiv -G\left(w_{est} - w_{nom}\right)$$

Note: u here refers to any or all of the controls u_{RB}, u_{DM}, or u_{FCA} :

$$u = \begin{bmatrix} u_{RB} \\ u_{DM} \\ u_{FCA} \end{bmatrix}$$

End-to-End WF Performance



- Putting it all together, the OPD prior to control is:

$$w_0 = \frac{\partial w}{\partial x} x_0 + \frac{\partial w}{\partial z} z_0 + \frac{\partial w}{\partial u} u_0 + w_{nom}$$

- WF sensing gives a noisy estimate of w:

$$w_{est} = w_0 + \delta w_{est}$$

- WF control feeds back the estimated OPD to drive w to w_{nom}:

$$u_1 = -G\left(w_{est} - w_{nom}\right) = -G\left(\frac{\partial w}{\partial x} x_0 + \frac{\partial w}{\partial z} z_0 + \frac{\partial w}{\partial u} u_0\right) \qquad G = \left(\frac{\partial w}{\partial u}\right)^{+}$$

- Substituting for w_{est} and u_1, and then rearranging terms:

$$w_1 = P_u \frac{\partial w}{\partial x} x_0 + P_u \frac{\partial w}{\partial z} z_0 + \frac{\partial w}{\partial u} G \delta w_{est} + \frac{\partial w}{\partial u} \delta u \qquad P_u = \left(I - \frac{\partial w}{\partial u} G\right)$$

- The "control projection matrix" P_u will be zero only if the various actuators have full controllability of the OPD at the sampled ray density
- The combined system WFE, post control, is the RMS of the OPD:

$$WFE_1 = \sqrt{\frac{1}{n}\left(\sum_j^{n_{ray}} w_{1_j}^2\right)}$$

This model gives the end-to-end WFE performance for a *single case*, a single realization of the various component errors and noises.

End-to-End Performance Statistics

- The expected value of WFE for many realizations of the component errors and noises can be computed using the linear model, based on the statistics of those errors and noises: their covariances
- In particular, the covariance of the end-to-end OPD is:

$$\mathrm{cov}(w) \equiv W = \mathrm{E}(w-\bar{w})^T(w-\bar{w})$$

$$W_1 = P_u \frac{\partial w}{\partial x} X_0 \frac{\partial w}{\partial x}^T P_u^T + P_u \frac{\partial w}{\partial z} Z_0 \frac{\partial w}{\partial z}^T P_u^T + \frac{\partial w}{\partial u} G W_{est} G^T \frac{\partial w}{\partial u}^T + \frac{\partial w}{\partial u} U \frac{\partial w}{\partial u}^T$$

- Here the X_0, Z_0, U, and W_{est} signify the covariance matrices of x_0, z_0, u, and w_{est}, respectively...
- ...and the expected value of the WFE, given these statistics and the linear model, is:

$$WFE = \sqrt{\frac{1}{n}\left(\sum_j^{n_{ray}} \sigma_{w_j}^2\right)} = \sqrt{\frac{1}{n}\mathrm{trace}(W)}$$

These matrices have large dimensions, n_{ray}-by-n_{DOF}, generally. But *WFE* is a scalar metric...

Lumping the Effects of Similar Terms

- Many of the component errors will be uncorrelated, with the same statistics (zero mean, same stdev): these can be grouped together
  - Initial segment misalignments can have the same stdev for each segment in each axis, so that the initial covariance is diagonal:

$$X_{\delta_{seg}} = \sigma^2_{x_{\delta_{seg}}} I$$

  - Plugging this into the W_1 equation, the contribution of segment misalignment to the total, post-control WFE is:

$$WFE_{x_{\theta_{seg}}} = \sigma_{x_{\theta_{seg}}} \alpha_{x_{\theta_{seg}}}$$, where:

$$\alpha_{x_{\theta_{seg}}} = \sqrt{\frac{1}{n}\operatorname{trace}\left(P_u \frac{\partial w}{\partial x_{\theta_{seg}}} \frac{\partial w}{\partial x_{\theta_{seg}}}^T P_u^T \right)}$$

This is a scalar expression for the (scalar) WFE due to the action of many different component errors, all of which have the same (scalar) initial stdev.

  - Every other term in the model can be treated the same way. For instance:

$$WFE_{u_{DM}} = \sigma_{u_{DM}} \alpha_{u_{DM}}$$

$$\alpha_{u_{DM}} = \sqrt{\frac{1}{n}\operatorname{trace}\left(\frac{\partial w}{\partial u_{DM}} \frac{\partial w}{\partial u_{DM}}^T \right)}$$

$$WFE_{w_{est}} = \sigma_{w_{est}} \alpha_{w_{est}}$$

$$\alpha_{w_{est}} = \sqrt{\frac{1}{n}\operatorname{trace}\left(\frac{\partial w}{\partial u} G G^T \frac{\partial w}{\partial u}^T \right)}$$

Combining: a Scalar End-to-End WFE Model

- Combining all such terms, this covariance analysis ends up with the classic RSS error model, but with coefficients based on detailed ray-trace calculations, and embedding accurately the effects of various controls:

$$WFE = \sqrt{\sigma^2_{x_{\theta_{seg}}} \alpha^2_{x_{\theta_{seg}}} + \cdots + \sigma^2_{z_{4_{seg}}} \alpha^2_{x_{4_{seg}}} + \cdots + \sigma^2_{w_{est}} \alpha^2_{w_{est}} + \cdots + \sigma^2_{u_{RB_{seg}}} \alpha^2_{u_{RB_{seg}}} + \cdots}$$

- Note should be taken of certain implicit assumptions:
  1. System is linear, and sensitivities are accurate
  2. Component errors are zero mean, normally distributed, and uncorrelated
- Generally, these assumptions will not be met – but fortunately, taken on the whole, they will be met closely enough
- *Also*: the α coefficients can be computed using Monte Carlo methods
  - Monte Carlo and Covariance Analysis agree when assumptions are met
  - Monte Carlo methods can better handle nonlinearities, correlated errors, and/or (some) non-normal distributions
  - Monte Carlo methods can be more computationally efficient, and can use experimental data if that is available

Spreadsheet Form: Concluding the Example

- **The scalar WFE model is easily realized in spreadsheet form, and can be linked to error block diagrams**
- **This form allows the system engineer to rapidly change error distributions, while accurately assessing the consequences**

9/1/08	Segmented Telescope WFE Budget	Initial Alignment				Metrology Control				Total WFE
	Degree of Freedom	Inputs		RMS WFE (nm)		Drift Inputs		RMS WFE (nm)		Post Metrology
		Values	Units	Pre WFC	Post WFC	Values	Units	Pre Metrology	Post Metrology	RMS WFE (nm)
Optical Telescope Assembly	PM Segment Tilt	2,000	urad	1,715,156	0.0	50	urad	42,879	0.0	0.0
	PM Segment Piston	1,000,000	nm	1,819,286	0.0	1,000	nm	1,819	0.0	0.0
	PM Segment Decenter	1,000,000	nm	47,080	0.3	1,000	nm	47	0.0	0.3
	PM Segment Clocking	3,000	urad	336	0.8	15	urad	2	0.9	1.2
	PM Low-order Figure (0-5 cycles/seg	300	nm	578	5.9	2	nm	4	3.9	7.0
	PM PSD Figure	6	nm	11	5.5	1	nm	1.8	1.8	5.8
		0.003	c/mm			0.003	c/mm			
	PM Segment Mandrels	408	nm	816	6.0					6.0
	PM Print Through	1,000	Pa	18	1.2					1.2
	PM Thermal	18.0	deg C	1,904	2.9	0.020	deg C	2	2.1	3.6
	PM Nanolaminate Strain	26	deg C	2,218	3.9	0.000	deg C	0	0.0	3.9
	SM Decenter	2,000,000	nm	258,751	0.0	2,500	nm	323	0.0	0.0
	SM Piston	2,000,000	nm	2,620,916	0.0	2,000	nm	2,621	0.0	0.0
	SM Clocking	2,500	urad	0	0.0	2,000	urad	0	0.0	0.0
	SM Tilt	5,000	urad	1,337,236	0.0	120	urad	32,094	0.0	0.0
	SM Low-Order Figure	30	nm	38	0.2	1	nm	1	1.3	1.3
	SM PSD Figure	15	nm	35	2.9	1	nm	1	1.4	3.2
		0.003	c/mm			0.003	c/mm			
Fold Flat	FF Decenter	2,000,000	nm	0	0.1	2,500	nm	0	0.0	0.1
	FF Piston	100,000	nm	7,452	0.4	2,000	nm	149	0.0	0.4
	FF Clocking	5,000	urad	0	0.0	2,500	urad	0	0.0	0.0
	FF Tilt	100	urad	3,539	0.6	20	urad	708	0.0	0.6
	FF Low-Order Figure	50	nm	64	0.4	1	nm	1	1.3	1.3
	FF PSD Figure	10	nm	10	2.0	1	nm	1	0.9	2.2
		0.005	c/mm			0.005	c/mm			
TM	TM Decenter	500,000	nm	4,813	0.3	2,500	nm	24	0.0	0.3
	TM Piston	200,000	nm	10,311	0.8	2,000	nm	103	0.0	0.8
	TM Clocking	5,000	urad	0	0.0	2,500	urad	0	0.0	0.0
	TM Tilt	100	urad	3,531	0.6	20	urad	706	0.2	0.6
	TM Low-Order Figure	50	nm	5	0.0	1	nm	0	0.1	0.1
	TM PSD Figure	33	nm	6	0.6	1	nm	0	0.2	0.6
		0.005	c/mm			0.005	c/mm			
FSM	FSM Decenter	2,000,000	nm	0	0.1	2,500	nm	0	0.0	0.1
	FSM Piston	200,000	nm	2	0.8	2,000	nm	0	0.0	0.8
	FSM Clocking	5,000	urad	0	0.0	2,500	urad	0	0.0	0.0
	FSM Tilt		urad			50	urad	1	0.0	0.0
	FSM Low-Order Figure	33	nm	7	0.0	1	nm	0	0.2	0.2
	FSM PSD Figure	33	nm	17	1.8	1	nm	1	0.5	1.9
		0.010	c/mm			0.010	c/mm			
Re-Imager	Decenter	100,000	nm	0	0.0	2,500	nm	0	0.0	0.0
	Piston	200,000	nm	37	0.0	1,000	nm	0	0.2	0.2
	RoC	0	nm	0	0.0	0	nm	0	0.0	0.0
	Tilt	2,000	urad	3	0.0	100	urad	0	0.2	0.2
	Low-Order Figure	33	nm	7	0.0	1	nm	0	0.2	0.2
	PSD Figure	33	nm	6	0.6	1	nm	0	0.0	0.6
		0.010	c/mm			0.010	c/mm			
Instrument Bench	Decenter	100,000	nm	0	0.0	2,500	nm	0	0.0	0.0
	Piston	200,000	nm	50	0.0	1,000	nm	0	0.2	0.2
	Tilt	2,000	urad	2	0.0	300	urad	0	0.3	0.3
	Low-Order Figure	33	nm	4	0.0	1	nm	0	0.1	0.1
	PSD Figure	33	nm	12	0.9	1	nm	0	0.4	1.0
		0.010	c/mm			0.010	c/mm			
	Mid-f error	0	nm	0	0.0					0.0
	Hi-f error	3	nm	3	3.0					3.0
	Other	4	nm	4	4.0					4.0
Metrology	Laser SNR		nm							
	Laser Frequency Drift		nm						6.0	6.0
	Beam Launcher Thermal Drift		nm							
	Misalignment									
	(TBD)									
WFC	WFS Error	3	nm	13	2.4		nm			2.4
	PM RB Actuator Error	2	nm	0	1.6		nm	0	0.0	1.6
	PM DM Actuator Error	2	V	0	0.0		units	0	0.0	0.0
	SM RB Actuator Error									
	PM thermal ΔT	0.000	°C	0	0.0	0.100	°C	7	6.0	6.0
WFE Summary	PM WFE			2,500,757	11.2			42,918	4.9	12.2
	Everything Else			2,953,737	7.3			32,218	8.9	11.5
	Design WFE	10	nm		10.0					10.0
	Payload WFE									19.5
	WFE Requirement									20.0
	WFE Margin									2.3%

Example Terms, and System Roll-Up

9/1/08	Segmented Telescope WFE Budget	Initial Alignment				Metrology Control				Total WFE
	Degree of Freedom	Inputs		RMS WFE (nm)		Drift Inputs		RMS WFE (nm)		Post Metrology
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	PM Segment Decenter	1,000,000	nm	47,080	0.3	1,000	nm	47	0.0	0.3
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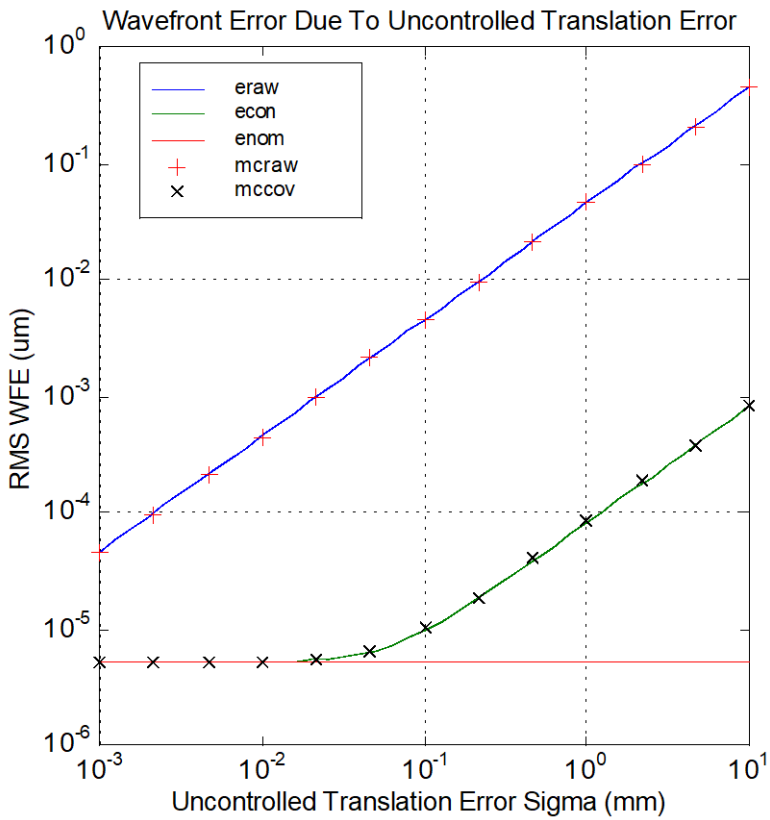
	PM PSD Figure	6	nm	11	5.5	1	nm	1.8	1.8	5.8
		0.003	c/mm			0.003	c/mm			
	PM Segment Mandrels	408	nm	816	6.0					6.0
	PM Print Through	1,000	Pa	18	1.2					1.2
	PM Thermal	18.0	deg C	1,904	2.9	0.020	deg C	2	2.1	3.6
	PM Nanolaminate Strain	26	deg C	2,218	3.9	0.000	deg C	0	0.0	3.9
	SM Decenter	2,000,000	nm	258,751	0.0	2,500	nm	323	0.0	0.0
	SM Piston	2,000,000	nm	2,620,916	0.0	2,000	nm	2,621	0.0	0.0
	SM Clocking	2,500	urad	0	0.0	2,000	urad	0	0.0	0.0
	SM Tilt	5,000	urad	1,337,236	0.0	120	urad	32,094	0.0	0.0
	SM Low-Order Figure	30	nm	38	0.2	1	nm	1	1.3	1.3
	SM PSD Figure	15	nm	35	2.9	1	nm	1	1.4	3.2
		0.003	c/mm			0.003	c/mm			
WFC	WFS Error	3	nm	13	2.4		nm			2.4
	PM RB Actuator Error	2	nm	0	1.6		nm	0	0.0	1.6
	PM DM Actuator Error	2	V	0	0.0		units	0	0.0	0.0
	SM RB Actuator Error									
	PM thermal ΔT	0.000	°C	0	0.0	0.100	°C	7	6.0	6.0
WFE Summary	PM WFE			2,500,757	11.2			42,918	4.9	12.2
	Everything Else			2,953,737	7.3			32,218	8.9	11.5
	Design WFE	10	nm		10.0					10.0
	Payload WFE									19.5
	WFE Requirement									20.0
	WFE Margin									2.3%

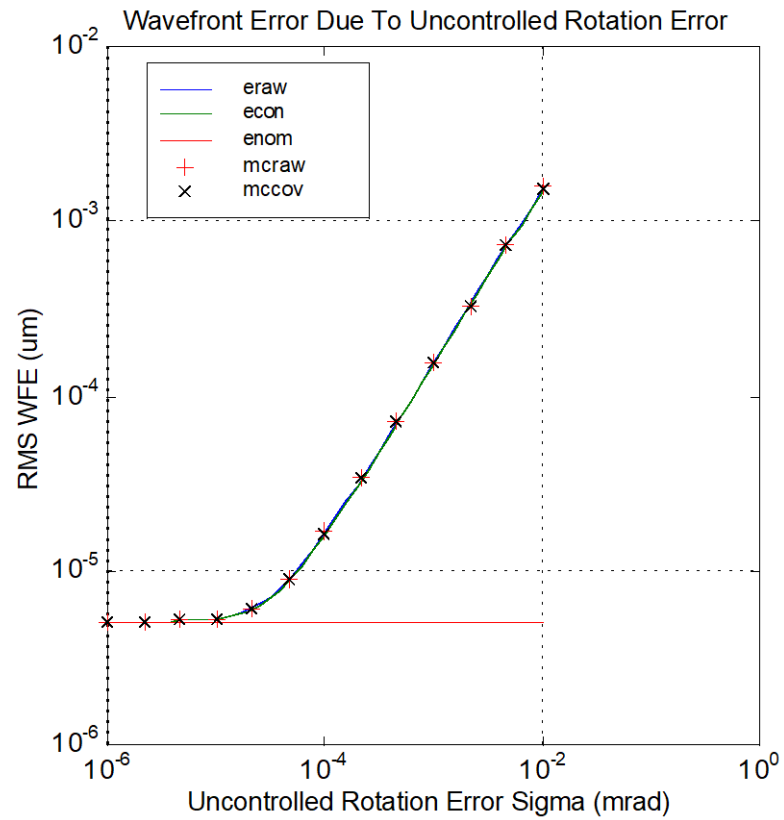
Bibliography

1. M. Postman and 67 others, “Advanced Technology Large Aperture Space Telescope (ATLAST): A Technology Roadmap for the Next Decade,” white paper submitted to the NRC ASTRO-2010 Survey (2009).
2. J. Dalcanton, S. Seager, S. Aigrain, S. Battel, D. Brandt, C. Conroy, L. Feinberg, S. Gezari, O. Guyon, W. Harris, C. Hirata, J. Mather, M. Postman, D. Redding, D. Schiminovich, H.P. Stahl, *From Cosmic Birth to Living Earths*, www.hdstvision.org, AURA (2015).
3. D. Redding and W. Breckenridge, “Optical Modeling for Dynamics and Control Analysis,” Journal of Guidance, Control and Dynamics, September 1991.
4. D. Redding and W. Breckenridge, “Linearized Ray-Trace Analysis,” Selected SPIE Papers on CD ROM, Volume 2, Lens Design, D. O'Shea, ed., 1998.
5. D. Redding, F. Shi, S. Basinger, D. Cohen, J. Green, A. Lowman and C. Ohara, “Wavefront Control for Large Space Optics,” IEEE Aerospace Conference (2003).

BACKUP

Covariance and Monte Carlo Agree





- **Compares linear covariance analysis to nonlinear Monte Carlo analysis, showing good agreement**
- **4DOF WF control is effective in compensating segment decenter translation error**
- **4DOF WF control is ineffective in compensating segment twist**
- **Floor is set by design error in these simplified cases**

Error Statistics (cont.)

Other component errors are highly correlated, such as figure error based on PSD model and sampled across mirror surfaces

$$X_{fig} = fcn(\omega_{1/2}, p, A)$$

$$X_{fig}(i,j) = C\ \rho(i,j)^{-p}$$

The figure covariance matrix X_{fig} can be directly generated using a generating function

X_{fig} has size N^4, yet reduces to a scalar contribution to the RSS model

$$\sigma_{fig}^2 = trace([I-C_uG]\ C_{fig}\ X_{fig}\ C_{fig}^T\ [I-C_uG]^T)$$

Rather than directly evaluate the covariance, it is computationally more efficient to perform *Monte Carlo simulations* to establish parametric dependencies for σ_{fig}^2

$$\sigma_{fig}^2 = fcn(\omega_{1/2}, p, A)$$

Another alternative is a direct PSD-based method...

More Wavefront Control Laws

- It can be very useful to damp the control response, by including a penalty c_{du} on the control change du, so that $J = 0.5\ (w^T w + c_{du}{}^T c_{du})$, and the control is:

$$du = -\left[\left(\frac{\partial w}{\partial u}\right)^T \frac{\partial w}{\partial u} + c_{du} I\right]^{-1} \left(\frac{\partial w}{\partial u}\right)^T \left(w_{est} - w_0\right) \qquad u_{LB} \le u \le u_{UB}, \text{ where } u = u_0 + du$$

- Another useful penalty can be imposed on the total u, to keep it near some preferred value u_r; in this case the control is:

$$du = -\left[\left(\frac{\partial w}{\partial u}\right)^T \frac{\partial w}{\partial u} + \left(c_{du} + c_u\right) I\right]^{-1} \left\{\left(\frac{\partial w}{\partial u}\right)^T \left(w_{est} - w_0\right) + c_u\left(u_0 - u_r\right)\right\}$$

- Weighting matrices on w, du and u can be used to shape the control: by frequency-weighting or masking the WF (M_w); by setting preferences on actuators to change (M_{du}); or to keep selected controls near the reference (M_u); in this case the control is:

$$du = -\left[\left(\frac{\partial w}{\partial u}\right)^T M_w^T M_w \frac{\partial w}{\partial u} + c_{du} M_{du}^T M_{du} + c_u M_u^T M_u\right]^{-1} \left\{\left(\frac{\partial w}{\partial u}\right)^T M_w^T M_w \left(w_{est} - w_0\right) + c_u M_u^T M_u \left(u_0 - u_r\right)\right\}$$

Multi-Field WFS&C

- **To best collimate the PM and SM of a telescope after launch, it is necessary to measure and correct the WF at at least 3 field points: center and 2 corners**
  - Not doing so runs the risk of having poor WF over much of the field!
- **The control laws are the same in this case, but the sensitivities and measurements are concatenated:**

$$\begin{bmatrix} w_{fov1} \\ \vdots \\ w_{fovn} \end{bmatrix} = \begin{bmatrix} \frac{\partial w}{\partial u}_{fov1} \\ \vdots \\ \frac{\partial w}{\partial u}_{fovn} \end{bmatrix} du + \begin{bmatrix} w_{estfov1} \\ \vdots \\ w_{estfovn} \end{bmatrix}$$

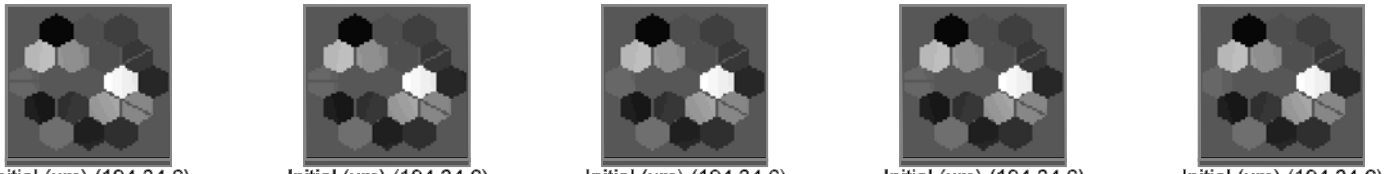
$$u_{LB} \le u \le u_{UB}, \text{ where } u = u_0 + du$$

- **The classic control is thus:**

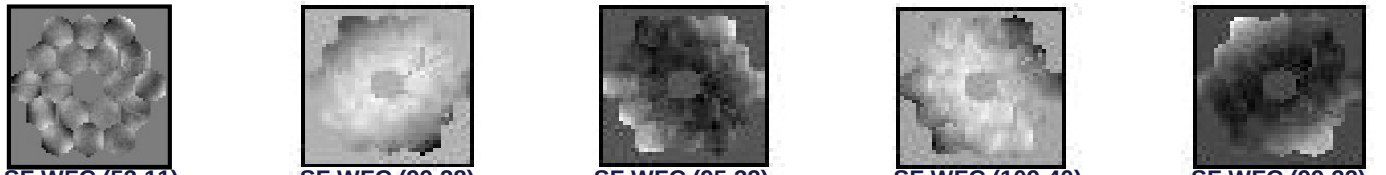
$$du = -\left[\begin{bmatrix} \frac{\partial w}{\partial u}_{fov1} \\ \vdots \\ \frac{\partial w}{\partial u}_{fovn} \end{bmatrix}^T \begin{bmatrix} \frac{\partial w}{\partial u}_{fov1} \\ \vdots \\ \frac{\partial w}{\partial u}_{fovn} \end{bmatrix} \right]^{-1} \begin{bmatrix} \frac{\partial w}{\partial u}_{fov1} \\ \vdots \\ \frac{\partial w}{\partial u}_{fovn} \end{bmatrix}^T \begin{bmatrix} w_{estfov1} - w_{0\,fov1} \\ \vdots \\ w_{estfovn} - w_{0\,fovn} \end{bmatrix}$$

Multi-Field WF Control Performance

Perturbed WFE at 5 field points (worst case of 20)

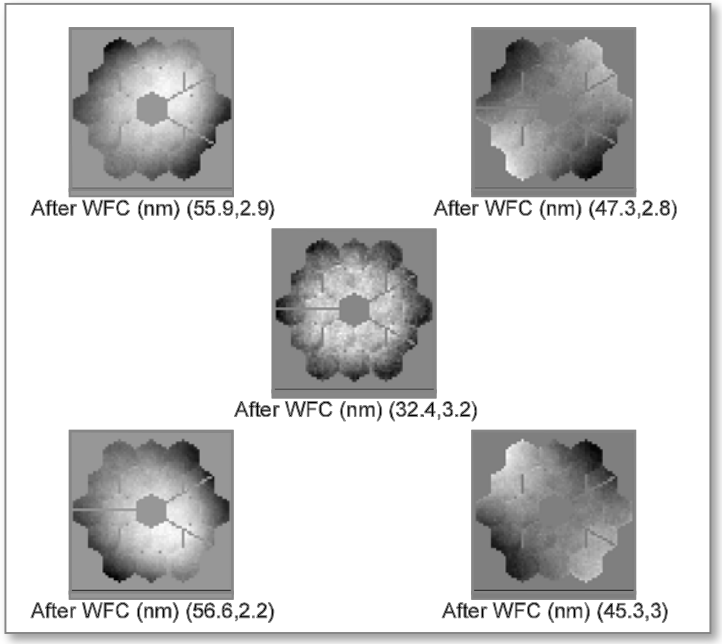


WFE after Single-Field WFSC, seen at 5 fields (nm RMS) (worst case of 20)



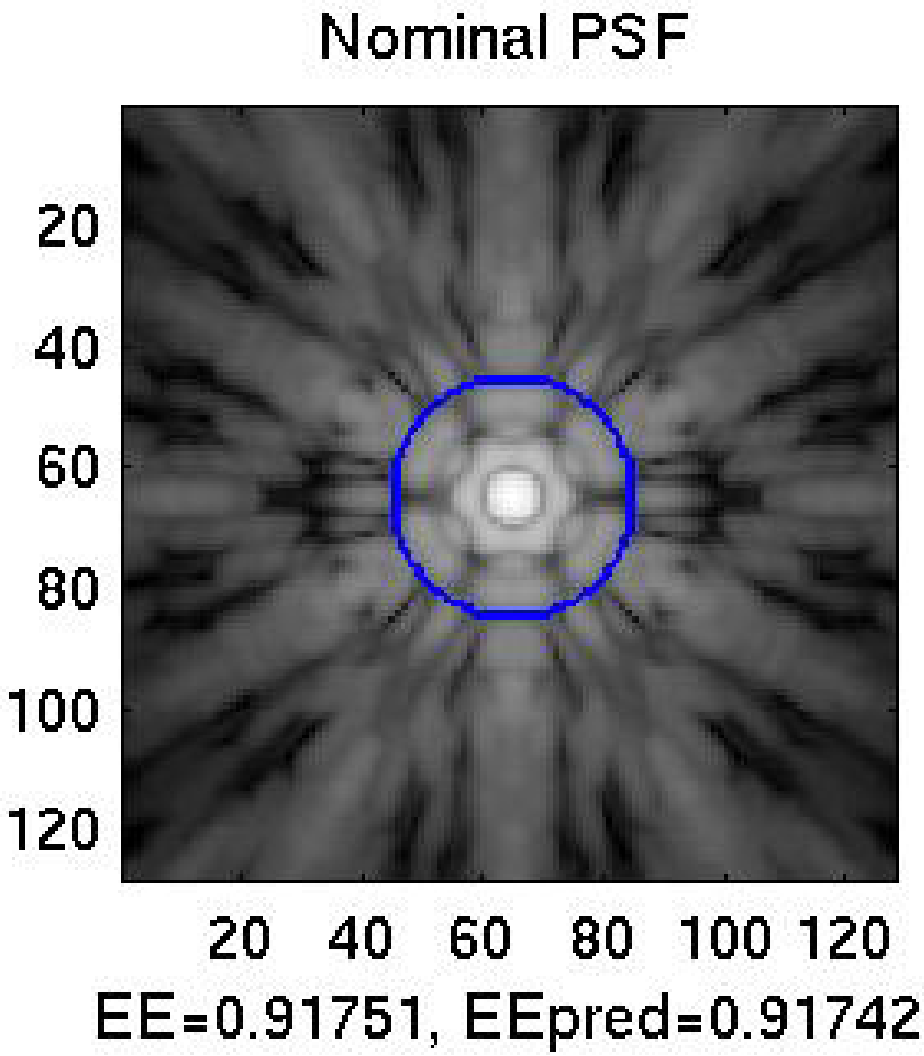
WFE following Multi-Field WFSC at 5 fields (worst case of 20)
Control: PM segment and SM 6DOF motion

OPD maps correspond to worse-case WFE of 20 trials. The (mean, std_dev) of residual WFE at each field position (nm) are shown.



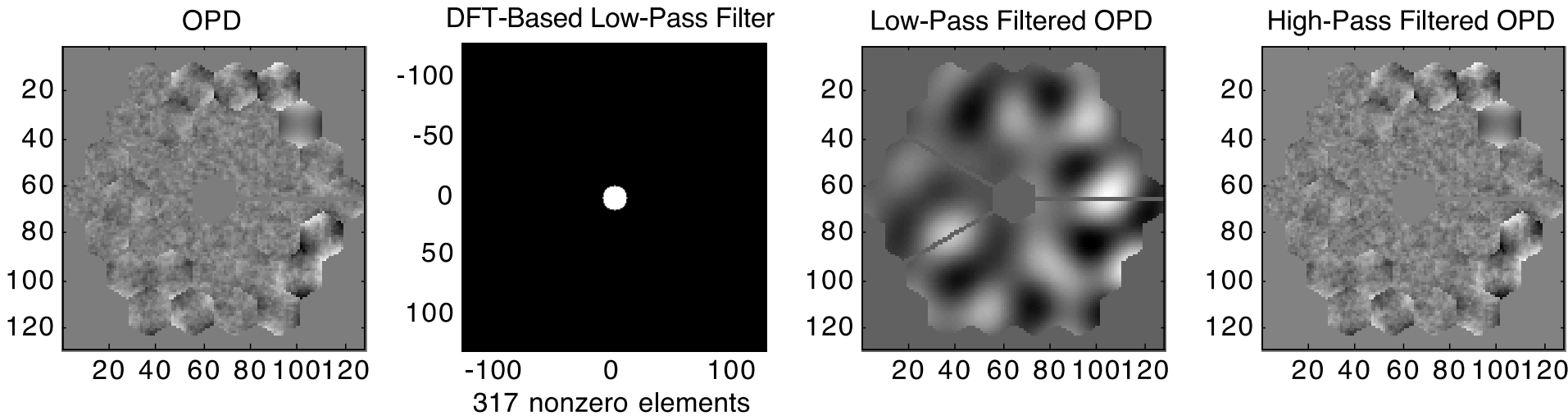
- **Single-field WFSC corrects the WF only for the field in which the data was taken**
- **By measuring at multiple field points and solving for the joint optimum, the telescope can be fully collimated**

PSF Morphology



- **PSF computed on 7.5 mas pixels**
- **Wavelength is 1 um**
- **R = 5, approximated with 10 spectral lines**
- **Blue ring is at 150 mas, corresponding to a spatial frequency of 5 cycles/aperture**

OPD Spatial Filters: Convolution Form



- **Filter is a sharp cutoff, passing only OPD spatial frequencies less than 5 cycles/aperture, which corresponds to 150 mas EE**
- **Convolution filter uses Fourier transform to select only the low spatial frequencies present in the OPD:**

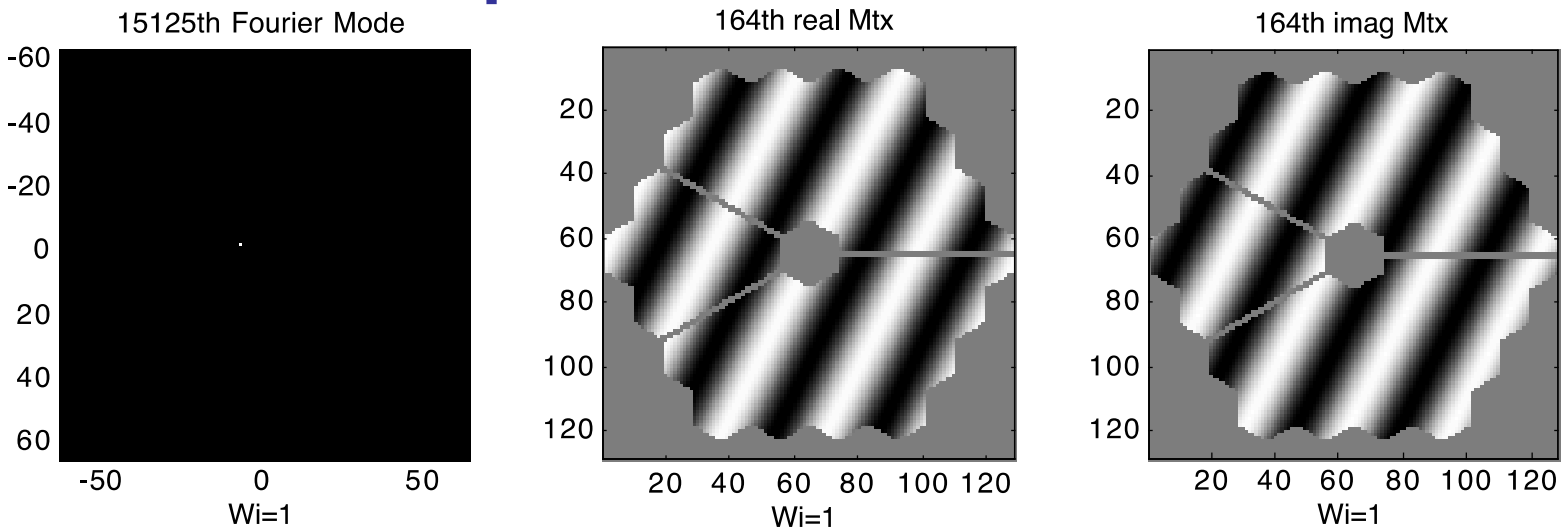
  Spectrum(OPD) = FFT(OPD)

  $\text{Spectrum}(\text{OPD}_{\text{Low-Pass}}) = \text{Spectrum}(\text{OPD})$.* Filter

  $\text{OPD}_{\text{Low-Pass}} = \text{FFT}^{-1}(\text{Spectrum}(\text{OPD}_{\text{Low-Pass}}))$.* Mask
- **High-pass OPD is the full OPD minus the low-pass OPD:**

  $\text{OPD}_{\text{Low-Pass}} = \text{OPD} - \text{OPD}_{\text{Low-Pass}}$

OPD Spatial Filters: Matrix Form



- **A pupil OPD matrix can be writen in vector form**

  w = Vector(OPD) = [column(OPD,1); column(OPD,2); …]

- **For each pixel in the filter bandpass, a complex "basis matrix" is computed and then vectorized**

  $\text{Spectrum}(\text{OPD}_{\text{Low-Pass}}) = \text{Spectrum}(\text{OPD}) .* \text{Filter}$

  $\text{Basis}_{ij} = \text{FFT}^{-1}(\text{Filter}(i,j)*\text{Delta}(i,j))$

  $C_{\text{low-pass}} = [\text{Vector}(\text{Basis}_1); \text{Vector}(\text{Basis}_2); \ldots]$

- **The matrix filters are computed from the matrix of bases**

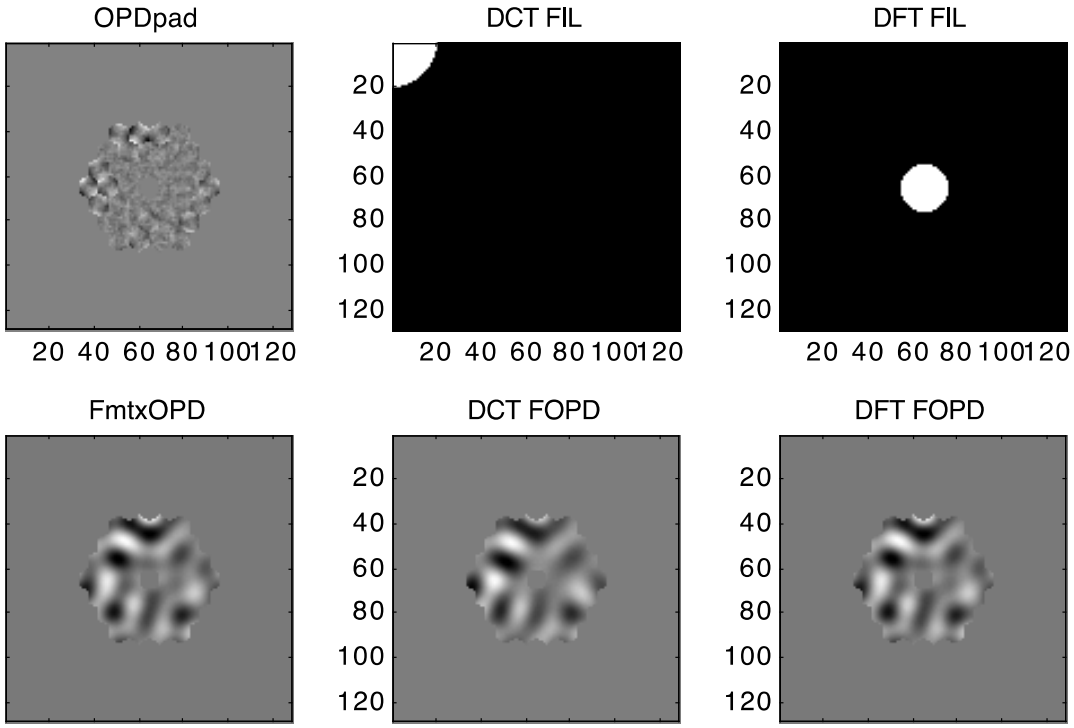
  $H_{\text{LowPass}} = C_{\text{low-pass}} * C^{+}_{\text{low-pass}}$ $\qquad$ $H_{\text{HighPass}} = I - C_{\text{low-pass}} * C^{+}_{\text{low-pass}}$

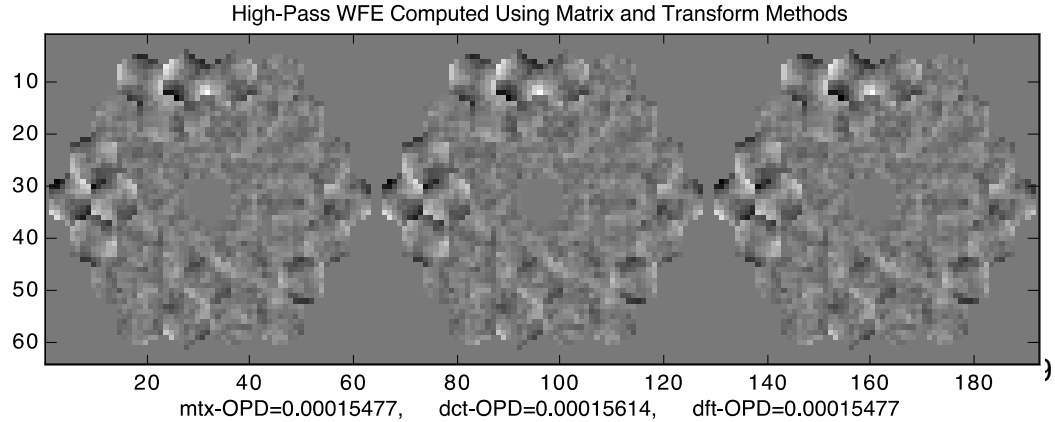
- **The filtered OPD is obtained via a matrix multiply**

  $\text{OPD}_{\text{HighPass}} = \text{Matrix}(H_{\text{HighPass}} * w)$

Matrix Form of High-Pass WF Filter

- **Raw OPD for a typical case is decomposed into low and high SF components**
  - High-pass = OPD - F_{lo}•OPD
- **Discrete Cosine and Discrete Fourier transform methods compared**
- **F_{mtx} is a matrix-multiply version of DFT filter**
- **Matrix filter is generated directly from transform filter**
  - Joe's EE filter



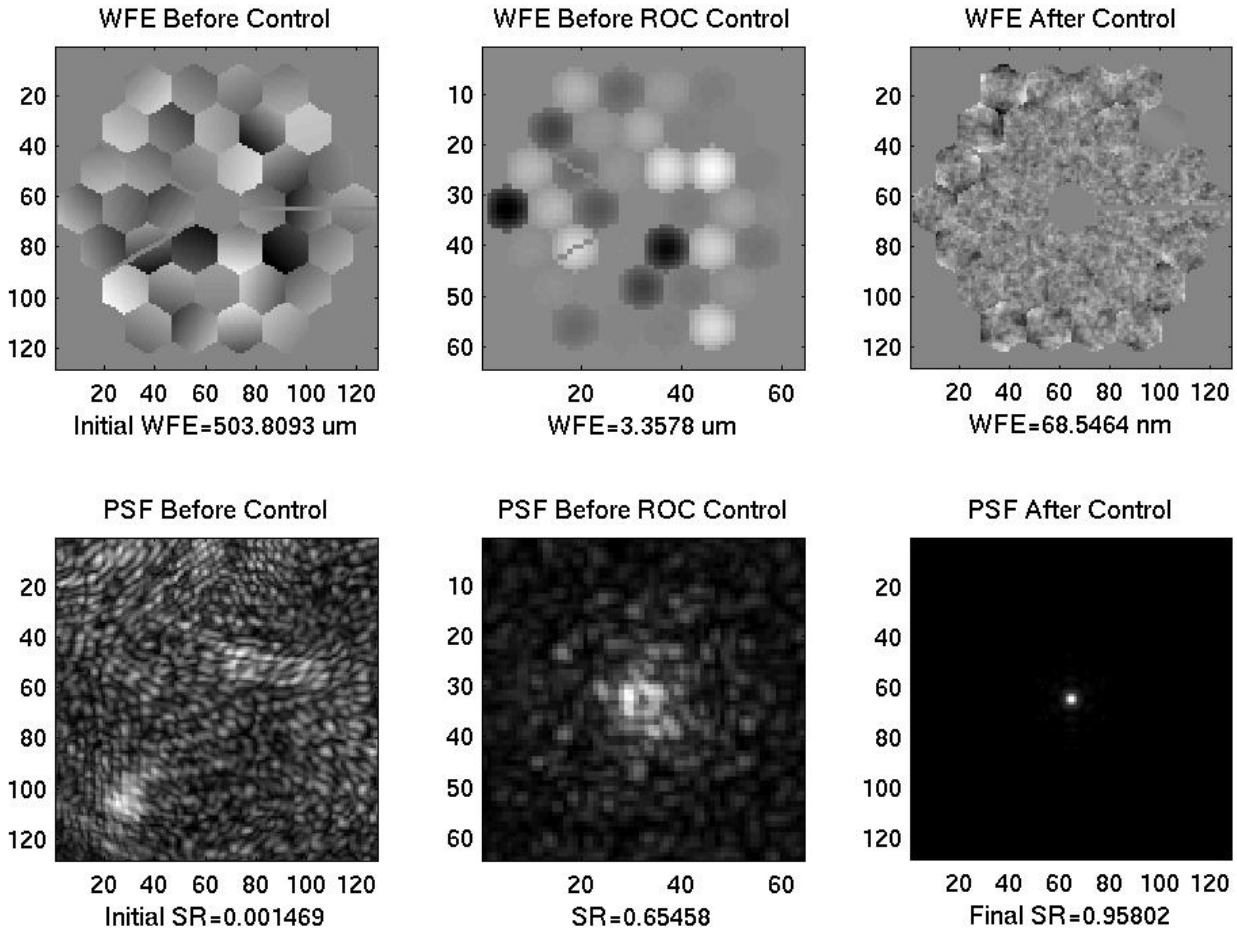


WF Control, Example 1

Initial errors (1-sigma):
- *Segment translation = 200 um*
- *Segment rotation = 300 urad*
- *RoC error = 1 um*

Segment figure PSD parameters:
- *Amplitude = 30 nm*
- *1/2 power freq = 0.003 cycles/mm*
- *Roll-off exponent = -3*

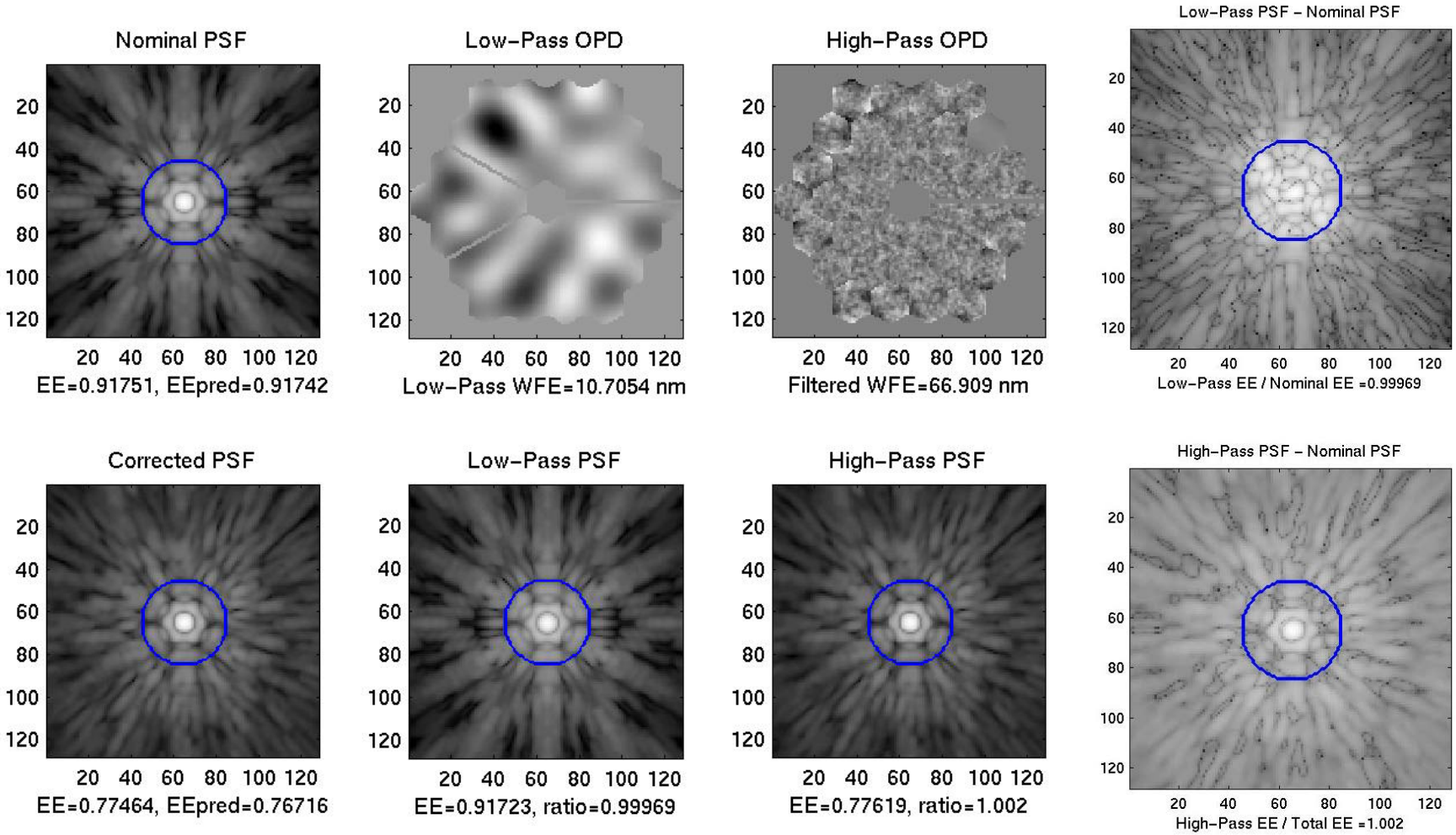
No WF sensing error
No WFC actuation error



files btcExample.m, btc.in, btc.f
5/13/02

- **Columns show OPD and PSF for 3 successive stages:**
  - Initial uncorrected optics
  - Following segment RB control (piston errors are left in anticipation of RoC correction, which has large piston components)
  - Following segment RB and RoC control

Encircled Energy and Band-Limited WFE for Example 1



- **Demonstrates that EE comes from high spatial-frequency WFE**
- **Filter has sharp roll-off at 5 cycles/aperture**

Model: htc.in
V. 5/7/02

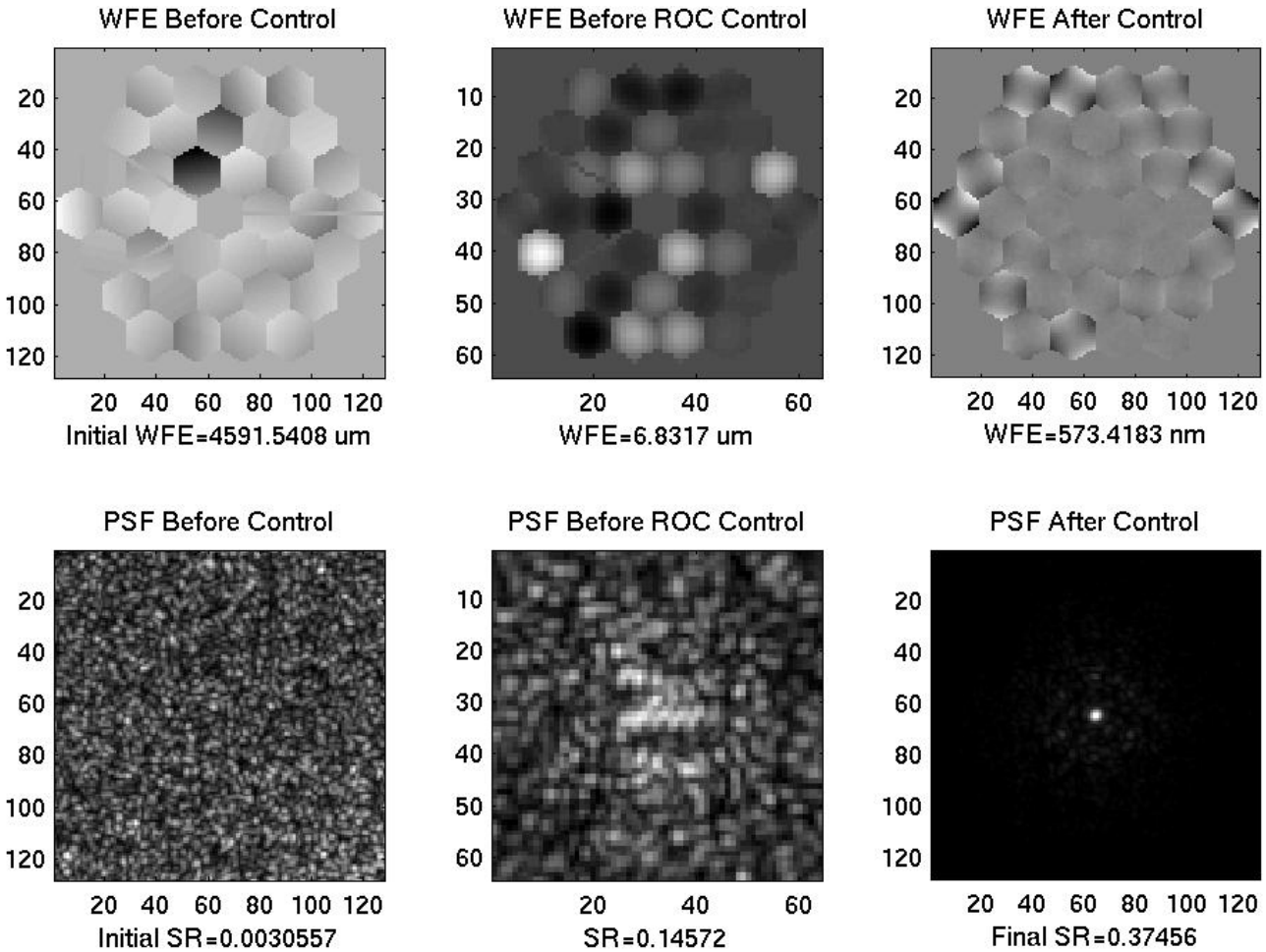
WFC Example 2

Initial errors (1-sigma):
- *Segment translation = 1 mm*
- *Segment rotation = 3 mrad*
- *RoC error = 5 um*

Segment figure PSD parameters:
- *Amplitude = 80 nm*
- *1/2 power freq = 0.003 cycles/mm*
- *Roll-off exponent = -3*

WF sensing error = 5 nm

WFC actuation errors
- *Segment rotation = 5 nrad*
- *Segment translation = 10 nm*

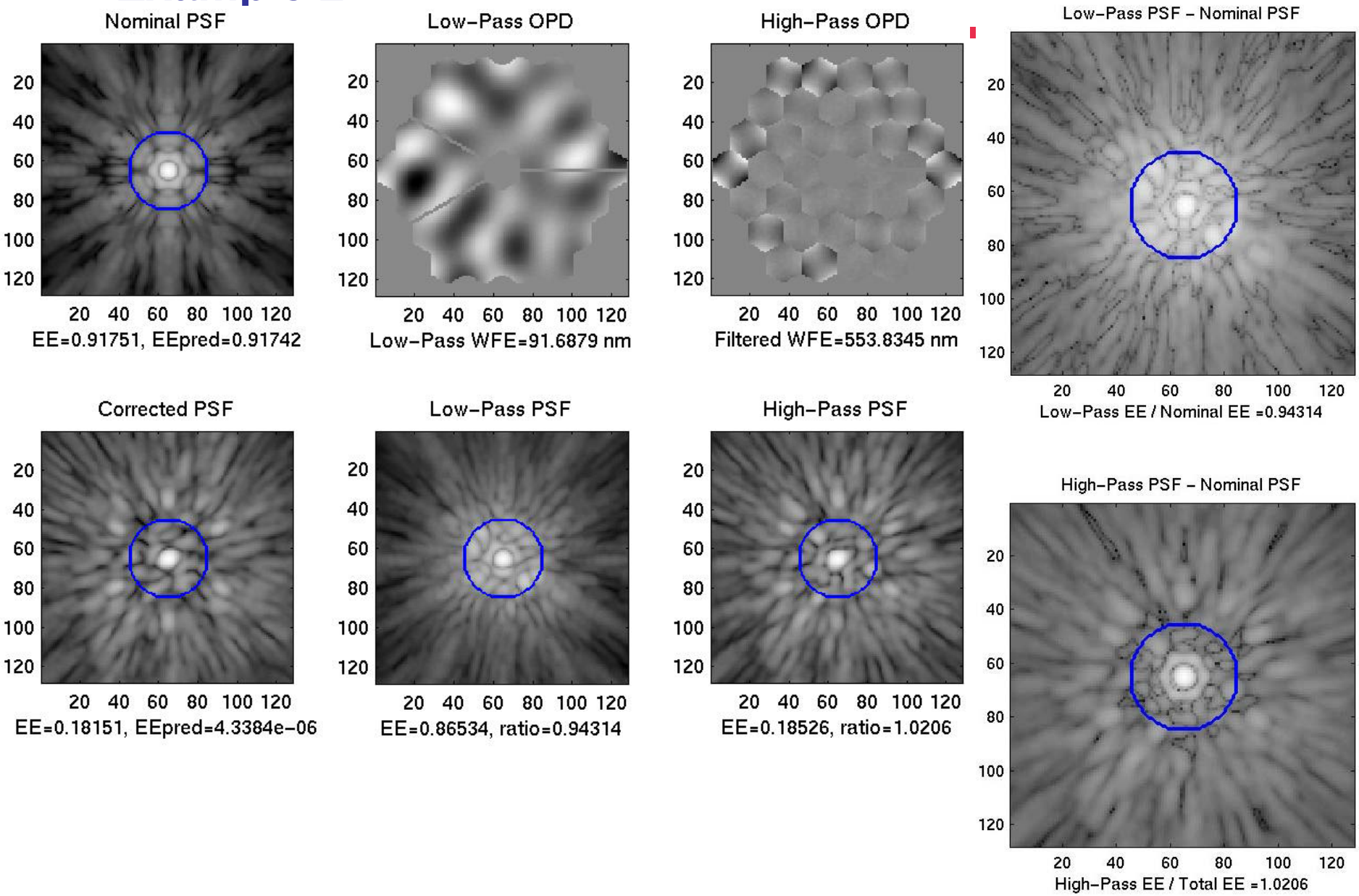


files btcExample.m, btc.in, btc.f
5/13/02

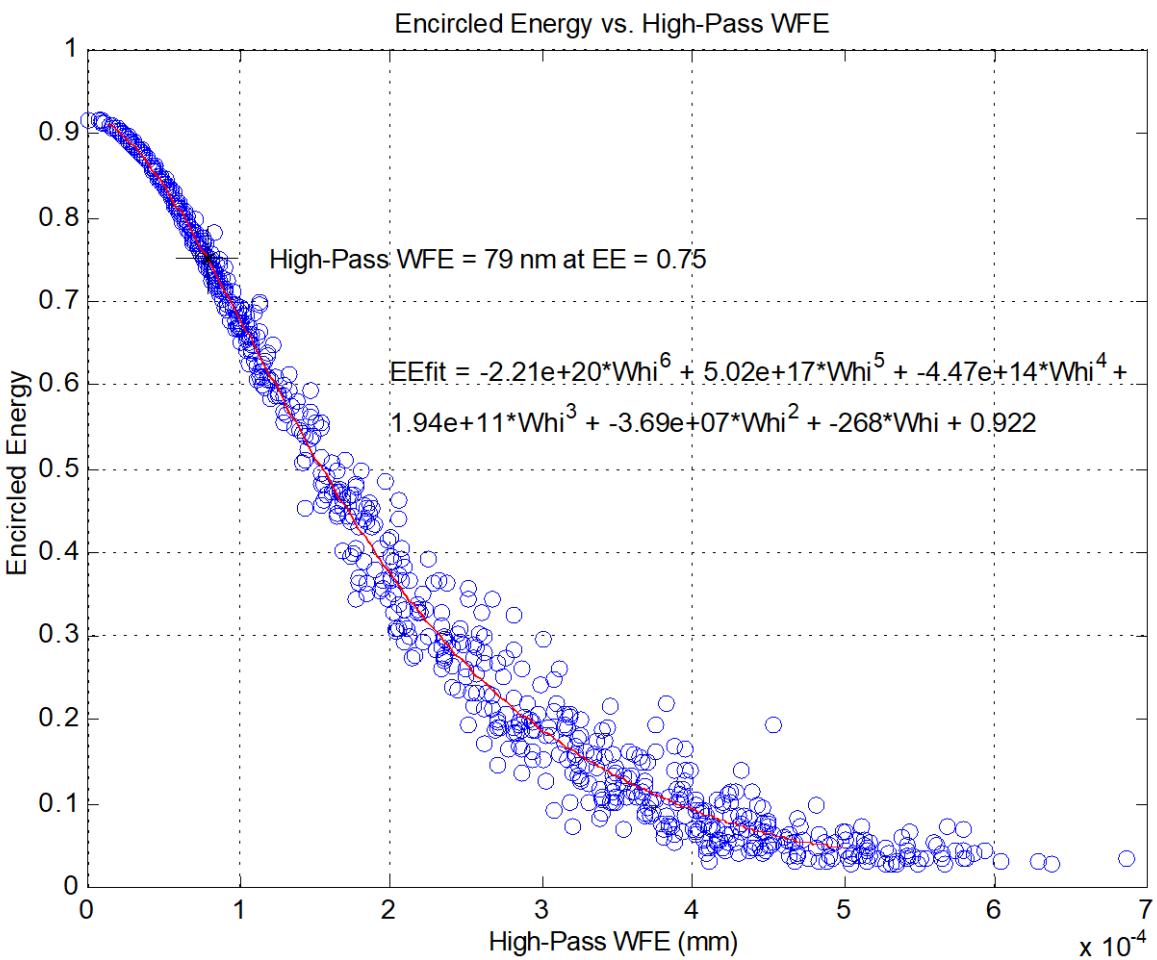
- **A more aberrated case...**

Model: btc.in
V. 5/7/02

Encircled Energy and Band-Limited WFE for Example 2

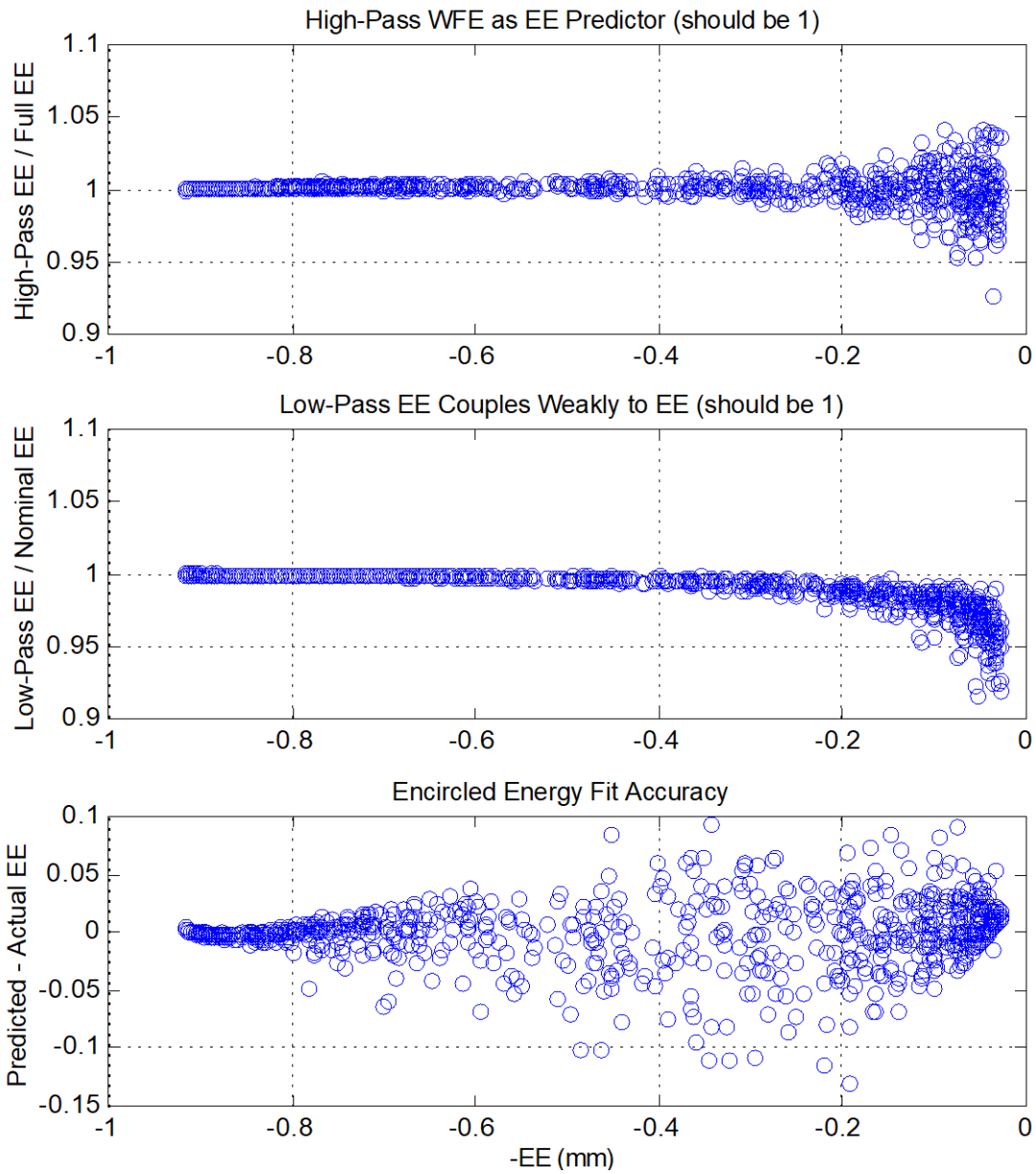


High-Pass WFE Predicts Encircled Energy



- **WFE generated using random initial errors as filtered by WFC system**
- ***EE objective met at 79 nm High-Pass WFE***

High-Pass WFE is an Accurate EE Predictor



- **Encircled Energy is degraded by scattering outside of PSF core**
- **This is caused by high spatial-frequency WF errors**
- **This chart confirms that high-pass WFE contributes nearly all the EE degradation in the region of interest**
- **EE is essentially unaffected by low-pass WFE in the region of interest**
- **Polynomial EE model is not as good a predictor**

Model: htc.in
V. 5/7/02