Technical University of Madrid
Space Dynamics Group

Introducing a degree of freedom in the family of generalized logarithmic spirals

Javier Roa and Jesús Peláez

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Context

Generalized logarithmic spirals

- Fully analytic solution.
- Two integrals of motion (energy+angular momentum).
- No degrees of freedom:

$$\mathbf{r} = \mathbf{f}(\theta; r_0, v_0, \theta_0, \psi_0)$$
$$t = g(\theta; r_0, v_0, \theta_0, \psi_0)$$

- Flexibility?
- Mission design?

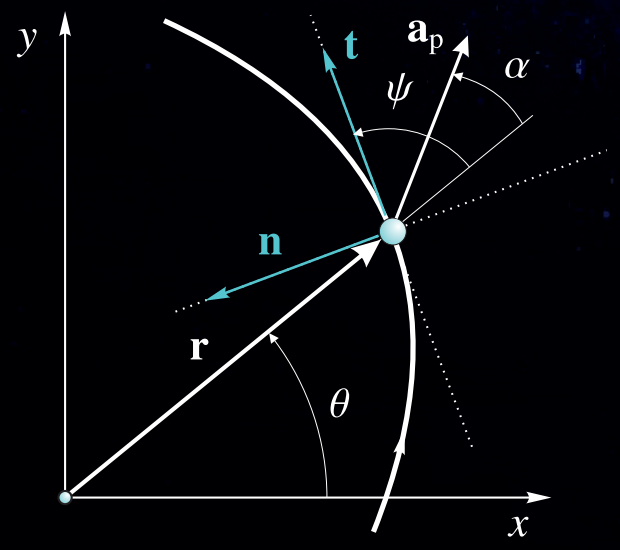
Results

Extended family of solutions

$$\mathbf{a}_p = \frac{\mu}{2r^2}\cos\psi\,\mathbf{t} \longrightarrow \mathbf{a}_p = \frac{\mu}{r^2}[\xi\cos\psi\,\mathbf{t} + (1-2\xi)\sin\psi\,\mathbf{n}]$$

- Fully analytic solution.
- Two integrals of motion.
- There is one degree of freedom.
- Improved flexibility for mission design.

Dynamics

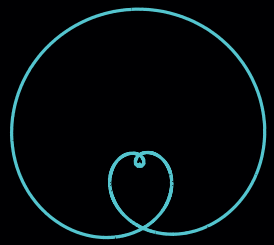


Equation of the energy

$$v^2 - \frac{2(1-\xi)}{r} = K_1$$

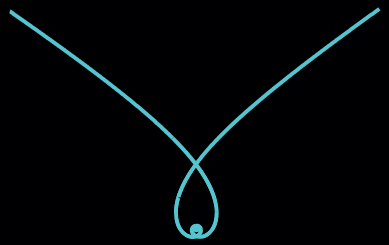
Equation of the angular momentum

$$rv^2 \sin\psi = K_2$$



Elliptic $K_1 < 0$

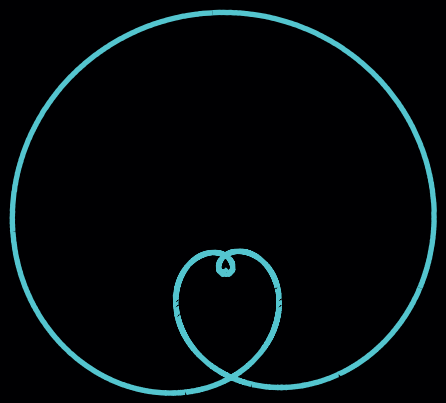
Parabolic $K_1 = 0$

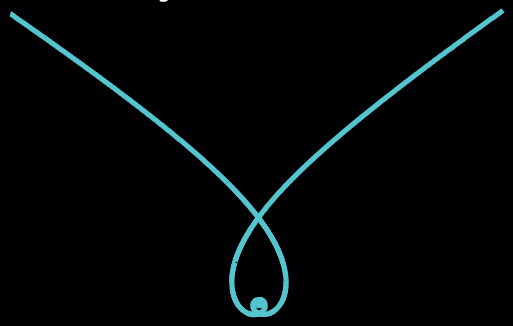


Hyperbolic $K_1 > 0$

The spirals

Elliptic $K_1 < 0$	Parabolic $K_1 = 0$	Hyperbolic $K_1 > 0$
Apoapsis	Logarithmic spiral	$K_1 = v_\infty^2 \equiv C_3$
$r_{\max} = \dfrac{2(1-\xi) - K_2}{(-K_1)}$	$v = \sqrt{\dfrac{2(1-\xi)}{r}}$	Periapsis
		$r_{\min} = \dfrac{K_2 - 2(1-\xi)}{K_1}$
Symmetric		Symmetric

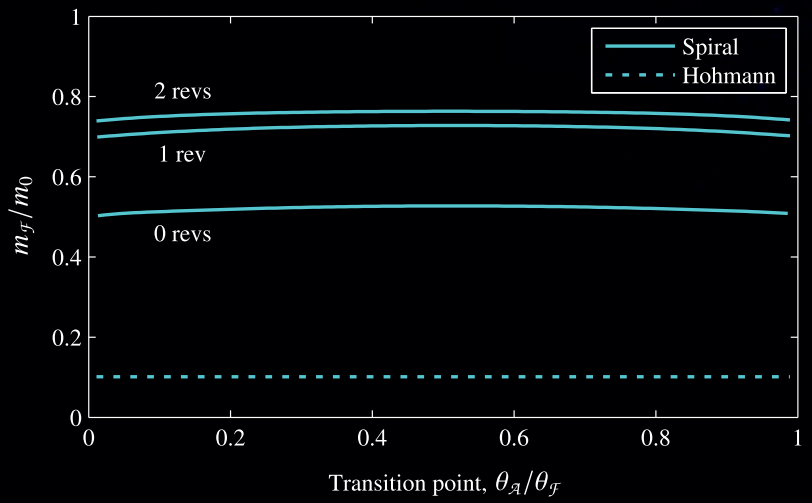


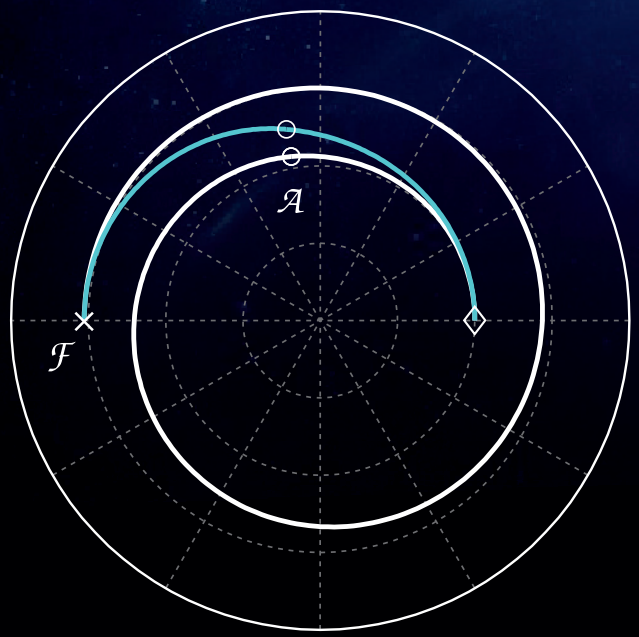


Circular to circular

"Patched spirals": Matching K_1 and K_2

- No need to change K_2.
- At least two spiral arcs.



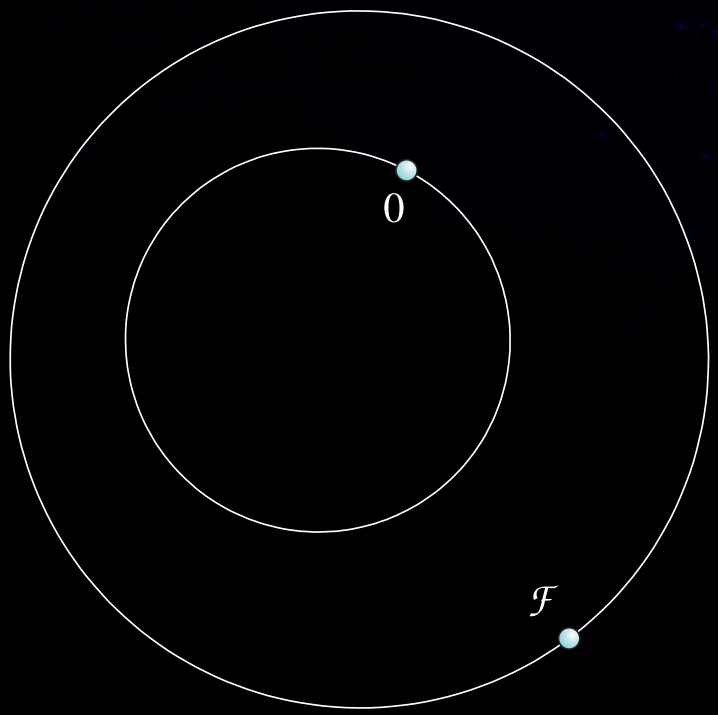


- One degree of freedom $\rightarrow \theta_{\mathcal{A}}$
- One transcendental equation:

$$r_{\mathcal{F}} = f(\theta_{\mathcal{F}}; \xi, r_0, v_0, \theta_0, \psi_0)$$

Arbitrary transfers

Required K_2 at departure/arrival points will not be the same



- $rv^2 \sin\psi = K_2$
- How to adjust K_2?
- Keplerian arc.

Matching conditions:
$\xi_2, \theta_{\mathcal{B}}$

- One equation $\rightarrow \xi_1$
- One DOF

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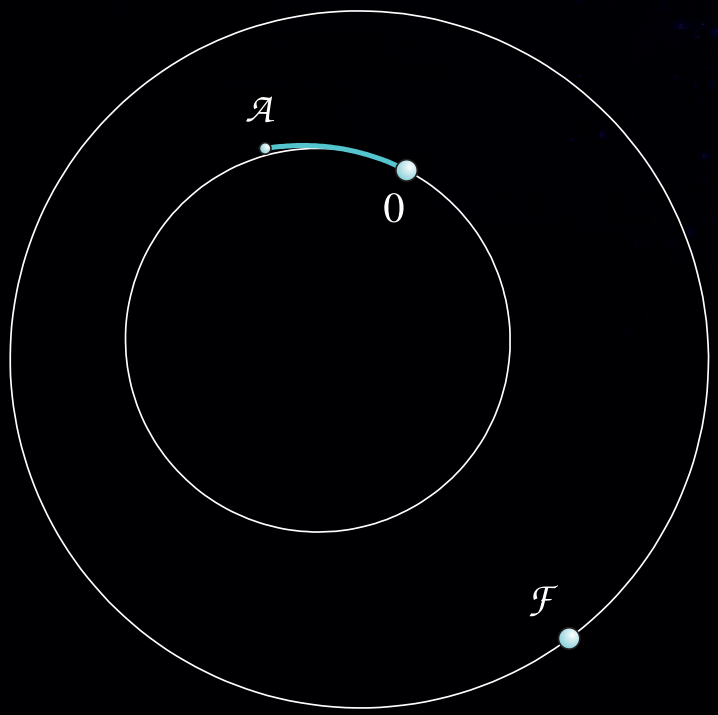

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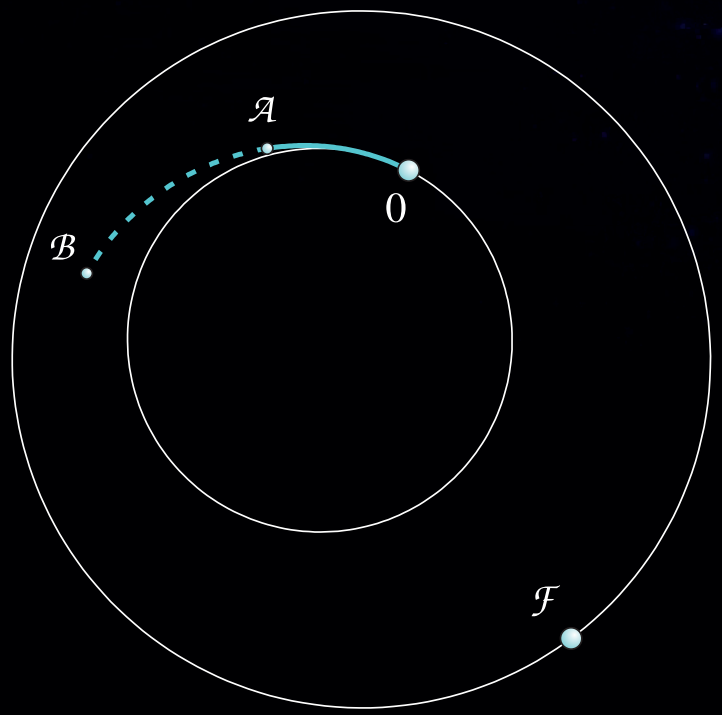

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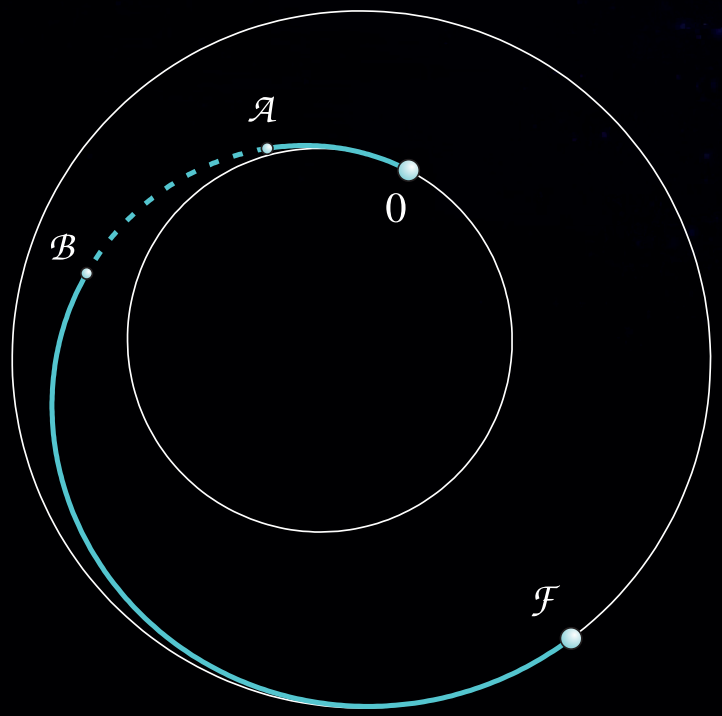

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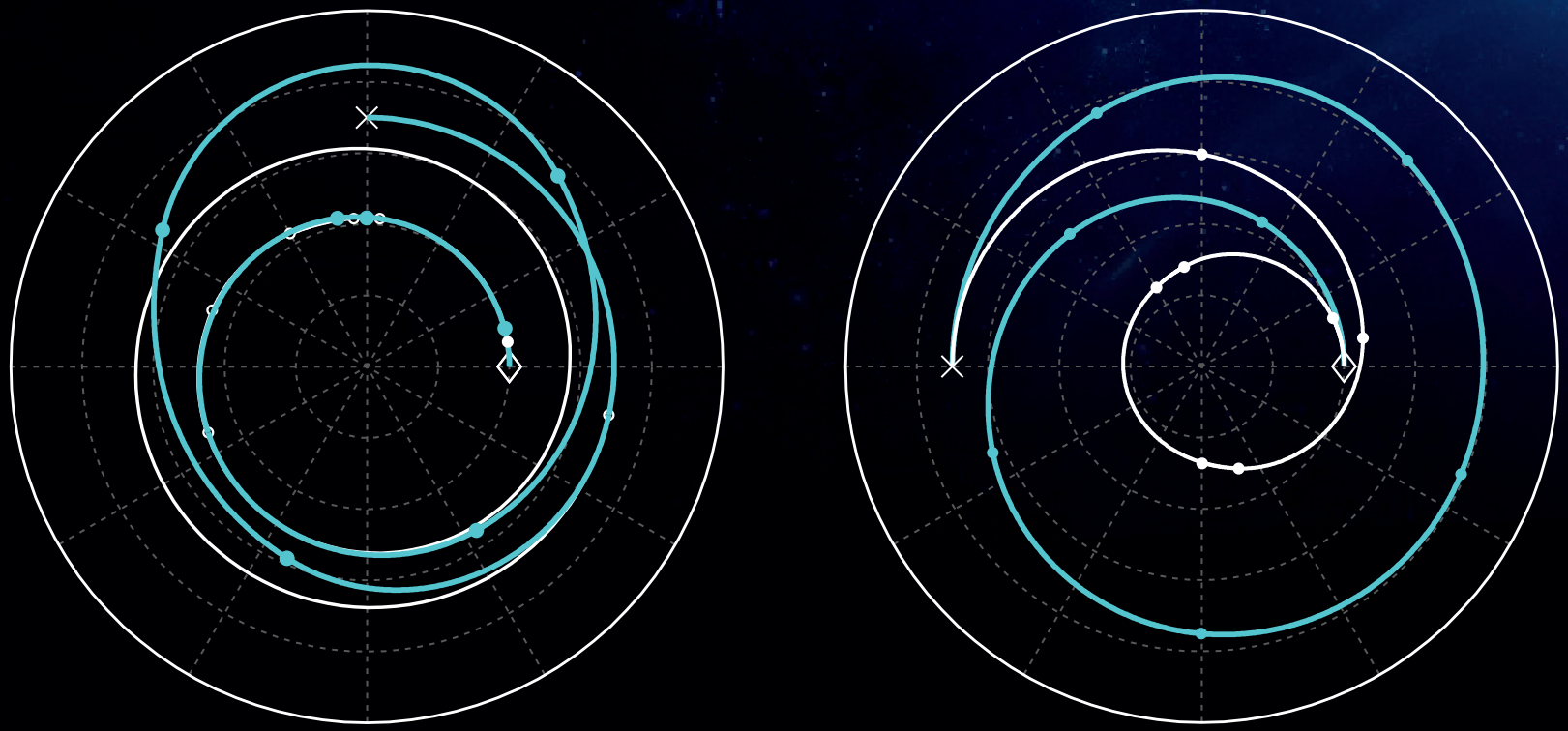
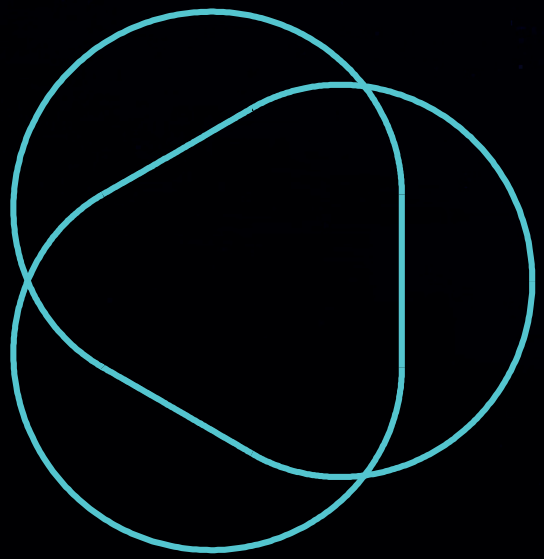
Improved flexibility



Periodic orbits



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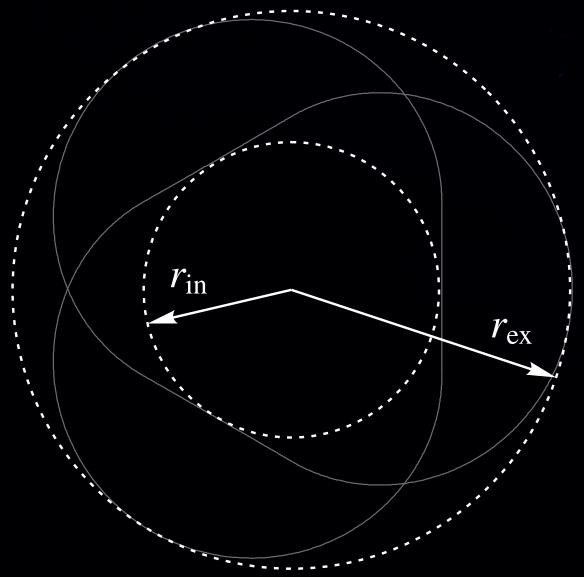

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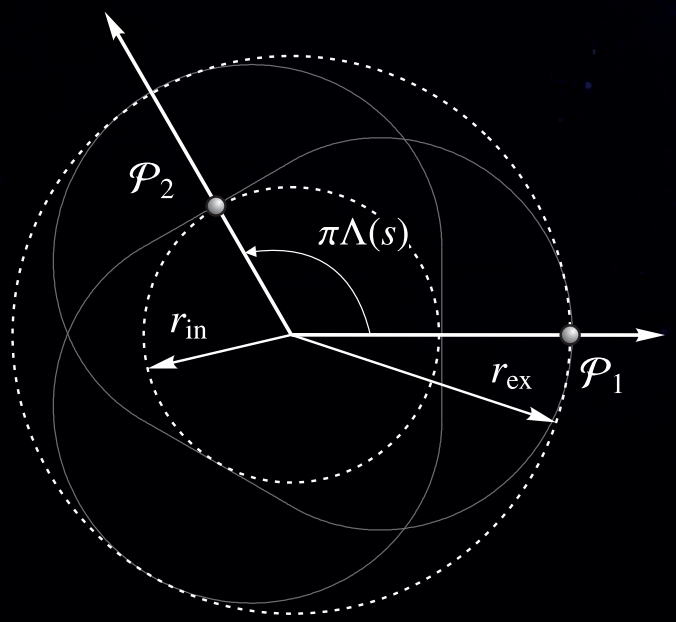

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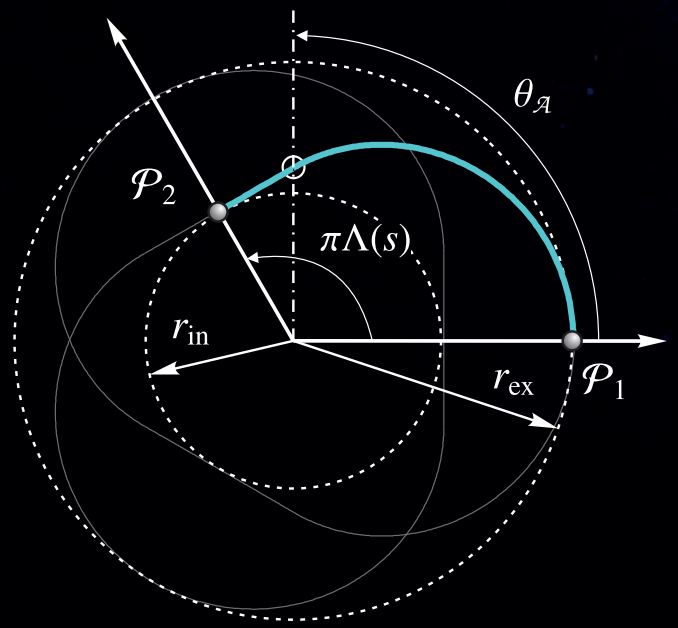

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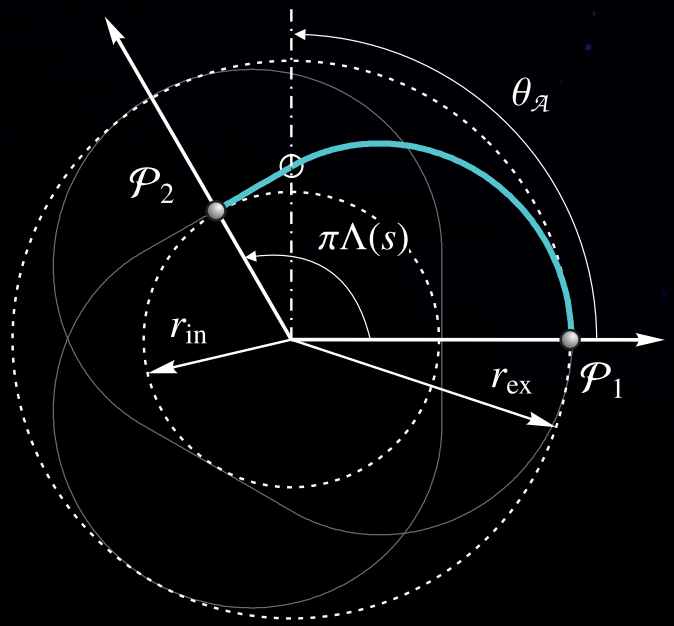
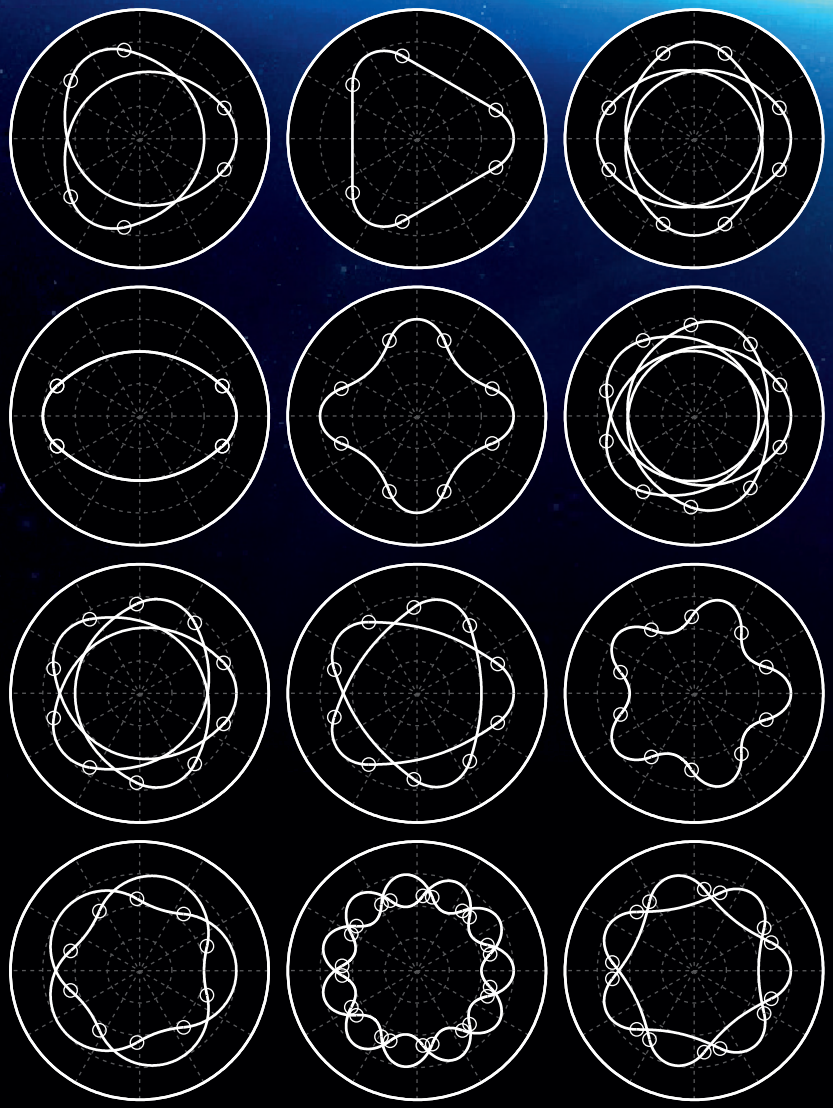
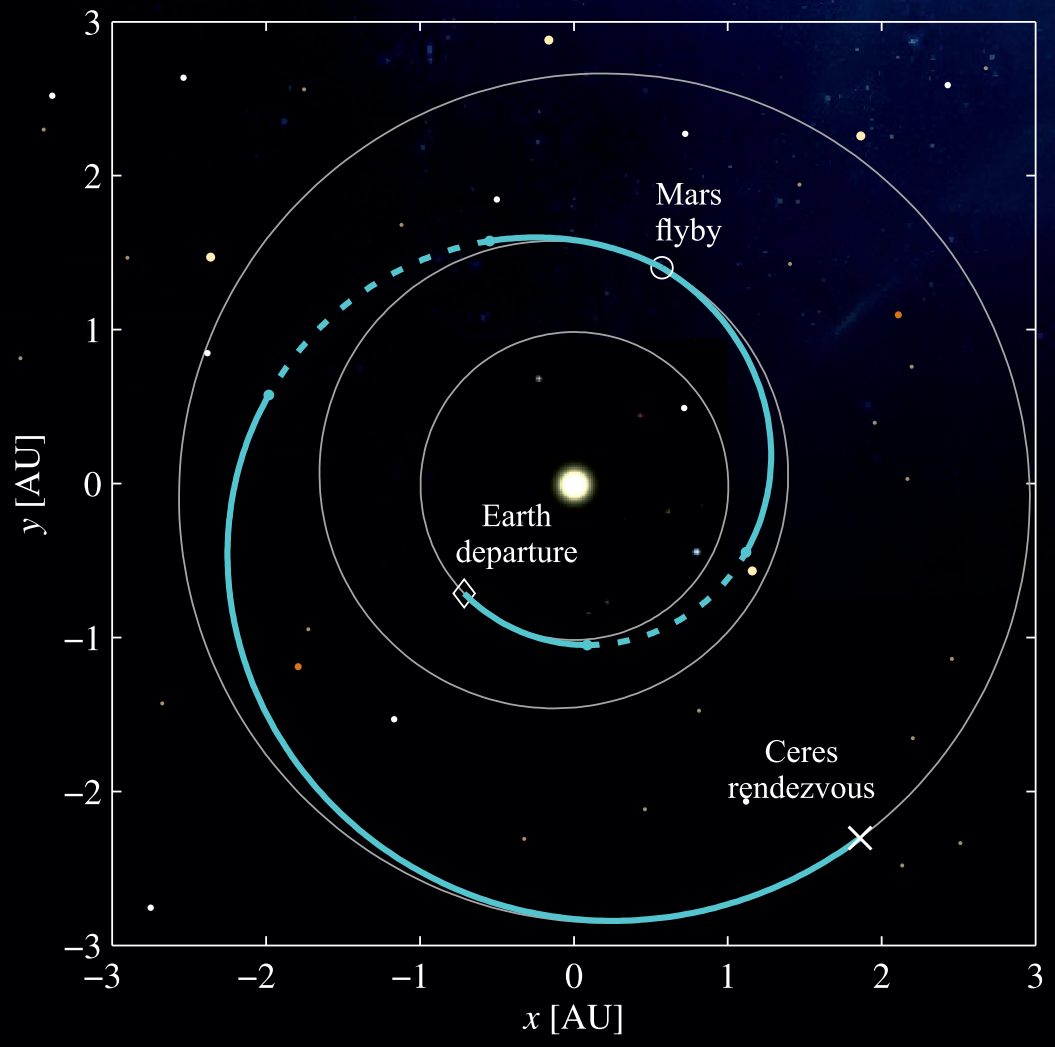




LT Gravity-Assist transfer

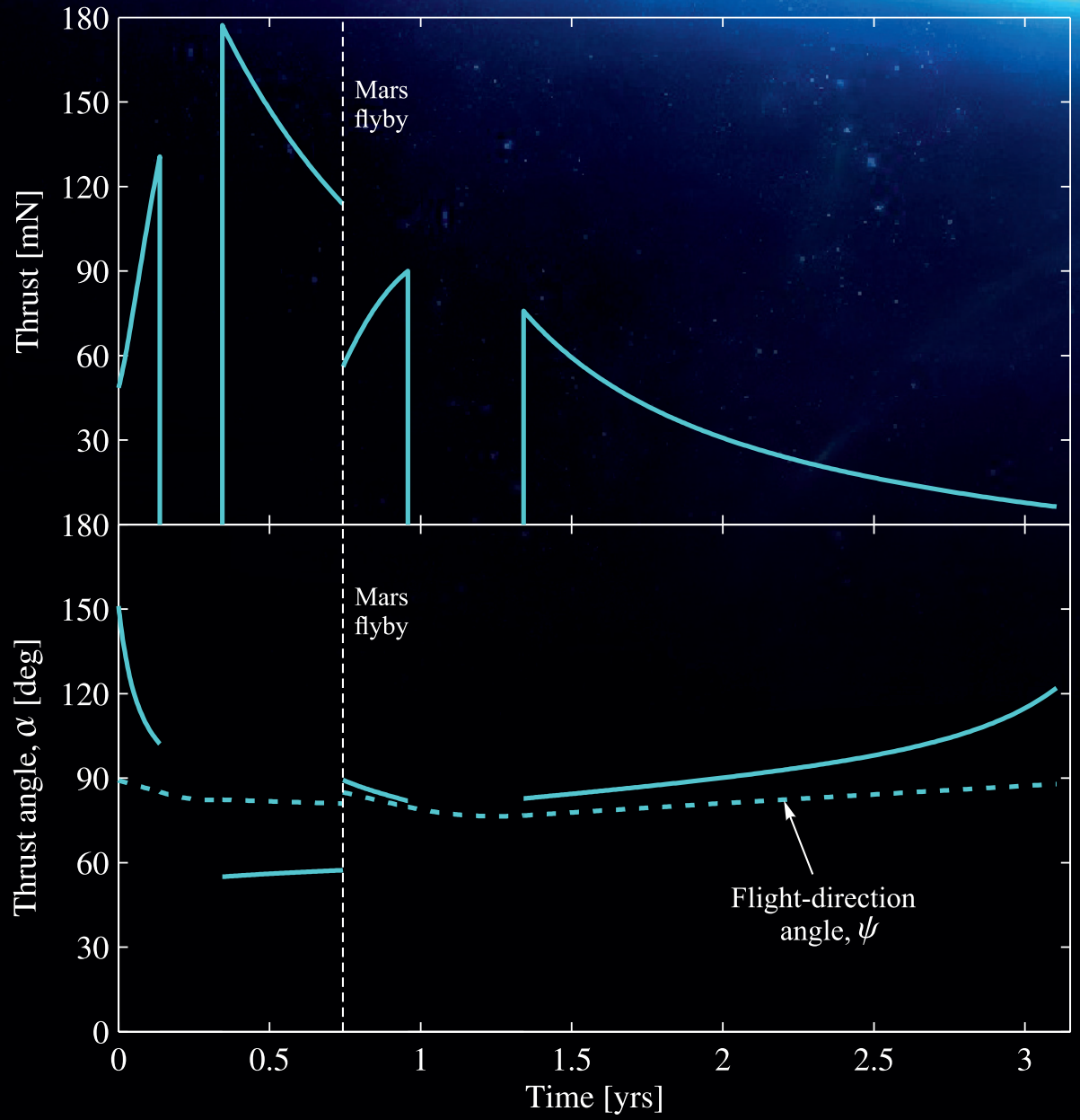
Earth-Mars-Ceres transfer



LT Gravity-Assist transfer II

	Units	CGLS	STOUR-LTGA	GALLOP
Launch v_∞	km/s	1.600	1.600	1.600
Mars flyby v_∞	km/s	1.590	1.435	1.920
Flyby altitude	km	562	5432	200
Mass fraction	–	0.256	0.256	0.234
Arrival v_∞	km/s	0.000	0.237	0.000
TOF Earth-Mars	days	271	271	271
TOF Mars-Ceres	days	862	862	862 (739)
TOF total	days	1133	1133	1133 (1010)

LT Gravity-Assist transfer III



Conclusions

- The flexibility of the GLS has been improved.
- Fully analytic solution is available.
- Results comparable to state of the art methods.
- Light computations.
- Promising for *preliminary* mission design.

Generalized logarithmic spirals yield a flexible method for low-thrust trajectory design

javier.roa@upm.es