A Trilateration Scheme for Relative Positioning

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***Abstract*[1]—We introduce a trilateration scheme that evaluates the 3-dimensional (3-D) relative position between a reference spacecraft and a target spacecraft using raw-range measurements from a distance baseline of known locations, which we call "anchors". The anchors can be antennas of a ground-based network (e.g., Deep Space Network (DSN) or Near Earth Network (NEN) stations), or satellites of a space-based network (e.g., global positioning system (GPS) or tracking and data relay satellite (TDRS)). We define raw-range as the range that includes all the systematic errors that occur during range measurements. A unique feature of this approach is that accurate relative position is derived from a "differencing function" of raw-range measurements of the reference spacecraft and target spacecraft, thereby eliminating most of the systematic errors, such as media effects, ephemeris errors, instrument delays, clock bias, etc. There can be an arbitrary number of target spacecraft, and relative positioning of target spacecraft with respect to the reference spacecraft can be done simultaneously.**

In this paper, we first assume an idealized system in which clocks on the reference and target spacecraft are synchronized, with clocks of the anchors synchronized as well.[2] We develop a novel iterative algorithm that computes the relative position of the target spacecraft with respect to the reference spacecraft.

We illustrate the relative positioning method using the scenario of a network of three ground stations (i.e., the anchors) at Goldstone, California, USA, Madrid, Spain, and Marlargue, Argentina tracking two spacecraft at geosynchronous orbit distance. We demonstrate that the algorithm converges to sub-meter accuracy in estimating the relative position, in the presence of random errors and systematic errors in raw-range measurements, and in the presence of angular errors in estimating the pointing vectors between the anchors and the reference spacecraft.

Next, we relax the requirement of perfect time synchronization between spacecraft, and show that by using an additional anchor, one can estimate and remove the clock biases between the reference and target spacecraft. We add a ground station at Kourou to the above example of three ground stations of Goldstone, Madrid, and Marlargue, and demonstrate that the updated algorithm also converges to meter-level accuracy (sub-meter in some cases) in the presence of clock biases in addition to the random errors, systematic errors, and angular errors as shown in the above case.

We compare this scheme with a similar trilateration scheme for relative positioning scheme first proposed by Montenbruck in 2002.

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1. INTRODUCTION AND MOTIVATION

The United States' global positioning system (GPS) consists of 31 satellites[3] in medium Earth orbit (MEO), which provide 24/7 global location and timing services for users on Earth's surface and in low Earth orbit (LEO). The current GPS provides a positioning accuracy of a few meters, and the differential GPS system provides 20–30 centimeters accuracy when users are in the vicinity of a reference station (within a few hundred kilometers).

Space vehicles operating above the LEO lose coverage of the GPS infrastructure at higher altitude, and have to rely on other means for tracking and navigation. Missions above MEO cannot rely on the Earth's GPS. For deep space missions, to perform tracking and navigation, an antenna of the Deep Space Network (DSN) performs 2-way ranging and Doppler measurements to estimate the spacecraft range and velocity [1]. This together with Delta Differential One Way Ranging (Delta-DOR) measurements [2] help to determine with high accuracy the trajectory of the spacecraft. This approach requires extensive calibration efforts, and it ties up at least one DSN antenna during the measurement campaign for each spacecraft.

[1] © 2016 California Institute of Technology. Government sponsorship acknowledged.

[2] The clocks between the spacecraft and the anchors do not need to be synchronized.

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[3] See https://en.wikipedia.org/wiki/Global_Positioning_System

There are missions and proposed mission concepts that involve a constellation of spacecraft performing science measurements at geosynchronous distance, lunar distance, or deep space distance [3][4][5]. This poses a challenge to the conventional way of determining the locations of all the spacecraft in the fleet. In this scenario, the exact locations of the spacecraft are typically not important, but rather their relative positions are required to be known to high precision, say between a mother spacecraft and its daughter spacecraft, so as to enable science measurements, e.g., interferometry, and to support spacecraft operations, e.g., approach and docking.

In this paper, we investigate a trilateration scheme for relative and simultaneous positioning scheme for a constellation of spacecraft that relies on a baseline of three or more "anchoring" spacecraft (e.g., GPS or tracking and data relay satellite (TDRS)) or ground antennas (e.g., DSN or Near Earth Network (NEN) stations) that perform raw-range measurements with the reference and target spacecraft. The key assumptions are as follows:

1. There is one reference spacecraft and multiple target spacecraft.

2. The clocks between the "anchors" are perfectly synchronized.

In this paper, we first assume that the clock between the reference spacecraft and target spacecraft are perfectly synchronized. We derive a novel iterative algorithm that computes the 3-dimensional (3-D) relative position between a reference spacecraft and a target spacecraft using the raw-range measurements between the spacecraft and three or more anchors. We then demonstrate the trilateration scheme for relative positioning by considering a baseline network of three ground stations at Goldstone, California, USA, Madrid, Spain, and Marlargue, Argentina, tracking a fleet of spacecraft at geosynchronous orbit (GEO) distance. The ground stations are approximately 8,000 kilometers apart from each other. We show that the algorithm converges to submeter accuracy in estimating the relative position between the reference and target spacecraft, in the presence of random errors and systematic errors in raw-range measurements, and in the presence of angular errors in estimating the pointing vectors between the anchors and the reference spacecraft.

Next, we relax the requirement of perfect time synchronization between spacecraft, and show that by using an additional anchor, one can estimate and remove the clock biases between the reference spacecraft and target spacecraft. We add a ground station at Kourou, French Guiana, to the above example of three ground stations of Goldstone, Madrid, and Marlargue, and demonstrate that the updated algorithm also converges to submeter accuracy in the presence of clock biases in addition to the systematic errors and angular errors as shown in the above case.

This scheme is unique in the following ways:

1. With three or more anchors, this approach can support simultaneous relative positioning between a reference spacecraft and an arbitrary number of target spacecraft.

2. The trilateration scheme for relative position is derived from a "differencing function" of raw-range measurements of the reference spacecraft and target spacecraft, thereby eliminating most of the systematic errors like clock offsets, media effects, ephemeris errors, instrument delays, etc. Note that this differencing function also calibrates out the clock bias between the spacecraft and the anchor.

3. We develop computationally efficient iterative schemes that ensure fast and accurate convergence of the relative position vector.

A similar trilateration scheme was first introduced by Montenbruck et al. in 2002 [6][7] that uses Earth's GPS to perform relative navigation, and variations and enhancements of this approach were reported in the literature as far as 2015 [8][9]. As pointed out in [7], this approach uses an "interferometric" approximation, and this confines the usefulness of this scheme to spacecraft distance of a few kilometers for applications that demand meter-level accuracy. The algorithm presented in this paper is an exact solver in the absence of errors, and this provides much higher accuracy in estimating the relative position, and extends the usefulness of this scheme to spacecraft separated by a hundred kilometers at GEO distance.

The rest of the paper is organized as follows: Section 2 assumes perfect time synchronization between the reference and the target spacecraft, and describes the problem formulation and architecture concept of relative positioning by a baseline of three or more anchors in the Euclidean space, and introduces an iterative algorithm that computes the 3-D relative position between the reference and the target spacecraft. Section 3 relaxes the time synchronization assumption, and extends the problem formulation to show that the clock bias between the reference spacecraft and a target spacecraft can be estimated and removed by using an additional anchor. Section 4 discusses the dominant error sources in the relative positioning schemes, and the methods to eliminate the errors. Section 5 described the simulated root-mean-square (RMS) error performance of the relative positioning schemes described in Section 2 (no clock bias) and Section 3 (with clock bias), using three ground stations and four ground stations, respectively, tracking a fleet of spacecraft at GEO distance. We also compare the relative positioning accuracy between our approach and Montenbruck's approach. Section 6 provides concluding remarks and future work.

2. PROBLEM FORMULATION AND ARCHITECTURE CONCEPT: NO CLOCK BIAS

In this section, we assume that the clocks on the reference spacecraft and the target spacecraft are perfectly

synchronized. Consider three points $\vec{G}_1$ (an anchor), $\vec{R}$ (the reference spacecraft), and $\vec{T}$ (the target spacecraft) that form a triangle Λ_1 in the Euclidean space as shown in Figure 1. We assume the coordinates of $\vec{G}_1$, and the coordinates of $\vec{R}$, and $\vec{T}$ are not precisely known. Let r_1 and r_1' be the raw-range measurements between $\vec{G}_1$ and $\vec{R}$, and between $\vec{G}_1$ and $\check{T}$, respectively. We define raw-range as the range that includes all the systematic errors that occur during range measurements. Let $\vec{U}_1 = \begin{pmatrix} u_{x1} \\ u_{y1} \\ u_{z1} \end{pmatrix}$ be the directional cosine of $\vec{R} - \vec{G}_1$, which we assume to know to a high degree of accuracy (discussed further in Section 4), and let $\vec{P} \cong \vec{T} - \vec{R} = \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix}$ be the relative position vector of interest.

We assume $r_1 >> \left\|\vec{P}_1\right\|$,[4] and $r_1' >> \left\|\vec{P}_1\right\|$. Let s_1 be the altitude of Λ_1 through $\vec{T}$. Denote the projection of $\vec{P}$ onto $\vec{R} - \vec{G}_1$ to be d_1, which can be expressed as the dot product between $\vec{U}_1$ and $\vec{P}$ (denoted by $\vec{U}_1 \circ \vec{P}$). Note that Λ_1 is made up of two right-angled triangles that share a common side s_1, where $s_1 \le \left\|\vec{P}_1\right\|$.

We construct the following relationships by applying the Pythagorean theorem on the two triangles:

$$s_1^2 = \left\|\vec{P}\right\|^2 - \left(\vec{U}_1 \circ \vec{P}\right)^2 \tag{1a}$$

$$d_1 = r_1 - (r_1'^2 - s_1^2)^{\frac{1}{2}} \tag{1b}$$

Note that d_1 results from a 'differencing' function of r_1 and r_1'. Similarly consider two additional known points $\vec{G}_2$ and $\vec{G}_3$ that form triangles Λ_2 and Λ_3 respectively with $\vec{R}$, and $\vec{T}$. Using similar derivation as discussed above, we have

$$s_2^2 = \left\|\vec{P}\right\|^2 - \left(\vec{U}_2 \circ \vec{P}\right)^2 \tag{2a}$$

$$d_2 = r_2 - (r_2'^2 - s_2^2)^{\frac{1}{2}} \tag{2b}$$

$$s_3^2 = \left\|\vec{P}\right\|^2 - \left(\vec{U}_3 \circ \vec{P}\right)^2 \tag{3a}$$

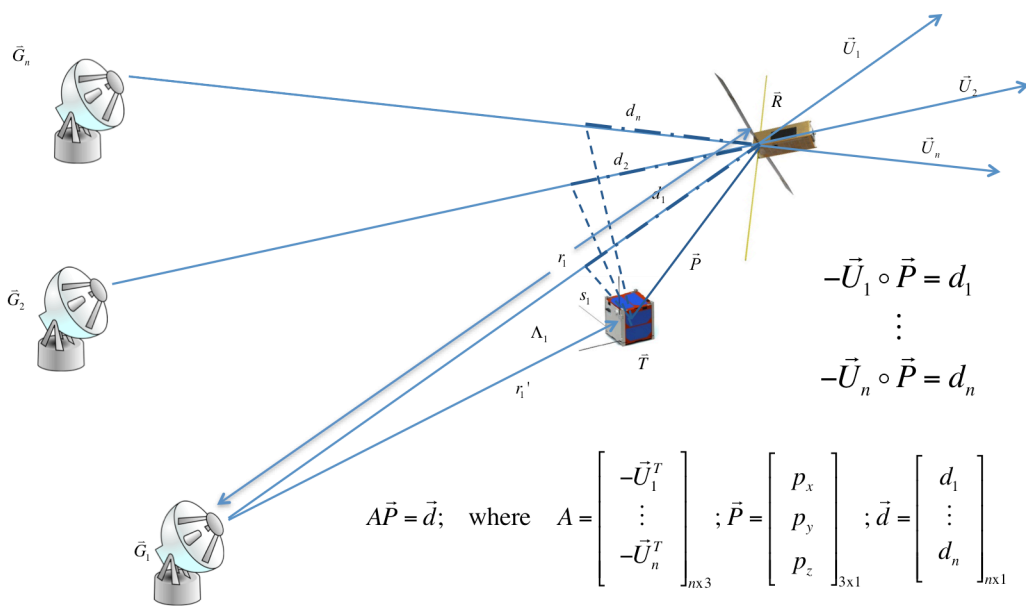

Figure 1. Geometry of the Problem Formulation: No Clock Bias

$$d_3 = r_3 - (r'^2_3 - s_3^2)^{\frac{1}{2}} \tag{3b}$$

We want to compute the vector $\vec{P}$ that has three unknowns. We form the matrix $A = \begin{bmatrix} -\vec{U}_1^T \\ -\vec{U}_2^T \\ -\vec{U}_3^T \end{bmatrix}$ and the vector $\vec{d} = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}$ such that $A\vec{P} = \vec{d}$, and

$$\vec{P} = A^{-1}\vec{d} \tag{4}$$

Equation (4) therefore forms an iterative relationship with equations (1), (2), and (3). In general, when there are n anchors, where $n \geq 3$, one can employ additional Pythagorean relationships as shown in the above equations, and compute the least mean square solution of an over-determined system as follows:

$$A = \begin{bmatrix} -\vec{U}_1^T \\ -\vec{U}_2^T \\ \vdots \\ -\vec{U}_n^T \end{bmatrix}_{nx3}, \qquad \vec{d} = \begin{pmatrix} d_1 \\ d_2 \\ \vdots \\ d_n \end{pmatrix}$$

$$\vec{P} = \left(A^T A\right)^{-1} A^T \vec{d} \tag{5}$$

Based on the above formulation, we construct an iterative method that guarantees convergence to the relative position vector $\vec{P}$ using raw-range measurements of $r_1, r_2, \ldots, r_n$ and $r_1', r_2', \ldots, r_n'$, where $n \geq 3$. We outline the method for the case $n = 3$:

Iterative Procedure

1. Initialization:
   a. Compute the directional cosines $\vec{U}_1$, $\vec{U}_2$, and $\vec{U}_3$, and construct the matrix A.
   b. Set $s_1 = s_2 = s_3 = 0$.
2. Compute d_1, d_2, and d_3 according to equations (1b), (2b), and (3b).
3. Compute $\vec{P} = A^{-1}\vec{d}$ according to equation (4).
4. Compute s_1^2, s_2^2, and s_3^2 according to equations (1a), (2a), and (3a).
5. Go to 2, and compute $\vec{P}$ until $\vec{P}$ converges.

We found that $\vec{P}$ typical converges within 5 or 6 iterations in our simulation.

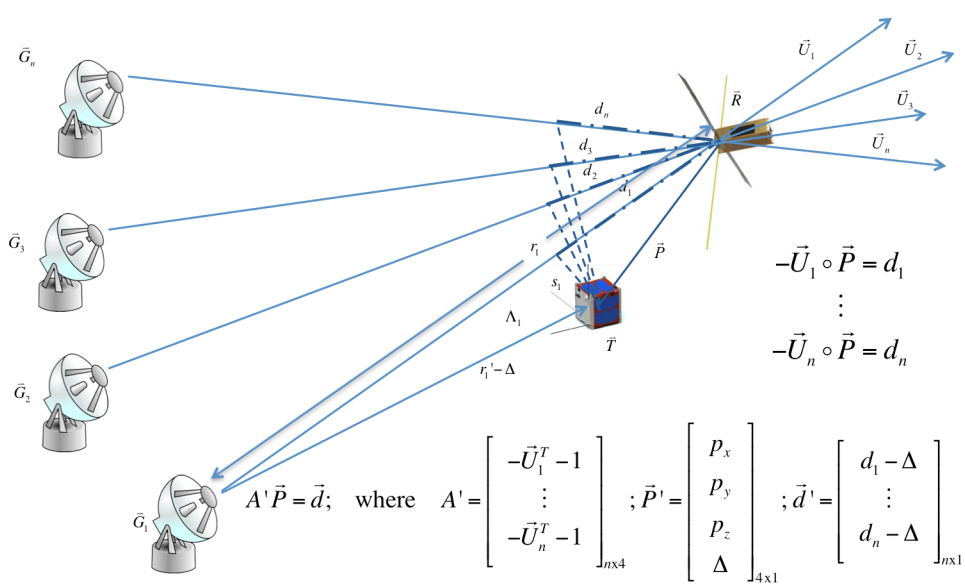

Figure 2. Geometry of the Problem Formulation: With Clock Bias

3. PROBLEM FORMULATION AND ARCHITECTURE CONCEPT: WITH CLOCK BIAS

In this section, we consider the presence of clock biases between the reference spacecraft and target spacecraft. Without loss of generality, we assume that the clocks of the reference spacecraft $\vec{R}$ and of the anchors are perfectly synchronized, and there is an additional anchor $\vec{G}_4$. We express the unknown clock bias of the target spacecraft $\vec{T}$ with respect to $\vec{R}$ as an unknown correction factor Δ in the raw-range measurements of $\vec{T}$. The problem formulation of this case is illustrated in Figure 2.

As in Section II, we assume i anchors, and $r_i >> \|\vec{P}\|$, and $r_i' >> \|\vec{P}\|$. We construct the following relationships by applying Pythagoras' Theorem on the two triangles Λ_1 and Λ_2 of Figure 2:

$$s_i^2 = \|\vec{P}\|^2 - \|\vec{U}_i \circ \vec{P}\|^2 \tag{6a}$$

$$d_i = r_i - ((r_i' - \Delta)^2 - s_i^2)^{\frac{1}{2}} \tag{6b}$$

For n = 4, we define the vector $\vec{P}' = \begin{bmatrix} p_x \\ p_y \\ p_z \\ \Delta \end{bmatrix}$, the matrix

$A' = \begin{bmatrix} -\vec{U}_1^T & -1 \\ -\vec{U}_2^T & -1 \\ -\vec{U}_3^T & -1 \\ -\vec{U}_4^T & -1 \end{bmatrix}$, and $\vec{d}' = \begin{bmatrix} d_1 - \Delta \\ d_2 - \Delta \\ d_3 - \Delta \\ d_4 - \Delta \end{bmatrix}$ such that

$$\vec{P}' = (A')^{-1}\vec{d}' \tag{7}$$

As in Section II, equation (7) forms an iterative relationship with equations (6). In general when there are n anchors, where $n \geq 4$, one can form additional Pythagorean relationships as shown in the above equations, and compute the least mean square solution of an over-determined system as follows:

$$A' = \begin{bmatrix} -\vec{U}_1^T & -1 \\ -\vec{U}_2^T & -1 \\ \vdots & \\ -\vec{U}_n^T & -1 \end{bmatrix}_{nx4} \text{ and } \vec{d}' = \begin{bmatrix} d_1 - \Delta \\ d_2 - \Delta \\ \vdots \\ d_n - \Delta \end{bmatrix}$$

$$\vec{P}' = (A'^T A')^{-1} A'^T \vec{d}' \tag{8}$$

Based on the above formulation, we construct an iterative method that guarantees convergence to the relative position vector $\vec{P}'$ using raw-range measurements of $r_1, r_2, \ldots, r_n$ and $r_1', r_2', \ldots, r_n'$, where $n \geq 4$. We outline the method for the case n = 4:

Iterative Procedure

1. Initialization:
   a. Compute the directional cosines $\vec{U}_1$, $\vec{U}_2$, $\vec{U}_3$, and $\vec{U}_4$, and construct the matrix A'.
   b. Set $s_1 = s_2 = s_3 = s_4 = 0$.
   c. Set Δ = 0.
2. Compute d_1, d_2, d_3, and d_4 according to equations (6b).
3. Compute $\vec{P}' = A'^{-1}\vec{d}'$ according to equation (7).
4. Compute s_1^2, s_2^2, s_3^2, and s_4^2 according to equations (6a).
5. Go to 2, and compute $\vec{P}'$ until $\vec{P}'$ converges.

Montenbruck et al. proposed a similar scheme in 2002 [6][7], and variations and enhancements of this approach were reported in the literature as far as 2015 [8][9]. Montenbruck's scheme has an underlying interferometric approximation in its key equation.[5] This allows a simple single-loop iteration to estimate the relative position and clock bias, and simplifies the signal processing and hardware required for relative position and velocity computations. This interferometric approximation is valid for signals from interstellar distance, for example in the case of X-ray pulsars, but can be problematic for GEO and lunar distance even for the case of no measurement errors. As a result, reports on the performance of Montenbruck's scheme are typically confined to spacecraft distance of within 10 km.

The iterative procedure discussed in this section (and Section 2) differs from Montenbruck's in the sense that it generates the exact solution of relative position and clock bias in the absence of measurement errors. It iterates in an alternating fashion with two sets of parameters: a) the relative position $\vec{P}'$ and clock bias Δ using equation (7), and b) the intermediate parameters s_i's using equation (6a). In the absence of errors, the alternating two-loop iterations enable the relative position $\vec{P}'$, clock bias Δ, and intermediate parameters s_i's converges to their respectively exact solutions.

[5] See equation (9) of [6], and equation (1) of [7].

4. ELIMINATION OF ERRORS IN RELATIVE POSITIONING SCHEMES

In the two variations of trilateration schemes for relative positioning described in Sections 2 and 3, the major error sources are a) estimation of directional cosines $\vec{U}_i$, b) random range measurement errors, c) systematic errors in the raw-range measurements, and d) clock bias between the reference and target spacecraft. In this section, we discuss how the trilateration schemes would eliminate or greatly attenuate these errors in the relative position estimations.

A. Estimation of Direction Cosine $\vec{U}_i$

$\vec{U}_1, \vec{U}_2, \ldots, \vec{U}_n$ as a function of time can be computed from the trajectory of the reference spacecraft and the location of the anchors $\vec{G}_1, \vec{G}_2, \ldots, \vec{G}_n$. The JPL Mission Design and Navigation Team has decades of experience in generating accurate spacecraft trajectory using a combination of DSN tracking measurements, high-fidelity modeling of celestial bodies and their dynamics, and advanced navigation tools [10][11][12][13]. The range accuracy of a spacecraft is 100-150 meters in a GEO orbit, and is 100–500 meters in a lunar orbit [14]. This translates into less than 0.1 millidegree angular errors in estimating the directional cosines $\vec{U}_1, \vec{U}_2, \ldots, \vec{U}_n$. We introduce an angular error of 0.1 millidegree in $\vec{U}_i$ in our simulations, and show that the errors in relative position estimations can still be of the order of decimeters (discussed in Section 5). We assume that the anchors and/or the spacecraft are provided with the predicted directional cosines $\vec{U}_1, \vec{U}_2, \ldots, \vec{U}_n$ prior to executing the relative position computations.

B. Random Range Measurement Errors

Today's GPS radios are equipped with carrier phase estimation function. In the presence of a reasonably strong sinusoidal signal, the radios have a precision of ½ to 1% of a cycle [15]. For L-band signals, this translates into an rms error of approximately 1.1 to 2.2 mm. To be conservative, we consider the cases of rms errors of 2.5 mm and 5.0 mm in random range measurement errors.[6] Range measurement is also affected by multipath. But in the subsequent performance simulations of three or four ground stations tracking a fleet of spacecraft in GEO, we ignore the multipath effects due to the high elevation angle of the ground antennas.

C. Systematic Errors in the Raw-Range Measurements

These are errors that are common in the estimation of r_i and r_i', which includes the common clock bias between the reference and target spacecraft and the anchors, media effects, ephemeris errors, instrument delays, etc. We discuss how the differencing scheme that computes $\vec{d}$ can remove most of the systematic errors in the raw-range measurements, and we illustrate the approach using Figure 1 in Section 2. Without loss of generality, we consider the measurements r_1 and r_1' in triangle Λ_1. Let's define the unknown true ranges to be $r_{1,True}$ and $r'_{1,True}$, and the unknown systematic error that is common to both $r_{1,True}$ and $r'_{1,True}$, to be σ. That is: $r_1 = r_{1,True} + \sigma$, and $r_1' = r'_{1,True} + \sigma$.

When there is no systematic error, we have

$$d_{1,True} = r_{1,True} - ((r'_{1,True})^2 - s_1^2)^{\frac{1}{2}} \tag{9}$$

When taking into account the systematic error σ, we have

$$d_1 = r_{1,True} + \sigma - ((r'_{1,True} + \sigma)^2 - s_1^2)^{\frac{1}{2}} \tag{10}$$

Note that $r'_{1,True} \gg s_1$, and also $r'_{1,True} + \sigma \gg s_1$. One can apply Taylor Series expansion to the square root terms in equations (9) and (10), and obtain the following expressions of $d_{1,True}$ and d_1:

$$d_{1,True} = r_{1,True} - r'_{1,True}\left(1 - \frac{1}{2}\frac{s_1^2}{(r'_{1,True})^2} - \frac{1}{8}\frac{s_1^4}{(r'_{1,True})^4} - \frac{1}{16}\frac{s_1^6}{(r'_{1,True})^6} - \cdots\right) \tag{11}$$

$$d_1 = r_{1,True} + \sigma - (r'_{1,True} + \sigma)\left(1 - \frac{1}{2}\frac{s_1^2}{(r'_{1,True} + \sigma)^2} - \frac{1}{8}\frac{s_1^4}{(r'_{1,True} + \sigma)^4} - \frac{1}{16}\frac{s_1^6}{(r'_{1,True} + \sigma)^6} - \cdots\right) \tag{12}$$

Subtracting equations (12) from (11), and discarding the higher order terms, the error in estimating $d_{1,True}$ in the presence of systematic error σ is of the order of $\frac{1}{2}\left(\frac{s_1}{r'_{1,True}}\right)^2 \sigma$.

The attenuation factor $\frac{1}{2}\left(\frac{s_1}{r'_{1,True}}\right)^2$ can be very small at GEO and lunar distance. We show simulation results of relative position accuracy under different values of σ (10 meters and 100 meters) in Section 5.

D. Clock Bias between the Reference Spacecraft and Target Spacecraft

In the problem formulation in Section 2, we assume perfect time synchronization between the reference and the target spacecraft, and perform relative position computations using three anchors. This assumption can be valid under a number of scenarios, for example, two spacecraft that perform dual

[6] Zero-mean Gaussian distribution.

one-way ranging (simultaneous ranging and time synchronization), simultaneous 2-way ranging where ranging signals are transmitted from the anchors and turned-around by the spacecraft, etc.

In the case when the reference spacecraft broadcasts clock signals to the target spacecraft, a reasonable time transfer scheme has an accuracy of 6 nanoseconds [16]. This translates into an error of 2 meters in relative position estimation.

In the problem formulation in Section 3, the clock bias can be completely removed using an additional anchor.

5. Relative Positioning Performance Simulations

A. Relative Positioning with Three Anchors – Perfect Time Synchronization

We illustrate the trilateration scheme for relative positioning derived in Section 2 using a baseline network of three ground stations at Goldstone, Madrid, and Marlargue tracking a reference spacecraft and a target spacecraft at GEO distance (36,000 km). These ground stations are approximately 8,000 kilometers apart from each other.

Figure 3 shows the 2-dimensional (2-D) coverage maps of the above ground network at GEO distance. Simultaneous tracking and relative positioning can only occur when the spacecraft are in the purple region, which can be rather limiting.

We consider the following operation scenarios:

1. We introduce an error of 0.1 millidegree in the estimation of directional cosines $\bar{U}_1$, $\bar{U}_2$, and $\bar{U}_3$.

2. We assume random range measurement errors of Gaussian distribution with zero mean and standard deviations of 2.5 mm and 5.0 mm.

3. Each raw-range measurement has a systematic error σ of 10 or 100 meters.

4. The distance between the reference spacecraft and the target spacecraft, namely $\|\bar{P}\|$, is 10 kilometers, 100 kilometers, or 500 kilometers.

We perform 10,000 simulations of the relative positioning algorithm based on random choices of $\bar{P}$ with respect to $\bar{R}$ for each combination of systematic error and spacecraft distance, and compute the root-mean-square-error (RMSE) in each case. The results for random rms measurement errors of 2.5 mm and 5.0 mm are tabulated in Tables 1 and 2, respectively.

We observe that the RMSE is independent on the magnitude of the systematic error σ, and scales linearly with the distance between spacecraft.

Table 1. RMSE at GEO distance (3 stations), random RMS error = 2.5 mm

	$\|\bar{P}\|$ = 10 km	$\|\bar{P}\|$ =100 km	$\|\bar{P}\|$ =500 km
σ = 10 m	4.69 cm	8.56 cm	4.90 cm
σ = 100 m	5.63 cm	6.70 cm	5.09 cm

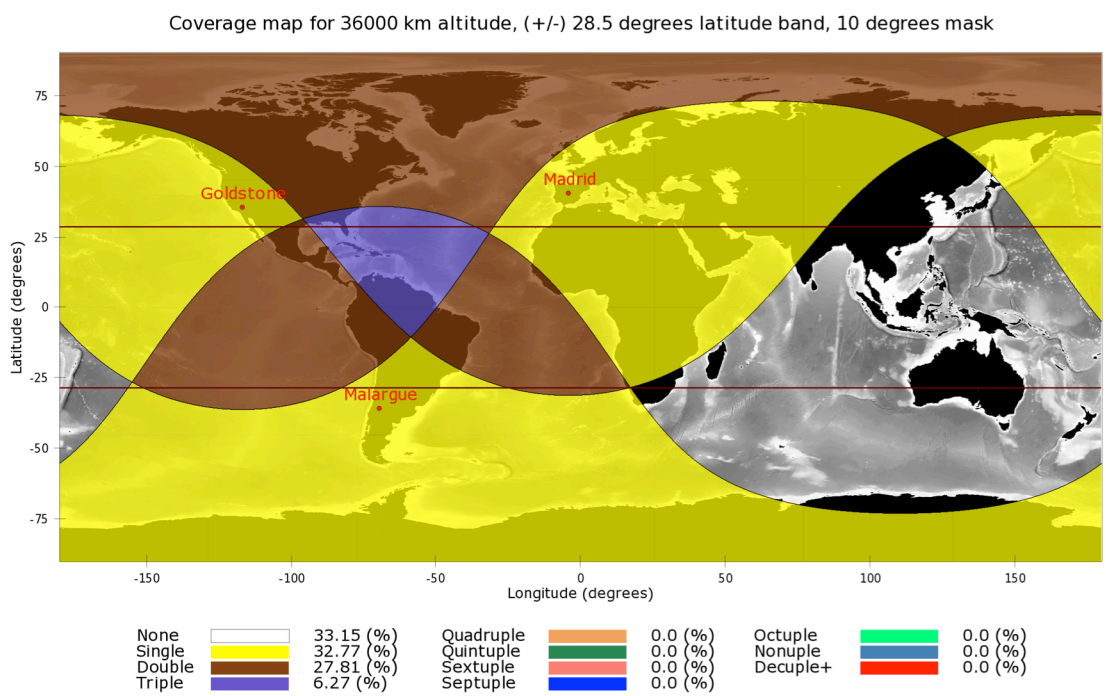

Figure 3. 2-D Coverage Map at GEO distance

Table 2. RMSE at GEO distance (3 stations), random RMS error = 5.0 mm

	$\|\vec{P}\|$ = 10 km	$\|\vec{P}\|$ =100 km	$\|\vec{P}\|$ =500 km
σ = 10 m	17.86 cm	9.90 cm	9.07 cm
σ = 100 m	9.81 cm	13.34 cm	13.83 cm

B. Relative Positioning with Four Anchors – With Clock Bias

Next, we apply the trilateration scheme for relative positioning derived in Section 3 (with clock bias between the reference and target spacecraft) to a baseline network of four ground stations at Goldstone, Madrid, Marlargue, and Kourou tracking a reference spacecraft and a target spacecraft at GEO distance.

Using the same assumptions on directional cosines errors, random measurement errors, and systematic errors as in Section 5.A, and adding the clock bias error Δ of 100 meters, 1,000 meters, and 10,000 meters, we simulate and compute the RMSE in each case and tabulate the results in Tables 3 and 4.

Using the same error assumptions, we simulate and compute the RMSE of Montenbruck's scheme at GEO and lunar distances, and tabulate the results in Tables 5 and 6.

Table 3. RMSE at GEO distance (4 stations), random RMS error = 2.5 mm

	$\|\vec{P}\|$ = 10 km	$\|\vec{P}\|$ =100 km	$\|\vec{P}\|$ =500 km
σ = 10 m, Δ = 100 m	57.77 cm	70.14 cm	53 m
σ = 100 m, Δ = 100 m	95.80 cm	49.30 cm	53 m
σ = 10 m, Δ = 1,000 m	52.64 cm	61.04 cm	54 m
σ = 100 m, Δ = 1,000 m	62.19 cm	123.68 cm	55 m
σ = 10 m, Δ = 10,000 m	53.39 cm	79.62 cm	54 m
σ = 100 m, Δ = 10,000 m	86.38 cm	94.82 cm	56 m

Table 4. RMSE at GEO distance (4 stations), random RMS error = 5.0 mm

	$\|\vec{P}\|$ = 10 km	$\|\vec{P}\|$ =100 km	$\|\vec{P}\|$ =500 km
σ = 10 m, Δ = 100 m	118.03 cm	97.83 cm	54 m
σ = 100 m, Δ = 100 m	149.18 cm	96.22 cm	54 m
σ = 10 m, Δ = 1,000 m	121.40 cm	106.63 cm	54 m
σ = 100 m, Δ = 1,000 m	97.36 cm	111.03 cm	52 m
σ = 10 m, Δ = 10,000 m	189.30 cm	114.80 cm	53 m
σ = 100 m, Δ = 10,000 m	200.00 cm	149.33 cm	54 m

Table 5. RMSE Using Montenbruck's Scheme, random errors = 2.5 mm

	$\|\vec{P}\|$ = 10 km	$\|\vec{P}\|$ =100 km	$\|\vec{P}\|$ =500 km
σ = 10 m, Δ = 100 m	16 m	1633 m	41578 m
σ = 100 m, Δ = 100 m	17 m	1633 m	41661 m
σ = 10 m, Δ = 1,000 m	16 m	1633 m	41599 m
σ = 100 m, Δ = 1,000 m	17 m	1631 m	41559 m
σ = 10 m, Δ = 10,000 m	16 m	1633 m	41602 m
σ = 100 m, Δ = 10,000 m	16 m	1633 m	41628 m

Table 6. RMSE Using Montenbruck's Scheme, random errors = 5.0 mm

	$\|\vec{P}\|$ = 10 km	$\|\vec{P}\|$ =100 km	$\|\vec{P}\|$ =500 km
σ = 10 m, Δ = 100 m	16 m	1632 m	41619 m
σ = 100 m, Δ = 100 m	17 m	1633 m	41629 m
σ = 10 m, Δ = 1,000 m	17 m	1633 m	41654 m
σ = 100 m, Δ = 1,000 m	18 m	1635 m	41688 m
σ = 10 m, Δ = 10,000 m	15 m	1631 m	41702 m
σ = 100 m, Δ = 10,000 m	17 m	1632 m	41628 m

6. Concluding Remarks and Future Work

In this paper, we introduce a trilateration scheme that evaluates the 3-D relative position between a reference spacecraft and a target spacecraft by differencing the raw-range measurements of the reference spacecraft and target spacecraft from a distance baseline of anchors. We consider the cases of a) using three or more anchors when the clocks on the reference point and the target points are perfectly synchronized, and b) using four for more anchors when there is a clock bias between the reference spacecraft and target spacecraft. We demonstrate that in both cases the algorithms converge to submeter accuracy at GEO and lunar distances when estimating the relative position. We assume the presence of common systematic errors in raw-range measurements, and angular error in estimating the pointing vectors between the anchors and the reference spacecraft. In case b) we also assume the presence of clock bias between the reference and the target spacecraft.

The plan for path-forward is as follows:

1. This paper provides the mathematical foundation of the trilateration scheme for relative positioning. We plan to investigate the detail implementation approaches.

2. We plan to consider new near-Earth and deep space mission concepts that can be enabled by this trilateration scheme for relative positioning.

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