Decomposition of Polarimetric Scattering Based on Spectral Decomposition Techniques

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INTRODUCTION

- Many algorithms exist to decompose scattering when full polarimetric measurements are made
- Most of these algorithms performs the decomposition in steps, which effectively assigns the maximum amount of scattering to the vegetation layer
- It has already been shown that when the algorithms do not ensure positive powers, the vegetation contribution is over-estimated
- In addition, some algorithms perform a de-orientation step on the overall measured matrix before performing the decomposition
  - This has been shown to decrease the vegetation contribution and increase double reflections
  - Is de-orienting the entire matrix appropriate from an electromagnetic modeling viewpoint?

EXAMPLE: NNED

$$\langle[\mathbf{C}]\rangle = P_V[C_V] + P_d[C_d(\alpha)] + P_s[C_s(\beta)] + [C_R]$$

- Assigns maximum possible to volume scattering term first before solving for the other parameters
- This may not give the optimal solution over all parameters

GENERAL APPROACH

- Chen, Wang, Xiao, Sato (2014)
- Perform global constrained optimization over all parameters in the model
  - Minimize L2-norm of remainder matrix
  - Nonnegativity constraints on each model term
- Can be applied to any scattering model
- But computationally more demanding
  - Also, numerical optimization yields local optima but unclear if it is guaranteed to converge to the correct global optimum

REVIEW: NNED PRINCIPLE

- All terms must have nonnegative eigenvalues to correspond to physical scatterers
- If we are confident in our knowledge of the covariance matrix $\langle[\mathbf{C}]\rangle$, and the remainder is due to some other physical scatterer not captured by the model, it should also satisfy the nonnegative eigenvalue constraint
- Assumes low noise/speckle

“OPTIMAL” NNED

- Similar to Chen, Wang, Xiao, Sato (2014)
- Global constrained optimization over all parameters
  - Minimize trace of remainder matrix
  - Nonnegativity constraints on each model term, including nonnegative eigenvalues on remainder
- Can be applied to any scattering model
- Again, computational issues, but good news in some important special cases:
  - Linear combination of fixed matrices
  - Freeman-Durden model and similar models

For linear combination of fixed matrices:

$$\langle [\mathbf{C}] \rangle = a_1[C_1] + a_2[C_2] + \cdots + a_n[C_n] + [C_R]$$

The optimization problem we solve is

$$\underset{a_1, \ldots, a_n}{\text{minimize:}} \quad \text{trace}\,[C_R]$$

$$\text{subject to:} \quad a_1 \geq 0, \ldots, a_n \geq 0$$

$$[C_R] \succcurlyeq [0]$$

This is a semidefinite program; reasonably efficient algorithms with guaranteed convergence to global optimum

For Freeman-Durden model:

$$\langle[\mathbf{C}]\rangle = P_V[C_V] + P_d[C_d(\alpha)] + P_s[C_s(\beta)] + [C_R]$$

$$[C_d(\alpha)] = \frac{1}{1+|\alpha|^2}\begin{bmatrix} |\alpha|^2 & 0 & \alpha \\ 0 & 0 & 0 \\ \alpha^* & 0 & 1 \end{bmatrix} \qquad [C_s(\beta)] = \frac{1}{1+|\beta|^2}\begin{bmatrix} |\beta|^2 & 0 & \beta \\ 0 & 0 & 0 \\ \beta^* & 0 & 1 \end{bmatrix}$$

$$\underset{P_V, P_d, P_s, \alpha, \beta}{\text{minimize:}} \quad \text{trace}\,[C_R]$$

$$\text{subject to:} \quad P_V \geq 0, P_d \geq 0, P_s \geq 0$$

$$[C_R] \succcurlyeq [0]$$

Re-write as:

$$\langle [\mathbf{C}] \rangle = P_V [C_V] + [M] + [C_R]$$

where $$[M] = \begin{bmatrix} M_{11} & 0 & M_{13} \\ 0 & 0 & 0 \\ M_{31} & 0 & M_{33} \end{bmatrix} = P_d [C_d(\alpha)] + P_s [C_s(\beta)]$$

The optimization problem we solve is

$$\min_{P_V, [M]} \quad \text{trace} \{ \langle [\mathbf{C}] \rangle - P_V [C_V] - [M] \}$$

subject to: $$P_V \geq 0, [M] \succcurlyeq [0]$$
$$\langle [\mathbf{C}] \rangle - P_V [C_V] - [M] \succcurlyeq [0]$$

Let $[Q] = \langle [\mathbf{C}] \rangle - P_V [C_V]$ $\quad [M] = \begin{bmatrix} M_{11} & 0 & M_{13} \\ 0 & 0 & 0 \\ M_{31} & 0 & M_{33} \end{bmatrix}$

And re-write the optimization problem as:

$$\min_{\substack{P_V \geq 0 \\ [Q] \succcurlyeq [0]}} \left\{ \begin{array}{ll} \min\limits_{[M]} & \text{trace}\,\{[Q] - [M]\} \\ \text{subject to:} & [M] \succcurlyeq [0], [Q] - [M] \succcurlyeq [0] \end{array} \right\}$$

Let $[Q] = \langle [\mathbf{C}] \rangle - P_V [C_V]$ $\quad [M] = \begin{bmatrix} M_{11} & 0 & M_{13} \\ 0 & 0 & 0 \\ M_{31} & 0 & M_{33} \end{bmatrix}$

And re-write the optimization problem as:

$$\min_{\substack{P_V \geq 0 \\ [Q] \succcurlyeq [0]}} \left\{ \begin{array}{ll} \min\limits_{[M]} & \text{trace}\,\{[Q] - [M]\} \\ \text{subject to:} & [M] \succcurlyeq [0], [Q] - [M] \succcurlyeq [0] \end{array} \right\}$$

Let $\vec{q} = \begin{bmatrix} Q_{12} \\ Q_{22} \\ Q_{32} \end{bmatrix}$

if $Q_{22} = 0$, solution is $[M] = [Q]$
if $Q_{22} > 0$, solution is

$$[M] = [Q] - \frac{1}{Q_{22}} \vec{q}\vec{q}^{\dagger}$$

Plugging in the analytical solution:

$$\min_{\substack{P_V \geq 0 \\ [Q] \succcurlyeq [0]}} \left\{ \begin{array}{ll} \min\limits_{[M]} & \text{trace}\,\{[Q]-[M]\} \\ \text{subject to:} & [M] \succcurlyeq [0], [Q]-[M] \succcurlyeq [0] \end{array} \right\}$$

$$= \min_{\substack{P_V \geq 0 \\ [Q] \succcurlyeq [0]}} \left\{ \frac{\vec{q}^{\dagger}\vec{q}}{Q_{22}} \right\} = \boxed{\begin{array}{l} \min\limits_{P_V} \left\{ \begin{array}{l} \text{rational function of } P_V \\ \text{(quadratic over linear)} \end{array} \right\} \\ \text{subject to:} \quad P_V \geq 0 \\ \langle [\mathbf{C}] \rangle - P_V [C_V] \succcurlyeq [0] \end{array}}$$

Solution is either

- at where derivative with respect to P_V is zero
- at the limit set by one of the constraints

Plug solution for P_V back into previous formula to find $[M]$

OPTIMAL NNED FOR FREEMAN-DURDEN MODEL

$$\langle[\mathbf{C}]\rangle = P_V[C_V] + [M] + [C_R]$$

$$[M] = \begin{bmatrix} M_{11} & 0 & M_{13} \\ 0 & 0 & 0 \\ M_{31} & 0 & M_{33} \end{bmatrix} = P_d[C_d(\alpha)] + P_s[C_s(\beta)]$$

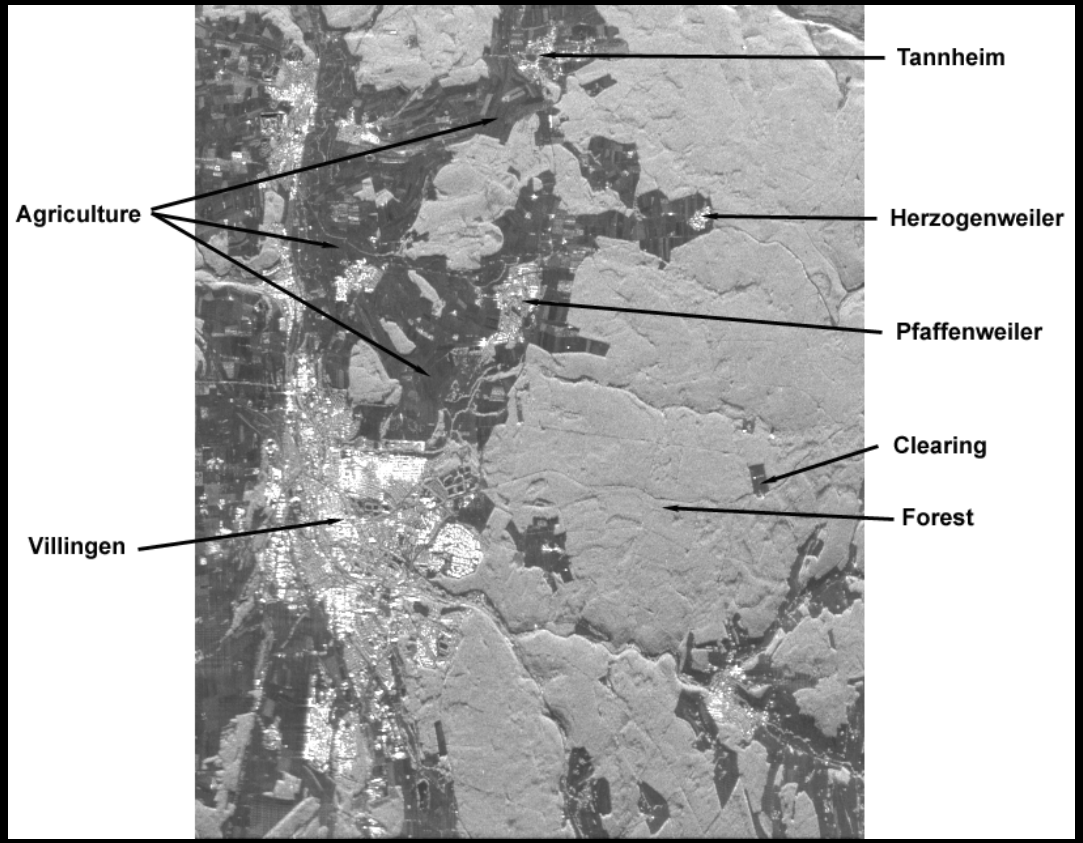
- Computationally efficient, globally optimal solution for P_V and $[M]$
- Can substitute any other choice of $[C_V]$
- No reflection symmetry assumption on $\langle[\mathbf{C}]\rangle$

OPTIMAL NNED FOR FREEMAN-DURDEN MODEL

$$[M] = \begin{bmatrix} M_{11} & 0 & M_{13} \\ 0 & 0 & 0 \\ M_{31} & 0 & M_{33} \end{bmatrix} = P_d[C_d(\alpha)] + P_s[C_s(\beta)]$$

- Surface- and double-bounce assignments from $[M]$ is inherently underdetermined; infinitely many possible valid choices
- Eigenvector decomposition is one choice
- Can also use original assignment scheme proposed by Freeman and Durden without problem of negative powers, since $[M] \succcurlyeq [0]$

TEST IMAGE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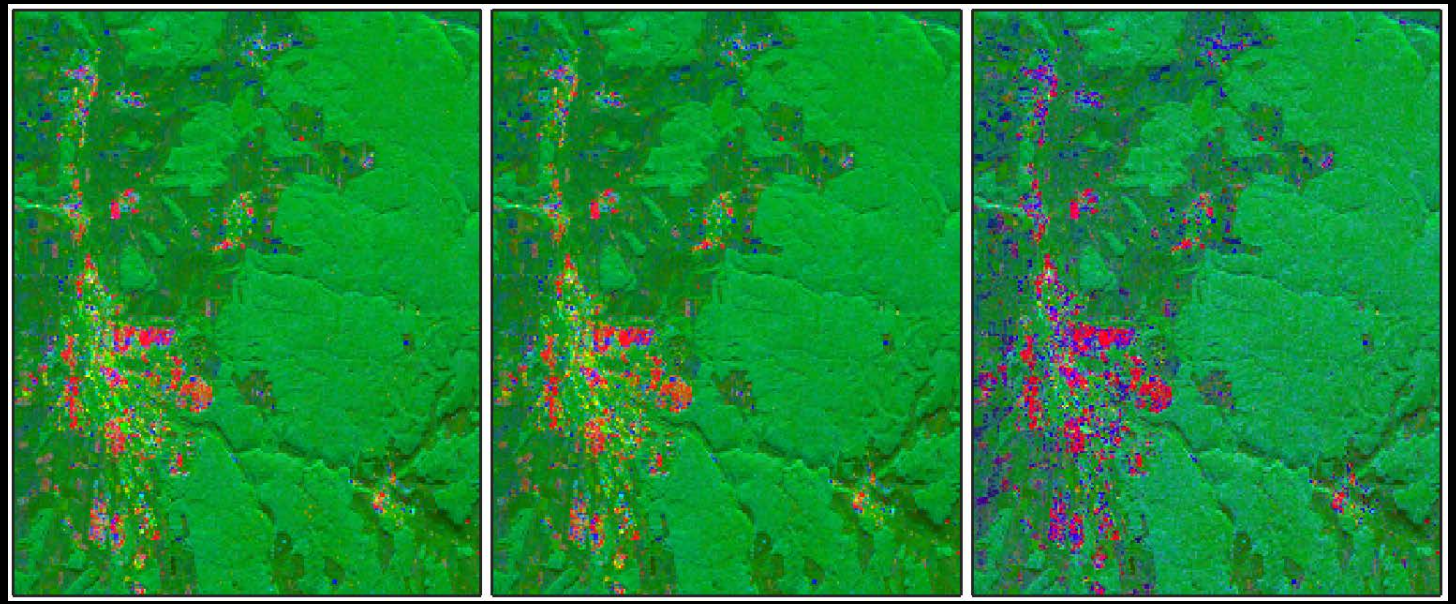
Agriculture
Herzogenweiler
Pfaffenweiler
Clearing
Forest
Villingen

C-BAND

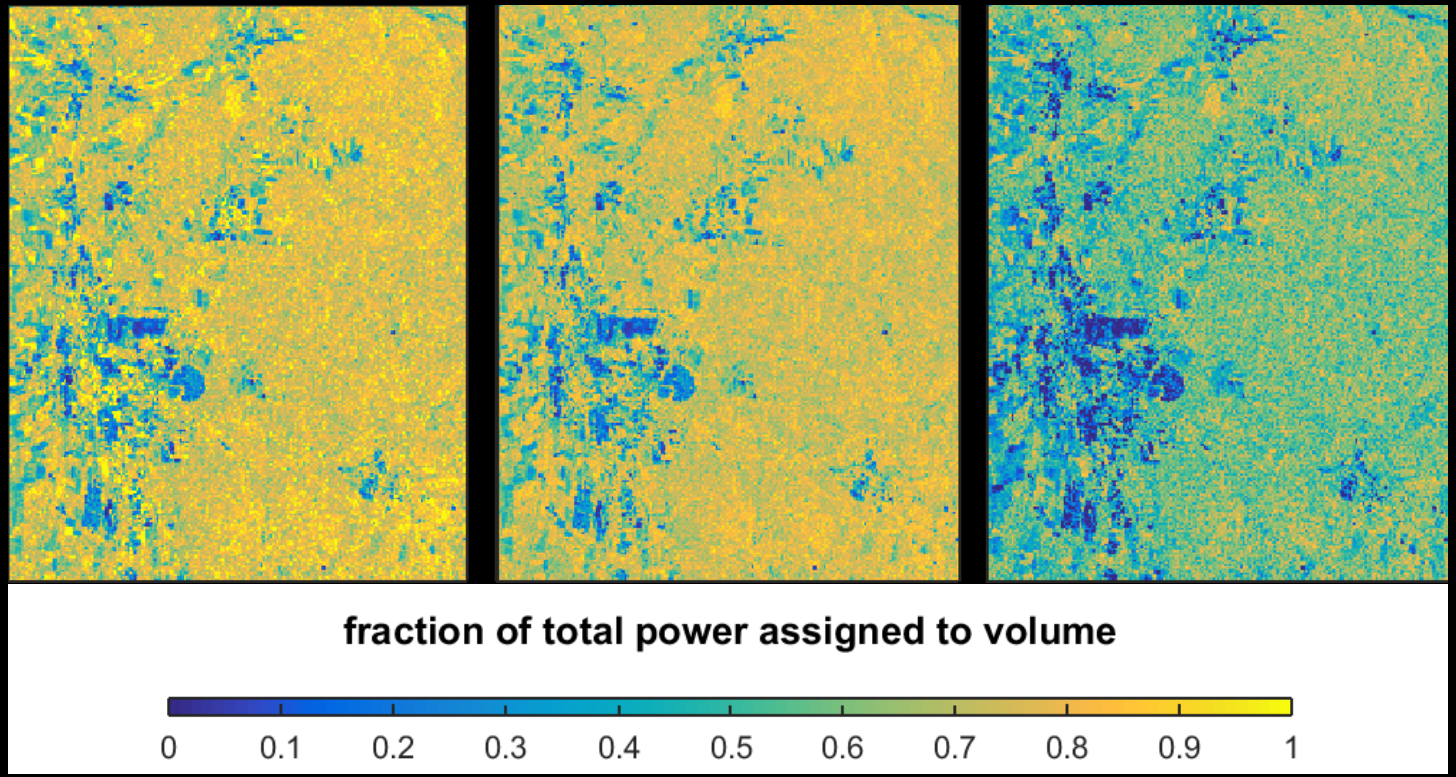


Freeman and Durden

NNED

optimal NNED

C-BAND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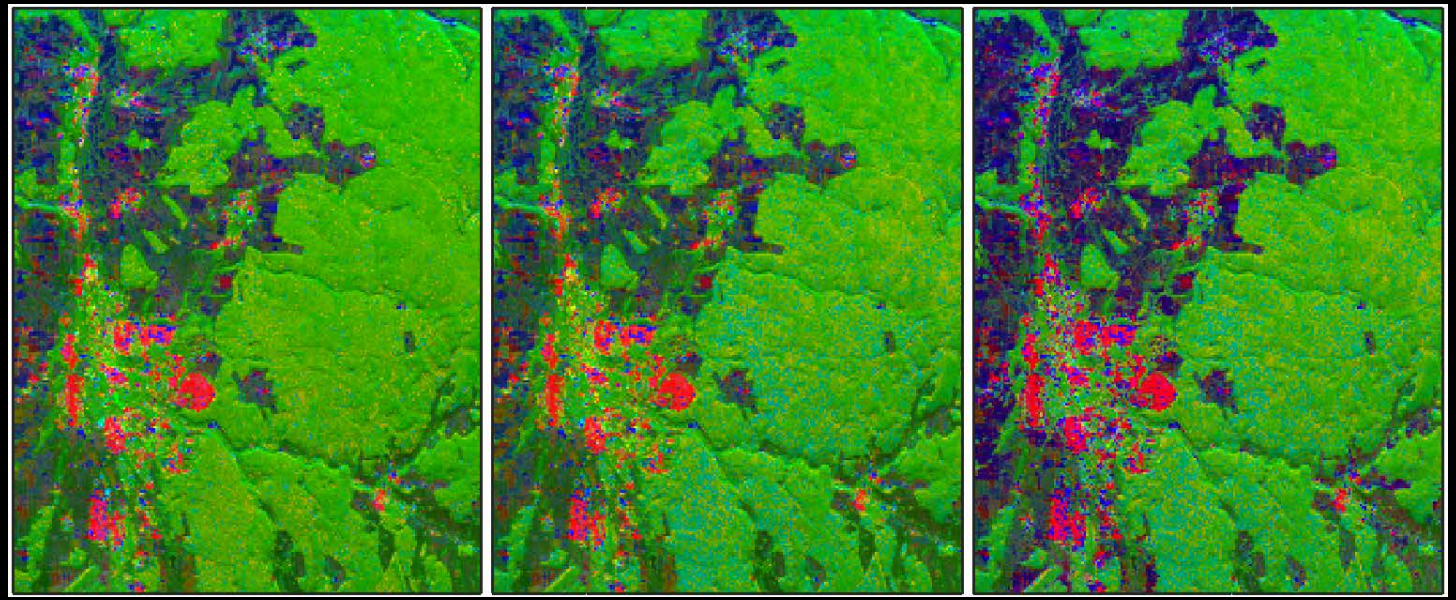
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

L-BAND

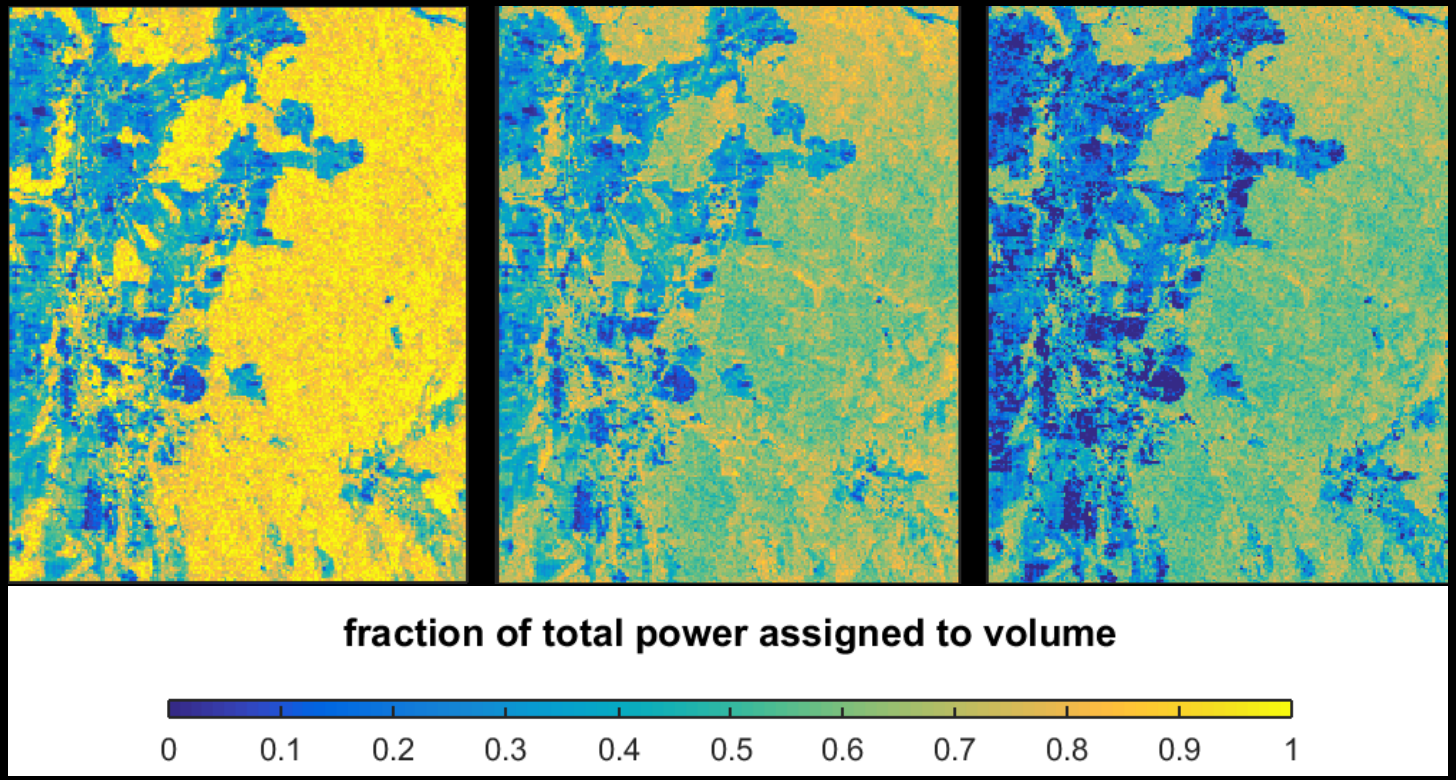


Freeman and Durden

NNED

optimal NNED

L-BAND


fraction of total power assigned to volume
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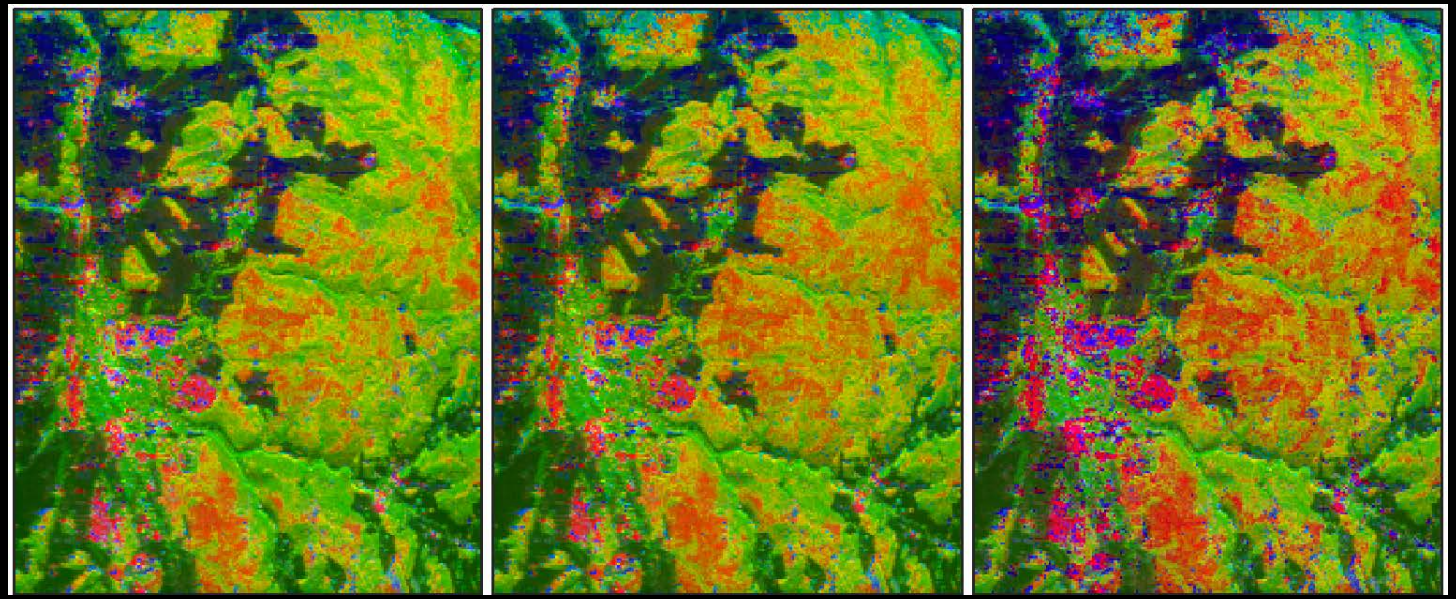
1

P-BAND

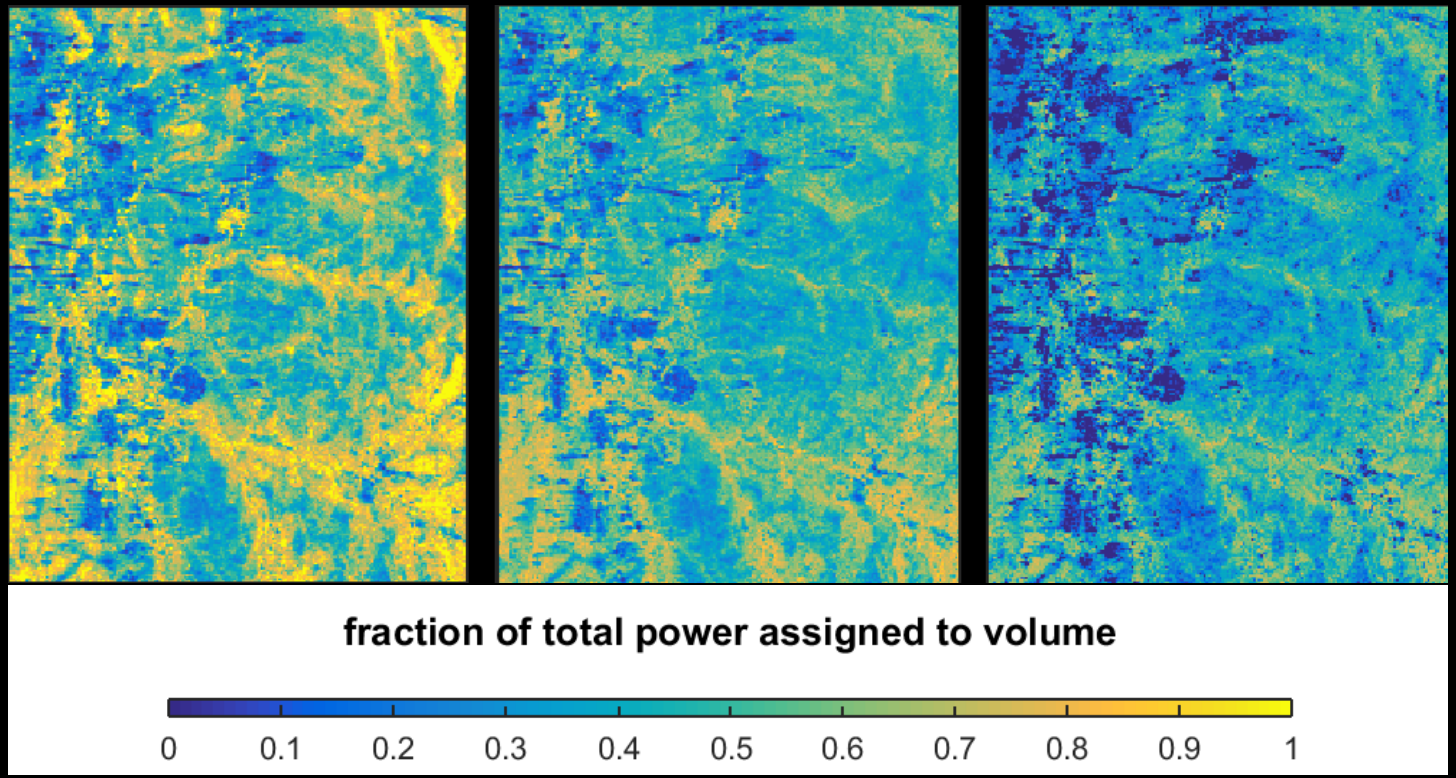


Freeman and Durden

NNED

optimal NNED

P-BAND


fraction of total power assigned to volume
0
0.1
0.2
0.3
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0.8
0.9
1

SUMMARY AND CONCLUSIONS

- We presented a general framework for model-based interpretation of observed polarimetric scattering, based on NNED principles
- Similar to Chen et al. (2014), the general inversion is formulated as a constrained optimization problem, but with different objective function and constraints
- Globally optimal solutions can be computed efficiently for important special cases:
  - Linear combination of fixed matrices
  - Freeman-Durden model (and variants)

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