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DYNAMICS OF ASTEROID 2006 RH120: PRE-CAPTURE AND ESCAPE PHASES

Brian D. Anderson[*] and Martin W. Lo[†]

Asteroid 2006 RH120 was the first natural object captured by the Earth to be observed called a Minimoon. In this work, we show that the invariant manifolds of the orbits around the L_1 and L_2 Lagrange points play a significant role in the capture of the asteroid around Earth and its eventual escape from the Earth approximately 1 year later. This is similar to the Temporary Capture of comets around Jupiter. We determined that the asteroid was in a 27:29 mean motion resonance with the Earth and approached the Earth through the stable manifold of an L_1 Northern Halo Orbit. After the Temporary Capture, the asteroid escaped the Earth through the unstable manifold of an L_2 Southern Halo orbit and into a 21:20 resonant orbit with the Earth. The asteroid travelled through a series of resonant orbits before and after the capture. These resonant transitions are similar to the orbits of Galileo and Cassini during their touring phase, using resonant orbits to reduce mission ΔV requirements.

INTRODUCTION

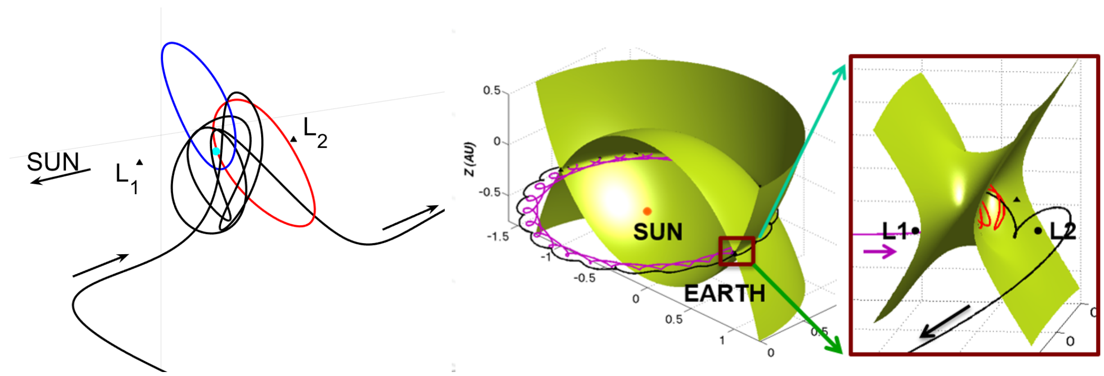
There are two types of moons, stable and temporary moons. Stable moons are in stable orbits around the planet; temporary moons are captured by the planet in highly chaotic orbits. The Outer Planets are known to have many temporary moons. In fact, the Jupiter Family of Comets like Oterma, Gehrels 3, Helin-Roman-Crockett were temporarily captured by Jupiter and then escaped. Shoemaker-Levy 9 was a temporary moon that eventually impacted Jupiter itself. In 2006, Near Earth Object (NEO) 2006 RH120 was captured into Earth orbit for nearly a year. It is the first temporary moon of the Earth to be observed. In previous work, we explained the dynamics of the Temporary Capture of Jupiter comets using invariant manifold theory.[1, 2] The same dynamics is also at work for the Temporary Capture of Earth's temporary moons. However, the small mass parameter of the Sun-Earth/Moon Barycenter Circular Restricted Three Body Problem as well as the large mass of Earth's Moon greatly complicate the Temporary Capture of NEOs around the Earth as compared to the capture of comets around Jupiter or Saturn.

Although Asteroid 2006 RH120 is Earth's first temporary moon to be observed, recent work by Granvik, Vaubillon, and Jedicke indicates that these moons may be abundant.[3] Their simulations showed that at any time there is at least one temporary moon of 1 meter diameter or larger orbiting the Earth. They named the Earth's temporary moons as Minimoons. If the prediction of this population of minimoons of the Earth is verified, this could open the door to many potentially interesting missions to NEOs right at our door step! Using CubeSats or SmallSats, these missions could be very low cost. Astronomers are currently busy at work to verify the existence

*Graduate Student, Department of Astronautical Engineering, University of Southern California. Affiliate, Jet Propulsion Laboratory, California Institute of Technology, bdanders@usc.edu.

†Technologist, Jet Propulsion Laboratory, California Institute of Technology, martin.w.lo@jpl.nasa.gov.

of this population of NEOs. Minimoons are difficult to detect because of their small size and their lower velocity profile which distinguishes them from the faster moving NEOs like Apophis. Given the potential for an abundance of these interesting Minimoons, a deeper understanding of their dynamics would help in locating them and in designing missions to explore them. In this paper, we examine the role that invariant manifolds of libration orbits around the Earth's L_1 and L_2 play in the capture and escape of Asteroid 2006 RH120. We defer the study of the asteroid's interaction with the Moon, which involves the Sun-Earth-Moon Four Body Problem, in later papers.



(a) (b)

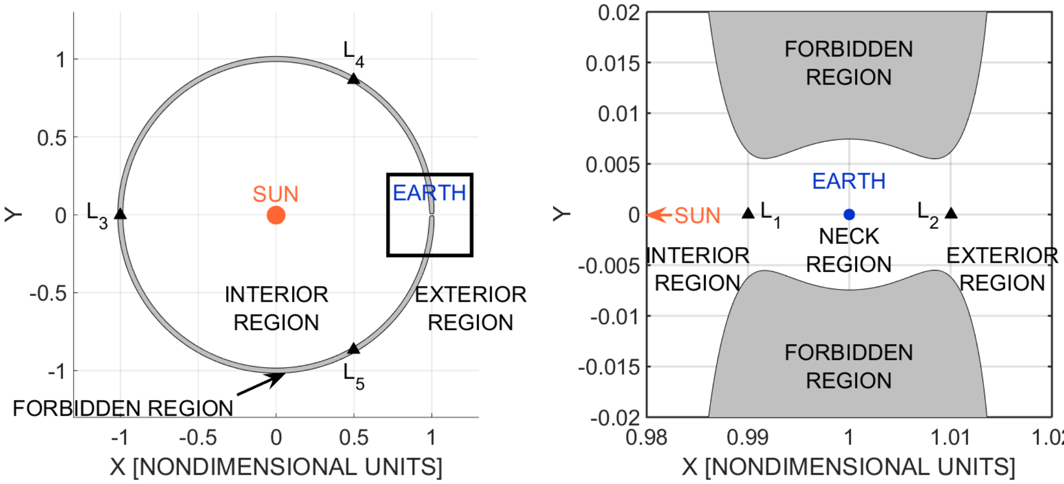
Figure 1. (a) The trajectory of Asteroid 2006 RH120 (black), the L_1 Northern Halo Orbit (blue) and the Southern Halo Orbit at L_2 (red). The invariant manifolds of these orbits controlled the motion of the asteroid's Temporary Capture and escape. (b) Global view of the asteroid orbit before capture (magenta) and after escape (black). The green surface is the Forbidden Region for the estimated Jacobi constant of the Pre-Capture Phase.

Figure 1(a) shows the black trajectory of Asteroid 2006 RH120 in Sun-Earth rotating frame, the blue Northern Halo orbit which controls the entry of the Asteroid into Temporary Capture by the Earth-Moon System; the red halo orbit at L_2 controls the exit of the asteroid into heliocentric orbit. Note that the L_1 Northern Halo Orbit has a smaller Jacobi constant than the L_2 Southern Halo Orbit. The interaction of the Moon reduced the energy of the asteroid as it orbited the Earth-Moon before escaping. Alternatively, the elliptic nature of Earth's orbit may be the reason that the Jacobi constant of the asteroid that we estimated is different before and after the Temporary Capture. The Jacobi constant is a quantity associated with the ideal system in the CRTBP and so introducing eccentricity to the Primaries' motion nullifies the assumptions necessary for this quantity to be a constant.[4, 5] Figure 1(b) is a global view of the asteroid trajectory for over 50 years. The magenta spiral curve is the resonant orbit which brought the asteroid to the Earth. The black spiral curve is the current orbit of the asteroid since it escaped the Earth in 2006. The close up view shows the orbit in the region around the Earth (the Earth is hidden by the surface of the Forbidden Region). The Temporary Capture Orbit is in red here and with a radius around 1 million km; it is much larger than the Moon's orbit. The many spiral loops of the trajectory prior to the capture and after the escape show that the asteroid is trapped in some mean motion resonance with the Earth.

BACKGROUND

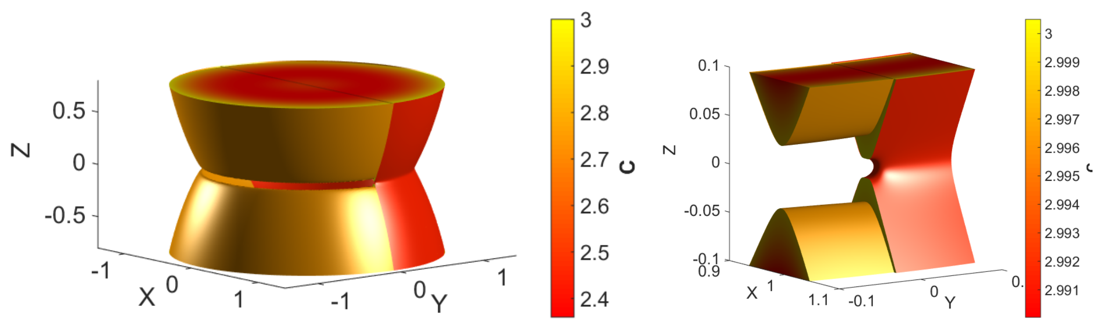
We model the dynamics of Minimoons using the Circular Restricted Three Body Problem (CRTBP). In this section, we provide a heuristic description of invariant manifolds of periodic orbits and their significance for the Temporary Capture problem. For more details, see Szebehely[4] and Koon, Lo, Marsden, Ross[6] and references therein for the equations of motion for the CRTBP and the theory of invariant manifolds of periodic orbits. The CRTBP utilizes a coordinate system that rotates along with two large bodies that orbit their common barycenter. This coordinate system is shown in Figure 2 for the Sun-Earth system. Although the two primary bodies can be any system of co-rotating celestial bodies, this work focuses on the Sun-Earth system so we will from here on refer to the primaries as the Sun and the Earth. Note that by "Earth" here we actually mean the Earth-Moon Barycenter orbiting the Sun in a circular orbit. In this reference frame, there are 5 equilibrium points known as Lagrange points labeled L_1 through L_5. L_1-L_3 lie on the x-axis while L_4 and L_5 are symmetrically placed on either side of the x-axis. All Lagrange points lie in the plane $z = 0$. This dynamical system has a constant of integration known as the Jacobi constant, which can at times be referred to as the "energy" in the literature as well. The energy-like behavior of the Jacobi constant is reversed however, as a low Jacobi constant is actually a high "mechanical energy". In this paper, for clarity, when we refer to "energy" it will always mean "mechanical energy", otherwise, we will refer to the Jacobi constant. An object with a given position and velocity in this frame that obeys the dynamics of the system will have a fixed Jacobi constant for all time. This Jacobi constant will determine the allowable regions of motion, and the regions where motion is not allowed which we call the "Forbidden Region"'. The Forbidden Region of an example Jacobi constant is shown in Figure 2. Since the system is spatial, the plots here only show a slice of a 3-dimensional Jacobi constant surface. For the level of the Jacobi constant plotted, we divide the region of allowable motion into 3 subregions separated by vertical planes through the L_1 and L_2 Lagrange points: the Interior Region, the Exterior Region and the Neck Region. The Interior Region is around the Sun inside the Earth's orbit and Forbidden Region. The Exterior Region is outside of the Earth's orbit and Forbidden Region. The Neck Region is the region containing the Earth and L_1 and L_2, connecting the Interior and Exterior Regions. For very low energy levels, all regions are completely cut off from each other. As the energy increases (Jacobi constant decreases), a hole opens up that allows for motion between the Interior and Neck Region through L_1. Further increase in the energy opens a hole to the Exterior Region. At an even higher energy, all in-plane motion near the Ecliptic is allowed. The Forbidden Region is in fact a 3 dimensional surface and for these higher energies, motion between the Interior and Exterior Region can still be unlikely to occur at any point along the Earth's orbit. Trajectories with non-trivial out of plane components will still be more likely to cross between regions where the Vertical opening between regions is the largest, such as the Neck Region near Earth. An example of the topological change in the Jacobi constant surface properties is seen in Figure 3. The low energy red surface only allows motion between Interior and Exterior Regions through the Neck Region near Earth, while the high higher surface in orange causes the top and bottom of the Forbidden Region to become two separate surfaces. The energy surfaces for the asteroid in question is shown in Figure 4. Two surfaces are indicated, since the estimated Jacobi constants for the Pre- and Post-Capture Phases are not the same. Note that the Neck Region for the Pre-Capture Jacobi constant is much more open and would more readily allow passage between the Interior and Exterior Regions. We will be studying this transit of the asteroid from the Interior Region, through the Neck Region and out to the Exterior Region.

It has been long observed that comets and asteroids can be temporarily captured by planets from time to time. A tremendous amount of work has been done to study this phenomenon with a vast literature. The most famous group of temporarily captured objects are the Jupiter Family of Comets, a few of which were mentioned earlier. The moons of Mars may be asteroids captured through a



(a) (b)

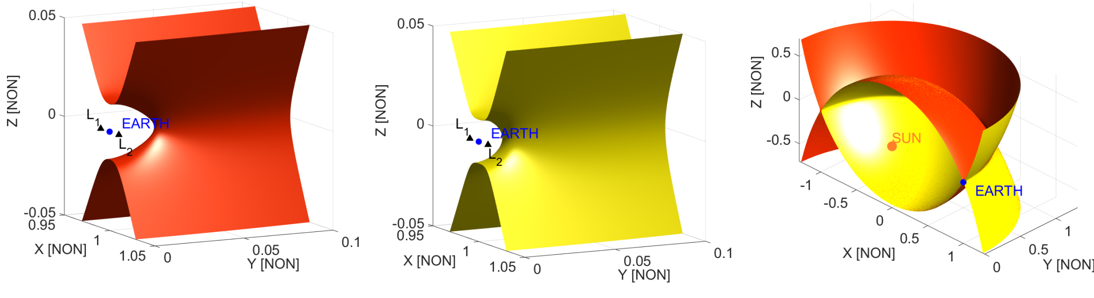
Figure 2. CRTBP coordinate frame with Lagrange point and region definitions (a) Large scale. (b) Boxed area in detail showing Neck Region.



(a) (b)

Figure 3. Illustrative example of Forbidden Region surfaces showing topological change for different Jacobi constants. The higher Jacobi constant (lower energy) in red only has a Neck Region near Earth, while the Forbidden Region with lower Jacobi constant (higher energy) in orange allows for motion in the entire plane $z = 0$. The low Jacobi constant surface is only shown for $y \geqslant 0$ in order for the high Jacobi constant surface to be visible. (a) shows the whole surfaces while (b) shows the surfaces near the Neck Region.

similar process. The transport mechanism is very complex. We know resonances play a significant role. The resonance transitions eventually bring the small body close to the L_1 or L_2 Lagrange points of the planet. At these locations, the invariant manifolds of libration orbits such as halo orbits or lissajous orbits are able to attract the small body and bring it into the region around the planet where it is temporarily captured. In the aforementioned papers, we showed how this occurs for Jupiter comets.[1, 2] The same process works for other planets as well. The significance of this approach to explain the Temporary Capture Phenomenon is that it helps us to visualize the capture process and this could help to predict the location of the NEOs before and after Temporary



(a) (b) (c)

Figure 4. Estimated Forbidden Region surfaces for Asteroid 2006 RH120 shown for the half plane $y \geqslant 0$ (a) Pre-Capture Phase. (b) Post-Capture Phase. (c) Large scale comparison of (a) and (b). Note that the surfaces are very similar for both Pre- and Post-Capture Phase, but that the opening is slightly larger for the Pre-Capture Phase, indicating a higher Jacobi constant or equivalently a smaller Jacobi constant. On a large scale, the surfaces are nearly indistinguishable. [NON] indicates nondimensional units.

Capture. Moreover, this could help us devise methods to prevent future impacts by dangerous NEOs, and it could help us design trajectories to visit the minimoons or change their energies and potentially capture them around the Moon or the Earth for many applications.

Figure 5 shows the invariant manifolds of Lyapunov orbits around L_1 and L_2 with the same Jacobi constant. The green curves forming tubes are on the stable manifolds which approach the Lyapunov orbits. The red curves forming tubes are on the unstable manifolds which leave the Lyapunov orbit.

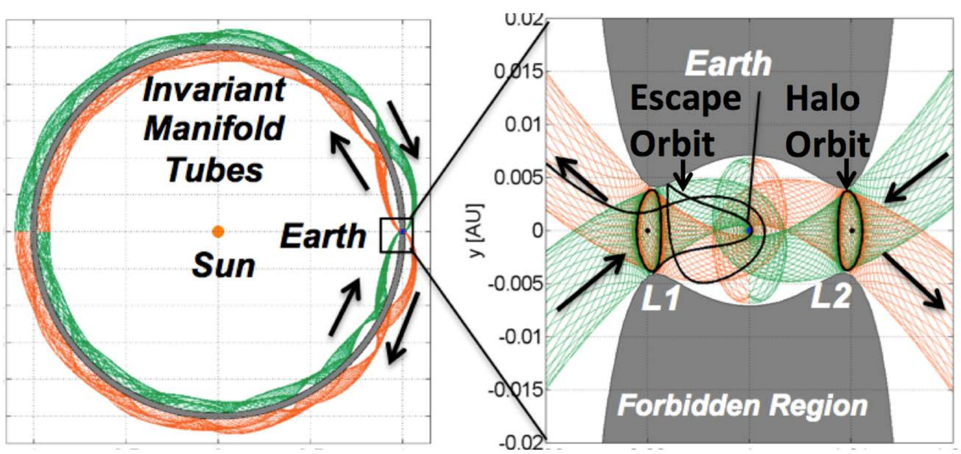
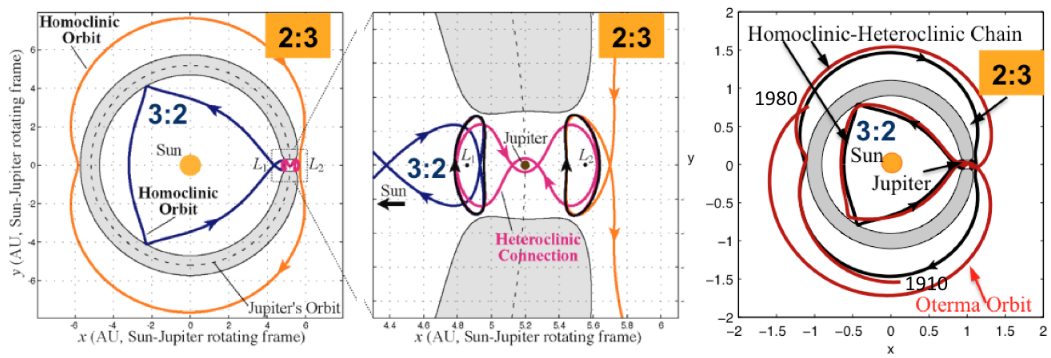


Figure 5. The invariant manifolds of the L_1 and L_2 Lyapunov orbits create the low energy routes for asteroids to approach Earth, impact the Earth or get temporarily captured by the Earth and then escape. The green trajectories approach the Lyapunov orbits, the red trajectories leaves the Lyapunov orbits, like the on-ramps and off-ramps of a highway. The tubes intersect to form a unique set of orbits called the Heteroclinic-Homoclinic Chain in Figure 6

The tubes intersect one another to produce a chain of special orbits called the Heteroclinic-Homoclinic Chain. This Chain provides a template for the motion of comets and asteroids at this energy level that are temporarily captured by Jupiter. Further more, this chain satisfies the Conley-

Moser Condition for the existence of chaos. Hence any object following this chain is in a chaotic orbit by this theorem.[7] Figures 6(a) and (b) show this chain for Jupiter at the Jacobi constant of the Comet Oterma, a well known member of the short period Jupiter Family of Comets. Figure 6(c) shows Oterma's orbit (red curve) closely following the one of Jupiters Heteroclinic-Homoclinic Chains (black curve). See Koon, Lo, Marsden, Ross for details on the computation and dynamics of the chain.[1,6]



(a) (b) (c)

Figure 6. (a) The Heteroclinic-Homoclinic Chain for Jupiter. The Homoclinic orbit (blue) in the Interior Region is following closely the 3:2 resonance with Jupiter. The Homoclinic orbit (gold) in the Exterior Region is shadowing the 2:3 resonance with Jupiter. (b) The two Lyapunov orbits (black) at L_1 and L_2 are the generators of this chain. The magenta trajectories connecting the Lyapunov orbits are the Heteroclinic orbits. (c) The orbit of Oterma (red) from 1910 to 1980 closely follows the chain (black) showing that it is a chaotic orbit by the Conley-Moser Condition.

The same dynamics is present at all other bodies in the Solar System. For example, Figure 7 shows a chain for the Sun-Earth system at the Jacobi constant of the Genesis trajectory. The Conley-Moser Condition is satisfied here also; this shows that the Genesis trajectory is chaotic which explains why this mission required so little ΔV, hence fuel, for its control throughout the mission.

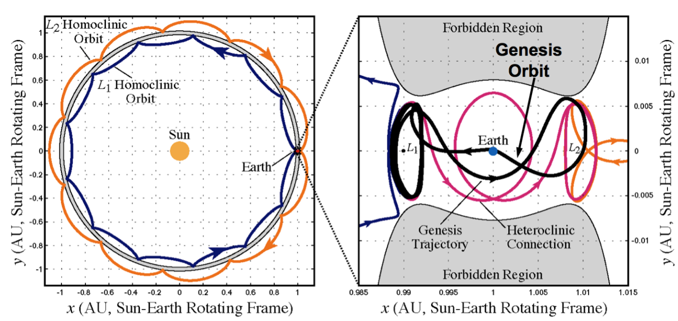


Figure 7. The figure on the right shows the Genesis trajectory in black and the Heteroclinic orbit connecting the two Lyapunov orbit at L_1 and L_2 in magenta. The Homoclinic orbit for the L_1 Lyapunov orbit is blue, for L_2, it's gold. The figure on the left shows how the Homoclinic orbits have many periods around the Sun before returning to the L_1 Lyapunov orbit. They are in fact shadowing resonant orbits.

In the Jupiter problem, the Homoclinic Orbits (which connects the L_1 Lyapunov Orbit to itself and similarly at L_2) are also shadowing the resonant orbits. For Jupiter, the Interior Resonance is 3:2 (the comet goes around the Sun 3 times for every 2 times Jupiter goes around the Sun), the Exterior Resonance is 2:3. For the Earth, typical of smaller planets, the resonances have many more revolutions. The unstable resonant orbits are the means by which asteroids and comets can quickly transport across the Solar System via Mean Motion Resonances which we call Resonant Transitions. This is precisely what is going on for resonant gravity assist maneuvers that has enabled missions from Voyager to Galileo and Cassini. See Anderson and Lo for details.[8,9] Secular resonances also play a crucial role in the dynamics of transport in the Solar System, but these require very long time span of many millions of years. In this paper, we will show the sequence of resonant and libration orbits used by Asteroid RH120 to approach the Earth for Temporary Capture, and then escape the Earth again through resonance transitions.

For comets Oterma and Genesis, we computed the chains in the 2D planar CR3BP model. Conley showed that for energies slightly above that of the L_1 and L_2 Lagrange points, the Lyapunov orbits control the dynamics around the planet.[10] Any object at these energies can only approach and escape the planet by going through the invariant manifolds of the Lyapunov orbits. This is because for the planar CRTBP the invariant manifolds are 2D tubes in 3D energy surfaces. Since there is a well-defined inside and outside topologically for a 2D tube in 3D space, this constrains the motion of comets and asteroids in the planar CRTBP as stated above.

However, actual comets and asteroids move in 3D trajectories. Howell, Marchand, and Lo showed that several of the comets in the Jupiter Family (Gehrels 3, Helin-Roman-Crockett) closely follow the invariant manifolds of Jupiter Halo Orbits during their Temporary Capture Phase.[2] This suggests that libration orbits around the L_1 and L_2 Lagrange points are gateways for the Temporary Capture Phenomenon. However, so far, this conjecture has not been proved except for a few specific comets. Even for the 2D case, the existence theorem of Conley does not specify the range of Jacobi constant for which it is true. For very high energies such as that of the Asteroid Apophis, halo orbits no longer exist. It is not known what libration orbits, if any, may control the dynamics of such asteroids and comets that approach the Earth or Jupiter. Recently, based on numerical study, Ren and Shan conjectured that the dynamics of objects approaching and departing a planet is controlled by the invariant manifolds of the Vertical Lyapunov Orbit and the Planar Lyapunov Orbit.[11] This is an exciting result which requires closer study and verification.

The main result of this paper is the demonstration that the trajectory of Asteroid 2006 RH120 is guided by a sequence of resonant and libration orbits and their invariant manifolds. This is typical of Minimoons. Hence by understanding the dynamics of these resonant transitions and temporary capture trajectories, we may be able to formulate theories and rules of thumb to locate, renedezvous, deflect, or capture NEOs at the energy levels of Minioons.

CRTBP AND EPHEMERIS MODEL

The Sun-Earth/Moon Barycenter Circular Restricted Three Body Problem (CRTBP) (see Reference 4) is a simplification of the motion of an object around the Sun and Earth. Therefore, analysis within this system requires some form of transformation from the real physical system. The Jet Propulsion Laboratory DE431 ephemeris will be used to represent the "real system" in this case.[12] In the CRTBP system, the distance between the primaries is assumed constant as is their rotation rate about the common barycenter. This would be the case if the two primary bodies were in circular orbits around their mutual barycenter and there are no other perturbing forces. In such a perfect system, units of length are nondimensionalized by this fixed distance and the units of time are nondimensionalized by setting the mean motion of the primaries to unity. Units of velocity and acceleration are derived from length and time. The frame of reference rotates along with the primaries such that the x-axis lies along the line from the Sun to the Earth and the z-axis is along the angular momentum of their motion. Although there is no perfect method for converting real trajectories into the CRTBP frame, the method used for this work allows for a rough analysis and is described in detail by Anderson and Lo.[13] Two methods are used for the work here, each with its advantages and disadvantages. Conversion Method 1 uses variable length and time units, representing the current position and motion of the primaries. The instantaneous distance between the primaries is selected as the distance unit, while the time unit is selected such that the instantaneous angular velocity of the primaries has unity magnitude. Conversion Method 2 uses fixed length and time units, chosen to represent the position and motion of the primaries at a specific reference time. The distance and time units are computed as in the Conversion Method 1, but the instantaneous conditions of the primaries at a fixed reference time is selected for units conversions. The common principles for frame conversion between both methods is the position of the right handed coordinate system. The x-axis is based on the instantaneous vector from the Sun to the Earth. The z-axis is aligned with the angular momentum vector of the motion of the Sun and Earth. The y-axis completes the right handed system and the origin is located at the barycenter of the two primaries.

The most natural conversion using Method 2 would be to use the mean semimajor axis and mean motion of the primaries, but this choice is not always the best one depending on the behavior one wants to analyze. For example, if Earth happens to be near its perihelion when an asteroid makes its closest approach, this conversion would place the asteroid around $x \approx 0.983$ which by theory is beyond L_1 in the CRTBP and would not be near Earth. Any analysis of the motion of an asteroid in the region between L_1 and L_2 would be better suited by using Conversion Method 1, as it would more accurately represent the motion of the asteroid relative to Earth. This analysis is however deferred to a later paper and the current work focuses on the Pre-Capture and Post-Capture Phases of Asteroid 2006 RH120. For this purpose, Conversion Method 2 is used extensively with the reference time chosen appropriately to allow analysis of the Asteroid's motion as it approaches L_1 and departs L_2, as well as the heliocentric motion, far from the gravitational influence of Earth. The equations for the conversions are given in the Appendix.

Conversion Method 1 has the advantage of producing an accurate visual representation of the positional trajectory in an approximate CRTBP frame of reference. Since the method displays the position of an object relative to the instantaneous position of Earth, the ellipticity effects of Earth's orbit are minimized. Based on this work, Conversion Method 1 produced a position plot that more closely follows the dynamics of an ideal CRTBP system. However, due to the oscillatory nature of the unit conversions, the converted velocity also displays oscillations. There are several coupled frequencies, causing the velocity to not behave as expected in an ideal CRTBP model. Since the Jacobi constant is computed from the position and velocity of an object, in this case the Jacobi constant exhibits oscillations on multiple frequencies. Due to the better behaved position conversion of Method 1, it is better suited for analyzing the long term resonances of the asteroid. However, the behavior of the Jacobi constant of Method 1 is unsuitable for analyzing the libration orbits for the approach and departure phases near Earth. Method 2 is more suitable in this case.

Conversion Method 2 removes the oscillations in the unit conversions and thus eliminates some of the frequencies in the Jacobi constant oscillation. The frequency that remains is an annual cycle. However, this method has the drawback that while it is a more accurate representation of the behavior close to the reference point, it may be poor far from the reference point. For this work, Conversion Method 2 is well suited for the analysis of the interaction between the invariant manifolds of the libration orbits and the asteroid. It is less suited for the long-term analysis of the Asteroid's resonances.

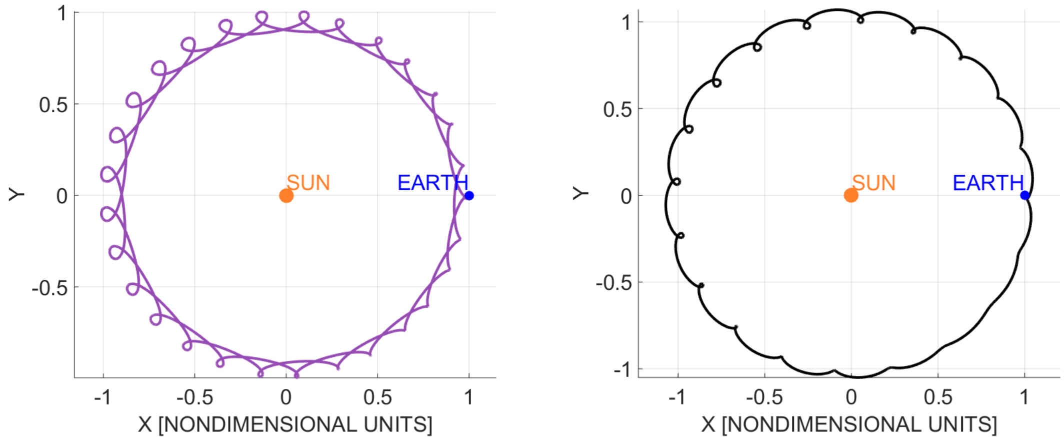
ASTEROID ORBIT

The Near Earth Object (NEO) 2006 RH120 is the first known natural celestial body to be temporarily captured by the Earth.[3, 14] The asteroid orbited Earth for approximately 1 year in a large, chaotic orbit beyond the orbit of the Moon, but between the Sun-Earth L_1 and L_2 Lagrange points. We call this the Temporary Capture (TC) phase. Before the TC Phase, Asteroid 2006 RH120 was on a heliocentric orbit inside the Earth's orbit. After the TC encounter, the asteroid transitioned to an orbit outside the Earth's orbit. The interaction of the asteroid with the Earth-Moon System is highly nonlinear and its orbit was significantly changed during the Temporary Capture Phase. Classical orbital elements varied wildly during this phase. In the Pre-Capture and Post-Capture Phases, the asteroid underwent a series of resonance hopping which is also exhibited by the Jupiter Family of Comets.

We use two methods to determine the resonances of Asteroid 2006 RH120. Resonance Method 1 used visual inspection of the trajectory in the rotating frame as follows. Each "loop" or oscillatory cycle of the Asteroid's path in the rotating frame represents one revolution around the Sun. Starting the analysis immediately before or after Temporary Capture, a propagation of the Asteroid's trajectory backward or forward in time respectively reveals details about its resonance cycle. After propagating the orbit for a certain amount of time, the asteroid returns to the vicinity of Earth, usually near L_1 or L_2. At this time, the number of heliocentric orbits of the asteroid is compared to the time elapsed in Earth years. The integer ratio of these two numbers will be the mean motion resonance of the cycle. Figure 8(a) shows the Pre-Capture orbit as converted to the rotating frame using Conversion Method 1 as described earlier. Analysis of the Pre-Capture orbit shows that a complete resonance cycle is completed in 27 years during which time the asteroid orbited the Sun 29 times. This suggests that the asteroid is initially in a 27:29 mean motion resonance with the Earth. Figure 8(b) shows the Post-Capture orbit converted to the rotating frame also using Conversion Method 1. Analysis of the Post-Capture orbit in the rotating frame shows that a complete resonance cycle is 21 years during which time the asteroid orbits the Sun 20 times. This suggests that the asteroid is in a 21:20 mean motion resonance with the Earth after TC. This initial analysis covers the time period April 1, 1979-November 1, 2028, which exhibits one full resonance cycle for both Pre-Capture and Post-Capture Phases.

The fundamental change in the orbit is clearly visible in the rotating frame as seen in Figure 8. If the same method of analysis is applied to an even longer time span, more interesting effects appear. As is seen in Figure 10(c), there are multiple rapid changes in the behavior of the Jacobi constant of Asteroid 2006 RH120. Upon closer correlation of these changes to the position of the asteroid, it is found that these events are the result of near-Earth approaches that do not result in Temporary Capture. At each Earth encounter, the asteroid changes resonance with the Earth. We will later correlate these changes to heteroclinic transitions between the resonant periodic orbits around the Sun in the CRTBP.

An alternate method for approximating the resonance of an object is through a simple Keplerian analysis. We call this Resonance Method 2. A period resonance in a pure 2-body situation can simply be described by the osculating period of an object relative to the period of some reference object (the Earth in this case). The semimajor axis of an object can be computed from its Kep-



(a) (b)

Figure 8. Asteroid 2006 RH120 trajectory converted to CRTBP rotating frame by Conversion Method 1 (a) Asteroid trajectory in Pre-Capture Phase. (b) Asteroid trajectory in Post-Capture Phase.

lerian energy, which in turn is determined directly from the object's position and velocity. In an unperturbed 2-body system, this energy should remain constant. But due to the multibody effects of the real physical system, the Two-Body energy changes with time. This allows us to determine the theoretical instantaneous period of an object over time. By focusing on parts of this period when it reaches a relatively constant value, a simple resonance analysis is possible. Figure 9 shows the semimajor axis of Asteroid 2006 RH120 over time. It is clear that when the asteroid is far enough away from Earth, the osculating semimajor axis is nearly constant. At these reference points, the Keplerian period indicates that the Pre-Capture resonance is 40:43 while the Post-Capture resonance is 21:20. These results agree with the previous results for the Post-Capture trajectory, but are slightly different for the Pre-Capture trajectory. It should be noted that the ratio 40:43 and 27:29 are very close, both are nearly 1:1 resonances, small differences become difficult to tell apart. The discrepancy between the results can be attributed to the more approximate nature of the Keplerian analysis. While the period of the asteroid stays nearly constant for a large part of its resonance cycle, the non-keplerian effects near the beginning and end of this cycle have an effect that causes the overall resonance cycle to be slightly different. Therefore, while Resonance Method 1 may be more time consuming, it is the preferred method for computing more accurate resonances.

As mentioned before, estimating the Jacobi constant of a real physical object will be largely affected by the choice of method for converting the trajectory into the ideal CRTBP frame. It becomes important to have a good estimate of the Jacobi constant when discussing the interaction between libration orbit manifolds and the trajectory. Every periodic orbit in the ideal CRTBP has a specific Jacobi constant and as such one should choose orbits that match the Jacobi constant of the asteroid. We chose to approximate the Jacobi constant of the asteroid at an appropriate reference point for studying its behavior. More specifically, Conversion Method 2 was used as explained previously and the reference point was chosen to be at the crossing of the L_1 plane for the Pre-Capture Phase and at the crossing the L_2 plane for the Post-Capture Phase. These reference points are indicated in Figure 10(a) while the Jacobi constant variation during the whole resonance cycle is shown in Figure 10(b). The planes through the Lagrange points are defined by the plane passing

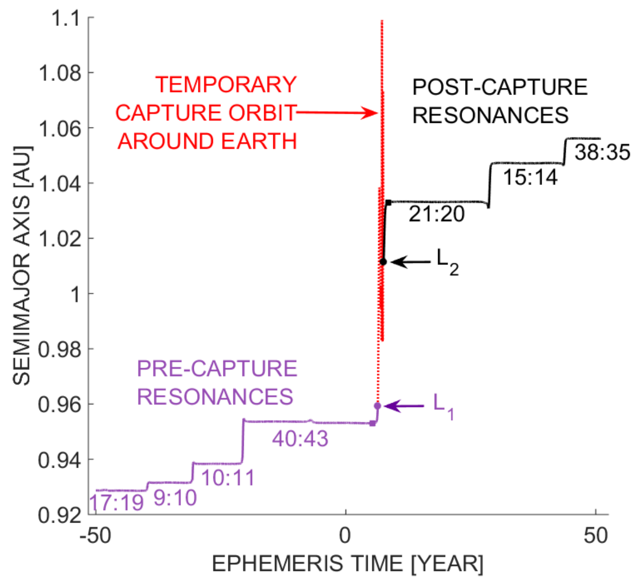


Figure 9. Osculating semimajor axis of Asteroid 2006 RH120 for the period 1950-2050. Resonance Method 2 was used to estimate period resonance ratios. The data shows multiple encounters and resonance transitions.

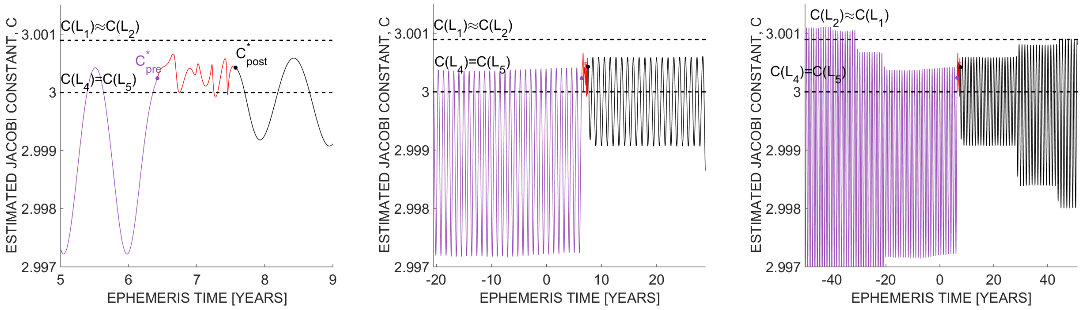
through the Lagrange point that is normal to the vector from the Sun to the Lagrange point. The position of the Lagrange points were obtained from JPL Ephemeris DE431 (as objects "391" and "392").[12] The Jacobi constants were estimated to be

$$C^*_{pre} = 3.000228226120707 \tag{1}$$

$$C^*_{post} = 3.000425683288712 \tag{2}$$

We realize the estimates cannot be accurate to 15 decimal places, but we have included the full value we used for finding orbits and surfaces in case anyone would like to reproduce some of our work.

If the time span under consideration is expanded to the full range provided for the object within the DE431 Ephemeris, 1950-2050, the Jacobi constant and semimajor axis both exhibit signs of repeated Earth encounters as seen in Figure 10(c) and Figure 9. Resonance Method 2 allows us to quickly approximate the resonances between each Earth encounter and are shown in Figure 9. The available data shows a trend of an increasing semimajor axis by repeated Earth encounters, with the most significant change occurring during the Temporary Capture. This behavior for an increasing semimajor axis is typical for Solar System dynamics. What is happening is that the invariant manifolds of the resonant orbits in the CRTBP intersect, thus allowing for natural transfers of the asteroid between each resonance set. This is a basic transport mechanism of the Interplanetary Superhighway. This can be compared to the behavior of the Jupiter Family of short period comets such as Oterma.[1] Further work is required to find the specific resonant orbits in the CRTBP and analyze their manifolds for heteroclinic connections. However, due to the large time periods involved, such an analysis in the ideal system may not have much application to the real physical system. Furthermore, the large time span between encounters allows for buildup of small perturbations and model inaccuracies.



(a) (b) (c)

Figure 10. Estimated Jacobi constant of Asteroid 2006 RH120. Pre- (purple) and Post-Capture (black) phases used Conversion Method 2 with units chosen at their respective reference points. The Temporary Capture Phase (red) used Conversion Method 1. (a) Near TC with reference points. (b) Showing 1 full resonance cycle on either side of TC. (c) Extended time showing several resonances 1950-2050. The dashed lines indicate Jacobi constant for the Lagrange points. Values under the upper black boundary indicate an open Neck Region near Earth, while values under the lower black boundary indicate when all motion in the plane is allowed.

EARTH APPROACH THROUGH L_1 MANIFOLDS

According to our theory, the asteroid approaches the Earth via the stable manifold of orbits around L_1. We computed 4 periodic orbits around L_1 and their invariant manifolds at the reference Jacobi constant of the Pre-Capture asteroid trajectory: a Planar Lyapunov orbit, a Vertical Lyapunov orbits, a northern halo orbit, and a southern halo orbit. We used a differential corrections algorithm which constrained the Jacobi constant to find these periodic orbits.[15] We found that the asteroid following most closely the stable manifold of a Northern Halo Orbit around L_1 shown in Figure 11.

EARTH DEPARTURE THROUGH L_2 MANIFOLDS

A similar analysis was performed on the escaping trajectory and orbits around L_2. In this case the unstable manifolds of L_2 periodic orbits were used to compare with the departing asteroid trajectory. For the reference Jacobi constant of the asteroid's Post-Capture Phase, 3 periodic orbits were found: a Planar Lyapunov orbit, a Vertical Lyapunov orbits, and a southern halo orbit. For the Post-Capture Phase, we found the southern halo orbit and its manifolds around L_2 to provide the best fit with the departing asteroid trajectory. The halo orbit and its unstable manifold are shown in Figure 12. Visual inspection of the first two plots shows the asteroid closely following the surface of the unstable manifold as seen in Figure 12(b) and (c).

EXAMPLE OF COMPLEX LUNAR INTERACTIONS WITH ASTEROID-LIKE TRAJECTORIES

Figure 13 shows a small portion of the unstable manifold (green curves) of the Genesis halo orbit (magenta orbit on the left) in the CRTBP rotating frame demonstrates the complex interactions with the Moon (in the gray orbit) and the Earth. Moving approximately from left to right, we see a Lunar Flyby has torn the manifold into two pieces. One piece of manifold resulted in orbits captured around the Moon like the Hiten trajectory. The other piece of the manifold resulted in an Earth Flyby where some of them became captured by the Earth (temporarily), others flyby the Earth once

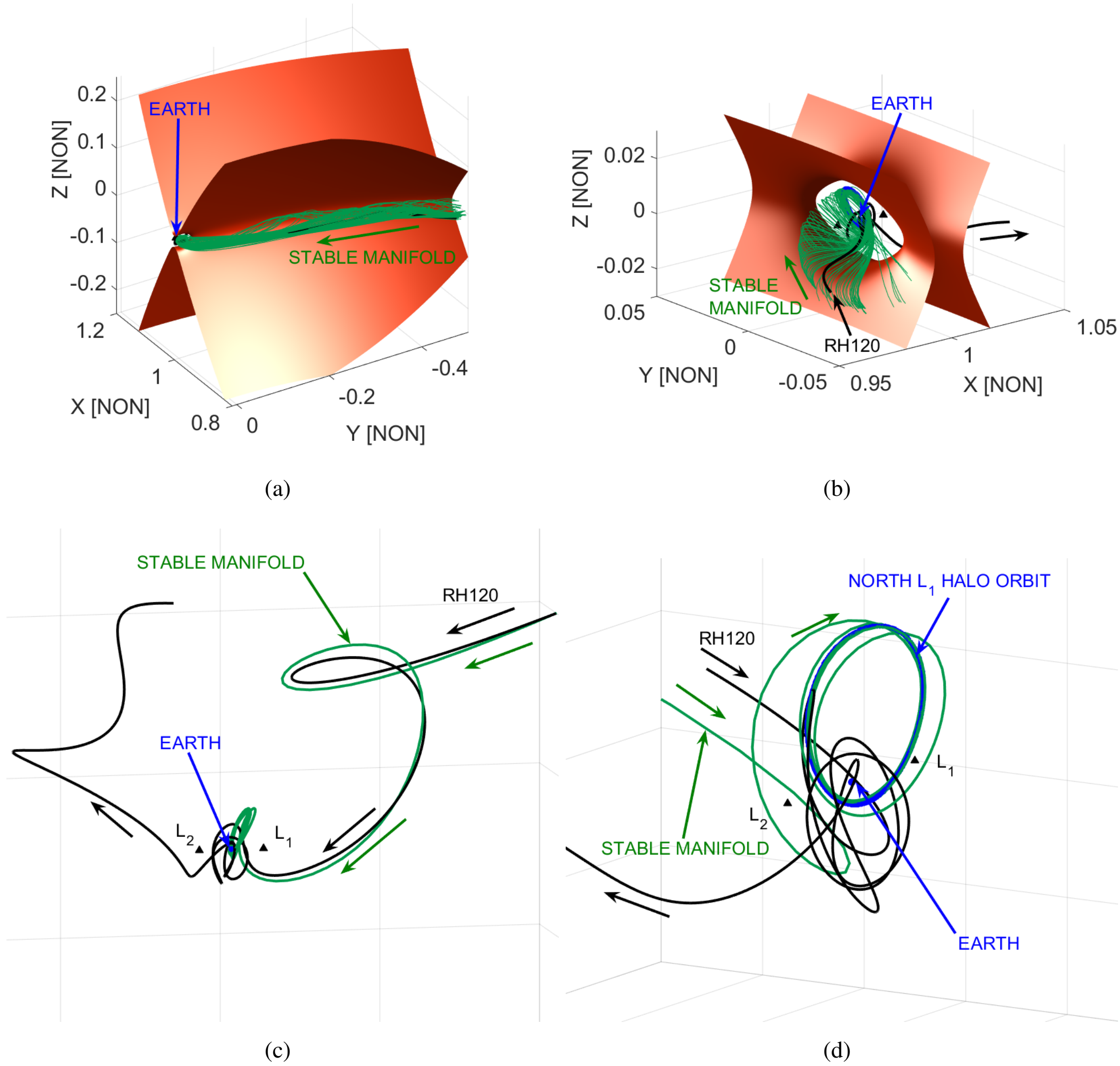
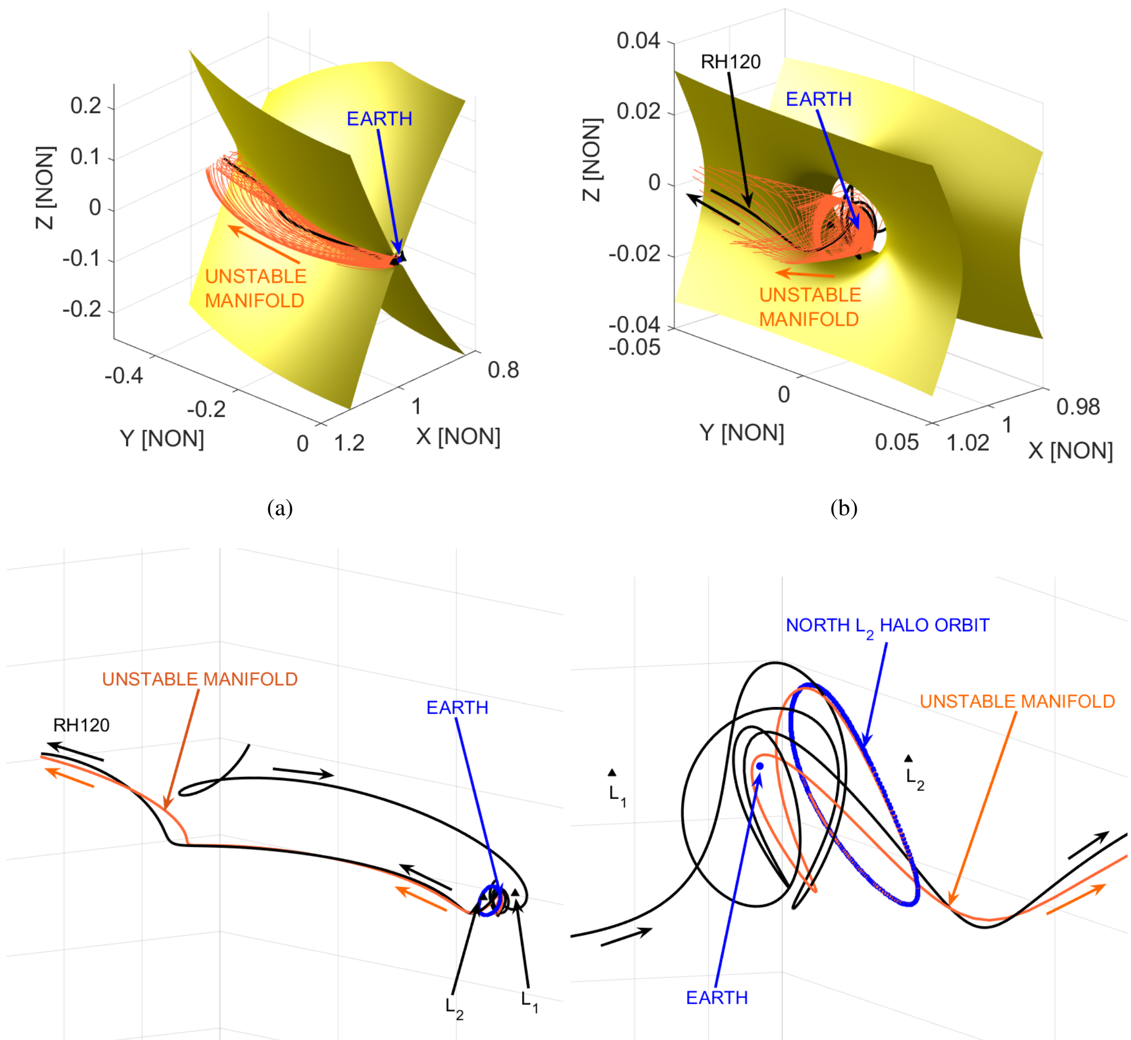


Figure 11. (a) The stable manifold (green) of a Northern Halo Orbit at L_1 attracting the asteroid (black) to approach Earth. The orange surface is Forbidden Region at the Pre-Capture energy. (b) Near Earth close-up. (c) A single trajectory on the manifold (green) selected for its similarity to the asteroid trajectory (black). (d) A close up view of the selected manifold trajectory (green), the Halo Orbit (blue) and asteroid trajectory (black). [NON] indicates nondimensional units.

and then escape via L_2 into a Spitzer-like Earth Trailer Orbit. In particular, one of the trajectories of the unstable manifold makes a wide excursion to L_2 and then impacts the Earth at the Utah Test and Training Range, sometime before 9 AM in September. This is the Genesis return trajectory to the Earth. Similarly, Near Earth Asteroids can approach the Earth from far away via the stable manifold of a halo orbit at L_1 and then follow its unstable manifold into the Neck Region around the Earth-Moon where this complex range of motions are possible. What it tells us is that the asteroid Minimoon can be captured around the Moon, or Earth, flyby the Earth and escape via L_2, or in the worst case, it may impact the Earth like the Genesis spacecraft. The analysis of the complex trajectory of Asteroid 2006 RH120 during its year-long capture phase requires the analysis of the



(a) (b)

(c) (d)

Figure 12. (a) The unstable manifold (orange) of a Southern Halo Orbit at L_2 guiding the asteroid (black) away from Earth with the plot of the estimated Post-Capture Forbidden Region (yellow). (b) Near Earth close-up. (c) A single trajectory on the unstable manifold (orange) selected for its similarity to the asteroid trajectory (black). (d) A close up view of the selected manifold trajectory (orange), the Halo Orbit (blue) and asteroid trajectory (black). [NON] indicates nondimensional units.

orbital motions near the Moon separately from the motions near the Earth. The invariant manifolds of some Lunar L_2 orbit are interacting with the invariant manifolds of the periodic orbits around the Sun-Earth L_1 and L_2 described earlier. The interaction with the Moon during the Temporary Capture Phase reduced the energy of the asteroid (raising its Jacobi constant) when it escaped the Earth via L_2. A detailed analysis of this work will be described in the next paper.

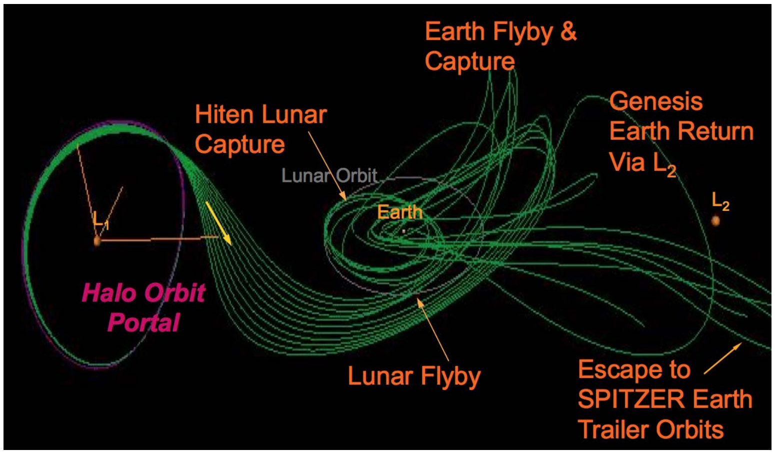


Figure 13. Shows a small subset of the invariant manifold for the Genesis trajectory. Minor perturbations of an initial state result in very chaotic subsequent trajectory evolutions.

CONCLUSION

Our goal is to show from this research that the dynamics of minimoons can be understood using the invariant manifolds of unstable orbits in the Sun-Earth-Moon System. Although the dynamics of minimoons is important in its own right, the real significance for understanding this dynamics is in the practical applications to science, engineering, and the industrialization of space. Knowledge and insight of the dynamics of minimoons would enable us to detect and locate them, potentially design means to deflect them. Since they may be abundant and are so easily accessible from the Earth or the Moon, understanding their dynamics could help us potentially design very cheap missions to rendezvous and land on them, if they are big enough, with robotic and human missions. This knowledge could help us potentially capture these minimoons into orbit around the Earth or the Moon with minimal effort. In the near future, we could mine the minimoons and begin a new industrial age in space. From the resources and wealth created from the commerce and industry in space, we could then truly begin to colonize the Moon and space beyond.

In this paper, we analyzed the dynamics of the Temporary Capture of the Minimoon 2006 RH120 during its Pre-Capture and Post-Capture Phases using the CRTBP. For the Pre-Capture Phase, we have shown that the asteroid trajectory is shadowing closely the trajectories of the invariant manifolds of a northern halo orbit around L_1. Similarly, for the Post-Capture Phase, we have shown that the asteroid trajectory is shadowing closely the trajectories of the invariant manifolds of a southern halo orbit around L_2.

We are currently studying the dynamics of the Capture Phase which requires a detailed analysis of the 4-body interaction of the Sun-Earth-Moon-Minimoon System. This is much more complicated. In Figure 14 we plotted the distance between the asteroid and the Moon during the Capture Phase in the Neck Region. It shows the asteroid moving in the Neck Region of the Earth where the invariant manifolds of the Sun-Earth and Earth-Moon 3 body systems intersect. These intersections create lobes that can trap the asteroid temporarily in highly chaotic chaotic orbits. Once we understand the dynamics of these Temporary Capture orbits, we could then begin thinking about low energy and low cost methods to convert a Temporary Capture orbit into a Long-Term Capture orbit.

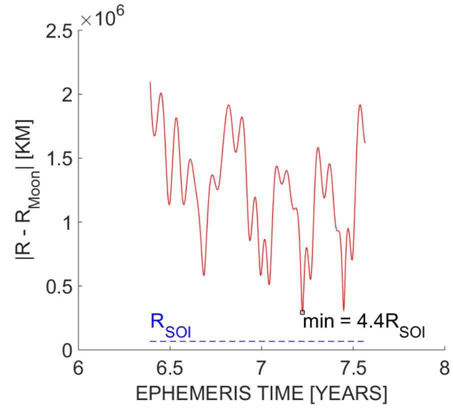


Figure 14. Distance between Asteroid 2006 RH120 and the Moon during Temporary Capture. The R_{SOI} is the radius of the Lunar Sphere of Influence.

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APPENDIX A: FRAME CONVERSION

Conversion Method 1: Variable Length and Time Units

The conversion method assumes the trajectory to be converted is an ephemeris state in the Ecliptic J2000 coordinate system relative to the larger primary body. In this case, the length unit is the instantaneous distance between the primaries and the time unit is such that the instantaneous angular velocity of the primaries motion is 1. The conversion method uses the following steps for each dimensional time, t_D:

1.) Use the ephemeris to determine the position, $\vec{R}$, and velocity, $\vec{V}$ of the smaller primary relative to the larger primary in the ecliptic J2000 coordinate frame.

2.) Set length unit.
$LU = |\vec{R}|$

3.) Compute the current angular velocity vector of the primaries' motion.
$\vec{\omega} = \frac{\vec{R} \times \vec{V}}{LU^2}$

4.) Set time and velocity units.
a) $TU = \frac{1}{|\vec{\omega}|}$
b) $VU = \frac{LU}{TU}$

5.) Select the 1st rotating frame axis as a unit vector along $\vec{R}$.
$\hat{e}_1 = \frac{\vec{R}}{|\vec{R}|}$

6.) Select the 3rd rotating frame axis as a unit vector along $\vec{\omega}$.
$\hat{e}_3 = \frac{\vec{\omega}}{|\vec{\omega}|}$

7.) Compute the final rotating axis to complete the right handed triad.
$\hat{e}_2 = \hat{e}_3 \times \hat{e}_1$

8.) Assemble the rotation matrix with each unit vector on a row.
$\mathbf{Q} = \begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \hat{e}_3 \end{bmatrix}$

9.) Rotate the position vector into the rotating frame.
$\vec{r}_D = Q\vec{r}_D^I$, where $\vec{r}_D^I$ is the position of the object in the inertial frame and dimensional units and $\vec{r}_D$ is the position in the rotating frame and dimensional units.

10.) Rotate the velocity vector into the rotating frame and subtract the velocity of the frame
$\vec{v}_D = Q\vec{v}_D^I - \vec{\omega} \times \vec{r}_D^I$, where $\vec{v}_D^I$ is the velocity of the object in the inertial frame and dimensional units and $\vec{v}_D$ is the velocity in the rotating frame and dimensional units.

11.) Convert to nondimensional units.
$\vec{r} = \frac{\vec{r}_D}{LU}$
$\vec{v} = \frac{\vec{v}_D}{VU}$, where $\vec{r}$ and $\vec{v}$ are the position and velocity in the rotating frame and nondimensional units.
$t = \frac{t_D - t_{D,0}}{TU}$, where $t_{D,0}$ is the initial time in dimensional units. This simply ensures that the nondimensional time starts at zero.

12.) Adjust position vector origin to the primaries' barycenter.
$\vec{r} = \vec{r} + \begin{bmatrix} \mu \\ 0 \\ 0 \end{bmatrix}$

Conversion Method 2: Fixed Length and Time Units

The conversion method assumes the trajectory to be converted is an ephemeris state in the ecliptic J2000 coordinate system relative to the larger primary. In this case, the length unit is selected to be the distance between the primaries at a selected reference time while the time unit is selected such that the angular velocity of the primaries' motion is unity at the selected reference time. One first selects the reference time $t_{D,ref}$ and determines the appropriate length and time units. The ephemeris is used to determine the position, $\vec{R}^* = \vec{R}(t_{D,ref})$, and velocity, $\vec{V}^* = \vec{V}(t_{D,ref})$, of the smaller primary relative to the larger primary in the ecliptic J2000 coordinate frame at this reference time. The units are then chosen as
$LU = |\vec{R}^*|$
$\vec{\omega}^* = \frac{\vec{R}^* \times \vec{V}^*}{LU^2}$
$TU = \frac{1}{|\vec{\omega}^*|}$
$VU = \frac{LU}{TU}$
The conversion method then uses the following steps for each dimensional time, t_D:

1.) Use the ephemeris to determine the position, $\vec{R}$, and velocity, $\vec{V}$ of the smaller primary relative to the larger primary in the ecliptic J2000 coordinate frame.

2.) Select the 1st rotating frame axis as a unit vector along $\vec{R}$.
$\hat{e}_1 = \frac{\vec{R}}{|\vec{R}|}$

3.) Select the 3rd rotating frame axis as a unit vector along $\vec{\omega}$.
$\hat{e}_3 = \frac{\vec{\omega}}{|\vec{\omega}|}$

4.) Compute the final rotating axis to complete the right handed triad.
$\hat{e}_2 = \hat{e}_3 \times \hat{e}_1$

5.) Assemble the rotation matrix with each unit vector on a row.

$$\mathbf{Q} = \begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \hat{e}_3 \end{bmatrix}$$

6.) Rotate the position vector into the rotating frame.
$\vec{r}_D = Q\vec{r}_D^I$, where $\vec{r}_D^I$ is the position of the object in the inertial frame and dimensional units and $\vec{r}_D$ is the position in the rotating frame and dimensional units.

7.) Rotate the velocity vector into the rotating frame and subtract the velocity of the frame
$\vec{v}_D = Q\vec{v}_D^I - \vec{\omega} \times \vec{r}_D^I$, where $\vec{v}_D^I$ is the velocity of the object in the inertial frame and dimensional units and $\vec{v}_D$ is the velocity in the rotating frame and dimensional units.

8.) Convert to nondimensional units.
$\vec{r} = \frac{\vec{r}_D}{LU}$
$\vec{v} = \frac{\vec{v}_D}{VU}$, where $\vec{r}$ and $\vec{v}$ are the position and velocity in the rotating frame and nondimensional units.
$t = \frac{t_D - t_{D,0}}{TU}$, where $t_{D,0}$ is the initial time in dimensional units. This simply ensures that the nondimensional time starts at zero.

9.) Adjust position vector origin to the primaries' barycenter.

$$\vec{r} = \vec{r} + \begin{bmatrix} \mu \\ 0 \\ 0 \end{bmatrix}$$

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