NISAR Spacecraft Environmental Torque Modeling During Operations

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Abstract

Environmental torques imparted on the NISAR spacecraft are modeled. The environmental torques comprise of the gravity gradient, atmospheric drag, and solar radiation pressure. These torques act as external disturbances on the spacecraft and cause momentum to accumulate. This momentum needs to be unloaded via attitude control hardware. In order to size the hardware, it is important to determine the environmental torques imparted on the spacecraft. The results indicate that the worst case gravity gradient, drag, and solar radiation pressure torques are 7.4x10^{-3} Nm, 6.01x10^{-3} Nm, and 14.03x10^{-3} Nm, respectively.

Nomenclature

A_k	=	spacecraft component surface area
Az	=	azimuth angle
$\hat{e}$	=	rotation vector
El	=	elevation angle
$\hat{n}_{gd}$	=	geodetic nadir vector
ϕ	=	rotation angle
${}^{A}R^{B}$	=	rotation matrix from B-frame to A-frame
$\vec{r}_{gc_k}$	=	spacecraft component geometric center position vector relative to SC-frame origin
SC-frame	=	spacecraft body-fixed frame
$\vec{\tau}_{drag}$	=	atmospheric drag torque
$\vec{\tau}_{gg}$	=	gravity gradient torque
$\vec{\tau}_{solar}$	=	solar radiation pressure torque
$\hat{u}_k$	=	spacecraft component unit normal
$\hat{v}_{SC}$	=	inertial spacecraft velocity direction
${}^{A}\vec{vec}$	=	vector expressed in A-frame

I. NISAR Spacecraft

The NASA-ISRO Synthetic Aperture Radar (NISAR) mission is a partnership between the NASA Jet Propulsion Laboratory (JPL) and the Indian Space Research Organization (ISRO). This proposed Earth-orbiting spacecraft is in the early stages of development and utilizes the Interferometric Synthetic Aperture Radar (InSAR) to make scientific measurements about various Earth processes [1]. The main processes of interest are: volcanic eruptions, tsunamis, ice-sheet collapse, and climate change. An artist's rendition for the fully deployed spacecraft in the Earth orbit is shown in Figure 1.

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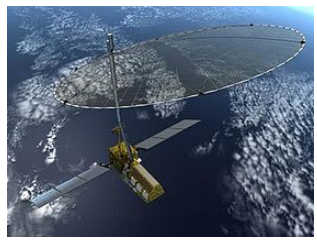

Figure 1: NISAR Spacecraft

During science operations, the external disturbances like gravity gradient, atmospheric drag, and solar radiation pressure torques cause momentum to accumulate on the spacecraft. This momentum is unloaded via the thrusters or magnetic torque rods. The NISAR pointing requirements do not allow thruster usage to unload momentum during the science operations. Hence, the unloading is performed using the torque rods. In order to size the torque rod hardware, it is important to determine the environmental torques imparted on the spacecraft. In this study, an environmental torque model is developed. The frame and vector defintions in II are used to develop the model. The spacecraft attitude variation model in III is developed to assess the worst case environmental torques. The spacecraft cross-sectional area model in IV provides the projected area to use in the drag and solar torque computations. Finally, the environmental torque model and results are given in V and VI.

II. Frame and Vector Definitions

The frame and vector definitions are illustrated in Figure 2.

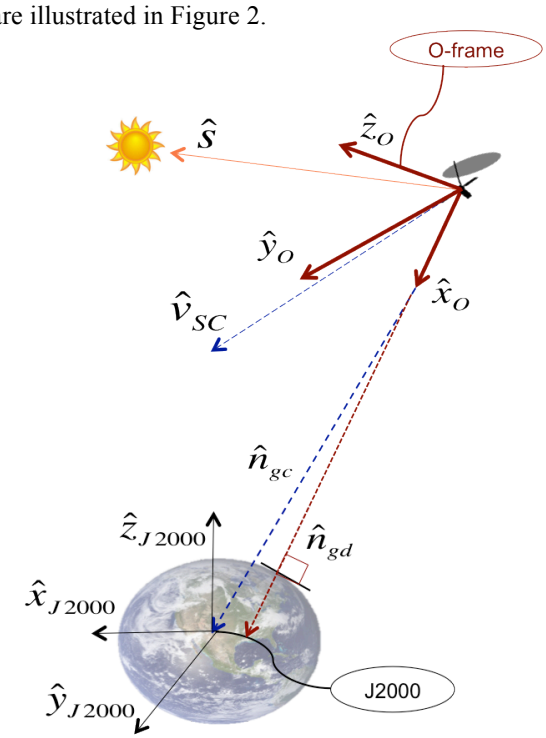

Figure 2: Frame and Vector Definitions

The inertial frame is J2000. The orbit-fixed, O-frame is defined as

$$O \rightarrow \left\{ \begin{array}{c} \hat{x}_O = \hat{n}_{gd} \\ \hat{y}_O = \hat{z}_O \times \hat{x}_O \\ \hat{z}_O = \dfrac{\hat{n}_{gd} \times \hat{v}_{SC}}{\left\| \hat{n}_{gd} \times \hat{v}_{SC} \right\|} \end{array} \right\} \tag{1}$$

where $\hat{n}_{gd}$ and $\hat{v}_{SC}$ are along the geodetic nadir and spacecraft inertial velocity directions, respectively. The roll axis, $\hat{y}_O$ is approximately along $\hat{v}_{SC}$. The sun inertial vector direction is $\hat{s}$. The SC-nominal (SC_{nom}) frame is a deviation from the O-frame defined with a 16.33° rotation about the $\hat{y}_O$ axis.

$$ {}^{SC_{nom}}R^{O} = \begin{bmatrix} \cos(16.33^\circ) & 0 & -\sin(16.33^\circ) \\ 0 & 1 & 0 \\ \sin(16.33^\circ) & 0 & \cos(16.33^\circ) \end{bmatrix} \tag{2} $$

During nominal operations, the spacecraft maintains a constant roll offset of 16.33° to meet the science instrument pointing objectives. Further spacecraft attitude deviations from the SC_{nom}-frame are represented by ${}^{SC}R^{SC_{nom}}$ to yield the spacecraft body-fixed, SC-frame.

III. Spacecraft Attitude Variation Model

The induced environmental torque bounds are computed by considering variations from the nominal spacecraft attitude. A variation from the nominal spacecraft attitude is defined with a rotation axis, $\hat{e}$ and rotation angle, ϕ, which are varied systematically to cover the desired set of attitude variations.

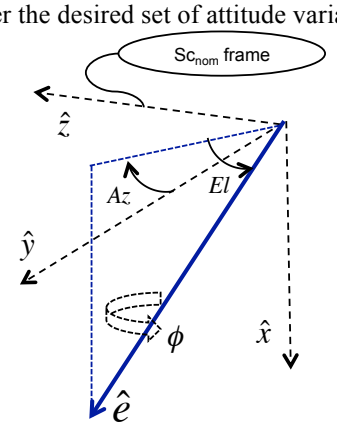

Figure 3: Rotation Axis and Angle Variation

Each rotation axis, $\hat{e}$ expressed in the SC_{nom}-frame is defined in terms of the azimuth, *Az* and elevation, *El* angle pair.

$$ {}^{SC_{nom}}\hat{e} = \begin{bmatrix} \sin(El) \\ \cos(El)\cdot\cos(Az) \\ \cos(El)\cdot\sin(Az) \end{bmatrix} \tag{3} $$

The *Az* is defined clockwise from $+\hat{y}$ axis. The +*El* is defined downward from the $\hat{y}\hat{z}$ plane. The set of $\hat{e}$ is obtained by varying *Az* from 0° to 360° and *El* from -90° to 90° in 2° intervals. Then the desired set of attitudes is completed by varying the rotation angle, ϕ from 0° to 360° in 2° intervals about each $\hat{e}$. The SC-frame variation relative to the SC_{nom}-frame is defined by

$$ {}^{SC}R^{SC_{nom}} = \begin{bmatrix} e_1^2(1-c\phi)+c\phi & e_1e_2(1-c\phi)+e_3s\phi & e_1e_3(1-c\phi)-e_2s\phi \\ e_2e_1(1-c\phi)-e_3s\phi & e_2^2(1-c\phi)+c\phi & e_2e_3(1-c\phi)+e_1s\phi \\ e_3e_1(1-c\phi)+e_2s\phi & e_3e_2(1-c\phi)-e_1s\phi & e_3^2(1-c\phi)+c\phi \end{bmatrix} \tag{4a}^2 $$

where $s\phi = \sin\phi, c\phi = \cos\phi, e_1 = e(1), e_2 = e(2), e_3 = e(3)$ and relative to the O-frame is defined by

$$ {}^{SC}R^{O} = {}^{SC}R^{SC_{nom}} \cdot {}^{SC_{nom}}R^{O} \tag{4b} $$

IV. Cross-sectional Area Model

In this study, the spacecraft is divided into six major elements as shown in Figure 4.

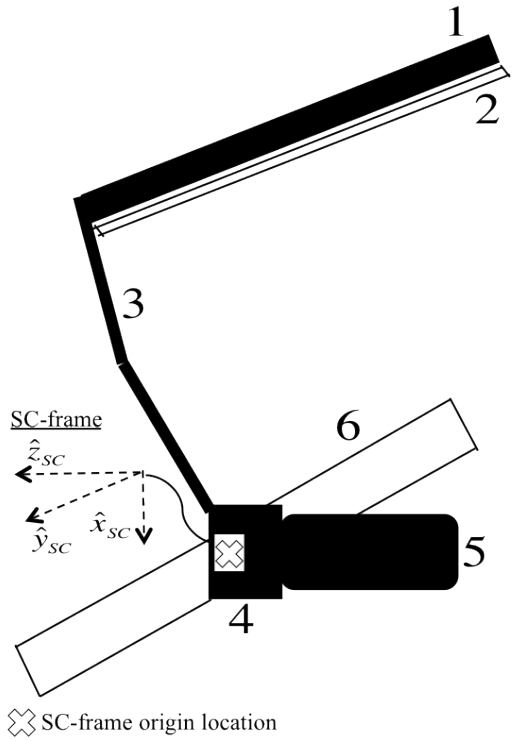

Figure 4: Spacecraft Components

Table 1 lists the elements and associated number of components.

Table 1: Spacecraft Element Model Type

	Element	Model Type	No. of Components
1	Reflector	Circular flat plate	2
2	Truss rods and nets	Rectangular plate	2
3	Boom	Cuboid	8
4	Spacecraft bus	Cuboid	6
5	Payload	Octagon	10
6	Solar array	Rectangular plate	4

The total number of components are 32. The total surface area, A_k, geometric center position vector, ${}^{SC}\vec{r}_{gc_k}$ relative to SC-frame origin, and unit normal, ${}^{SC}\hat{u}_k$ in SC-frame (where k = 1 to 32) for each component is determined from the spacecraft .nx models.

In order to compute the total cross-sectional area of the spacecraft, the area projected on the plane defined by $\hat{e}$ of the each individual component is considered. If ${}^{SC}R^{SC_{nom}} \cdot {}^{SC_{nom}}\hat{e} \bullet {}^{SC}\hat{u}_k > 0$ for the kth component, then the component surface area is

$$ A_{kc} = A_k \cdot \left({}^{SC}\hat{e} \bullet {}^{SC}\hat{u}_k \right) \tag{5a} $$

and the total spacecraft surface area becomes

$$ A = \sum_{k=1}^{32} A_{kc} \tag{5b} $$

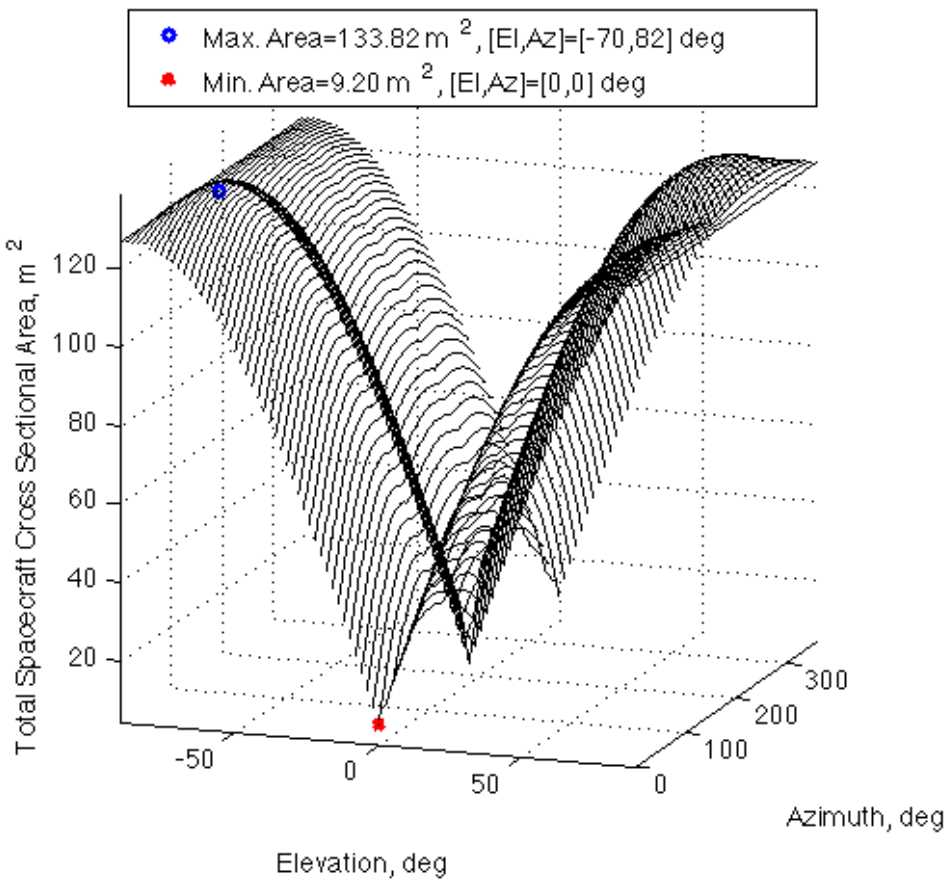

Figure 5: Total Spacecraft Cross-Sectional Area

The maximum area of 133.82 m^2 corresponds to $\hat{e}$ perpendicular to the reflector flat plate surface (Figure 4) and minimum area of 9.2 m^2 corresponds to $\hat{e}$ edge-on to the reflector flat plate surface as expected. In this computation, the shadowing effects are considered negligible. The maximum error in the cross-sectional area due to this assumption is 8 m^2 resulting in a torque error of only 0.0003 Nm. This error is insignificant during science operations.

V. Environmental Torque Model

The gravity gradient, atmospheric drag, and solar radiation pressure environmental torques imparted at the spacecraft center of mass are given by

$$ {}^{SC}\vec{\tau}_{gg} = \frac{3\mu_{earth}}{R_c^3} \cdot \left({}^{SC}R^{O} \cdot {}^{O}\hat{n}_{gd} \right) \times {}^{SC}I_{sc} \cdot \left({}^{SC}R^{O} \cdot {}^{O}\hat{n}_{gd} \right) \tag{6a} $$

$$ {}^{SC}\vec{\tau}_{drag} = \sum_{k=1}^{32} {}^{SC}\left(\vec{r}_{gc_k} - \vec{r}_{sc_{com}} \right) \times -\frac{1}{2}\rho C_d v_{sc}^2 A_{kc} \, {}^{SC}\hat{e} \tag{6b} $$

$$ {}^{SC}\vec{\tau}_{solar} = \sum_{k=1}^{32} {}^{SC}\left(\vec{r}_{gc_k} - \vec{r}_{sc_{com}} \right) \times C_f P A_{kc} \cdot {}^{SC}\hat{e} \tag{6c} $$

where ${}^{SC}R^{O}$ and ${}^{O}\hat{n}_{gd}$ are described in II, ${}^{SC}\hat{e}$ in III, and $\vec{r}_{gc_k}$ and A_{kc} in IV. The ${}^{SC}\hat{e}$ in Eq. (6b) is parallel to the inertial velocity direction, ${}^{SC}\hat{v}_{SC}$ and ${}^{SC}\hat{e}$ in Eq. (6c) is parallel to the sun direction, ${}^{SC}\hat{s}_{SC}$. The other parameters are described in Table 2 [3 and 4].

Table 2: Model Parameters

Parameters	
μ_{earth}	398600 km^3/s^2
R_c, geodetic altitude from Earth center	7106 km (mean)
${}^{SC}I_{SC}$, spacecraft moment of inertia	$\begin{bmatrix} 9061.1 & -3.6276 & -734.03 \\ -3.6276 & 13209 & -1.422 \\ -734.03 & -1.422 & 10572 \end{bmatrix} kgm^2$
${}^{SC}\vec{r}_{sc_{com}}$, spacecraft center of mass from SC-frame origin	[-0.384, 0, -1.71] m
ρ, atmospheric density at R_c	0.69 x 10^{-13} kg/m^3
C_d, coefficient of drag	2
${}^{SC}v$, orbital velocity magnitude	7.47 km/s (mean)
C_f, coefficient of reflectivity	2
P, solar pressure at 1 AU	4.5267 x 10^{-6} W/m^2

The torque model assumes that the center of pressure is located at the geometric center and the spacecraft is a perfect reflector (mirror) to yield a coefficient of reflectivity of 2.

VI. Results

The total gravity gradient torque imparted at the spacecraft center of mass for the desired attitude variations is given in Figure 6.

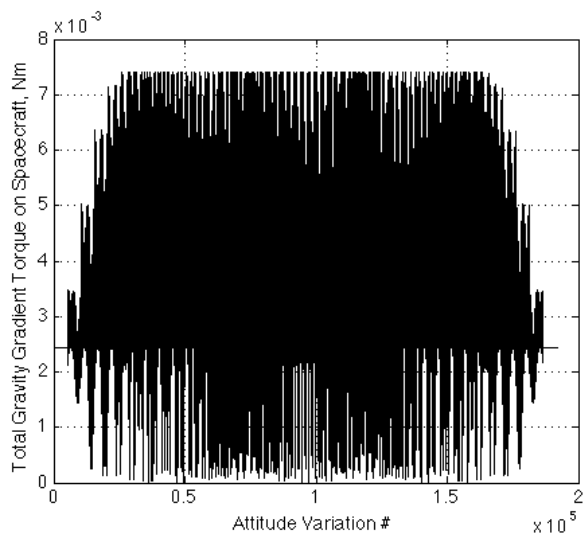

Figure 6: Total Gravity Gradient Torque Imparted on the Spacecraft

The maximum gravity gradient torque is 7.4x10^{-3} Nm and the minimum is 0.013x10^{-3} Nm. The attitude variation number (#) is a function of a [*El*, *Az*, ϕ] pair where *El* ranges from -90° to 90°, *Az* from 0° to 360°, and ϕ from 0° to 360° as described in III. The total drag and solar torques imparted at the spacecraft center of mass for $\hat{e}$, which represents the velocity and sun vector directions are given in Figure 7.

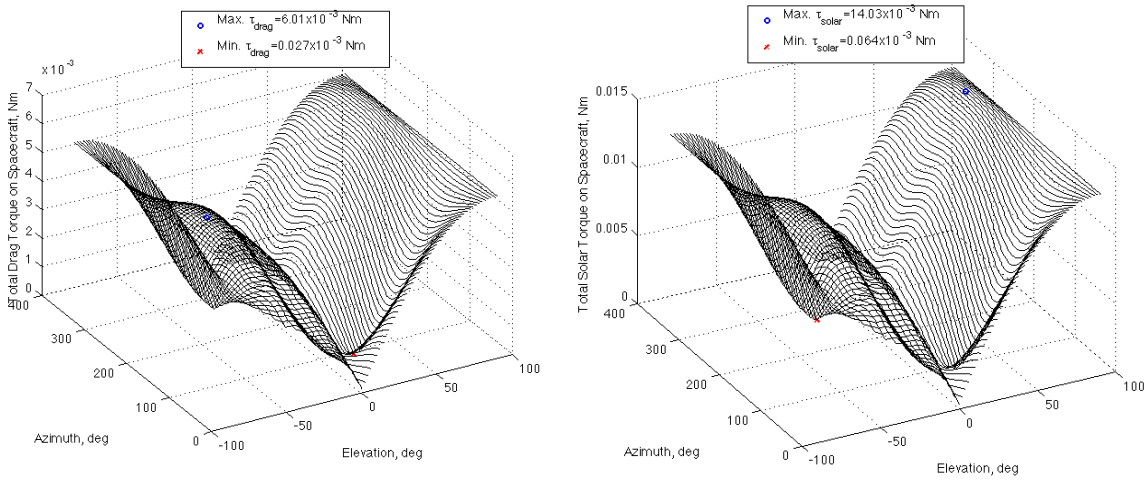

Figure 7: Total Drag and Solar Torques Imparted on the Spacecraft

The torque signature is similar to the total cross-sectional area curve in Figure 5 as expected. Table 3 provides a summary of the maximum and minimum torques imparted at the center of mass of the spacecraft expressed in the SC-frame.

Table 3: Environmental Torque Summary

	Minimum	**Maximum**
${}^{SC}\vec{\tau}_{gg}$, gravity gradient, Nm	0.013x10^{-3}	7.4x10^{-3}
${}^{SC}\vec{\tau}_{drag}$, atmospheric drag, Nm	0.027x10^{-3}	6.01x10^{-3}
${}^{SC}\vec{\tau}_{solar}$, solar radiation pressure, Nm	0.064x10^{-3}	14.03x10^{-3}

VII. Conclusion

In this paper, the gravity gradient, drag, and solar radiation pressure torques imparted on the NISAR spacecraft are evaluated. The spacecraft attitude variation model is used to determine the environmental torque bounds. The cross-sectional area model is used to compute the total spacecraft area projection on a plane represented by the velocity and sun vector directions. The worst case environmental torques are 7.4x10^{-3} Nm for gravity gradient, 6.01x10^{-3} Nm for drag, and 14.03x10^{-3} Nm for solar radiation pressure. These magnitudes can be compared with the magnetic torque rod capability in unloading the accumulated momentum.

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References

[1]Eisen, H., Jai, B., Rosen, P., Veilleux, L., and Xaypraseuth, P., "Conceptual Development of the DESDynl Mission," *Proceedings of the IEEE Aerospace Conference*, Big Sky, MT, 2012.

[2]Schaub, H., and Junkins, J., *Analytical Mechanics of Space Systems*, 2nd ed., AIAA Education Series, AIAA, Virginia, 2009, pp. 98, 188-191.

[3]Kinsey, R., Bacaloni, M., and Walterscheid, R., "Preliminary Aerodynamic Model of SMAP Spacecraft," Aerospace Corporation, 2010.

[4]Rizvi, F., "Solar Sail Coning Control to Induce Orbital Effects in Spinning Versus Non-Spinning Sails," *Proceedings of Third International Symposium on Solar Sailing*, edited by M. Macdonald, University of Strathclyde, Glasgow, 2013, pp. 737-754.