Growth and Saturation of Ion Acoustic Waves in Hall Thrusters

Ira Katz[1], Alejandro Lopez Ortega, [2], Benjamin A. Jorns [3], and Ioannis G. Mikellides [4]
Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA, 91107, USA

It has been recognized for over a decade that scattering from ExB Hall current driven ion acoustic waves is the probable cause of the anomalous cross field electron transport in Hall thrusters. Generation of these waves has been shown numerically, supported with analysis, and observed through experiment However, trying to implement a simple formulation based on single fluid ion acoustic waves in a 3-D Hall thruster code without considering Landau damping was only marginally successful; a completely self-consistent solution was not found unless the waves were restricted to grow only downstream of a fixed location in the channel. When Landau damping is included, main ion beam acoustic wave amplitude in the Near Plume reduced so much that the calculated electrics fields are orders of magnitude greater than measured, and self-consistently calculated potential profiles don't resemble those of actual Hall thrusters.

In this paper we present for the first time a conjecture that the electron transport in Hall thrusters is controlled by ion acoustic waves in two distinct ion populations. The first, and largest population, is the main beam, an ion fluid generated upstream of the Acceleration Region. These ions experience electrons with largest ExB drift, but because beam ions have been heated during the ionization process, Landau damping limits the growth of ion acoustic waves. The second population is composed of ions generated at low potentials farther downstream that don't thermalize with the fast moving ion beam. The existence of this separate population has been recognized for decades as the source of the high angle ions in Hall thruster plumes. Because these ions aren't heated and have velocities much different from main beam ions, the amplitude of ion acoustic waves in this population is not Landau damped and grows to saturation. Scattering from these waves reduces the Hall parameter in the near plume where slow ions make up a significant fraction of the population.

Nomenclature

A	=	channel cross section area
B	=	magnetic field
c_s	=	ion acoustic speed
E	=	electric field
e	=	electron charge
Γ	=	growth rate

[1] Supervisor, Electric Propulsion Group, 4800 Oak Grove Drive, Pasadena, CA, 91109, Mail Stop 125-109, Senior Member AIAA.
[2] Member of the Technical Staff, Electric Propulsion Group, 4800 Oak Grove Drive, Pasadena, CA, 91109, Mail Stop 125-109, Member AIAA.
[3] Member of the Technical Staff, Electric Propulsion Group, 4800 Oak Grove Drive, Pasadena, CA, 91109, Mail Stop 125-109, Member AIAA.
[4] Principal Engineer, Electric Propulsion Group, 4800 Oak Grove Drive, Pasadena, CA, 91109, Mail Stop 125-109, Associate Fellow AIAA.
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γ	=	growth rate
λ_D	=	Debye length
k	=	wave number
ℓ	=	thermalization path length
m_e	=	electron mass
m_i	=	ion mass
n	=	electron density
$\dot{n}$	=	ionization rate
ν_e	=	electron scattering frequency
ϕ	=	potential
q	=	ion charge
σ	=	conductivity
T	=	temperature
t	=	time
u	=	flow velocity
W	=	wave energy
Ω	=	Hall parameter
ω_{ce}	=	electron cyclotron frequency

I. Introduction

IN 2004 Adam, Heron, and Laval [1], based on the results of a 2-D, Z-θ particle simulation, proposed that turbulence generated by the electron cyclotron drift instability causes the enhanced cross-field electron transport observed in Hall thrusters. This concept was supported by both analysis [2] and measurement [3, 4]. A decade later Coche and Garrigues [5] found similar results using a different 2-D, Z-θ particle simulation code. Last year we presented results [6] from the first attempt to incorporate a self-consistent model of electron transport based on this instability in a 2-D R-Z code, Hall2De [7] that used a wave action formulation to relate wave energy to enhanced scattering [8]. A similar approach has been used successfully to implement a self-consistent model of ion acoustic turbulence enhanced collision frequency in a 2-D, R-Z, code that models hollow cathode physics [9, 10].

The implementation ion acoustic waves in Hall2De without considering Landau damping was only marginally successful. A completely self-consistent solution was not found unless the waves were restricted to grow only downstream of a fixed location in the channel. However, ion temperatures inferred from Laser Induced Flourescence (LIF) measurements [11] are high enough that Landau damping is important. In the few cases where numerical solutions could be found, the inclusion of Landau damping in Hall2De reduced the acoustic wave amplitude in the Near Plume so much that the calculated electrics fields in that region were an order of magnitude greater than measured, and self-consistently calculated potential profiles bore little resemblance to those of actual Hall thrusters. Examination of the ad-hoc collision frequency used in Hall2De required to fit measured plasma data, leads to the conclusion that anomalous scattering in the acceleration region is different from that downstream of the exit plane. In this paper we present a conjecture and supporting self-consistent 1-D code calculations that the electron transport in Hall thrusters is controlled by ExB driven ion acoustic waves in two distinct ion populations. The difference in electron between the two regions is due to different ion populations, not different waves or instabilities.

II. Hall Thruster Regions

While the cross field azimuthal electron drift, the Hall current, is fast enough to excite ion acoustic waves, implementation in multi-dimensional simulation codes, while promising, have failed to reproduce the characteristic potential profiles observed in Hall thrusters. Figure 1 shows results of a Hall2De [7] simulation of the H6 thruster (without magnetic shielding, called H6US in our previous articles) performed using an ad hoc "anomalous" collision frequency. The ad-hoc profile was adjusted to best fit the all the experimental data including plasma parameters and thrust. This ad-hoc profile differs slightly from that used in previously reported calculations [6]. The operating condition chosen is 300V discharge voltage and 20A discharge current. The beam ion current is about 15A. Figure 1 shows values along the channel centerline from the 2-D simulation of the calculated electric potential and the classical (f_ei and f_en in the plot) and "anomalous" scattering frequencies (f_Anom). Also plotted is the sum of all the electron scattering frequencies (f_e). The potential structure shows the three distinct regions characteristic of Hall thrusters.

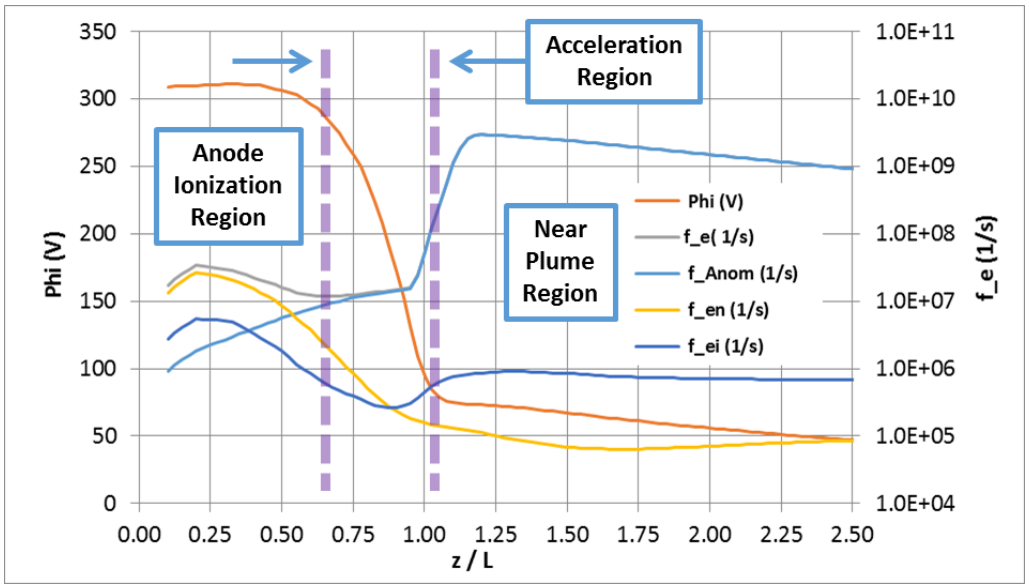

Figure 1. The electric potential and electron scattering frequencies along the channel centerline of the Hall2De H6 Hall thruster simulation.

The first, the Anode Ionization Region, typically extends from the anode downstream to near the end of the discharge channel. It is characterized by an almost constant potential (weak electric field). Experimentally, since a typical Hall thruster has most of the ions accelerated with about 90% of the anode potential, we know that most of the propellant gas is ionized in this region. The plasma density also peaks in this region.

Just downstream is the Acceleration Region, characterized by a strong electric field. The potential in this region starts at near the anode potential to and drops to a few tens of volts in just a centimeter or two. From Ohm's law we know that the plasma conductivity is lowest in this region. Since in Hall thrusters the applied magnetic field is much larger than any self-field, using the definition of the Hall parameter,

$$\Omega \equiv \frac{\omega_{ce}}{\nu_e}, \tag{1}$$

we can rewrite the plasma conductivity,

$$\sigma = \frac{n e^2}{m_e \nu_e (1+\Omega^2)}, \tag{2}$$

in terms of the density and the Hall parameter,

$$\sigma = \frac{n e^2 \Omega}{m_e \omega_{ce} (1+\Omega^2)}. \tag{3}$$

This formulation is very convenient in Hall thrusters, since we know the magnitude of the magnetic field, and thus the electron cyclotron frequency everywhere. Notice that the conductivity is maximum for Ω=1. For higher Hall parameters the magnetic field limits the current; for low Hall parameters, classical resistivity limits the current. Neglecting pressure terms, the electron current density from Ohm's law is

$$j_e = \sigma E = \frac{n e^2 \Omega}{m_e \omega_{ce} (1+\Omega^2)} E, \tag{4}$$

From Hall2De results, the maximum Hall parameter in the Acceleration Region is several hundred, and the peak electric field,

$$E_{\max} \approx 40{,}000\ \frac{V}{m}, \tag{5}$$

is an order of magnitude larger than in the Anode Ionization Region. This large electric field results in a large azimuthal electron drift

$$u_{drift} = \frac{E}{B}, \tag{6}$$

much larger than the ion sound speed,

$$c_s = \sqrt{\frac{eT_e}{m_i}}, \tag{7}$$

as shown in Figure 2.

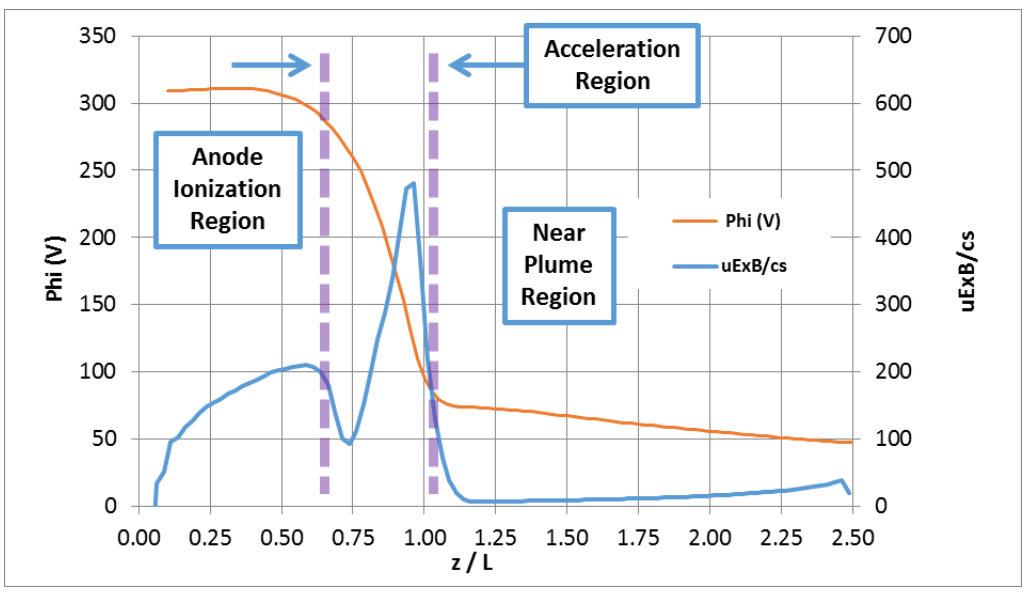

Figure 2. In the Acceleration Region the azimuthal ExB drift velocity is hundreds of times faster than ion sound speed.

Immediately downstream, in the Near Plume Region, the electric field drops dramatically by a couple orders of magnitude. The potential drops tens of volts in the order of ten centimeters. As shown in Reference 6, the Hall parameter in the Near Plume region must be less than 10. The electron cyclotron frequency in the near plume is the order of

$$\omega_{ce} \approx 10^9\, s^{-1}, \tag{8}$$

As seen in Fig. 1, this is more than three orders of magnitude larger the classical collision frequencies in the near plume. Only anomalous scattering can account for the observed low electric fields and Hall parameters.

III. Scattering by Ion Acoustic Waves

Several papers have discussed how the azimuth ExB electron drift leads to the growth of ion acoustic waves [1-6]. Ion acoustic waves with a substantial azimuthal wave vector component can scatter electrons across field lines and enhance transport. Following Ref. 8, the anomalous scattering frequency is proportional to the acoustic wave energy density.

$$\nu_{an} = \omega_{pe} \frac{W}{nT_e} , \tag{9}$$

where.

$$W \equiv \frac{n}{T_e} \sum_k (e\phi_k)^2 , \tag{10}$$

The wave energy density evolution in an inhomogeneous plasma, is described by the plasma wave kinetic equation

$$\frac{\partial W}{\partial t} + \frac{\partial}{\partial \vec{r}} \cdot \left[\mathbf{u}_i W \right] = 2\gamma W , \tag{11}$$

where $\mathbf{u}_i$ is the ion velocity vector and the growth rate, γ, has both drift induced destabilizing and Landau damping terms

$$\begin{aligned} \gamma &= \gamma^+ - \gamma^-_{Landau} \\ \gamma^+ &= \sqrt{\frac{\pi}{8}} k c_s \left(\sqrt{\frac{m_e}{m_i}} \left(\frac{u_{drift}}{c_s} - \frac{1}{\sqrt{1+k^2\lambda_D^2}} \right) \right) \frac{1}{\left(1+k^2\lambda_D^2\right)^{3/2}} \\ \gamma^-_{Landau} &= \sqrt{\frac{\pi}{8}} k\, c_s \left(\left(\frac{T_e}{T_i} \right)^{3/2} \exp\left(-\frac{T_e}{2T_i\left(1+k^2\lambda_D^2\right)} \right) \right) \frac{1}{\left(1+k^2\lambda_D^2\right)^{3/2}} \end{aligned} . \tag{12}$$

The destabilizing term has a maximum for a wave number shown in Eq. 13.

$$k = \frac{1}{\sqrt{2}\,\lambda_D} \tag{13}$$

The growth rate at this wave number is

$$\begin{aligned} \gamma &= \gamma^+ - \gamma^-_{Landau} \\ \gamma^+ &= \frac{\sqrt{\pi}}{3^{3/2}\lambda_D} c_s \sqrt{\frac{m_e}{m_i}} \left(\frac{u_{drift}}{c_s} - \frac{1}{\sqrt{1.5}} \right) \\ \gamma^-_{Landau} &= \frac{\sqrt{\pi}}{3^{3/2}\lambda_D} c_s \left(\frac{T_e}{T_i} \right)^{3/2} \exp\left(-\frac{T_e}{3T_i} \right) \end{aligned} . \tag{14}$$

The destabilizing term peaks in the middle of the Acceleration Region

$$2\gamma^{+}_{\max} \approx 8\times 10^{7}\, s^{-1}. \tag{15}$$

Since the ion velocities at the end of the Acceleration Regions are ~ 20,000 m/s, the scale length for wave growth in the ion beam is less than a millimeter.

$$\frac{u_i}{\gamma^{+}_{\max}} \approx \frac{2\times 10^{4}}{8\times 10^{7}} = 0.25\, mm. \tag{16}$$

This points out an apparent contradiction. The ion acoustic wave growth peaks in the middle of the Acceleration Region and the growth is so fast that it saturates in a few millimeters. This would suggest a high ion acoustic wave energy density and subsequent high anomalous scattering. However, in the Acceleration Region the anomalous scattering is the smallest and the Hall parameter the largest. How can both statements be true? The answer is in the Landau damping term.

If the growth and damping terms are both large, we can use the "Marginal Stability" approximation introduced by Manheimer and Boris [11] to evaluate Equation 11.

$$\frac{\partial W}{\partial t} + \frac{\partial}{\partial \vec{r}} \cdot \left[\mathbf{u}_i W\right] = 2\gamma W \approx 0. \tag{17}$$

For ion acoustic waves where the ExB drift velocity is much greater than the ion acoustic speed and neglecting collisional damping terms, good approximations in the Acceleration Region, Equation 17 reduces to

$$\begin{aligned} &\gamma = \gamma^{+} - \gamma^{-}_{Landau} \approx 0 \\ &\frac{\sqrt{\pi}}{3^{3/2}\lambda_D} c_s \sqrt{\frac{m_e}{m_i}} \left(\frac{u_{drift}}{c_s} - \frac{1}{\sqrt{1.5}} \right) = \frac{\sqrt{\pi}}{3^{3/2}\lambda_D} c_s \left(\frac{T_e}{T_i} \right)^{3/2} \exp\left(-\frac{T_e}{3T_i} \right) \cdot \end{aligned} \tag{18}$$

Given the other plasma parameters, this equation can be used to find the ion temperatures such that Landau damping balances ion acoustic wave growth

$$\left(\frac{T_e}{T_i} \right)^{3/2} \exp\left(-\frac{T_e}{3T_i} \right) = \sqrt{\frac{m_e}{m_i}} \left(\frac{u_{drift}}{c_s} - \frac{1}{\sqrt{1.5}} \right). \tag{19}$$

Figure 3 shows the ion temperature profile calculated from Eq, 19 using parameters from the Hall2De calculations, compared with ion temperature estimated from published LIF data [11]. The points labeled Ti LiF(eV) was obtained by visual estimating the width of the published beam ion velocity distribution functions from a figure in Ref. [11]. The published velocity distribution function (VDF) was clearly non-Maxwellian for a single point in the Acceleration Region.

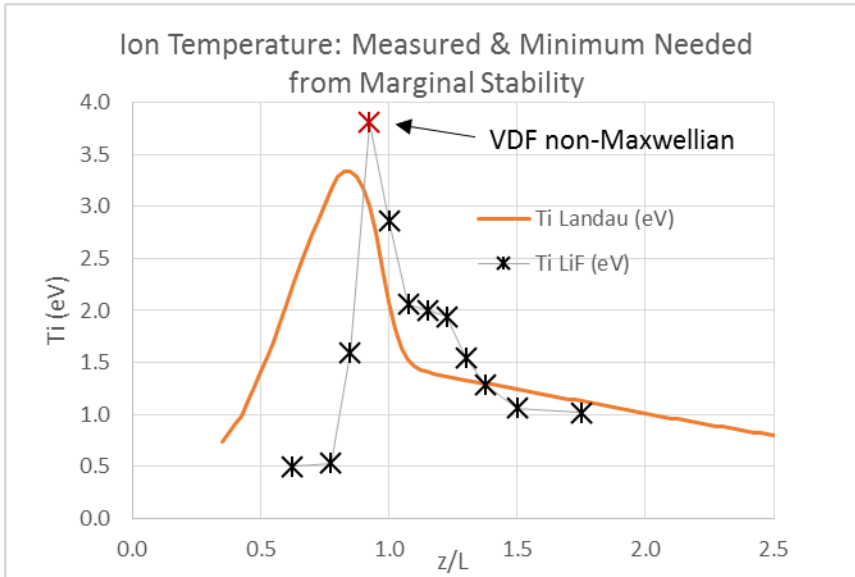

Figure 3. Beam ion temperatures profile from published LIF measurements compared with the ion temperatures needed for Landau damping to balance ion acoustic growth.

We draw two conclusions from Figure 3. The first is that in the downstream section of the Acceleration Region, the measured beam ion temperatures are large enough for Landau damping to limit the growth of ion acoustic waves. This explains how it possible for the ExB drift to peak in the Acceleration Region, while the anomalous scattering from ExB driven ion acoustic waves remains low. Recognizing the importance ot the ion temperature, an ion energy equation was added to Hall2De.

The second conclusion is that the measured beam ion temperatures in the Near Plume Region are also large enough to limit the growth of ion acoustic waves in the main beam. However, it is in this Region, based on the measured low electric fields, where the anomalous scattering must be most important. This discrepancy shows a gap in the physical understanding of wave induced enhanced transport as it applies to Hall thrusters. The calculated and measured ion temperatures will cause substantial ion Landau damping of ion acoustic waves in the Near Plume, but at the same time anomalous scattering must be extremely high.

We propose to resolve this apparent paradox by the following conjecture. Upstream of the acceleration region, the plasma ions that form the main ion beam act as a fluid. The ion velocity distribution functions are Maxwellian and ion Landau damping suppresses ion acoustic wave growth as expected. Newly generated ions are rapidly assimilated into the Maxwellian distribution.

As fluid ions fall down the potential they are accelerated and the energy difference between main beam ions and newly generated ions increases. The classical ion-ion scattering cross section decreases as one over the square of the energy difference. Eventually, the cross section is so small that the new ions aren't assimilated into the main beam fluid, but act as a slower, separate, non-Maxwellian ion population. Since the electric field is axisymmetric, the ions in this new population have very small azimuthal velocities. Their axial velocities are also much less than that of main beam ions. As a result, if ion sound waves are generated in this second population, it is possible that very few main beam or secondary population ions will be resonant with the wave phase velocity, and, consequently, Landau damping will be minimal. In the following section we implement a simple anomalous scattering formulation based upon this two ion population conjecture and marginal stability, in a 1-D Hall thruster code. The results of self-consistent calculations using this formulation are remarkably similar to those obtained using an ad-hoc anomalous scattering profile in the same code

IV. 1-D Hall Thruster Code

As a test bed to investigate anomalous transport physics we developed Hall1De, a simplified, 1-D version of Hall2De [7]. Hall1De solves Ohm's law, and the conservation equations (mass, momentum and total energy) for singly charged ions with spatial variation only in the axial dimension, for a single, main beam, fluid. The width of the main beam is confined to the channel upstream of the exit plane. Downstream of the exit plane it is assumed to expand with a 25° half angle.

Ion temperatures are calculated using a total ion energy equation,

$$\frac{\partial}{\partial t}\left(n\frac{3q}{2m_i}T_i\right)+\nabla\cdot\left(n\mathbf{u}\frac{3q}{2m_i}T\right)+p\nabla\cdot\mathbf{u}=\dot{n}\left(\frac{3q}{2m_i}T_n+\left|\mathbf{u}_i-\mathbf{u}_n\right|^2\right)+n\nu_{in}\left(\frac{3q}{2m_i}\left(T_n-T_i\right)+\left|\mathbf{u}_i-\mathbf{u}_n\right|^2\right). \quad (20)$$

Ionization and ion-neutral scattering add energy as the velocity difference between ions and neutrals. To account for the reduced ion-ion collision frequency when the velocity difference is large, the heating from the ionization term is reduced as follows:

$$\dot{n}_i\left|\mathbf{u}_i-\mathbf{u}_n\right|^2 \Rightarrow (1-\exp(-\nu_{ii}\tau))\ \dot{n}_i\left|\mathbf{u}_i-\mathbf{u}_n\right|^2,\ \tau=\frac{\ell}{u_n}, \quad (21)$$

where υ_{ii} is the scattering frequency associated with slowing down of a fast ion, u_n is the neutral gas thermal speed, and ℓ is a characteristic path length available for thermalization. In the calculations presented below it is chosen to be about one centimeter.

$$\ell \approx 1\text{cm}. \quad (22)$$

The test case for the 1-D code is the same 6 kW thruster as used in the Hall2De results discussed earlier. The 1-D codes assumes a uniform density across the channel width, over estimating wall losses compared with a modern Hall thruster with focusing magnetic fields. This results in the 1-D code predicting much lower peak electron temperatures than those calculated using Hall2De To overcome this, ion losses to the channel walls in the 1-D code were reduced from Bohm by 90% so that the temperatures in the 1-D and 2-D codes were about the same.

$$\Gamma_{wall} = f_{wall}\, n_i\, u_{Bohm}, \qquad f_{wall} \approx 0.1 \quad (23)$$

Potentials and electron temperatures calculated using the 1-D code are shown Figure 4. Classical electron neutral scattering dominates upstream in the Anode-Ionization region. Downstream in the Near Plume, the anomalous scattering frequency is the electron cyclotron frequency, the scattering frequency that sets the Hall parameter to unity and maximizes the plasma conductivity. For a short distance in the Acceleration Region, just upstream of the exit plane (z/L=1) the anomalous scattering frequency varies slowly at an intermediate value. The value in that short region controls the total current and the peak electric fields.

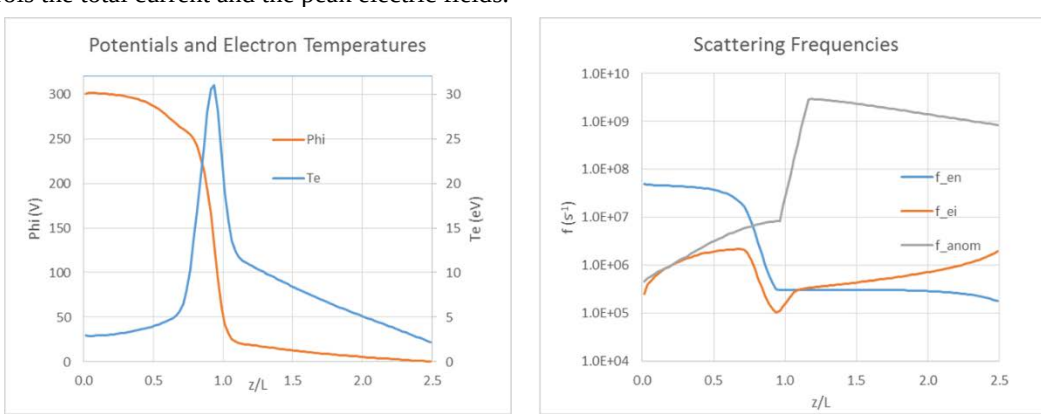

Figure 4 Potentials and electron temperatures calculated using Hall1De and an ad-hoc collision frequency profile.

V. Self-Consistent Anomalous Scattering Implemented in the 1-D Code

The self-consistent anomalous scattering frequency consists of contributions from the main beam and from a secondary population of ions generated downstream. In the calculation below, ions generated where the potential is

greater than 100 V are added to the main beam fluid; ions generated where the potential is less than 100 V are treated as part of a second population.

$$\dot{n}_2 = \dot{n}(\phi < 100V) \tag{24}$$

This approach, following a second populations of ions defined by ions generated below a fixed potential, has long been recognized as necessary to describe the Hall thruster far field plume [13,14], and previously implemented before in Hall2De[15] & Hall2DePlume[16]. Earlier authors ascribed the source of this population to charge exchange in the main beam, but more recent work [16] has shown that electron impact ionization of neutrals may be even a more significant source in the near plume

Hall1De follows the single fluid that makes up the main beam. In the code, we estimate the density of the second population using mass continuity assuming the ions stream axially with kinetic energy gained from the potential difference btween the present cell and the cell where they were generated, and, downstream of the exit plane they spread at the same angle as the main beam, a 25° half angle,

$$\begin{aligned} & n\,u\,A = V\,\dot{n} \\ & n_2(j) = \frac{1}{A_j}\sum_{k=1,\,j-1}\dot{n}_2(k)V_k\frac{1}{\sqrt{\dfrac{2(\phi_k-\phi_j)}{m_i}}}\exp\left(-\frac{\nu_{ii}(k)\ell}{u_n}\right), \end{aligned} \tag{25}$$

where V_k is the volume of cell k, and A_j is the cross sectional area of cell j. The last term in Eq 25 reduces the second ion population by those ions that equilibrate with the main beam (Eq. 21).

The anomalous scattering frequency associated with main beam ions, n_1, is found from a simplified Ohm's law,

$$\begin{aligned} & \sigma \equiv \frac{e^2 n_e}{m_e \nu_e} \\ & j_e = \sigma\frac{1}{1+\Omega_e^2}E, \;\; \Omega_e \equiv \frac{\omega_{ce}}{\nu_e} >> 1, \qquad \omega_{ce} = \frac{eB}{m_e} \\ & j_e = \frac{e^2 n_e \nu_e}{m_e \omega_{ce}^2}E \\ & u_{drift} = \frac{E}{B} = j_e\frac{m_e\omega_{ce}^2}{e^2 n_e \nu_e B} = j_e\frac{\omega_{ce}}{e\,n_e\nu_e} = u_e\frac{\omega_{ce}}{\nu_e} \end{aligned}, \tag{26}$$

and Eq. 19,

$$\begin{aligned} & \gamma = \gamma^+ - \gamma^-_{Landau} \\ & u_{drift} = u_e\frac{\omega_{ce}}{\nu_e} = u_e^{th}\left(\frac{T_e}{T_i^{n_1}}\right)^{3/2}\exp\left(-\frac{T_e}{3T_i^{n_1}}\right) + \frac{c_s}{\sqrt{1.5}}, \qquad u_e^{th} \equiv \sqrt{\frac{eT_e}{m_e}} \\ & \nu_{AN}^{n_1} = \omega_{ce}\left(\frac{u_e}{u_e^{th}}\right)\left(\frac{T_i^{n_1}}{T_e}\right)^{3/2}\exp\left(\frac{T_e}{3T_i^{n_1}}\right), \qquad u_{drift} >> c_s \\ & \nu_{AN}^{n_1} = \omega_{ce}M_e\left(\frac{T_i^{n_1}}{T_e}\right)^{3/2}\exp\left(\frac{T_e}{3T_i^{n_1}}\right), \qquad M_e \equiv \frac{u_e}{u_e^{th}} \end{aligned}. \tag{27}$$

It is apparent from Eq. 27, that the anomalous scattering frequency depends critically on the ion - electron temperature ratio, since all other wave damping mechanisms are ignored. This assumption is particularly valid in the Acceleration Region, where the destabilizing frequency exceeds 10^7 s^{-1}.

We implement the two ion population conjecture by assuming that there is no significant damping of ion acoustic waves in the low energy population. That is ion acoustic waves in this population are saturated,

$$\begin{aligned} n_2(x,t) &= n_2 \cos kx \cos \omega t \\ \phi_k &= \frac{n_2 T_e}{n_0}, \qquad n = n_1 + n_2 \\ W^{n_2} &= \frac{n}{T_e}\phi_k^2 = \frac{T_e}{n} n_2^2 \qquad , \\ \nu_{AN}^{n_2} &= \omega_{pe}\frac{n_2^2}{n^2} \end{aligned} \tag{28}$$

and that scattering from the two populations is additive.

$$\nu_{AN} = \nu_{AN}^{n_1} + \nu_{AN}^{n_2} . \tag{29}$$

The anomalous scattering frequency is limited to the electron cyclotron frequency. For scattering frequencies higher, than the electron cyclotron frequency the concept of an ExB drift velocity is no longer valid.

$$\nu_{AN} = \max(\nu_{AN}, \omega_{ce}) \tag{30}$$

VI. Self Consistent 1-D Code Results

Starting from the ad-hoc collision frequency results (Fig. 4.), a self consistent solution was found by direct iteration. The self-consistent anomalous collision profile was blended with the ad-hoc profile over the course of a millisecond (25,000 code cycles) per major iteration. Then a new collision frequency profile was found from these results and they blended with the results of the previous major iteration. The calculated anode current converged to within 0.01% in 10 major iterations.

The self-consistent solution is shown in Figure 5. The potentials and electron temperatures are almost identical with those of the ad-hoc solution (Fig. 4). The anomalous collision frequency calculated self-consistently plotted along with the ad-hoc collision frequency is also shown in Fig. 4. The thermalization path length, 1 cm, was adjusted to make the anode current, 19.35A close to 20.0A calculated using the ad-hoc profile.

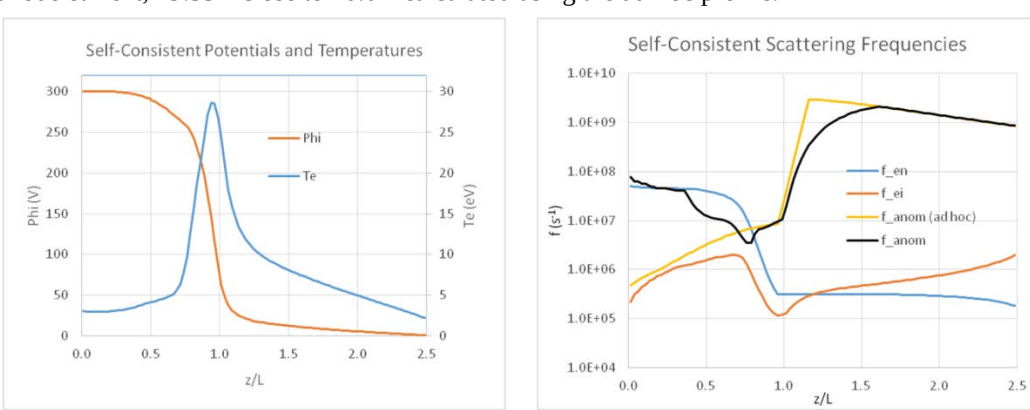

Figure 5. Potentials and electron temperatures from Hall1De with a self-consistent collision frequency.

While the shape of the anomalous collision frequency profile differs from the ad-hoc formulation, three basic features are the same. First, in the upstream portion of the Acceleration Region, the electron-neutral collision

frequency dominates both the self-consistent and ad-hoc collision frequencies. The contributions to the self-consistent electron scattering frequency and the calculated Hall parameter are shown in Figure 6. There is a small region near the anode, where the self-consistent anomalous scattering is comparable to the electron neutral scattering, but the Hall parameter in this region is small, and the electric fields in both solutions are quite small.

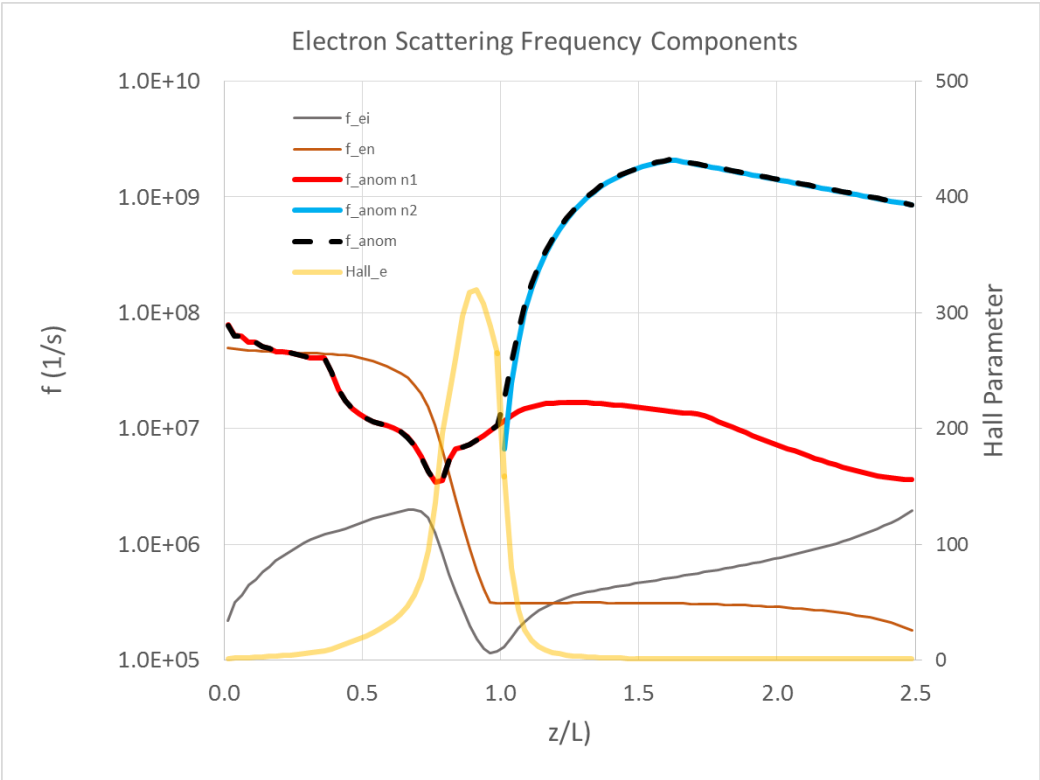

Figure 6. Components of the self-consistent electron scattering frequency from the 1-D code and the calculated Hall parameter.

The second feature the two anomalous formulations have in common is the shape and value of the plateau in last 20% of the channel. The amplitude of the anomalous scattering in this short region controls the magnitude of the electron current and the peak electric field. Even though the ExB electron drift peaks in this short region, because the calculated ion temperatures are significant, the self-consistent scattering frequency from main beam ions is held down by ion Landau damping. As seen in Figure 5, the Hall parameter actually peaks in this region, albeit more than an order of magnitude below what it would be in the absence of scattering by ion acoustic waves. This is a significant finding: even though the electron drift that drives ion acoustic waves peak in this region, Landau damping is also high and prevents the waves from growing large enough to further reduce the Hall parameter.

The third feature is the rapid rise in anomalous collision frequency in the Near Plume Region that reduces the Hall parameter, and thus the electric fields, by orders of magnitude compared with the Acceleration Region. In the self-consistent model, this rise is all due to waves in the slow ion population. Even in the Near Plume, ion acoustic waves in the main beam undergo substantial Landau damping. This can be seen clearly in Figure 6. In the Near Plume, the contribution to the scattering by main beam ions falls two orders of magnitude below that from the slow ion population. Without the scattering by the second population, electric fields in the Near Plume would be orders of magnitude higher than observed, and the potential drop in the Acceleration Region would be greatly reduced.

As shown in Fig. 8, the potential and electron temperature are almost identical to those calculated with the ad-hoc profiles

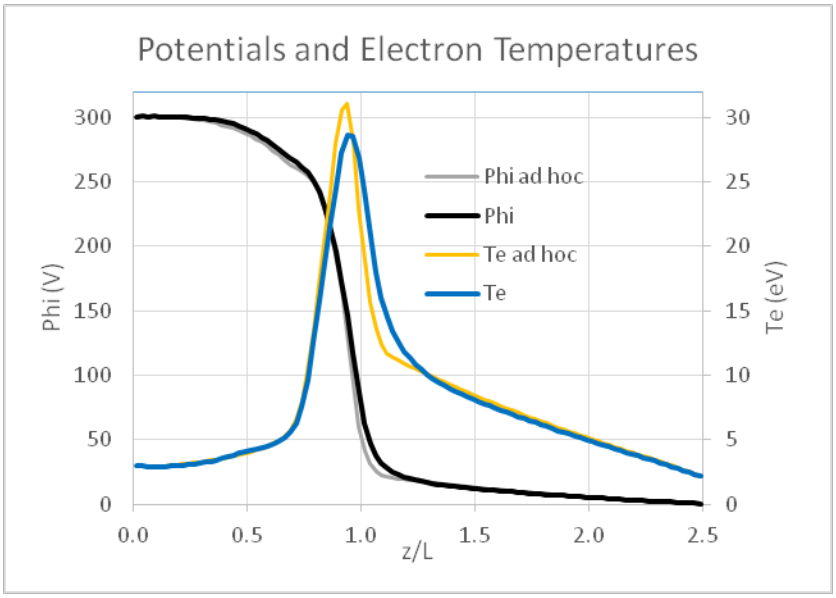

Figure 7. Components of the self-consistent electron scattering frequency from the 1-D code and the calculated Hall parameter.

However, as seen in Fig. 8, the self-consistently calculated ion temperatures are almost a factor of two below the measured values everywhere downstream of the Anode Ionization region. That is, the calculated ion temperatures are low everywhere the anomalous collision frequency is important.

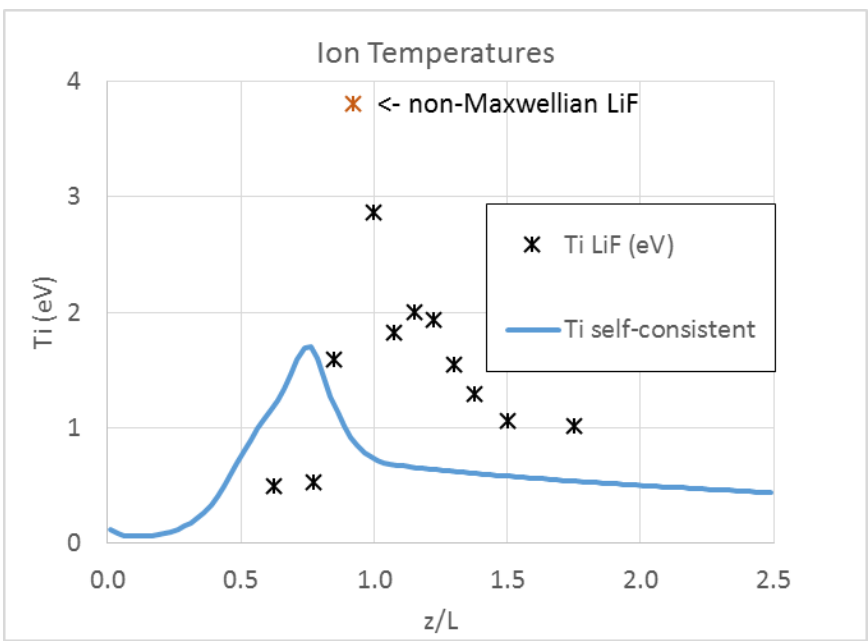

Figure 8. Self-consistently calculated ion temperatures fall well below measured values in the Acceleration and Near Plume Regions.

VII. Ion Acoustic Waves in a Two Ion Beam Plasma

The two ion population model implemented in the 1-D code assumes that ions waves in the two populations act independently of each other. However, they form a single plasma and ions from both populations are subject to the same electric fields. As a first step to check our assumption, we have derived the dispersion relation for a collisionless plasma with two drifting Maxwellian ion populations.

$$
\begin{aligned}
0 = k^2 - \omega_{pi}^2 \Bigg(& -\frac{m_i}{T_e} + \eta \frac{k^2}{\omega - k \cdot u_i^s} + (1-\eta) \frac{k^2}{\omega - k \cdot u_i^f} \Bigg) \\
& - i \left(\frac{\pi}{2} \right)^{1/2} \Bigg[-\frac{\omega / k - \hat{k} \cdot u_e}{\lambda_D^2 \, u_{the}} + \eta \omega_{pi}^2 \left(\frac{m_i}{T_i^s} \right)^{3/2} \left(\frac{\omega - k \cdot u_i^s}{k} \right) \exp\left(-m_i \frac{\left(\omega / k - \hat{k} \cdot u_i^s \right)^2}{2 T_i^s} \right) \\
& + (1-\eta) \omega_{pi}^2 \left(\frac{m_i}{T_i^f} \right)^{3/2} \left(\frac{\omega - k \cdot u_i^f}{k} \right) \exp\left(-m_i \frac{\left(\omega / k - \hat{k} \cdot u_i^f \right)^2}{2 T_i^f} \right) \Bigg]
\end{aligned}
\tag{31}
$$

The real part of the dispersion relation is a quartic. Numerical solutions using computed Near Plume plasma parameters show that there are two sets of waves. The first pair have phase velocities equal to the fast population drift $\pm c_s$. The second wave pair have much slower phase velocities. As a result, the exponent in the fast ion Landau term is much more negative for the slow waves, and thus the Landau damping of the slow waves due to the fast ion temperature is greatly reduced. This is consistent with the assumption made in the 1-D code. A more complete and rigorous analysis will be the subject of a future paper

VIII. Conclusions

It has been recognized for over a decade that scattering by ExB Hall current driven ion acoustic waves is the probable cause of the anomalous cross field electron transport in Hall thrusters. Generation of such waves has been shown numerically [1, 5], supported with analysis [2,4], and observed through experiment [3]. However, trying to implement a simple formulation based on single fluid ion acoustic waves in Hall2De without considering Landau damping was only marginally successful; we were not able to find a completely self-consistent solution unless the waves were prevented from growing upstream of a fixed location. Even worse, in the few cases where solutions were found, the inclusion of Landau damping reduced the acoustic wave amplitude in the Near Plume so much that the calculated electrics fields in that region an order of magnitude greater than measured, and calculated self-consistently potentials bore little resemblance to those of actual Hall thrusters.

In this paper we have conjectured that the electron transport in Hall thrusters is controlled by waves in two distinct ion populations. The first, and largest population is the main beam, an ion fluid generated upstream of the Acceleration Region. Although they experience the fastest electron ExB drift, because the ions have been heated during the ionization process, Landau damping limits the growth of ion acoustic waves. The second population consists of ions generated at low potentials that don't thermalize with fast moving beam ions. The existence of this separate population has been recognized for decades as the source of the high angle ions in Hall thruster plumes [13-16]. Because these ions aren't heated and have velocities much different from main beam ions, the amplitude of ion acoustic waves in this population is not Landau damped. The result is low Hall parameters in the near plume where this population is significant.

In hindsight, if we examine the ah-hoc collision frequency used in Hall2De that was adjusted to fit all the measured data, it is obvious that there are two different sources of anomalous scattering. In Fig. 9 we overlay on the graph in Fig. 1 an extrapolation of the upstream ad-hoc collision frequency. It is clear that a second, larger source of anomalous scattering arises and dominates the scattering in the near plume, consistent with our conjecture.

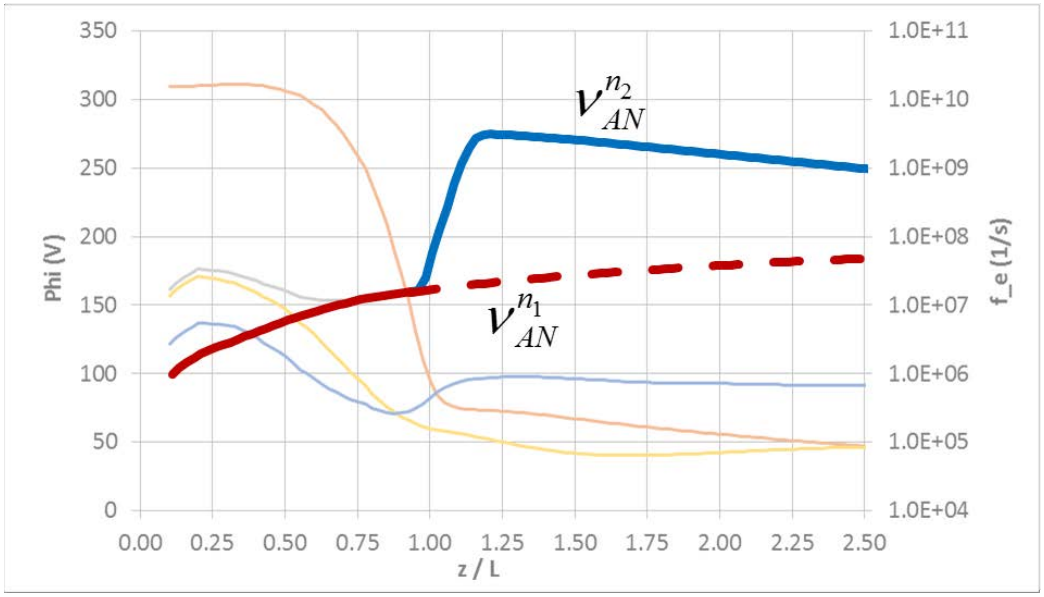

Figure 9. Data in Figure 1 replotted to emphasize that a second, larger source of anomalous scattering dominates the scattering in the near plume.

The two ion population conjecture was tested by implementing it in a simplified 1-D Hall thruster code. The code calculated the electron scattering frequency self-consistently, albeit using the steady state marginal stability approximation, Eq. 19, rather than a time dependent convection equation. The only wave damping included was Landau damping of waves in the main beam. The plasma parameters and thrust from the self-consistent 1-D calculations were very close to those calculated earlier in the same code with an ad-hoc collision frequency profile. The self-consistent potential profile has the characteristics of a typical Hall thruster. It had low electric fields upstream in the Anode Ionization Region, a very strong electric field in the Acceleration Region, and an electric field down by orders of magnitude in the Near Plume.

The dispersion relation for a two ion beam plasma was presented. The dispersion relation allows two pairs of waves, one pair associated with the fast ions, the other with the slow ions. The difference in phase velocity between these pairs of waves acts to suppress Landau damping between them. The saturated amplitude of the undamped waves limits to that used in the self-consistent calculation, Eq. 28.

While these first results are promising, a great deal of additional work is needed to determine if ion acoustic waves in two distinct ion populations control the electron transport in Hall thrusters. First, there must be a more rigorous analytical investigation into wave growth in the near plume ion populations to provide the theoretical foundation for future algorithm development. Second, is the difficult task of developing a robust, self-consistent, time dependent, formulation that can be incorporated in a 2-D code such as Hall2De. The formulation should have additional physics, including other damping mechanisms and wave ion heating, missing in the 1-D model presented.

The dependence on wave number was not studied during this investigation. The particular choice of wave number chosen for the numerical simulaitions given in Eq. 13 does not maximize the growth when Landau damping is important; it maximizes at a smaller wave number that reduces that Landau damping term. However, as shown above in Fig. 8, where the anomalous collision frequency dominates, the calculated ion temperatures are about half the measured values. The wave number choice overestimates Landau damping; the error in the calculated ion temperature underestimates the damping. Resolving both of these issues needs to be addressed in future work.

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