Viscous Instability Triggered by Layered Accretion in Protoplanetary Disks

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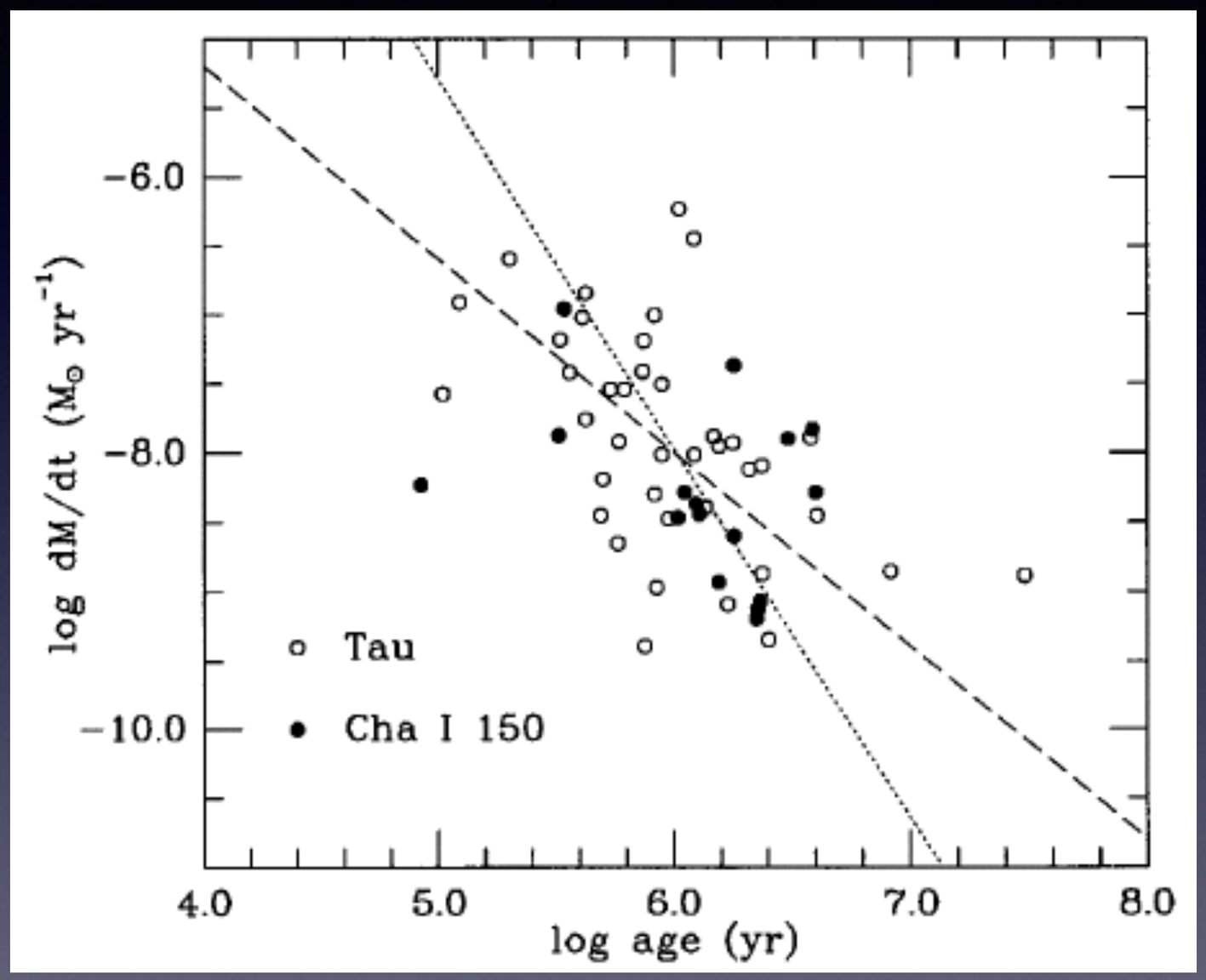
ApJ, 815, 99 2015

Disk Properties



Hartmann et al 1998

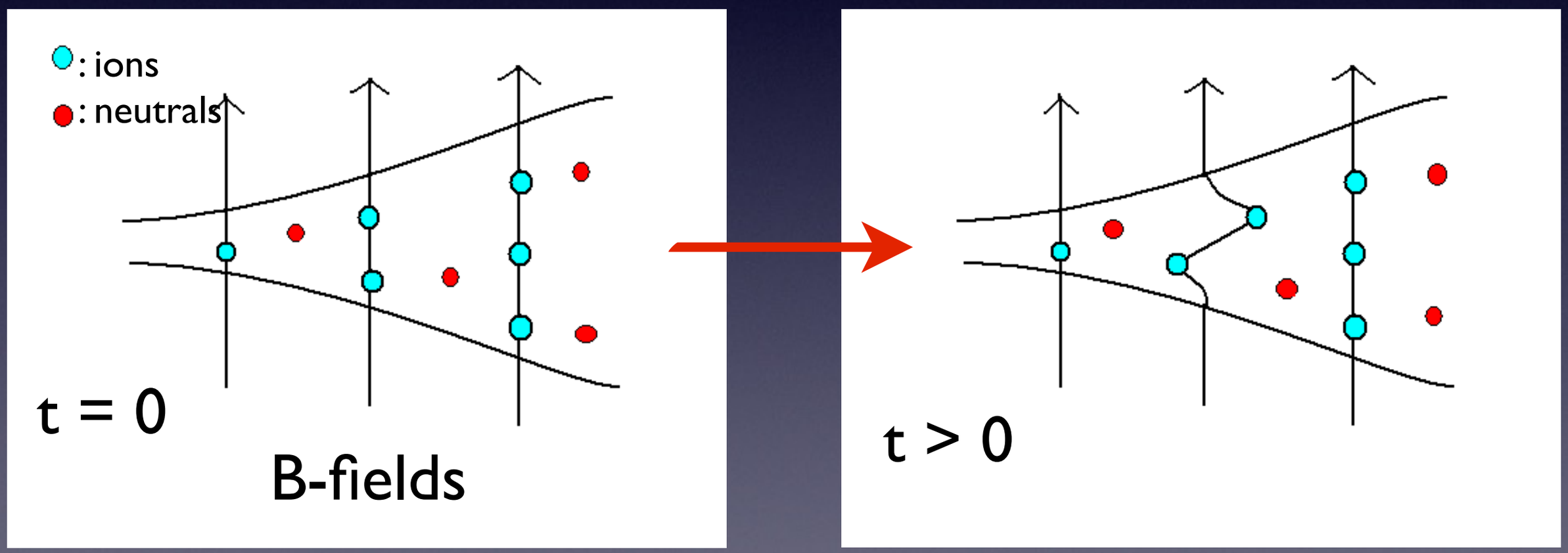
Disk accretion is too high to be explained by molecular viscosity

Disk turbulence that can cause anomalous viscosity is needed

Origins of Disk Turbulence

MagnetoRotational Instabilities (MRIs)

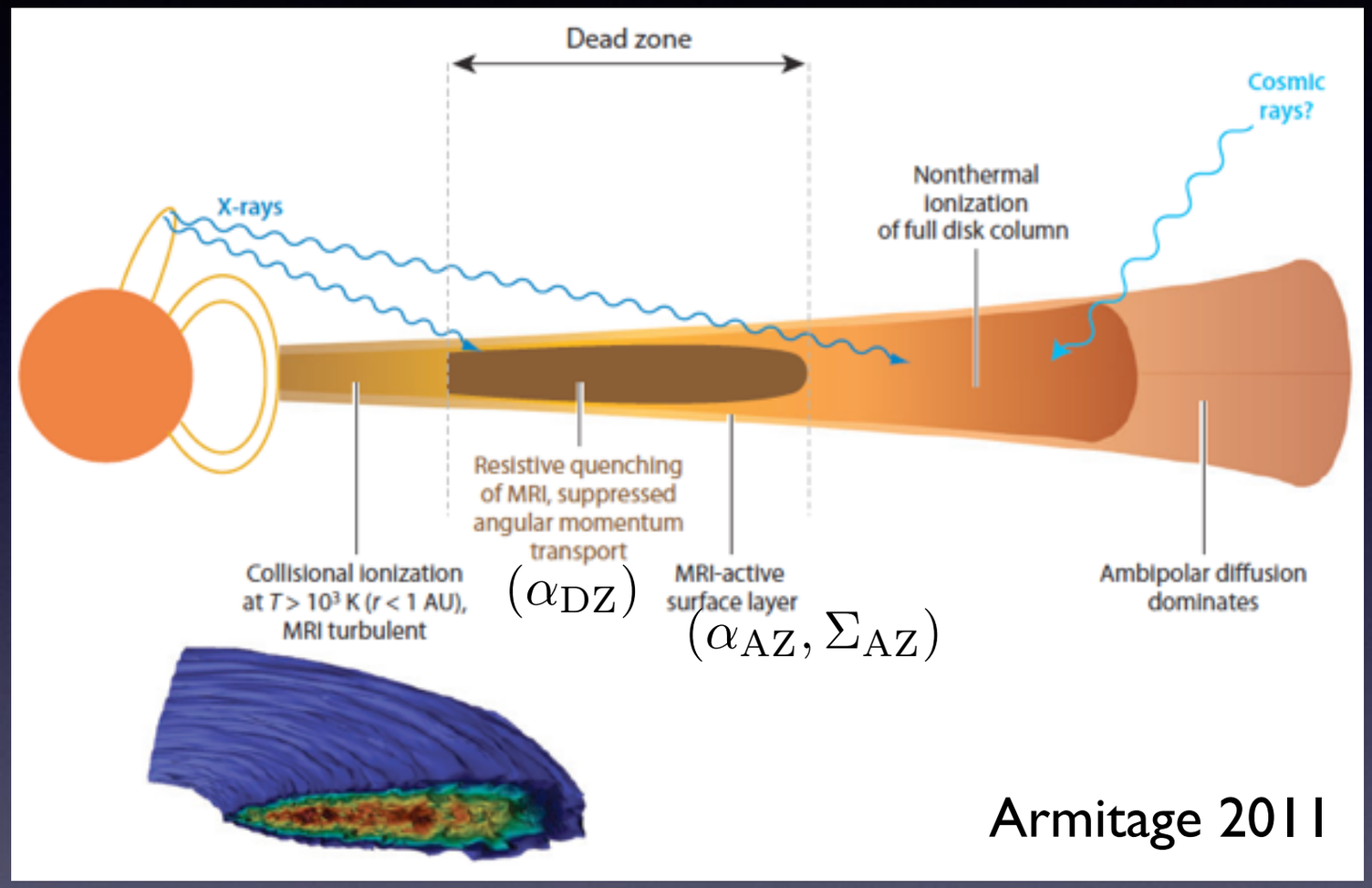
e.g. Balbus & Hawley 1991a, b, 1998, Fleming & Stone 2003, Nelson & Papaloizou 2004



B-fields transport angular momentum

Classical Picture of Layered Accretion

e.g.,Gammie 1996



Armitage 2011

$\alpha \to \alpha_{\rm AZ}$ if $\Sigma < \Sigma_{\rm crit}$

: $\Sigma = \Sigma_{\rm AZ}$

in the outer disk

$\alpha \to \alpha_{\rm cl}$ if $\Sigma > \Sigma_{\rm crit}$

: $\Sigma_{\rm AZ} = \Sigma_{\rm crit}$

in the inner disk

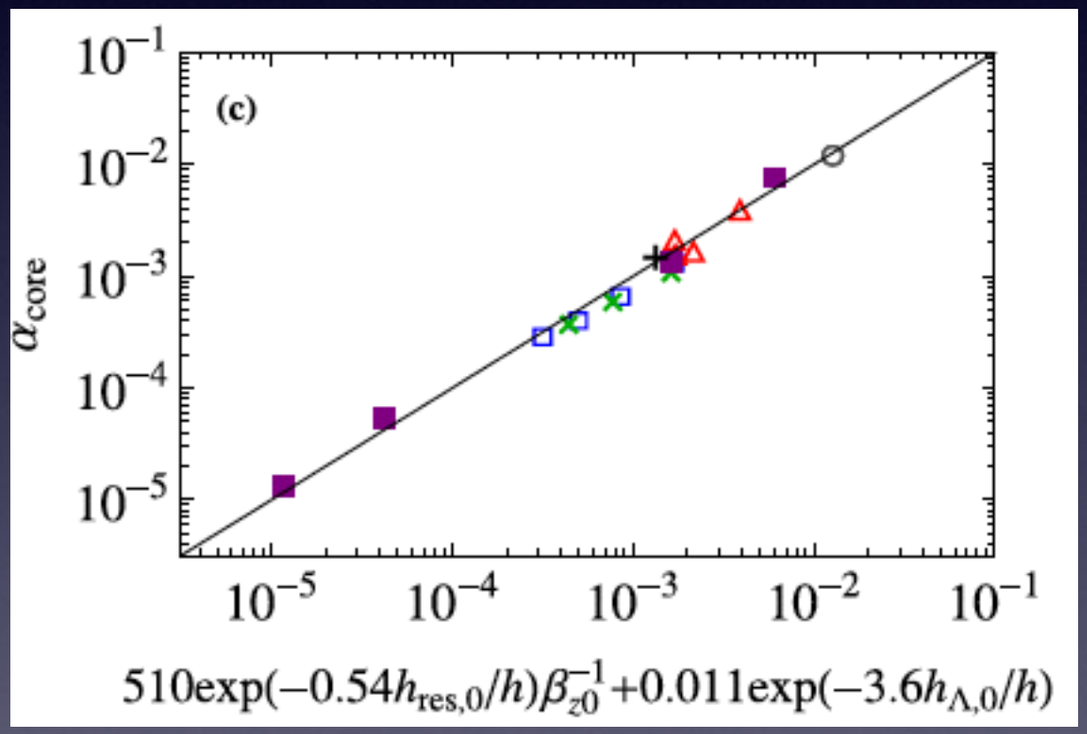
$$\alpha_{\rm cl} \equiv \frac{\Sigma_{\rm AZ}\alpha_{\rm AZ} + (\Sigma - \Sigma_{\rm AZ})\alpha_{\rm DZ}}{\Sigma} \simeq \alpha_{\rm DZ} + \alpha_{\rm AZ}\frac{\Sigma_{\rm AZ}}{\Sigma}$$

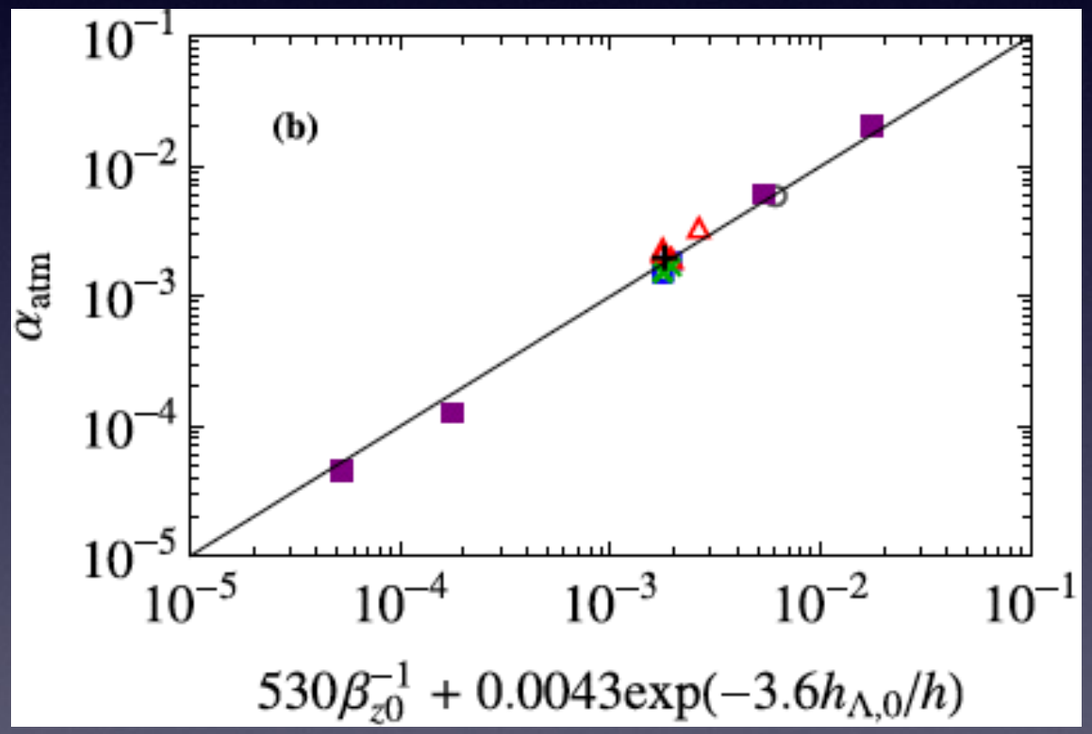
e.g., Kretke & Lin 2007, Hasegawa & Pudritz 2011b

An Empirical Formula for α

Okuzumi & Hirose 2011

vertically stratified MHD simulations
with ohmic diffusion (as a non-ideal MHD effect)





An Empirical Formula for α

Okuzumi & Hirose 2011

vertically stratified MHD simulations
with ohmic diffusion (as a non-ideal MHD effect)

$$\alpha_{\rm OH} \simeq \alpha_{\rm floor}\left[\left(\frac{\beta_z}{\beta_{z0}}\right)^{-1} + \exp\left(-\delta\frac{h_{\rm AZ}}{h}\right)\right]$$

$\alpha_{\rm floor} \simeq 10^{-2}$: a reference value for fully turbulent disks, $\beta_z \equiv \frac{8\pi\rho_{mid}c_s^2}{<B_z>^2}$

$\beta_{z0} \simeq 10^4$: a normalization factor, $\delta = 3.6$: a fitting parameter

$h_{\rm AZ}$: vertical extent of the active zone, h : gas pressure scale height

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1. Maxwell stress $(\text{or } \alpha) \propto < B_z >^2$ e.g., Hawley et al 1995

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1. Maxwell stress $(\text{or } \alpha) \propto < B_z >^2$ e.g., Hawley et al 1995

2. dead zones disappear when disks are fully turbulent
$(\alpha_{\rm OH} \to \alpha_{\rm floor} \text{ when } h_{\rm AZ} \to 0)$

$\alpha_{\rm floor} \simeq 10^{-2}$: a reference value for fully turbulent disks, $\beta_z \equiv \frac{8\pi\rho_{mid}c_s^2}{< B_z >^2}$

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New Picture of Layered Accretion

$$\alpha_{\rm OH} \simeq \alpha_{\rm floor}\left[\left(\frac{\beta_z}{\beta_{z0}}\right)^{-1} + \exp\left(-\delta\frac{h_{\rm AZ}}{h}\right)\right]$$

New Picture of Layered Accretion

$$\alpha_{\rm OH} \simeq \alpha_{\rm floor}\left[\left(\frac{\beta_z}{\beta_{z0}}\right)^{-1} + \exp\left(-\delta\frac{h_{\rm AZ}}{h}\right)\right]$$

$$\frac{\Sigma_{\rm AZ}}{\Sigma} = {\rm erfc}\left(\frac{h_{\rm AZ}}{h}\right)$$

$$\alpha_{\rm floor} \equiv \alpha_{\rm AZ}$$

$$\alpha_{\rm DZ} \equiv \alpha_{\rm AZ}\frac{\beta_{z0} < B_z >_0^2}{4\sqrt{(2\pi)}\Sigma_{\rm crit}c_s\Omega}$$

$$\alpha_{\rm sim} \equiv \alpha_{\rm DZ}\left(\frac{< B_z >}{< B_z >_0}\right)^2\left(\frac{\Sigma}{\Sigma_{\rm crit}}\right)^{-1} + \alpha_{\rm AZ}\left[k_1\left(\frac{\Sigma_{\rm AZ}}{\Sigma}\right) + k_2\left(\frac{\Sigma_{\rm AZ}}{\Sigma}\right)^q\right]$$

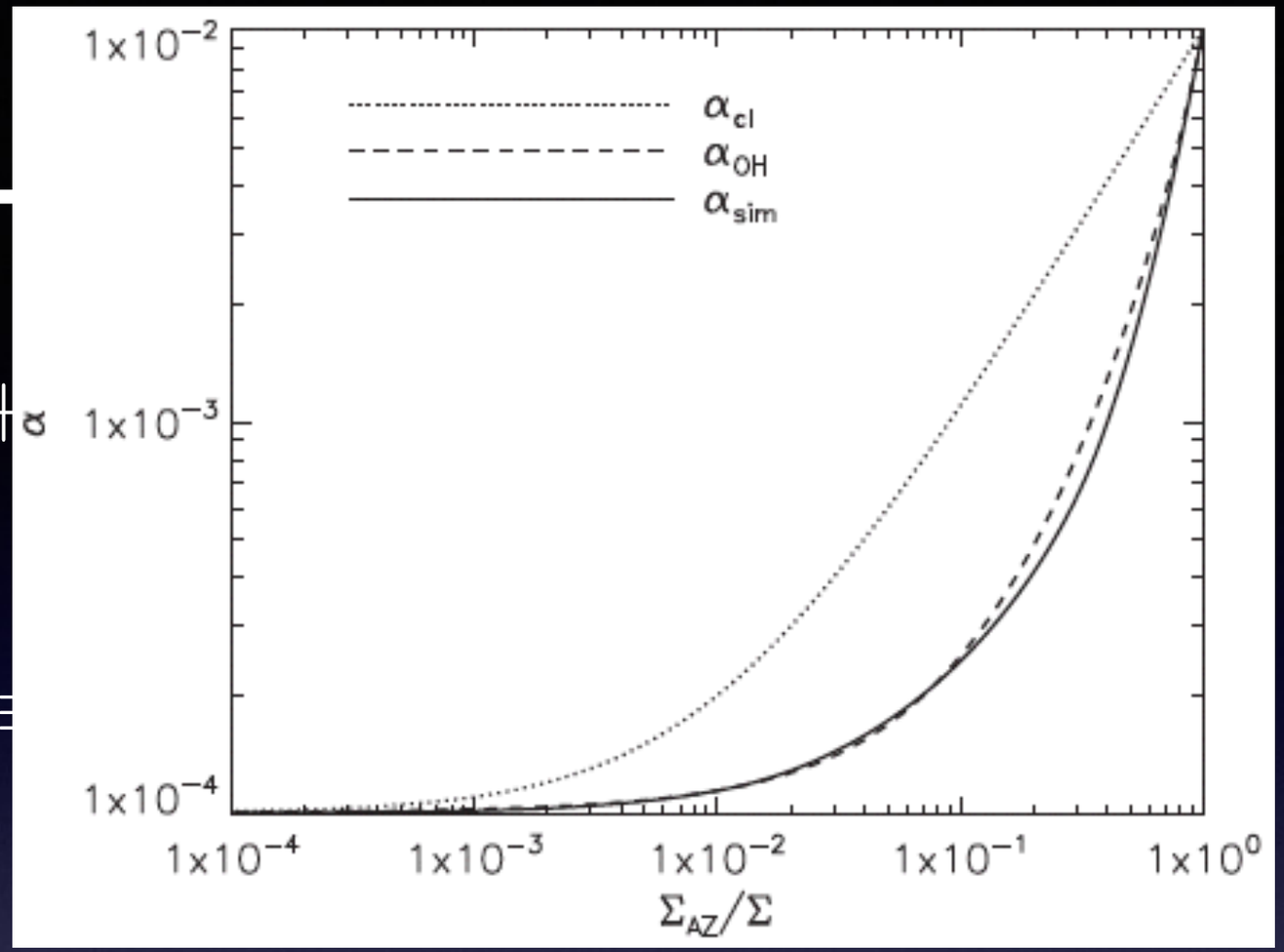
$k_1 = 1/7$, $k_2 = 6/7$, and $q = 3.6$

New Picture of L

$$\alpha_{\rm OH} \simeq \alpha_{\rm floor} \left[\left(\frac{\beta_z}{\beta_{z0}} \right)^{-1} + \right.$$

$$\frac{\Sigma_{\rm AZ}}{\Sigma} = {\rm erfc}\left(\frac{h_{\rm AZ}}{h} \right)$$

$$\alpha_{\rm floor} \equiv$$



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1. it can reproduce $\alpha_{\rm OH}$ very well

$$k_1 = 1/7,\ k_2 = 6/7,\ \text{and}\ q = 3.6$$

New Picture of L

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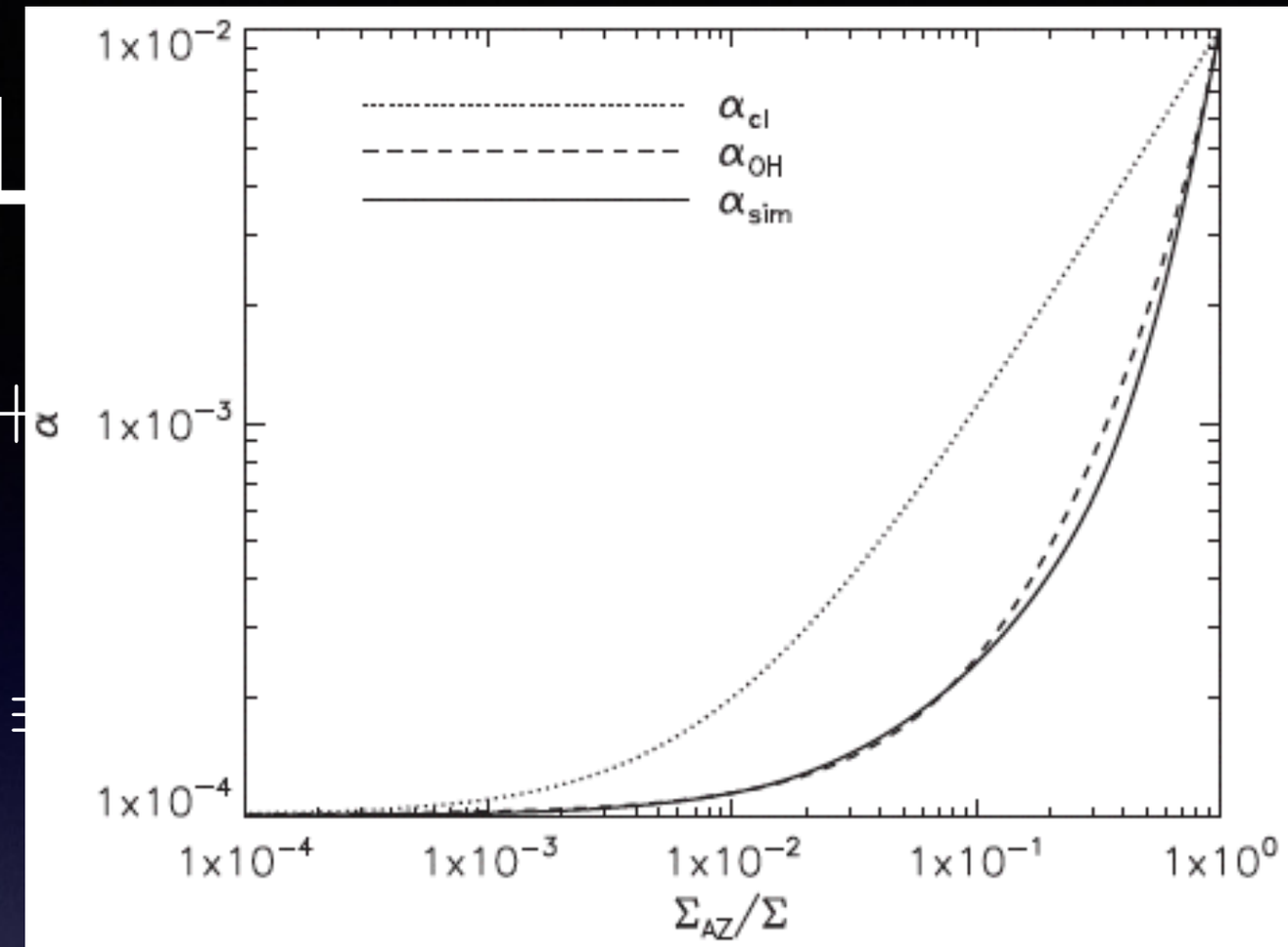

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1. it can reproduce $\alpha_{\rm OH}$ very well
2. it can be viewed as a general extension of $\alpha_{\rm cl}$

$$\alpha_{\rm cl} \simeq \alpha_{\rm DZ} + \alpha_{\rm AZ}\frac{\Sigma_{\rm AZ}}{\Sigma}$$

$$k_1 = 1/7,\ k_2 = 6/7,\ \text{and}\ q = 3.6$$

Classical

$$\alpha_{\rm cl} \simeq \alpha_{\rm DZ} + \alpha_{\rm AZ}\frac{\Sigma_{\rm AZ}}{\Sigma}$$

New

$$\alpha_{\rm sim} = \alpha_{\rm DZ}\left(\frac{< B_z >}{< B_z >_0}\right)^2\left(\frac{\Sigma}{\Sigma_{\rm crit}}\right)^{-1} + \alpha_{\rm AZ}\left[k_1\left(\frac{\Sigma_{\rm AZ}}{\Sigma}\right) + k_2\left(\frac{\Sigma_{\rm AZ}}{\Sigma}\right)^q\right]$$

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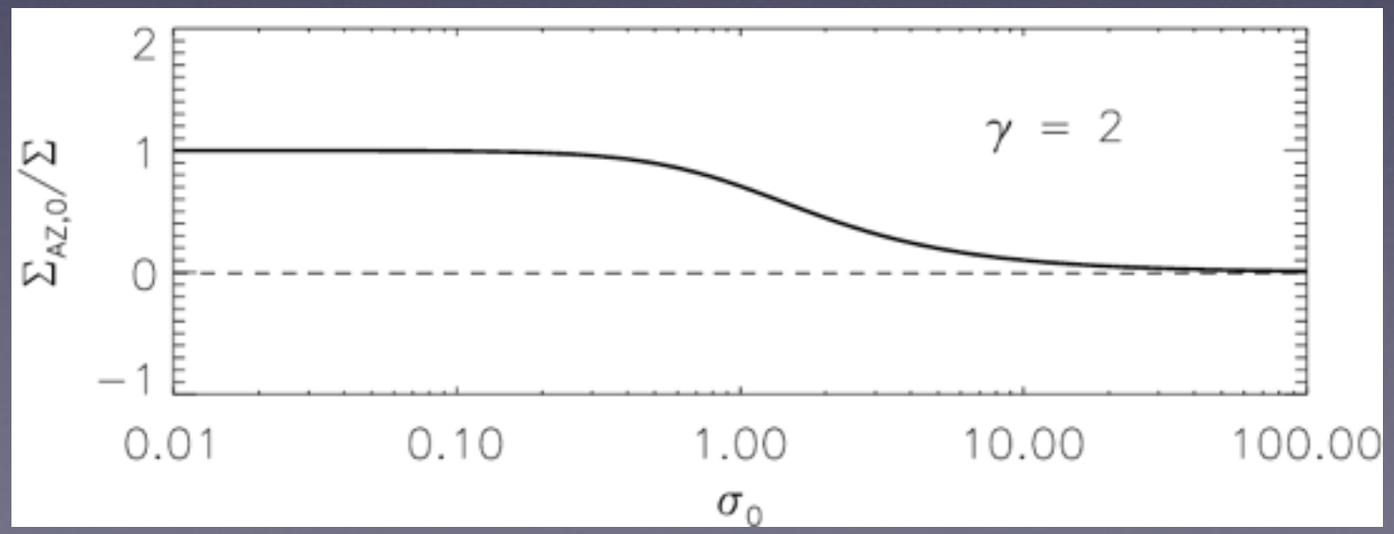
Linear Stability Analysis

$$\frac{\Sigma_{\rm AZ}}{\Sigma} \simeq \frac{\min[\Sigma_{\rm crit}, \Sigma]}{\Sigma} \approx (\sigma^\gamma + 1)^{-1/\gamma} \text{ with } \sigma = \frac{\Sigma}{\Sigma_{\rm crit}} \;\&\; \gamma = 2$$



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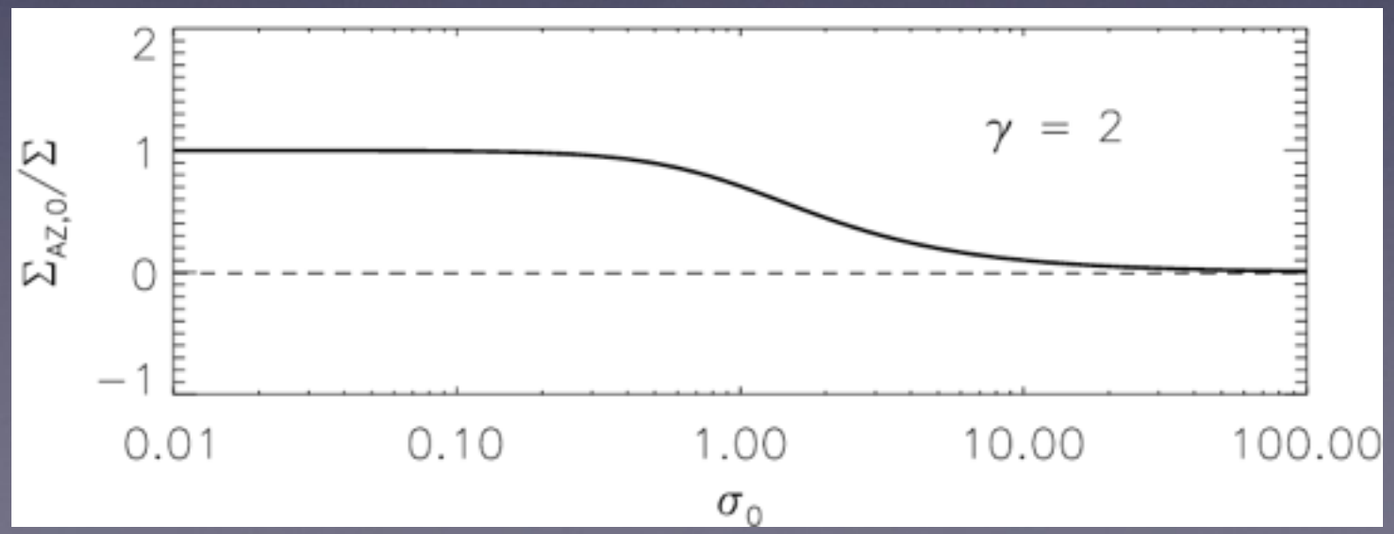
$$\sigma = \sigma_0 + \sigma_1 \qquad \alpha_{\rm sta} \equiv \alpha(\sigma_0) + \left.\frac{\partial\alpha}{\partial\sigma}\right|_{\sigma_0}\sigma_0 < 0$$

$$\alpha^{\rm cl}_{\rm sta} = \alpha_{\rm DZ} + \frac{\alpha_{\rm AZ}}{(\sigma_0^\gamma + 1)^{1+1/\gamma}} > 0$$



New

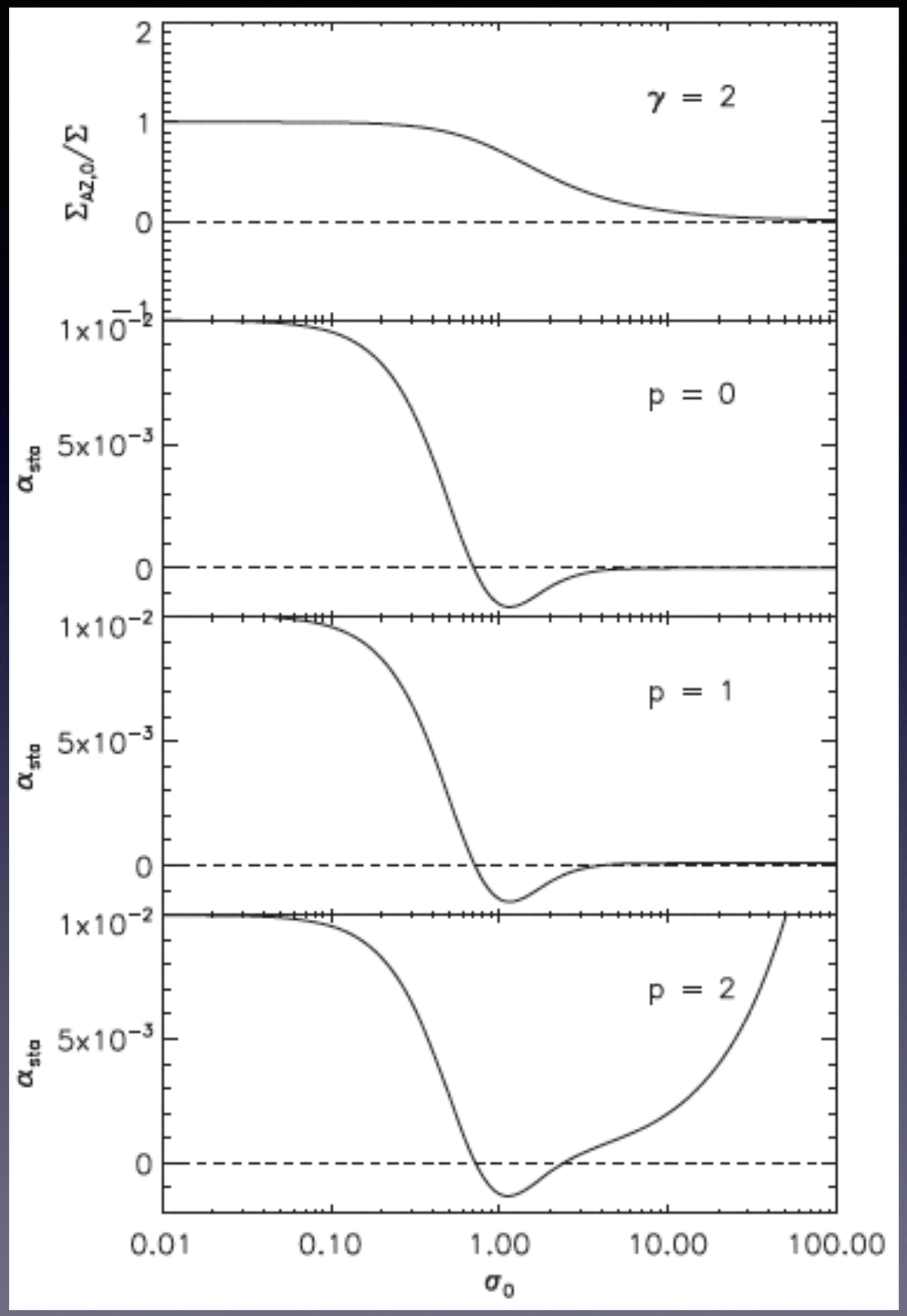
$$\alpha_{\rm sim} = \alpha_{\rm DZ}\left(\frac{< B_z >}{< B_z >_0}\right)^2\left(\frac{\Sigma}{\Sigma_{\rm crit}}\right)^{-1} + \alpha_{\rm AZ}\left[k_1\left(\frac{\Sigma_{\rm AZ}}{\Sigma}\right) + k_2\left(\frac{\Sigma_{\rm AZ}}{\Sigma}\right)^q\right]$$

$$< B_z >^2 = K(r)\Sigma(r)^p$$

$$B_z = \text{const. for } p = 0$$

$$\beta_z = \text{const. for } p = 1$$

$$\Sigma/B_z = \text{const. for } p = 2$$



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$$\Sigma/B_z = \text{const. for } p = 2$$

disks with new α become unstable!!

Classical

$$\alpha_{\rm cl} \simeq \alpha_{\rm DZ} + \alpha_{\rm AZ}\frac{\Sigma_{\rm AZ}}{\Sigma}$$

New

$$\alpha_{\rm sim} = \alpha_{\rm DZ}\left(\frac{< B_z >}{< B_z >_0}\right)^2\left(\frac{\Sigma}{\Sigma_{\rm crit}}\right)^{-1} + \alpha_{\rm AZ}\left[k_1\left(\frac{\Sigma_{\rm AZ}}{\Sigma}\right) + k_2\left(\frac{\Sigma_{\rm AZ}}{\Sigma}\right)^q\right]$$

Physical Interpretation

Classical

$$\alpha_{\rm cl} \simeq \alpha_{\rm DZ} + \alpha_{\rm AZ}\frac{\Sigma_{\rm AZ}}{\Sigma}$$

New

$$\alpha_{\rm sim} = \alpha_{\rm DZ}\left(\frac{< B_z >}{< B_z >_0}\right)^2\left(\frac{\Sigma}{\Sigma_{\rm crit}}\right)^{-1} + \alpha_{\rm AZ}\left[k_1\left(\frac{\Sigma_{\rm AZ}}{\Sigma}\right) + k_2\left(\frac{\Sigma_{\rm AZ}}{\Sigma}\right)^q\right]$$

Physical Interpretation

$$\dot{M} \propto \alpha\Sigma$$

$$\dot{M}_{\rm cl} \propto \Sigma$$

$$\dot{M}_{\rm sim} \propto f(\Sigma)$$

Classical

$$\alpha_{\rm cl} \simeq \alpha_{\rm DZ} + \alpha_{\rm AZ}\frac{\Sigma_{\rm AZ}}{\Sigma}$$

New

$$\alpha_{\rm sim} = \alpha_{\rm DZ}\left(\frac{< B_z >}{< B_z >_0}\right)^2\left(\frac{\Sigma}{\Sigma_{\rm crit}}\right)^{-1} + \alpha_{\rm AZ}\left[k_1\left(\frac{\Sigma_{\rm AZ}}{\Sigma}\right) + k_2\left(\frac{\Sigma_{\rm AZ}}{\Sigma}\right)^q\right]$$

Physical Interpretation

$$\dot{M} \propto \alpha\Sigma$$

$$\dot{M}_{\rm cl} \propto \Sigma$$

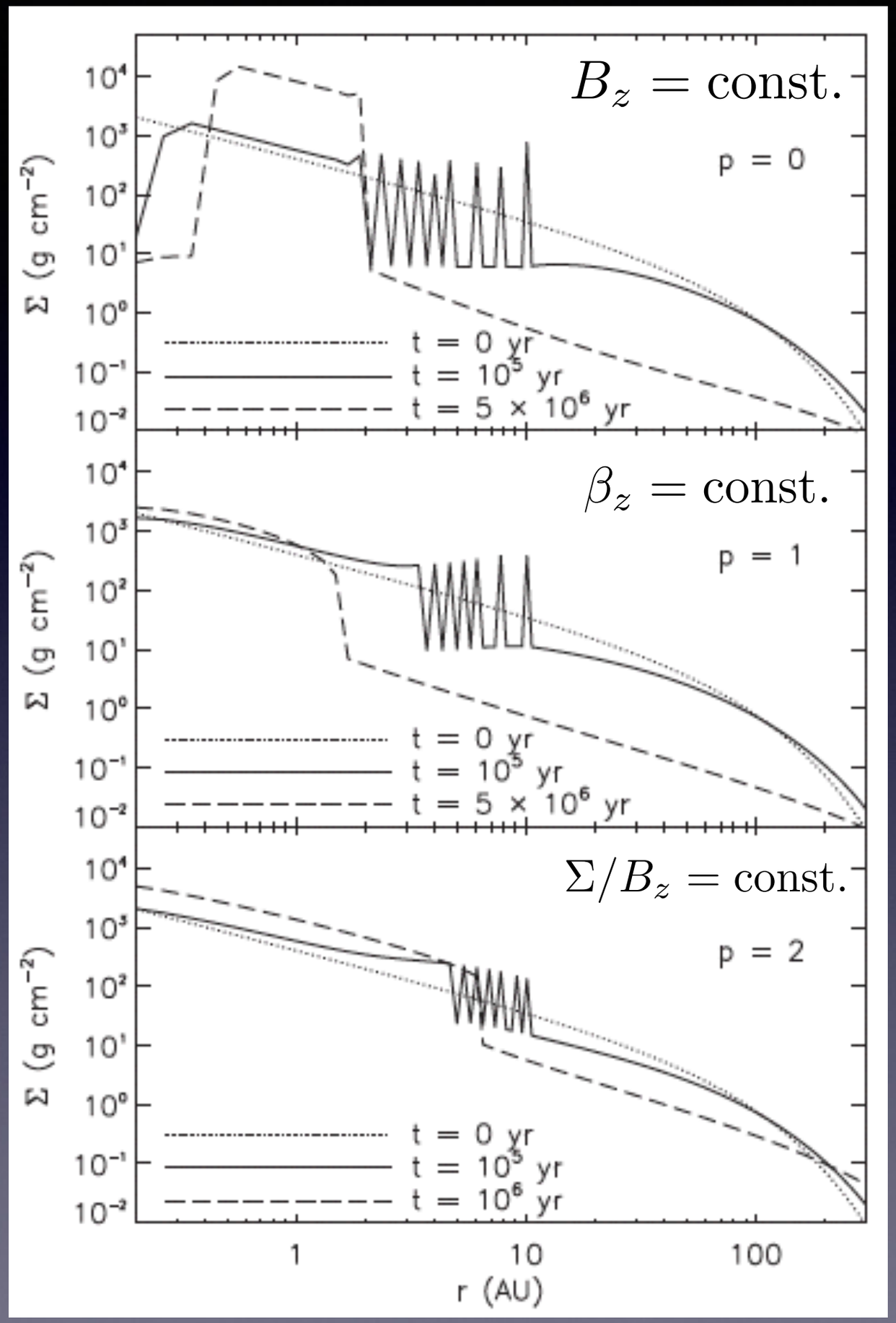
$$\dot{M}_{\rm sim} \propto f(\Sigma)$$

0

$$\alpha_{\rm OH} \simeq \alpha_{\rm floor}\left[\left(\frac{\beta_z}{\beta_{z0}}\right)^{-1} + \exp\left(-\delta\frac{h_{\rm AZ}}{h}\right)\right]$$

$$\dot{M}_{\rm OH} \propto < B_z >^2$$

the surface density can fluctuate for a given $\dot{M}$



Long-term Evolution

The viscous instability grows at $t < 10^5 \mathrm{yr}$

A prominent sawtooth-like structure emerges at the outer dead zone edge

The structure is transient, since it's gone at $t > 10^6 \mathrm{yr}$

The exact shape of the structure should be examined in more detailed simulations

Summary

Hasegawa & Takeuchi (2015) ApJ, 815, 99

- Protoplanetary disks are the birthplace of planets
- Turbulence is needed to account for the observed disk accretion rate, and magneto rotational instabilities (MRIs) are promising processes for that
- Classical picture of layered accretion ends up with disks that are stable against a viscous instability
- New picture of layered accretion triggers a viscous instability that is caused by the disk accretion rate which is a function of B-fields (not the surface density)
- 1D viscous evolution of disks confirms the instability
- More detailed numerical simulations are needed to fully examine the consequence of the instability on disk evolution and the subsequent planet formation