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DETERMINATION OF CERES PHYSICAL PARAMETERS USING RADIOMETRIC AND OPTICAL DATA

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The Dawn spacecraft was launched on September 27th, 2007. Its mission is to rendezvous with and observe the two largest bodies in the main asteroid belt, Vesta and Ceres. It has completed over a year's worth of direct observations of Vesta from early 2011 through late 2012. In the spring of 2015, the Dawn spacecraft entered orbit around the asteroid Ceres for the start of what is expected to be more than a year of science operations. The science data collected from this encounter consist of infrared (IR) images and spectra, visible images through a number of color filters, gamma ray detections and measurements of the Ceres gravity field. These data will be collected during several science phases: an Approach phase (1500000-4860 km from Ceres), a Survey orbit (4860 km radius), a High Altitude Mapping Orbit (HAMO) (1940 km radius) and a Low Altitude Mapping Orbit (LAMO) (855 km radius). The Approach phase included three Rotational Characterization (RC) opportunities.

Designing each science orbit and successfully transferring into that orbit requires a sufficiently accurate estimate of Ceres physical parameters (body fixed frame, GM and harmonics). This paper focuses on work performed to estimate Ceres physical parameters using Deep Space Network (DSN) radiometric tracking data and optical measurements derived from science camera imagery. This paper describes planning for the data acquisition, as well as processing techniques and methodology. The trajectories predicted by the gravity field estimations are also compared with the actual as-flown trajectories. Observations of the gravity at high altitudes are found to be sufficient to design precision orbits at lower altitudes. Follow-up analysis after successfully reaching LAMO is included, as is a discussion of lessons learned..

INTRODUCTION

The Dawn spacecraft was launched on September 27th, 2007 as the ninth mission of NASA's Discovery Program[1,2]. The primary mission is to rendezvous with and observe the two largest bodies in the asteroid belt, Vesta and Ceres, in hopes that they will yield insights into the formation of planetoids during the early eras of our solar system. The arrival at Vesta began in mid-2011, with the departure in late-2012. The rendezvous with Ceres started in early 2015 (see Figure 1) and the spacecraft is still in orbit as of January 2016.

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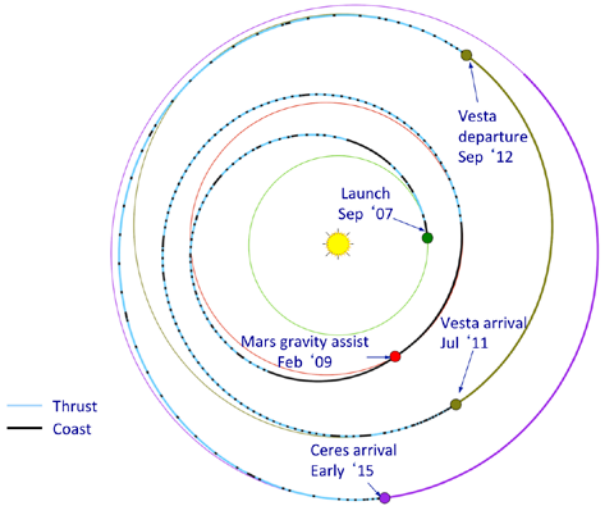

Figure 1: Heliocentric view of Dawn mission interplanetary trajectories spanning 2007 to 2015.

Spacecraft Overview

A Dawn spacecraft schematic is shown in Figure 2, along with the spacecraft body coordinate frame.

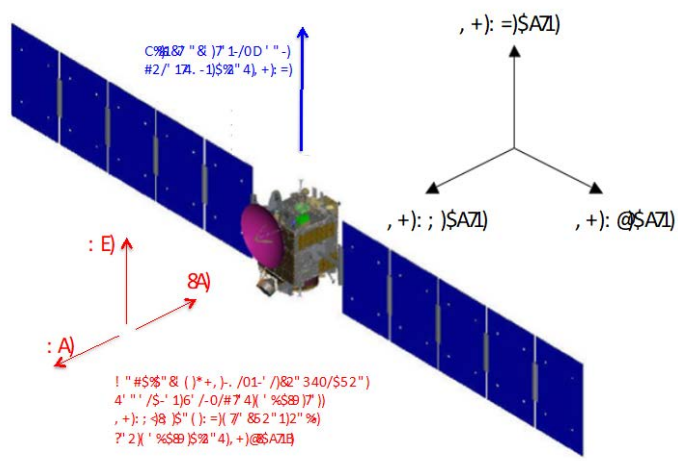

Figure 2. The Dawn spacecraft. All boresights for science instruments are aligned along the +Z-axis. RCS thrusters point in the –X, +X and +Z directions.

The spacecraft science suite is comprised of three instruments, all of which are aligned along the spacecraft +Z axis:

- The visible light Framing Camera (FC)
- The Visible and Infrared Spectrometer (VIR)
- The Gamma Ray and Neutron Detector (GRaND)

Propulsion is provided by one of three Ion Propulsion Subsystem (IPS) thrusters, each capable of producing 91 mN of thrust when 2.5 kW of excess power is available from the solar panels. However, at Ceres to Sun distances the available excess power is in the range of 0.6 to 1.0 kW, which only enables thrust magnitudes of order 25-30 mN. The IPS was used to match Dawn's heliocentric trajectory with Ceres' orbit around the sun. Once the spacecraft was captured by Ceres, the IPS was used to change orbit altitudes.

The spacecraft's Attitude Control Subsystem (ACS) provides three-axis stabilized control of Dawn during all nominal mission phases. Reaction Wheel Assemblies (RWAs) and a partially coupled hydrazine-fueled Reaction Control System (RCS) are available for attitude actuation. When three of the four RWAs are in use, momentum dumps are scheduled every 1 to 3 days.

Changes in ACS Operations

As of January 2016, only two of the four RWAs on the spacecraft are functional; one RWA failed in 2010 while on the way to Vesta and a second RWA failed in 2012 after departing Vesta. Three functional wheels are required for nominal RWA control at Vesta. Having only two functional wheels required development of a "Hybrid" ACS mode, which uses the remaining two wheels concurrently with the RCS to control attitude. This new mode is only in use during science operations at the lowest mission altitude (December 2015 and onward). During the entirety of the cruise to Ceres and the preceding LAMO operation, ACS controlled attitude in an "All-RCS" mode, in which only the RCS was used to control the attitude. In the event of the failure of one of the two remaining wheels, the remainder of science operations will be conducted in RCS mode.

As a result of the change in attitude control actuation, the fuel for the RCS system hydrazine became a key factor for the Ceres mission plan. This impacts operations by limiting the number of turns to Earth for high-rate data playback. Also, the resultant continual use of the RCS impacts the accuracy of the trajectory estimation.

Ceres Overview

The dwarf planet Ceres is located in the main asteroid belt between Mars and Jupiter. It orbits the Sun once every 4.60 years, has a rotational period of 9.07 hours, an average radius of 470 km and an estimated GM of 62.63 km^3/s^2. For this GM and the above orbit radii, the Dawn science orbit periods were of order 3.11 days, 18.87 hours and 5.47 hours for Survey, HAMO and LAMO, respectively. The period for LAMO (see Figure 3) places the Dawn spacecraft inside the Ceres 1:1 orbit period/rotation resonance.

Science Overview

The science plan[1,2] involves the acquisition and transmission of these types of scientific data:

- Visible imagery
- Infrared spectroscopy
- Gamma ray and emitted neutron counts
- Gravity science observations

The mission is divided into several science phases, listed in Table 1. Figure 3 shows the four science phases that consist of at least one full polar orbit around Ceres. Between each of these science phases was an orbit transfer phase, during which time the orbit altitude was lowered using the thrust from the IPS. These transfers took several weeks, with the IPS operating over 90% of that time. The Navigation team designed each orbit and its transfer, using the latest available data from the previous mission phase to refine the models of the Ceres gravity and orientation.

Table 1: Ceres Science Phases.

Phase	Distance from Ceres (km)	Orbit period	Dates	ACS mode	# Of orbits
Approach	1500000 to 4860	N/A	Dec, 2014 to Mar, 2015	All-RCS	-
RC1	87000	N/A	Feb 12th, 2015	All-RCS	-
RC2	49000	N/A	Feb 19th, 2015	All-RCS	-
RC3	14100	~14 days	Apr 23rd – May 9th, 2015	All-RCS	~1
Survey	4860	~3 days	June 3rd, 2015 – July 15st, 2015	All-RCS	8
High Altitude Mapping Orbit (HAMO)	1940	~19 hours	August 13th, 2015 – October 23rd, 2015	All-RCS	80
Low Altitude Mapping Orbit (LAMO)	855	~5 hours	December 7th, 2015, - EOM	Hybrid	400+

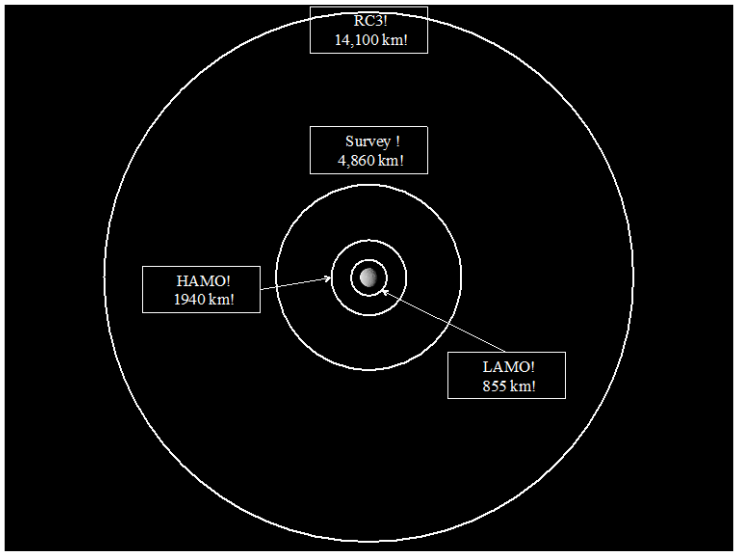

Figure 3: The four science orbits for the Ceres mission (all polar).

NAVIGATION

The Dawn Mission is supported by the Dawn Navigation team, comprised of the Mission and Maneuver Design (MD) team, the Orbit Determination (OD) team and the Optical Navigation (Opnav) team.

Each of the science phases listed in Table 1 is performed while in a polar, circular orbit that was designed to meet science requirements of orbit period, groundtrack spacing, altitude, inclination and sun beta angles. The design of each orbit is developed by the MD team. The MD team needs an estimation of the Ceres body fixed frame orientation, Ceres' mass parameter (GM) and gravity harmonics field in order to design the reference orbit for each science phase. The OD team obtains estimates of these Ceres physical parameters using radiometric data (Doppler and range) from the Deep Space Network (DSN) and optical data processed by the Opnav team. The DSN provides approximately 16 hour/day of coverage during all science phases, and 16 hours/week during low thrust transfers between phases. The Opnav team provides the optical data, which is reduced from science images.

Figure 4 shows a mosaic of images of Ceres taken from the Dawn framing camera. The Opnav team uses a process called stereo photoclinometry (SPC) to process all available images to construct a shape model of Ceres and a database of landmarks[3]. Each individual image is then registered against the landmarks to create a multitude of vectors, which can then be used in the OD process. The increased resolution at each successively lower orbit allows for a denser grid of landmarks to be generated. For RC1, RC2, RC3, Survey, HAMO and LAMO the number of landmarks in the grid was 158, 384, 849, 3714, 17866 and 86628, respectively. Further information on the fitting of the optical data, including figures showing a grid layout, can be found in Kennedy et. al[4].

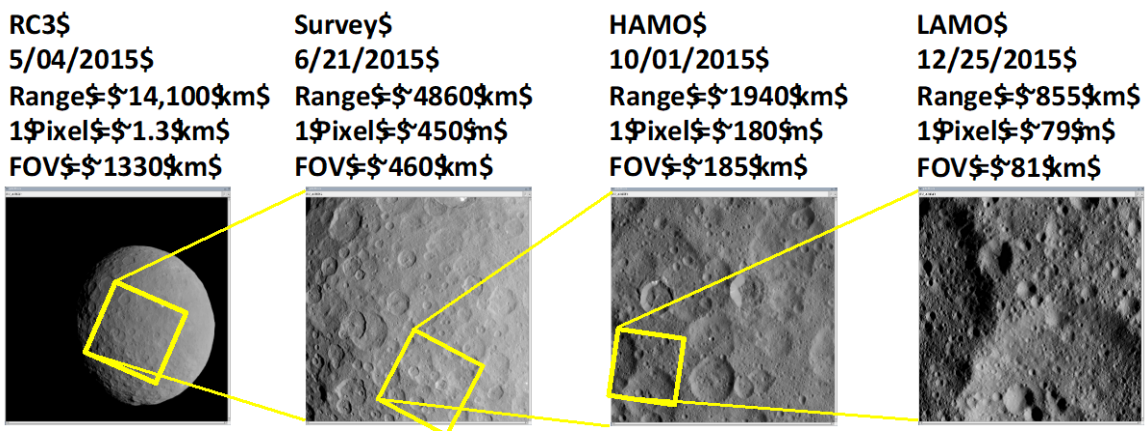

Figure 4: Images of Ceres using Dawn's Framing Camera at 4 resolutions.

Given the radiometric and Opnav data, the OD team performs a least-squares fit to minimize the sum-of-squares between the observed and predicted values of the data. The parameters adjusted in the fit include, in addition to the epoch state (position and velocity) of the spacecraft, various model parameters which describe the trajectory and the observation data. The fit is performed with JPL's MONTE institutional software, using a batch SRIF (square root information filter) method. Additional description of the Dawn OD processes can be found in Kennedy et al.[4,5] and Abrahamson et al[6].

The dynamic force parameters in the filter include non-gravitational effects, such as solar radiation pressure and thrust from the RCS and IPS. All these forces are estimated in the filter. The gravitational forces include the central body GM (Ceres in this case), the GMs of all the planets and sun, and the non-sphericity of the central body. Of these, only the central body GM and non-spherical gravity are estimated, with the gravity field modeled using the standard spherical harmonic representation described by Kaula[7]. Modeling and estimating the non-uniformity of the central body gravity requires knowledge of the body's pole orientation, prime meridian, and spin rate. The estimation of these parameters for Ceres and how it affects the OD and maneuver design process is the central focus of this paper. Note that Ceres is in principal axis rotation, so the pole and spin rate are assumed to be fixed through the time of interest.

A complicating factor in estimating Ceres' physical parameters are the thrust forces from the RCS and IPS. The RCS thruster firings can number several hundred per day, with pulses averaging 20-200 microns/second (~0.0003 mm/s^2). The IPS delivers 2.7 meters/second each day (0.03 mm/s^2). The acceleration due to the Ceres gravity ranges from 0.3 mm/s^2 in RC3 up to 50 mm/s^2 in LAMO, but the more subtle gravitational accelerations due to the non-uniform gravity field of Ceres are much less than 1% of the bulk gravity and these can be comparable to the effects of the RCS and the IPS.

Complementary Data Types

Both radiometric and optical data types are necessary to realize the complete set of Ceres parameters. The optical data provide insight into the Ceres GM, pole orientation, prime meridian and rotation rate, but practically no information on the Ceres harmonics. Radiometric data provides insight into Ceres harmonics and GM, but almost no appreciable information on the Ceres pole, meridian or rate until HAMO and LAMO altitude. Even at those altitudes, optical data still provides more information about the orientation than radiometric data.

The ability of radiometric data to resolve pole orientation is based on how well the zonal components in the harmonics field are resolved. The meridian and rate can also be sensed when the sectoral and tesseral components resolve. Dawn OD uses the optical data to establish the orientation both for targeting the science orbits and to provide a more accurate *a priori* model of the orientation of the spherical harmonics. See Table 2 for a comparison of Ceres orientation confidence between radiometric solutions and merged radio/optical solutions. In Table 2, the uncertainties at LAMO are higher than for HAMO because they are based on only two weeks of available data.

Table 2: Ceres frame knowledge when using only radiometric data vs. using radiometric and optical data. Knowledge of meridian at epoch can be inferred from rate knowledge.

	Pole R.A uncertainty (º)		Pole Dec. uncertainty (º)		Rate uncertainty (º/year)	
Phase	Radio only	Merged	Radio only	Merged	Radio only	Merged
RC3	N/A	0.007	N/A	0.004	N/A	0.08
Survey	1.0	0.002	0.06	0.0016	30	0.04
HAMO	0.24	0.0007	0.012	0.0003	1.5	0.011
LAMO(2 wks.)	0.02	0.0016	0.004	0.0004	2.0	0.07

The ability of optical data to resolve the GM is based on the angular information available in the direction vectors that comprise the optical data. In Kennedy[4] it was reported that the Vesta GM values from OD solutions containing both optical and radio data had statistically significant differences from the Vesta GM values from OD solutions containing just radio data. Part of this discrepancy is due to the treatment of the landmark database as truth, resulting in an optimistic covariance. Also, if the initial landmark database is wrong due to errors in the establishing spacecraft trajectory, then these errors reflect themselves in later solutions as an incorrect GM. By estimating either the locations of individual landmarks, or estimating a radial scaling factor on all vectors in the database, the resulting GM uncertainty is more relaxed and the value better reflects a combination of radiometric and optical data. For the Ceres campaign, estimation of a landmark scale factor is regularly performed to identify potential scale factor errors in the landmarks database.

PARAMETER PREDICTIONS

The OD team is responsible for providing the MD team with a prediction of Ceres parameters for the design of four science phases. The parameters for Ceres include the Ceres body frame (pole orientation, meridian, rate), the Ceres GM and the spherical harmonics coefficients that represent the Ceres gravity field. Table 3 shows the delivery date, and data arc used for the design each phase.

Table 3: Parameter deliveries for Science orbit design

Phase being designed	Delivery Date of estimates	Data arc
RC3	March 18th, 2015	Dec 6th, 2014 – Mar 18th, 2015
Survey	May 5th, 2015	Feb 19th, 2015 – May 5th, 2015
HAMO	July 9th, 2015	June 3rd, 2015 – July 9th, 2015
LAMO	October 1st, 2015	Aug 13th, 2015 – Oct 1st, 2015

In preparation for Ceres operations, the OD team performed numerous covariance studies in order to determine how effective the OD team would be in determining the actual Ceres parameters at these points in the mission. An OD covariance study takes a simulation of the data in the expected data arc and runs it through the same SRIF used in operations. The formal uncertainties for the parameters and the correlations between those parameters are then exported as a full covariance to the MD team, who use it to seed a Monte Carlo (MC) study of the low-thrust transfer into the next orbit[8]. In order to demonstrate margin against unexpected parameter changes, these full covariances are inflated by a factor of five before being used in the MD studies. This factor of five was based on study of past performance at Vesta[5] and the engineering judgment of the MD and OD team leads.

Sizing the Fields Using Kaula's Power Law

Determining the appropriate size of the harmonics field for the MD covariances relies on an application of Kaula's power law[9]. Kaula derived this power law to describe the shape and magnitude of the power spectrum for the spherical harmonics coefficients that model the Earth's gravity field. For the Earth, the expected value for a parameter of degree-N is $1e\text{-}5/N^2$. The expected power spectrum for spherical non-Earth bodies can be derived from the Earth's power spectrum by applying a scale factor, based on body mass and radius. The value of this scale factor for

Ceres is 0.0086 (Konopliv et al.[10]). If the results of a covariance study show that the formal uncertainties for the coefficients of degree-N are meaningfully better than $0.0086/N^2$, then the OD team is justified in estimating a field up to that degree. Based on covariance studies before the encounter with Ceres, the Dawn OD team expected to see only GM in RC3, up to degree/order 3 in Survey, up to degree/order 5 in HAMO, and up to degree/order 10 in LAMO.

Parameter Prediction Performance

The effectiveness of the predictions can be gauged by comparing them with the current best estimates using LAMO data up to 12/31/15, shown in Table 4. Delivery values of Ceres parameters for each science orbit design are listed in Table 5. Also listed are the formal uncertainties based on the delivery solutions, the assumed uncertainties used for the covariance studies and the statistical significance of the error. The assumed uncertainties are not the formal sigma from a filter run, but the 5x inflated value that reflects expected errors, as described earlier. For brevity, C, S and higher-order J parameters are not listed in Tables 4 or 5.

Table 4: Current Best Estimates (CBEs) of key parameters (LAMO data only, up to 12/31/15).

Parameter	Value (radio-only)	Value (radio+optical)
GM	62.63 ± 0.12 km^3/s^2	62.6307 ± 0.0009 km^3/s^2
Ceres Pole Right Ascension	291.44 ± 0.02º	291.419 ± 0.002º
Ceres Pole Declination	66.762 ± 0.004º	66.7616 ± 0.0004º
J[2]	1.15114e-2 ± 7e-7	1.15101 ± 3e-7
J[3]	-3.96e-5 ± 2e-7	-3.95e-5 ± 2e-7
J[4]	-5.408e-4 ± 3e-7	-5.404e-4 ± 3e-7
J[5]	3e-7 ± 3e-7	1e-7 ± 3e-7
J[6]	2.07e-5 ± 4e-7	2.10e-5 ± 5e-7

In Table 5, the radio solutions show a poor confidence in GM. This statistical uncertainty is mainly due to the presence of numerous RCS pulses (several hundred per day), and the presence of a stochastic error model to estimate the pulse-to-pulse variation. If the spacecraft still had use of three RWAs (see earlier section on changes in ACS actuation) the resulting absence of the frequent thruster pulses would allow a radiometric solution to claim an improved GM uncertainty of 0.02 km^3/s^2.

Table 5: OD prediction performance for Ceres parameters. Bolded values are significant prediction errors that are discussed in the text.

Parameter	Predicted Value (radio+optical)	Error vs. CBE (radio-only)	Error vs. CBE (radio+optical)	Assumed uncertainty for MC runs	Ratio of Error to Assumed uncertainty
For RC3 Design (based on RC1 and RC2 data)					
GM (km^3/s^2)	62.640 ± 0.013	-0.01	-0.0093	0.018	-0.52
Pole R.A. (º)	291.73 ± 0.17	-0.29	-0.311	0.54	-0.58
Pole Dec. (º)	66.97 ± 0.14	-0.208	-0.2084	0.29	-0.72
For Survey Design (based on RC3 data)					
GM (km^3/s^2)	62.6287 ± 0.0008	+0.0013	+0.002	0.08	0.03
Pole R.A. (º)	291.646 ± 0.007	-0.206	**-0.227**	0.05	**-4.54**
Pole Dec. (º)	66.762 ± 0.004	+0.000	+0.0004	0.03	0.01
J2	7.6e-3 ± 9e-4	+3.9114e-3	+3.9101e-3	6e-3	0.65
J3	0.0 ± 3e-4	-3.96e-5	-3.95e-5	3.2e-2	-0.00
For HAMO Design (based on Survey data)					
GM (km^3/s^2)	62.63877 ± 0.00005	-0.00877	-0.00807	0.006	-1.345
Pole R.A. (º)	291.416 ± 0.002	-0.024	-0.003	0.006	-0.50
Pole Dec. (º)	66.7613 ± 0.0016	+0.0007	+0.0003	0.003	0.10
J2	1.156e-2 ± 4e-5	-4.86e-5	-4.99e-5	6e-4	0.08
J3	-3e-5 ± 1.1e-4	-9.6e-6	-9.5e-6	8e-4	-0.01
J4	-2.7e-4 ± 4.8e-4	-2.708e-4	-2.704e-4	7e-4	-0.39
For LAMO Design (based on HAMO data)					
GM (km^3/s^2)	62.63197 ± 0.00015	-0.00197	-0.00127	0.02	0.06
Pole R.A. (º)	291.4157 ± 0.0007	-0.0243	**-0.0033**	0.0009	**3.67**
Pole Dec. (º)	66.7563 ± 0.0003	+0.0057	**+0.0053**	0.0004	**13.25**
J2	1.1506e-2 ± 1e-6	+5.4e-6	+4.1e-6	8e-6	0.51
J3	-4.96e-5 ± 1.4e-6	+1e-5	+1.01e-5	9e-6	1.12
J4	-5.25e-4 ± 3e-6	-1.58e-5	-1.54e-5	3e-5	0.51
J5	2e-5 ± 1e-5	-1.97e-5	-1.99e-5	1e-4	0.20

Also in Table 5, the radio solutions show that the pole can be determined with high accuracy, even without the benefit of optical data, given a sufficiently low altitude. This high accuracy is due to the sensing of the zonal spherical harmonics, which allow observations of a spacecraft in a

polar orbit to determine the orientation of the zonal field, and therefore of the pole of the orbited body. Radio solutions did not begin to sense the pole beyond the apriori uncertainty until the HAMO altitude, but the formal confidence was proportionally weaker than when optical data is used. In LAMO, the formal pole knowledge using optical data is ten times better than the knowledge using radio-only data, but when using only HAMO data the optical knowledge is over 100 times better than the radio knowledge of the pole. At Vesta the pole estimation knowledge using radio data was stronger, due to the lower LAMO altitude[4]. It was also noticed that the pole knowledge using optical data was overly optimistic, based on the sheer volume of optical data being processed[4]. This optimistic pole estimation remains true for the Ceres mission. It was partly due to this optimistic filter knowledge that the Ceres MC studies used inflated covariances to provide margin against statistically significant errors in parameter estimation[8].

The harmonics values in Table 5 are determined almost entirely by the radio data. Covariance studies using optical data show that J2 cannot practically be sensed beyond its apriori uncertainty by using only optical data, in any of the Dawn science orbits. However, when radio data and optical data are used in a merged solution, their complementary observations allow the filter to express a more confident J2 solution. GM values in the merged solutions also show a greater confidence, due to the camera's 5º field of view and a well-established landmark database, which strongly imply an observation of distance to Ceres. Dawn has no altimeter or other instrument capable of measuring distance, so these optical data are the only way the filter gets a direct sense of the distance between Dawn and Ceres, allowing a GM to be better established by having a better fix on the orbit altitude. However, since the landmark database is not "truth" in a strict sense, and is modeled from earlier (possibly incorrect) trajectories, the GM observations are strongly influenced by those earlier trajectories. Taking images at off-nadir angles adds geometric strength to the observations, which can mitigate the effect of an ambiguous trajectory aliasing in the landmark database. At Vesta[5] it was observed that radio-only and merged solutions produced significantly different values for GM. This discrepancy has been partially mitigated in Ceres operations by performing more iterations between the Opnav and OD teams, in which the landmarks and optical data are re-processed by the Opnav team using a trajectories from the OD team that fit an earlier version of the optical data. Estimation of a single scaling factor on the entire landmarks database was also performed. In practice, it was found that iterating between Opnav and OD teams were most effective in reconciling GM values. Still, statistically significant differences between GM values in radio and merged solutions remain.

For most of the parameters listed in Table 5, the delivered values did not exceed the CBE values by more than the allowed MC uncertainties. The only exceptions in Table 5 were some of the Pole estimates, which sometimes fluctuated at statistically significant levels. Even the five-fold inflation of the formal filtered uncertainties did not always encompass this level of error. For example, the error in the pole declination predicted for the LAMO orbit was over ten times higher than that allowed for in the MC studies, which amounts to a fifty-fold deviation from the formal uncertainty. Fortunately, the mission requirements on pole knowledge for LAMO (primarily having to do with beta angle drift) were not as stringent as the MC allowances, and primarily have to do with beta angle drift. The beta angle is the angle between the orbit plane and the vector from Ceres to the Sun. Low beta angles result in solar occulations. The 0.0053º error realized in the reference orbit results in a beta angle prediction error of less than 0.1º after spending one full year in the LAMO orbit, so the error in inclination prediction is not an issue.

Not listed in the Table 5 are the C and S coefficients. Among those, no coefficients used in the design of Survey exceeded their expected error assumed for the MD Monte Carlo study. For the design of HAMO, only S4,3 and C4,4 exceeded their expected errors, with S4,3 exceeding by more than two. During the transfer to LAMO, only C5,5 and S5,5 exceeded their expected error.

Although these parameters did not violate the Monte Carlo's three-sigma covariance (three times the errors allowed for by the MD and OD team), these were more than five sigma errors for the OD estimation (more than ten sigma in the case of S4,3). When performing OD to estimate a gravity field on relatively short timeline, statistically significant errors in estimation such as these should be expected, and accounted for in the mission plan. For Dawn, these were accounted for by demonstrating successful Monte Carlo studies based on five times the formal OD covariance[8].

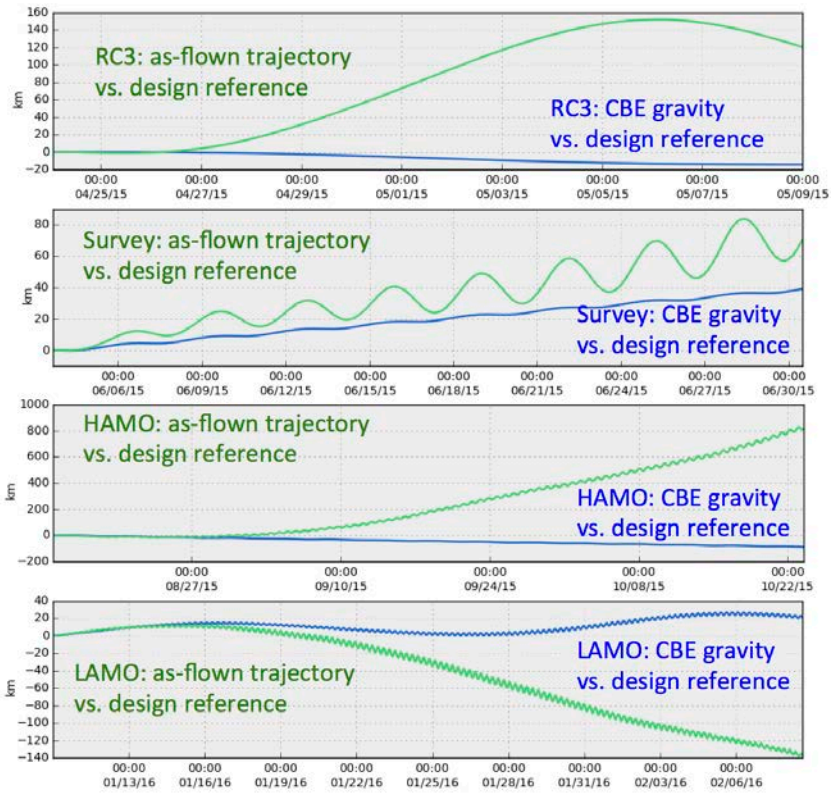

Figure 5: Trajectory differences for RC3, Survey, HAMO and LAMO phases. CBE gravity propagation (blue) and trajectory reconstruction with RCS activity and gravity (green) shown vs. reference design trajectory.

Another metric of the performance of the predictions is shown in Figure 5, which shows a comparison of trajectory propagations through different Ceres gravity fields. Trajectories for each science orbit were propagated through the CBE gravity field for Ceres, as of 12/31/15. Trajectories were also propagated through the reference fields at the same orbits. The blue traces in Figure 5 show the difference between reference design trajectory and a trajectory propagated through the CBE gravity field, at each science altitude. Agreement is quite good, showing that errors in gravity prediction had minimal effect on orbit phasing. As a point of comparison, the reconstructions of the as-flown trajectories were also compared to the reference; these are the green traces in the same Figure 5. The reconstructions reflect the impact of RCS activity, which is not modeled in the design of the reference orbit. The large discrepancies show that the unmodeled thruster firing was a much larger contributor to downtrack error than gravity prediction errors in RC3, HAMO and LAMO. Gravity errors and RCS activity appear to be equivalent error sources for Survey, with neither one being substantial; their net impact caused a 7 minute phasing error after one month in Survey. Since LAMO is still in progress as of December 2015 and had only recently begun, the LAMO reconstruction is actually not a reconstruction, but is instead a predict-

ed trajectory that models future predicted RCS activity. The resultant growth in phasing error (in degrees/day) is shown in Table 6 for all four comparisons.

Table 6: Trajectory downtrack errors.

Phase	Span	Error due to Gravity (°/day)	Error due to Gravity and RCS perturbation (°/day)
RC3	April 24th – May 9th, 2015	0.004	0.044
Survey	June 3rd – June 30th, 2015	0.016	0.035
HAMO	August 14th - October 23rd, 2015	0.038	0.358
LAMO	January 10th – February 8th, 2015	0.065	0.332

Estimation of Gravity During Low Thrust

Between science phases, the Dawn spacecraft was in a transfer orbit designed to insert the spacecraft into the next science orbit. During that period, the IPS was almost continuously in operation, and direct spacecraft observations were limited to one high-gain and one low-gain DSN tracking pass per week. During the low-gain pass the spacecraft IPS remained in operation and high-fidelity telemetry of the RCS activity was unavailable. This mode of operation was a trade to conserve hydrazine by limiting the overall duration of the transfer orbit and minimizing the number of attitude slews, at the expense of a higher quantity and quality of radiometric tracking data.

Determining gravity with the limited observations, at a time when the gravity field resolution was continuously increasing, was a challenge to the OD team. After fitting each tracking pass, the OD team provided the MD team with an updated gravity field that was used to design the IPS profile for the following week. Between HAMO and LAMO, there were seven deterministic thrust designs plus one TCM, designated segments 1 through 8. A comparison of these gravity estimations was performed by propagating a LAMO trajectory through each field and comparing the differences in the trajectories. Figure 6 shows this comparison, with the reference being a trajectory propagated through the CBE gravity field.

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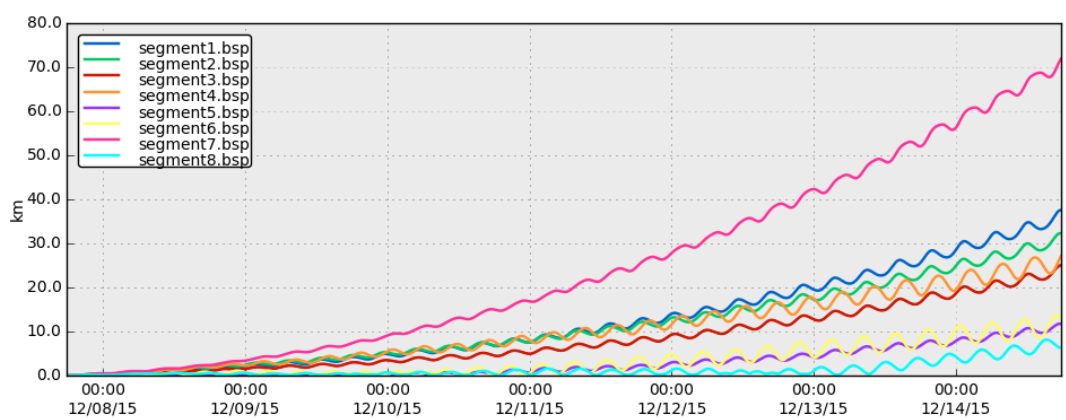

Figure 6: Comparison of trajectories that model gravity fields observed during the HAMO to LAMO transfer.

Figure 6 shows that the first six gravity deliveries for the transfer into LAMO estimated consistent gravity fields. The field delivered for the design of the thrust for segment 7 (the pink line in Figure 6), shows an outlier performance that was later found to be caused by underestimation of the gravity field; the OD team was still estimating only a 6-degree/order field at an altitude where degrees seven and higher were beginning to assert themselves. The OD solutions for the TCM design (segment 8) were expanded to estimate an 8-degree/order field, and this resulted in a better prediction that showed the best agreement with the Current Best Estimate (CBE) gravity field, out of all the fields estimated during the transfer.

At the end of the transfer into LAMO, analysis showed that failure to estimate spherical harmonic coefficients higher than degree/order-6 had resulted in the gravity signal going almost entirely into the estimations for parameters C5,5 and S5,5. These values shifted enough to create a field estimate that did not predict as well as the other field estimates.

Seeing the Effects of Individual Coefficients

Further investigation was merited, and OD analysts looked at the gravity field elements inside of the State Transition Matrix (STM). These elements are partial derivatives that mathematically describe the sensitivity of the spacecraft position to each coefficient in the gravity field. The partial derivatives are propagated along with the spacecraft trajectory, and their values change over the course of the mission. These changing values over time were combined to create a heat map, seen here as Figure 7. The colors in the figure correspond to the relative sensitivity of downtrack position to gravity field harmonics on a logarithmic scale to show the "driving" parameters in the field while still preserving subtle structures among the other parameters. From top to bottom, coefficients of degree-2 through degree-8 are shown. The value of the partial derivatives for each degree is multiplied by the Kaula power rule for Ceres[10] to allow for the expected lower impact of higher order terms. In Figure 7, time increases from left to right, and the partial derivative values change as the trajectory drops Dawn lower in altitude. The evolving values are seen during the transfers into Survey, HAMO and LAMO. For brevity and clarity, the actual time spent in Survey and HAMO is not shown in Figure 7. Many interesting artifacts can be seen in this plot, such as the signature of various resonances, which can be seen as temporary increases in sensitivity as Dawn descends. Also, the similarity of C and S parameters of a given degree and order can be seen. Since C and S harmonics are differentiated only by the arbitrary selection of the prime meridian, this similarity is not surprising. Most striking in Figure 7 are the relative strength of the partial derivative values for C5,5; S5,5; C7,5 and S 7,5. These terms begin to dominate the downtrack position sensitivity to the gravity field just as Dawn reaches the LAMO altitude, which is very close to the point in the transfer where estimating only a degree/order-6 field was found to be insufficient in modeling the effects of the Ceres gravity.

Also apparent in Figure 7 is the strength of the partial derivatives for C4,3 and S4,3 as the trajectory nears the LAMO altitude. These two coefficients had earlier been identified by the MD team as causing large inclination changes when nearing the end of the transfer from HAMO to LAMO. During some samples of their Monte Carlo run, large changes to C4,3 or S4,3 were occasionally sampled from the truth covariance, and if large enough they were found to cause large inclination changes in the last week of the transfer. Clearly, analysis of the STM in this fashion is useful in identifying the relative importance of coefficients when planning a transfer to a lower orbit. Those parameters which have the most impact on the trajectory should be the focus of additional analysis.

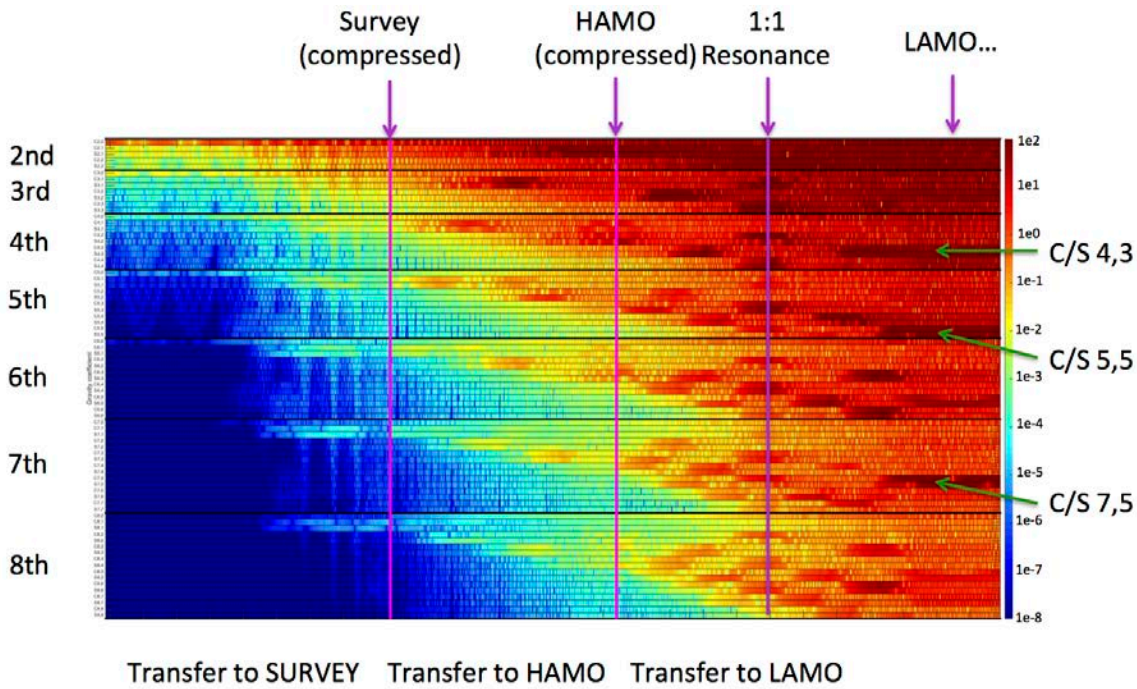

Figure 7: Heat map showing the relative sensitivity of Dawn's downtrack position to specific coefficients of the Ceres gravity field, over the course of the Ceres mission.

The observations about sensitivity to C4,3, S4,3, C5,5 and S5,5 were corroborated by a study of Kaula's form[7] of the Lagrange's Planetary Equations (LPEs). These equations may be used to compute the linearized sensitivity of classical orbital elements to specific gravitational harmonics. For brevity, a single equation (for semi-major axis) is presented here as Equation 1. Here, F is a function of inclination, G is a function of eccentricity, S is a combination of the C and S harmonics terms and l, m, p and q are indices in the infinite summation that represents the gravity field potential. This equation and the ones for the other conics have a common denominator, presented here as a standalone term in Equation 2. The value of Equation 2 was evaluated over a 10x10 field, assuming a frozen orbit with no beta angle change and a fixed Ceres rotation rate, and plotted on a logarithmic scale over Dawn's orbital period (decreasing let to right) in Figure 8. As the value shown in Equation 2 goes to zero, the effect on the orbital element increases, since the term appears in the denominator of the LPEs. In Figure 8, the orbital resonances (most notably the 1:1 resonance, in magenta) are seen as downward spikes, where the denominator is driven to zero for various combinations (l, m, p, q) of the gravity field of Ceres. The LAMO altitude is indicated as the cyan line, which is very near the 5:3 resonance. The perturbing effects of the 5,5 and 7,5 terms (and others) are noted. Also highlighted in Figure 8 is the resonance where the 4,3 terms are perturbing. Figure 8 provides analytical support to the relative sensitivity of Dawn's downtrack position to the (5,5) and (7,5) harmonic terms. Since the LAMO orbit period is quite near values at which the (5,5) and (7,5) terms cause the denominator of the LPEs to approach zero, the relative change in orbital elements due to those harmonics is significantly higher than other terms are at that specific altitude.

$$\Delta a_{lmpq} = \mu R^l \frac{2F_{lmp}G_{lpq}(l-2p+q)S_{lmpq}}{na^{l+2}\left[(l-2p)\dot{\omega} + (l-2p+q)\dot{M} + m(\dot{\Omega}-\dot{\theta})\right]} \qquad (1)$$

$$(l-2p)\dot{\omega} + (l-2p+q)\dot{M} + m(\dot{\Omega} - \dot{\theta}) \tag{2}$$

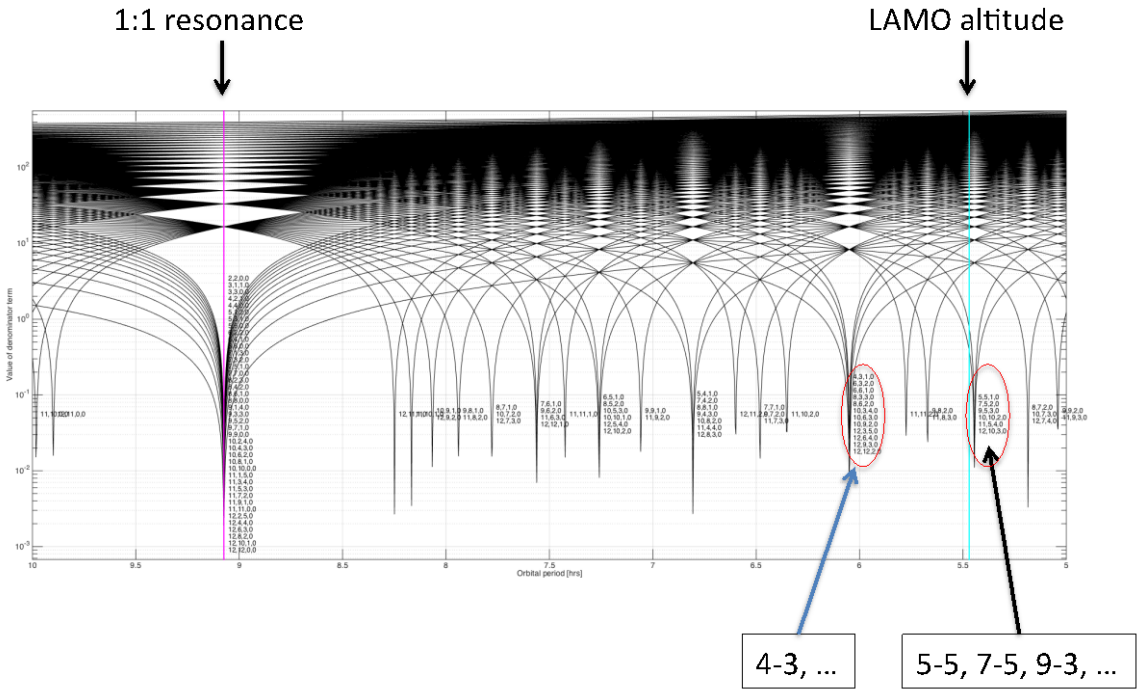

Figure 8: Plot of modified LPEs, showing sensitivity of conic elements to individual gravity field parameters.

CONCLUSIONS

As of January 2016, all of the Ceres science orbits have been entered into successfully, and the mission requirements for science data collection have so far been met.

At Ceres, observations of the gravity at high altitudes are sufficient to design precision orbits at lower altitudes. In fact, the gravity field determined in HAMO was so close to the best estimates of the field in LAMO that the reference LAMO trajectory is not in need of a re-design. Observations of the spacecraft over several weeks in a polar, circular orbit have enabled the continued use of the original reference. The only significant errors are due to RCS activity, which can be corrected with the planned OMMs. Obtaining an accurate estimate of the Ceres pole orientation was successful to a degree that satisfied mission requirements, but the covariances for the preceding pole solutions were very optimistic. When considering the impact of pole estimation on mission design, allow for larger covariances, possibly up to a factor of ten times the formal sigma. Similarly, when considering GM and gravity field uncertainties in the design of a mission, the formal OD covariances should be increased by at least a factor of five for the Monte Carlo analyses.

When descending into the Ceres gravity well during the last orbit transfer, OD efforts to resolve gravity quickly with the limited amounts tracking data were adequate, but in one case the estimated gravity field was out of family. Taking more time to collect tracking data, or tracking while the IPS is not perturbing the spacecraft would likely have helped with estimating a more

accurate field. Also, estimating a field with a higher degree was shown to have produced a more in accurate result with the data collected at the point in the transfer. The improved resolution of the field occurred earlier than expected by earlier covariance studies based on Kaula's power law[7], so in practice one should estimate at least one to two orders higher than covariance suggest one can confidently observe.

We did not use the STM analysis to identify which specific parameters in the field have the most impact on the trajectory until after reaching LAMO. Analysis of this type would have been useful earlier in the mission planning stages to identify specific coefficients that would have large impact on the trajectory at critical times. Using Kaula's form of the LPEs[7] to identify perturbing parameters at the various resonances also proved useful, as this was a more analytical approach.

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