Statistical ARQ Link Analysis and Planning for Dynamic Links

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Outline of Talk

Part 1: Theory

- **Characteristics of ARQ Transmission Mechanism**
- **Summary of prior ARQ Link Analysis (Constant SNR)**
- **Summary of Link Uncertainty Analysis (Concept of Margin)**
- **Incorporation of Link Uncertainty in ARQ Link Analysis**

Part 2: Illustration With (1024, ½) LDPC Code

- **"Fast-Varying" ARQ Channel – E.G. Deep Space Link**
- **"Slowing-Varying" ARQ Channel – E.G. Proximity Link**
- **Comparison of Results and Insights**
- **Conclusion**

Note: the subject on ARQ latency is discussed
In the paper, but not in the talk

Characteristics of ARQ Transmission Mechanism

- Standard communications assumes data are sent once – “sent-once” link
- Bit-error-rate (BER) or frame-error-rate (FER), and link margin are common metrics that characterize the quality of a link, and used to determine supportable data rate
- A paradox - with ARQ, when messages are corrupted during transmission, they can be re-sent multiple times until they are received and acknowledged. Thus ARQ link is therefore “error-free”
- ARQ transmission mechanism is discrete in nature
  - Transmit and re-transmit of the same codeblock until it is correctly received and acknowledged
- There is a latency between consecutive transmissions of the same codeblock, ranging from milliseconds for proximity links (hundred’s of km) to 10-20 minutes for deep space links (hundred’s of millions of km)

Summary of Prior ARQ Link Analysis [1][2]

- Effective data rate for "send-once" link
  - Loss data is non-recoverable

$$R_{eff} = R_b(1 - P_{bk})$$

- Effective data rate for ARQ link using Selective Repeat protocol
  - Loss data is recoverable; assume lossless acknowledgement links

$$R_{eff} = R_b(1 - P_{bk})$$

- Effective data rate for ARQ link using Go-Back-N protocol is lower-bounded by
  - Assume lossless acknowledgement links

$$R_{eff} \geq R_b \frac{1 - P_{bk}}{1 + (N-1)P_{bk}}$$

- Extend link analysis to ARQ link: express R_{eff} as a function of R_b, N, P_{ack}, code performance model f(.), and link analysis output C (P_d/No)

Tunable

ARQ protocol used

Link analysis output that include all contributions of the link

$$R_{eff} = R_b\left(1 + \frac{N(1-(1-f(\frac{C}{R_b}))(1-P_{ack}))}{(1-f(\frac{C}{R_b}))(1-P_{ack})}\right)^{-1}$$

$$R_b \leq \frac{\frac{G}{T} EIRP}{k L_o L_s (\frac{E_b}{N_o})_{Th} M}$$

Code performance curve

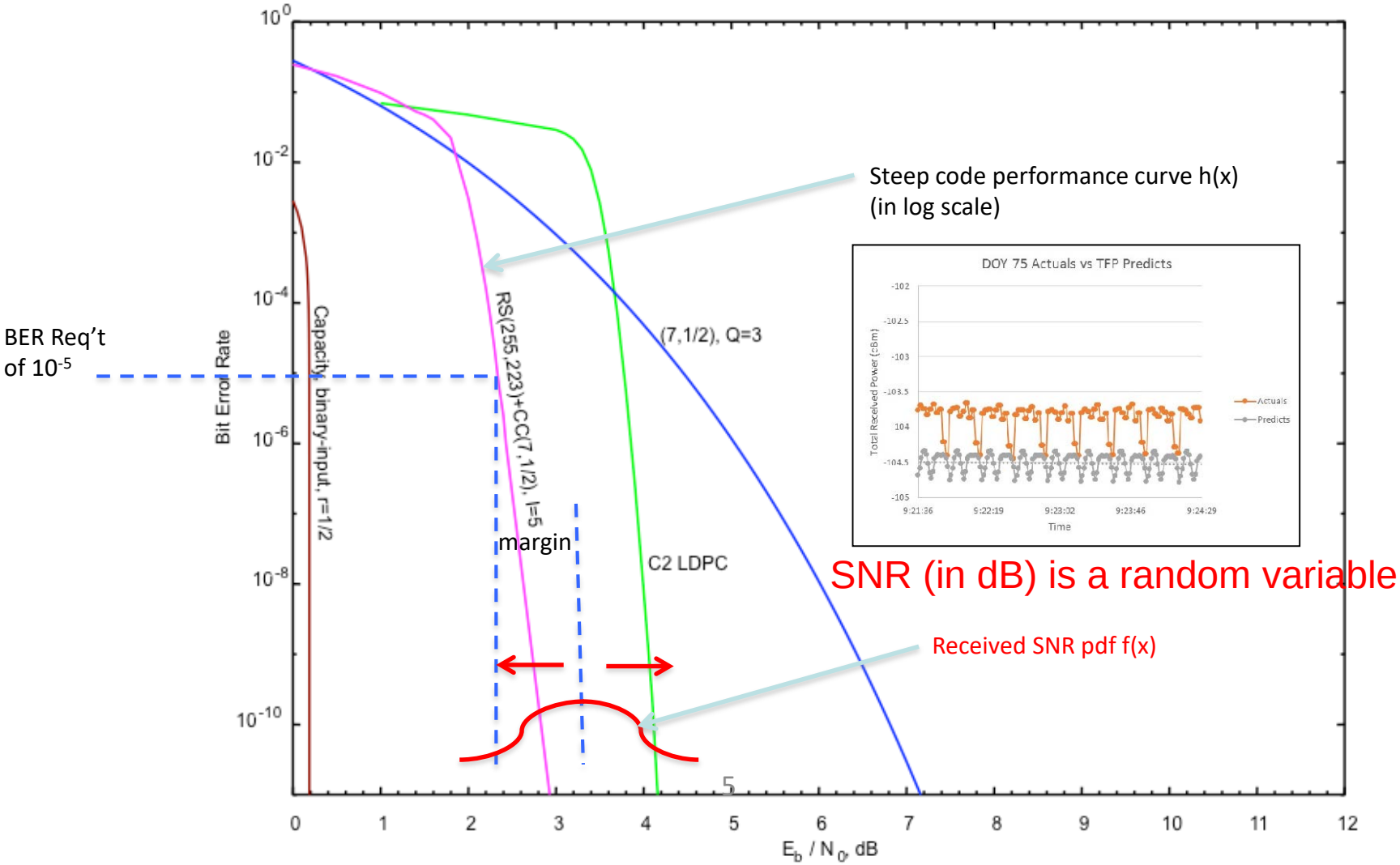
Contribution from ack channel

- Prior analysis assumes constant signal-to-noise ratio

Summary of Link Uncertainty Analysis [3]

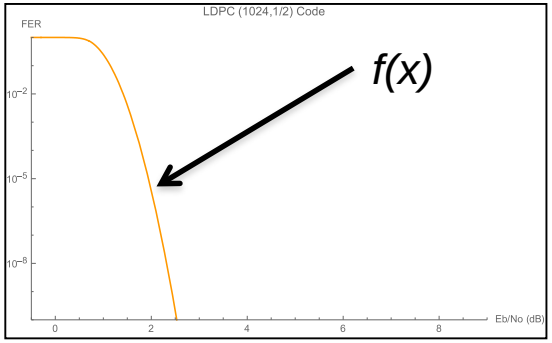
- Margin and link variability and its relationship with the code performance curve

Link performance Curves (Courtesy of J. Hamkins)



Summary of Link Uncertainty Analysis [3]

- *f(x)* is generated by curve-fitting the code performance simulation data points
- *h(x|.)* can be of the forms



(Gaussian Case [1][2]*)

$$h(x \mid m;\sigma;L_K) = \sum_{i=10}^{CD} P_i \frac{1}{\sqrt{2\pi\sigma}} e^{\frac{-(x-\hat{m}+\bar{l}-l_i)^2}{2\sigma^2}}$$

(Non-Gaussian Case [4]*)

- Average error rate $\bar{e}(x,\sigma)$ is given by

$$\bar{e}(x,\sigma) = \int_{-\infty}^{+\infty} f(y) h(y \mid x;\ \sigma)\, dy$$

- Define margin M as additional SNR required to maintain an average error rate ε. For a given s_1 and s_2 such that
  - ε = $h(s_1) = \bar{e}(s_2,\sigma)$

$$\boxed{M = s_2 - s_1}$$

* See references in paper

Summary of Link Uncertainty Analysis [3]

Link-adjusted code performance curve

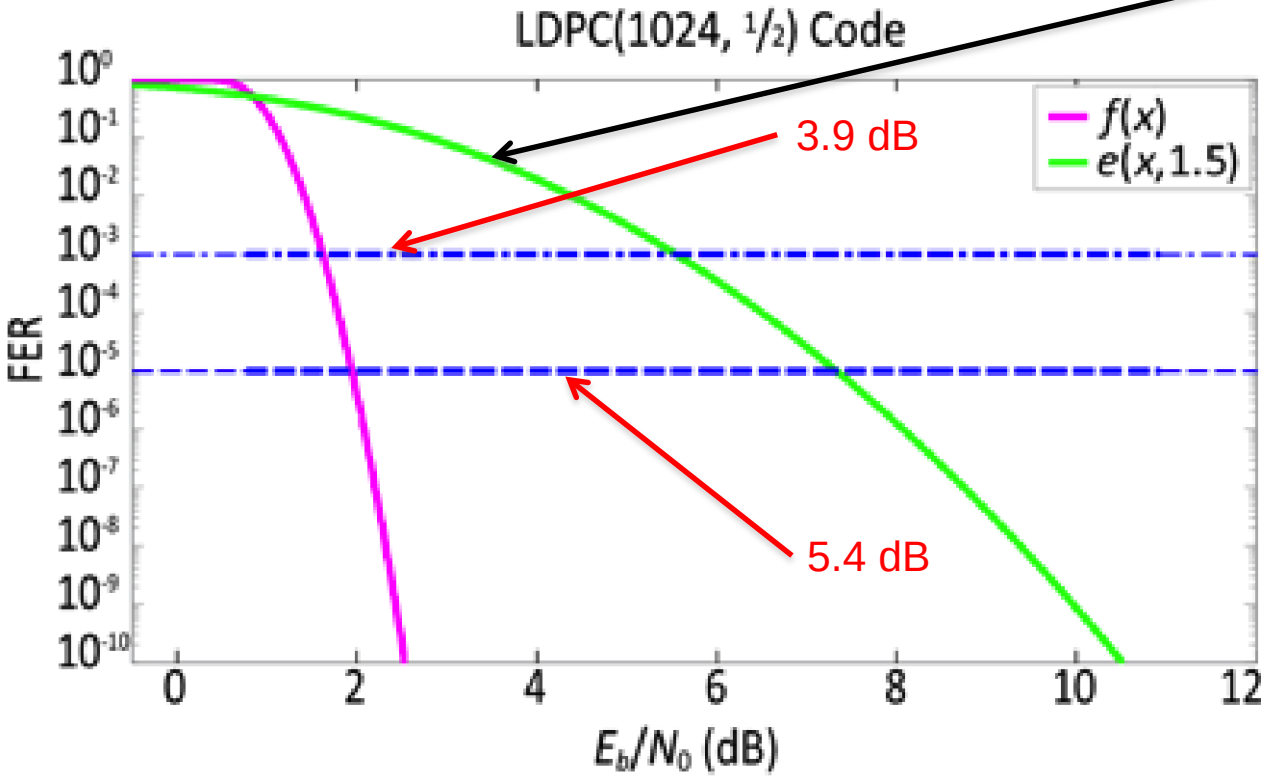
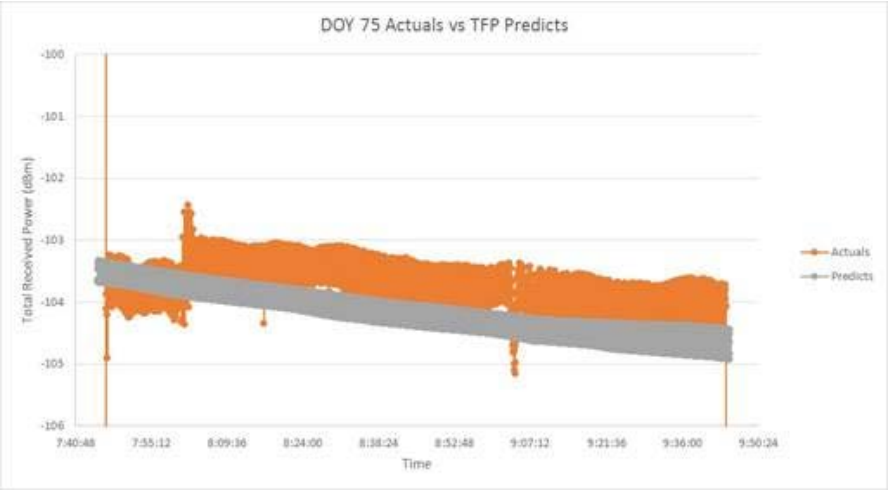


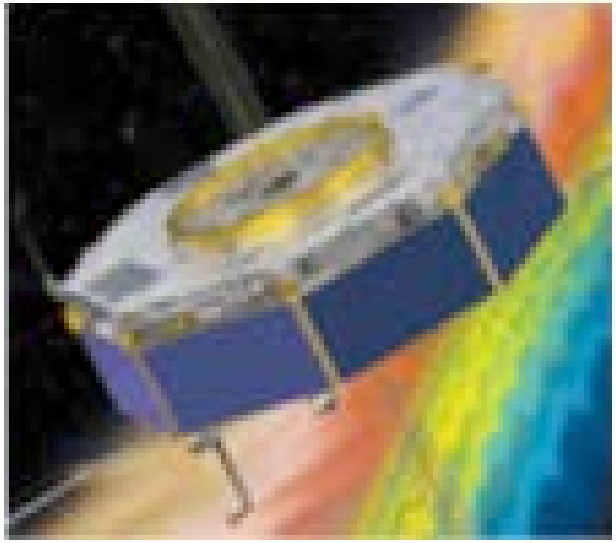
Table 1. Coding Performance for LDPC (1024, ½) Code—FER versus E_b/N_0

Coding Performance Type	FER	E_b/N_0 (dB)
Constant SNR	10^{-3}	1.62
Constant SNR	10^{-5}	1.94
Link Adjusted ($\sigma = 1.5$)	10^{-3}	5.54
Link Adjusted ($\sigma = 1.5$)	10^{-5}	7.33

Incorporation of Link Uncertainty in ARQ Link Analysis

- In communication, received power fluctuations are functions of natural and man-made effects with different time-scale (time-constant, coherency time)
  - Weather effects, pointing losses, axial-ratio mismatch, spinner/non-ideal antenna patterns, etc.
  - Example of received power fluctuation, MMS received power (5/16/2015)



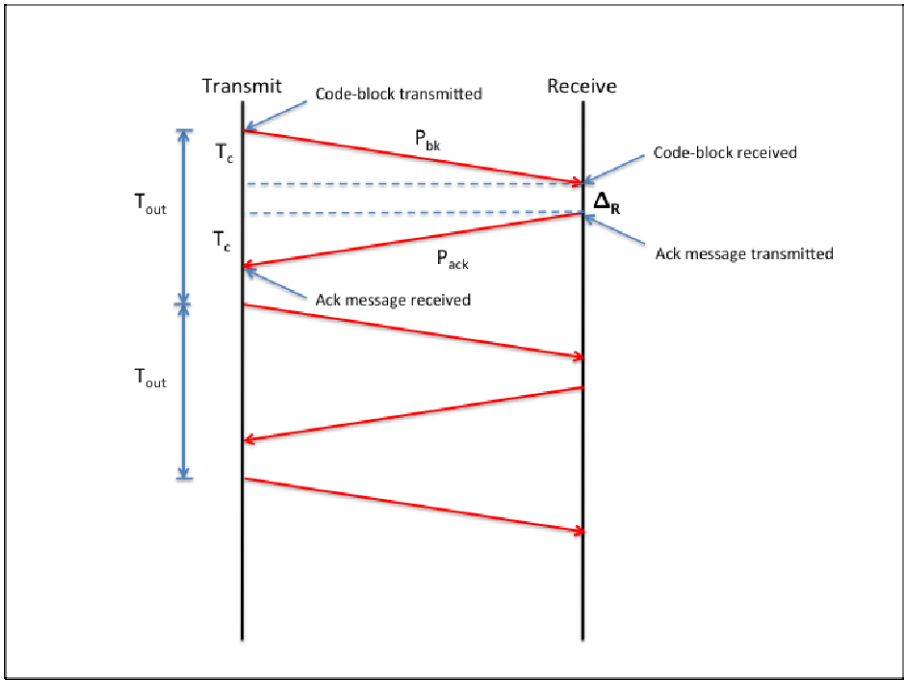


Random effect

Pseudo-random effects

Incorporation of Link Uncertainty in ARQ Link Analysis

- ARQ acknowledgement time ≥ 2-way light time
- For deep space communication at Mars distance, one-way light time is 10 – 20 min
- For Mars proximity communication between a rover and an orbiter (~100's km), one-way light time is a few millisecond
- Consider two limiting cases
  - "Fast-varying" ARQ channel: when SNR's between subsequent re-transmission are independent, e.g. deep space link
  - "Slow-varying" ARQ channel: when SNR's between subsequent re-transmission are identical, e.g. proximity link



- Define "effective SNR" $\left(\frac{E_b}{N_0}\right)_{eff}$ to be the average energy per information bit to noise spectrum ratio, including the energy spent on non-decodable frames
- Assume SNR has a Gaussian distribution $f(x \mid m;\sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{(x-m)^2}{2\sigma^2}}$

Fast-Varying ARQ Channel, e.g. Deep Space Link

- SNR's between subsequent re-transmission are independent
- Link-adjusted frame error rate (of each transmission event) is given by

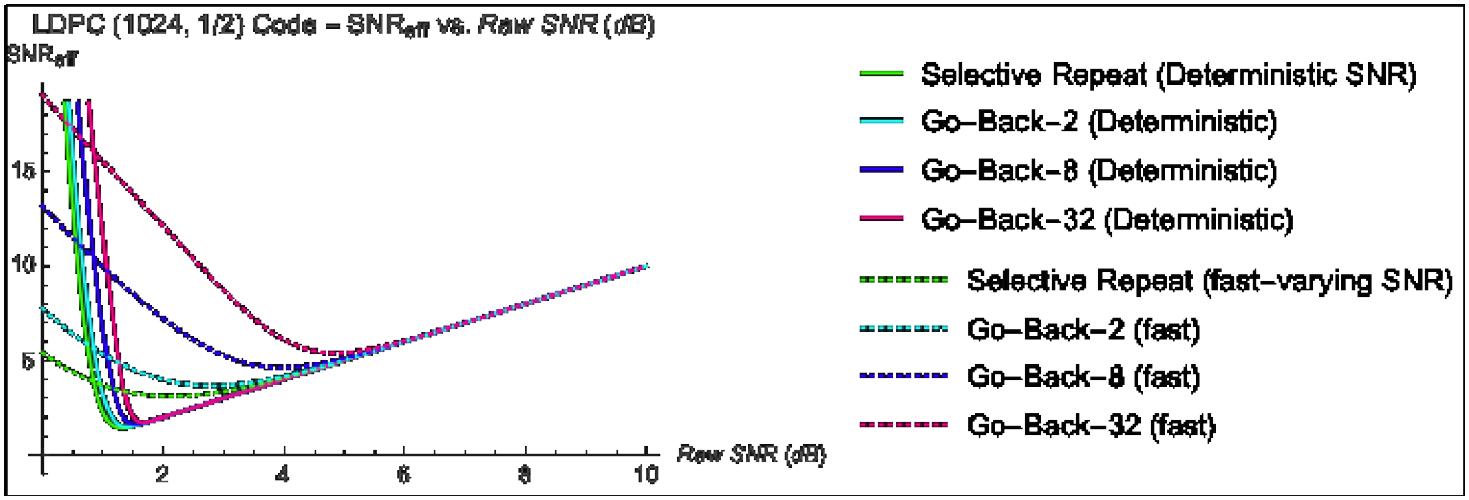
$$\bar{e}(x,\sigma)=\int_{-\infty}^{+\infty} h(y) f(y \mid x;\ \sigma)\, dy$$

- Effective SNR (in dB) is given by

- ARQ link performance for fast-varying SNR case

Table 2. Lowest Effective SNR for ARQ Protocols for "Fast-varying" Case

Deterministic/ Varying	Protocol Type	Effective SNR (dB)	Raw SNR (dB)
Deterministic	Selective Repeat	1.428	1.32
Deterministic	Go-Back-2	1.5	1.4
Deterministic	Go-Back-8	1.628	1.541
Deterministic	Go-Back-32	1.741	1.66
Fast-Varying	Selective Repeat	3.099	2.09
Fast-Varying	Go-Back-2	3.669	2.78
Fast-Varying	Go-Back-8	4.626	3.963
Fast-Varying	Go-Back-32	5.402	4.84



Slow-Varying ARQ Channel, e.g. Proximity Link

- Same SNR for each re-transmission until a code frame is correctly decoded
- Effective SNR (in dB) is given by

- ARQ link performance for fast-varying SNR case

Table 3. Lowest Effective SNR for ARQ Protocols for "Slow-varying" Case

Deterministic/ Varying	Protocol Type	Effective SNR (dB)	Raw SNR (dB)
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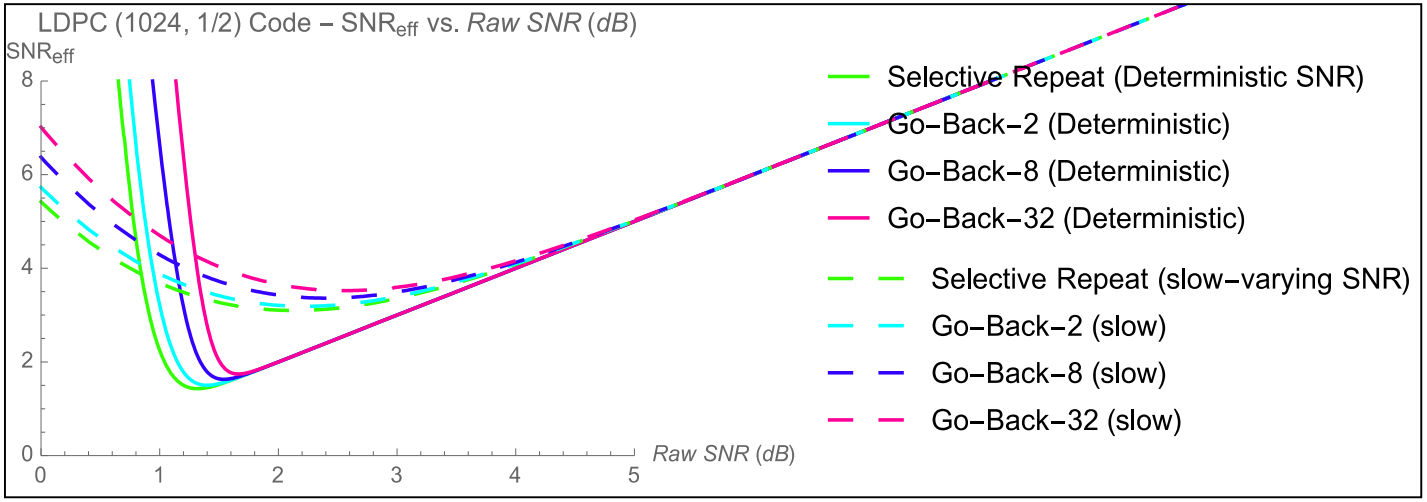
Deterministic	Go-Back-32	1.741	1.66
Slow-varying	Selective Repeat	3.099	2.09
Slow-varying	Go-Back-2	3.188	2.19
Slow-varying	Go-Back-8	3.362	2.397
Slow-varying	Go-Back-32	3.523	2.52



Comparison of Results and Insights

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- For "sent-once" link in Ka-band environment, rule-of-thumb margin of 3 dB is not enough
  - Need 3.9 dB margin for FER=10^{-3}
  - Need 5.4 dB margin for FER=10^{-5}
- Advantage of ARQ in Ka-band environment
  - Lossless
  - Lower SNR – 3.1 dB compared to 5.5 or 7.3
- Link inefficiency on using Go-Back-N in proximity link is only a few tenths of a dB

Table 2. Lowest Effective SNR for ARQ Protocols for "Fast-varying" Case

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Conclusion

- This paper incorporates link uncertainty into the analysis of ARQ Links

- Due to the discrete nature of ARQ transmission mechanism, and comparing the ARQ acknowledgement time with the time-scale of link variability, we introduce two limiting cases of ARQ channel:
  - "Fast-varying" ARQ channel – e.g. deep space link
  - "Slow-varying" ARQ channel – e.g. proximity link

- We compare "sent-once" link and ARQ links with different protocols and under different dynamic environment, in terms of "effective SNR" $\left(\frac{E_b}{N_0}\right)_{eff}$, which is the average energy per information bit to noise spectrum ratio, including the energy spent on non-decodable frames

- We show that ARQ link provide "lossless" communications with lower SNR (2-4 dB) compared to sent-once link, but in the expense of
  - Higher complexity: acknowledgment channel (feedback), data management, etc.
  - Higher latency (discussed in references and in the paper)

References

- [1] K. Cheung, “Problem Formulation and Analysis of the 1-Hop ARQ Links,” *Interplanetary Network Progress Report*, Vol. 42-194, pp. 1-15, August 15, 2013.

- [2] K. Cheung, C. Lau, C. Lee, “Link Analysis for Space Communication Links Using ARQ Protocol,” *IEEE Aerospace Conference 2014,* Big Sky, Montana, March 2014.

- [3] K. Cheung, “The Role of Margin in Link Planning and Optimization,” *IEEE Aerospace Conference 2015,* Big Sky, Montana, March 2015.

- [4] K. Cheung, “Statistical Ka-Band Link Analysis,” *SpaceOps 2014,* Pasadena, California, May 2014.

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