Trajectory Dispersion Control for the Cassini Grand Finale Mission

Mau Wong*, Yungsun Hahn, Duane Roth, and Mar Vaquero
Jet Propulsion Laboratory, California Institute of Technology
4800 Oak Grove Drive, Pasadena, CA 91109 USA
** Corresponding author. Tel: 1-818-354-7405; Email: mau.c.wong@jpl.nasa.gov*

Abstract: *The Cassini Grand Finale Mission, which consists of 22 ballistic orbits, will begin on April 22, 2017 after the last targeted Titan flyby. It will end on September 15, 2017 when the spacecraft dives into Saturn's atmosphere and be permanently captured. High volumes of unique science data from various onboard instruments are expected from the mission. To ensure its success and facilitate science planning, the trajectory dispersion needs to be controlled below 250 km (root-mean-square spatial deviation at the 68th percentile level) for a few segments of trajectory in the mission. This paper reports the formulation and solution of this dispersion-control problem. We consider various sources of uncertainties including flyby error, orbit determination error, maneuver execution error, thruster firing control error, and uncertainty in Saturn's atmospheric model. A non-linear Monte Carlo Trajectory Dispersion tool is developed and employed for the analysis. It is found that a total of three Orbit Trim Maneuvers with a 99% ΔV usage of less than 2 m/s will adequately control the trajectory.*

Keywords: *Cassini, trajectory dispersion, trajectory control, orbit determination, maneuver.*

1. Introduction

After a decade of exploring and collecting valuable scientific data with unprecedented details of the Saturnian system, the highly successful Cassini mission is slated to enter the last phase of its 20-year interplanetary endeavor – the Grand Finale Mission *(also known as the Proximal Orbits, as depicted in Fig. 1)*. It will begin with one last targeted, close flyby of Titan (T126; 979 km altitude) on April 22, 2017 that sets up the subsequent 22.5 highly inclined (~62 degrees), short period (~6.5 days), ballistic orbits each passing through the gap between the inner D-ring and the upper atmosphere of Saturn. The unique geometry of these orbits will afford scientists opportunities, previously unavailable, to further study the intricacies of the Saturnian system. For instance, the last 5 of these 22 orbits will pass through regions where the atmospheric density is ~ 10^9 cm^{-3} (0.01 nbar) to allow for direct sampling of the species in Saturn's thermosphere. After these close flybys, the Cassini spacecraft will dive deeper into Saturn's atmosphere and be permanently captured in September 2017.

As the science team began planning observations for proximal mission, they quickly realized that their data return could be vastly improved if pointing and timing errors were reduced such that events of interest can be identified and located with high precision. This realization led to a request for the navigation team to control the trajectory, thereby limiting dispersions from the reference trajectory used to plan and design science observations.

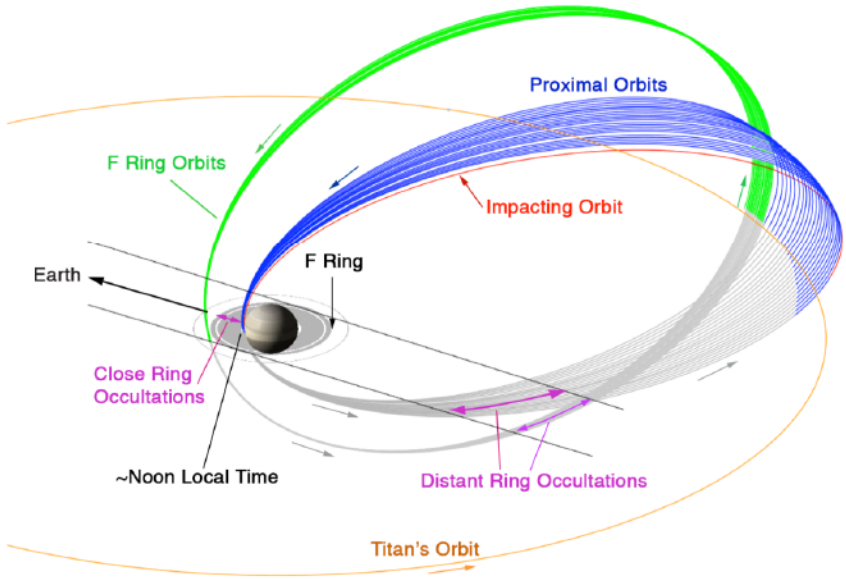

Figure 1: Schematic of the Proximal Orbits (blue). The red segment denotes the last, impacting orbit. Also shown are the F-ring orbits (green) that precede the proximal orbits.

To achieve the goal of controlling the trajectory dispersions, statistical maneuvers are required to account for navigation uncertainties including flyby, orbit determination, and maneuver execution errors, which are normally unavoidable in operations. In addition, because of the diverse, tightly packed science observations planned during the Grand Finale mission, many reaction control system (RCS) thruster activities (> 70) are needed to accordingly adjust the orientation of the spacecraft with efficiency. Some of these RCS activities will take place near periapses and have non-trivial uncertainties (~ 70 mm/s per axis) that can contribute significantly to the overall trajectory dispersions. Furthermore, the trajectory perturbations due to aerodynamic forces the spacecraft experiences during its close skimming of Saturn's atmosphere have been shown to be non-negligible. It is the responsibility of the Cassini Navigation Team to analyze and design a control strategy so that the Cassini Grand Finale Mission can be flown with relatively small dispersions to ensure the safety of the spacecraft, comply with the planetary protection requirements, and accommodate various requests from the science team. These conditions can be summarized as follows:

- Achieve Saturn atmospheric capture at the end of mission (R < 60,848 km) to avoid possible future collision with and contamination of natural satellites
- Maintain spacecraft's controllability by staying above the tumble altitude (R > 61,700 km)
- Control the trajectory dispersions (68th percentile RSS deviation) to be under 250 km for 3 selected segments (to be discussed below) of the mission to aid science planning and sequencing
- Minimize the number of maneuvers required and the propellant usage; refrain from placing maneuvers at locations with high science activities (e.g. within 24 hours of a periapsis)

In the Prime, Equinox, and Solstice tours of the last eleven years, up to three maneuvers could be used to target every close satellite flyby that served as discrete control points of the trajectory. In

the Grand Finale mission, however, the navigation goal of controlling the overall dispersion of the orbits will have to be achieved with no more than a few maneuvers. Because of the mission's unique characteristics, existing statistical analysis tools need to be modified and new ones developed to physically model the many coupling, inter-dependent sources of uncertainties, and to accurately compute the resulting perturbed and controlled trajectories. Our approach is to first use results from orbit determination (OD) covariance analysis as the estimates for various sources of uncontrolled uncertainties, which will then be fed into the non-linear trajectory dispersion tool to calculate the maneuvers needed to satisfy the aforementioned requested conditions.

2. Orbit Determination (OD) Analysis

The first step in controlling the proximal mission trajectory was to determine the expected trajectory dispersions without control. To accomplish this, a covariance analysis was performed with simulated Doppler and range tracking data. Data weights and filter configuration paralleled the current operations configuration as much as possible. Considered errors included the ephemeris and masses of Saturn and its satellites, Saturn's pole orientation, tracking station locations, Earth's polar motion, and media effects. Estimated typical operational parameters included the spacecraft epoch state, OTM execution errors, three ΔV's per orbit about Saturn to account for spacecraft momentum management (assumed to be at apoapsis, 12 hours before periapsis, and 12 hours after periapsis), seven intervals of thruster activity needed for specific science observations (surrounding proximal periapses 2, 15, 18, 19, 20, 21, and 22), a low level acceleration to account for asymmetric thermal radiation from Cassini's power source and for solar pressure, and a range bias. Deviations from the typical operational configuration were implemented because Cassini will be much closer to Saturn at periapsis than at any other time during the entire mission. Saturn's harmonic gravity field coefficients through thirteenth order were estimated, whereas the filter was sensitive to only even components through eighth order in previous operations. Saturn's ring mass was estimated separately from Saturn's mass since the spacecraft flies through the gap between Saturn's cloud tops and rings for the first time. And finally, Saturn's atmospheric density profile was estimated since the spacecraft will experience drag from the planet for the first time, especially during the last five periapses and atmospheric entry.

Proximal mission dispersions are computed by filtering simulated tracking data from the arc epoch of April 9, 2017 to the data cutoff for the last control point prior to T126 and then mapping the resulting covariance past T126. The last control point is OTM 469, scheduled to execute April 18 18:11 SCET, 3.5 days before T126. The corresponding data cutoff is at the end of the tracking pass that precedes OTM 469, April 16 23:00 UTC. One-sigma dispersions for the subsequent proximal orbits about their reference trajectory as a function of time are shown in Fig. 2. With peaks at periapses and troughs at apoapses, along-track dispersions are largest by far, indicating that the dispersions are primarily related to orbit period differences. By the 22nd periapsis, the 7757 km along-track dispersion is roughly equivalent to dispersion in periapsis time of 226 seconds.

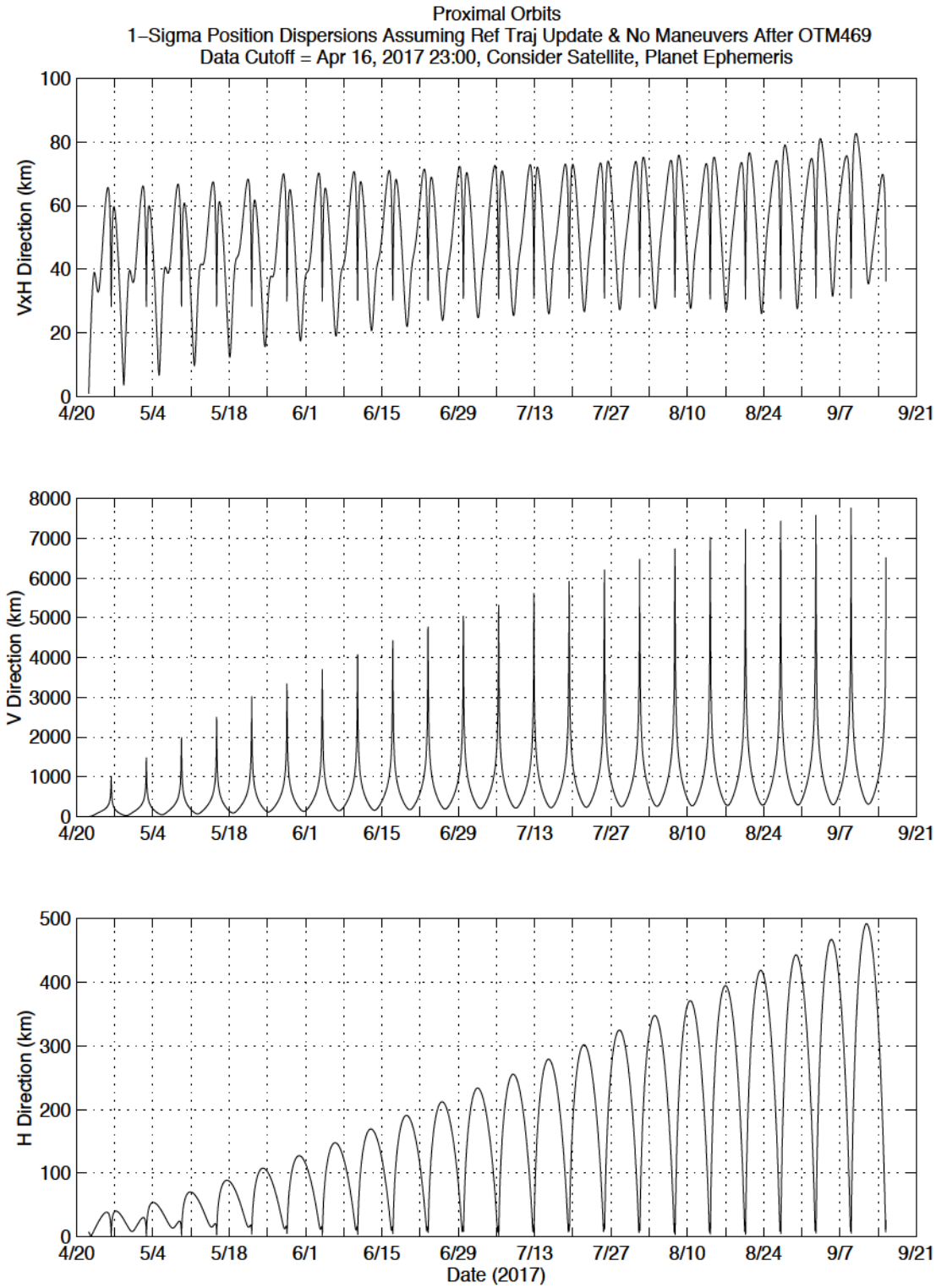

Figure 2: 1σ dispersions of the 22 proximal orbits in three spatial dimensions after T126 flyby. V denotes along-track direction, H denotes angular momentum direction, and VxH completes the orthogonal coordinate system.

Computation of dispersions is not only useful as the starting point for trajectory control analysis, but also indicates bounds for the trajectory if a failure occurs causing loss of spacecraft control. In particular, spacecraft disposal must be assured. Figure 3 shows radial uncertainties at each periapsis. In the unlikely event that Cassini is unable to perform maneuvers after T126, atmospheric entry on September 15 is still statistically guaranteed. The radial uncertainty is only 30 km, and Cassini's periapsis radius is several hundred kilometers below the capture radius when no atmospheric drag is modeled. Any drag encountered by the spacecraft will cause Cassini to fall even lower than the capture radius.

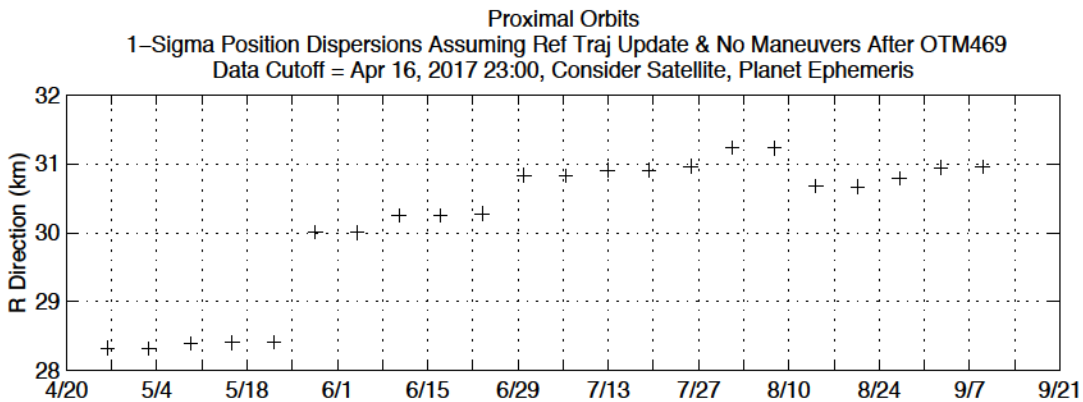

Figure 3: 1σ radial dispersions at periapses of the 22 proximal orbits after T126 flyby.

To reduce dispersions, additional orbit trim maneuvers, or control points, must be inserted after T126. Orbit determination knowledge available at the time of each OTM design is needed to re-set propagation of uncertainties after OTM execution. The process is the same as described previously, but the tracking data cutoff is extended to the next control point. Knowledge uncertainties for OTM 470, the T126 cleanup maneuver, are shown in Fig. 4. The tracking data arc has been extended to April 24, 00:14 UTC, for the maneuver executing at April 24 17:52 SCET.

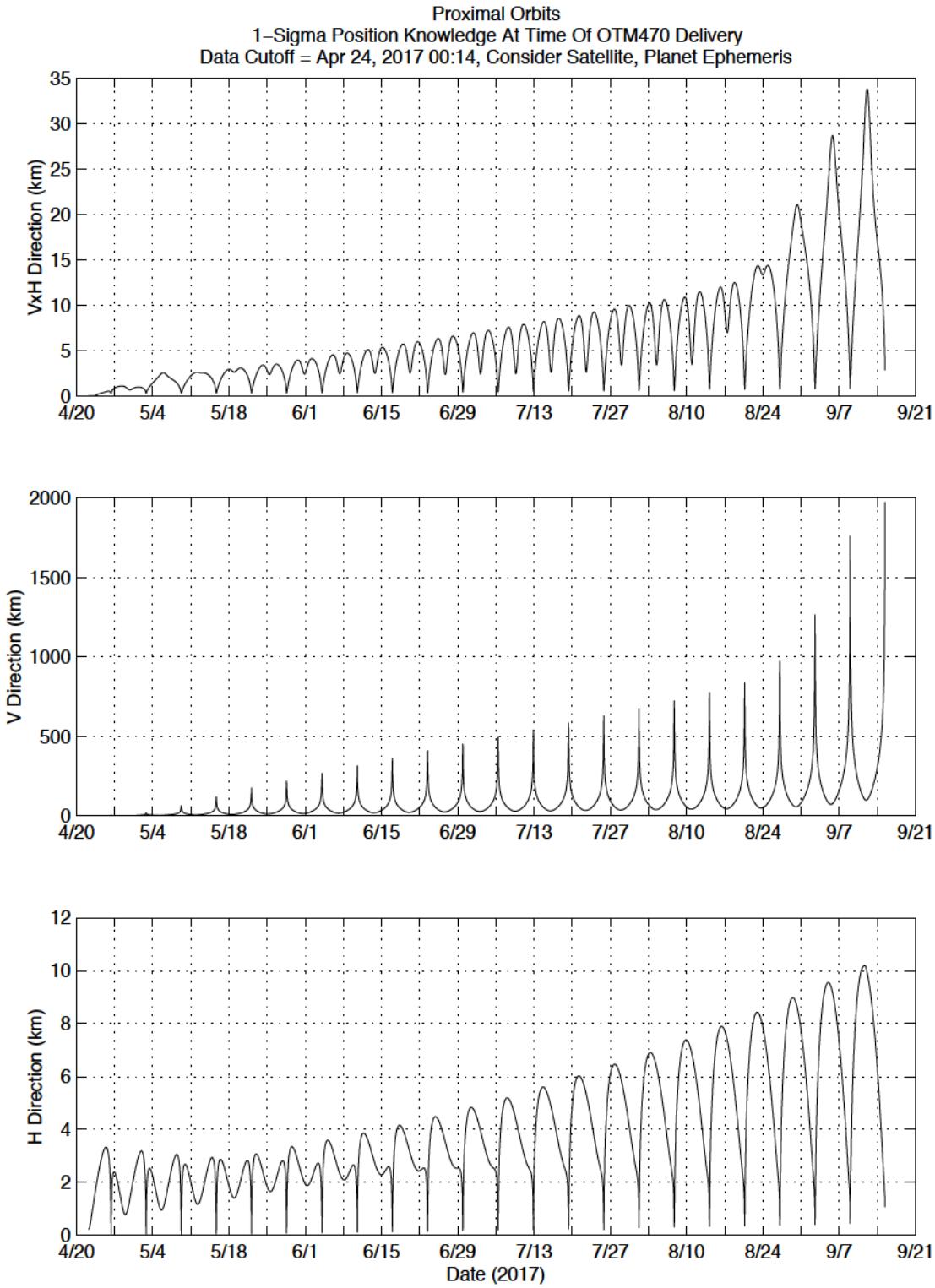

Figure 4: 1σ dispersions in three spatial dimensions starting with knowledge uncertainties at OTM470.

While the dispersion possibilities remain as large as shown previously, the additional knowledge after T126 narrows down the uncertainty of determining the actual trajectory. For

example, by the 22nd periapsis, the post-T126 knowledge uncertainties only grow to 1800 km along-track at the 1-σ confidence level. The steep decline in uncertainties from using an OTM 470 vs. OTM 469 data cutoff implies that the target miss at T126 is the primary cause of the large downstream dispersions seen in Figure 2. Note that in Fig.4 the relative rapid rise of uncertainties in the last few orbits is due to the effects of the uncertainties in atmospheric drag force, which are also present in the results shown in Fig. 2 but dominated by the T126 flyby uncertainties.

3. Control Model Description and Results

The goal of our dispersion-control analysis is to find a maneuver strategy that can reduce the expected dispersions. Specifically, periapsis-3, -14, and -16 (P3, P14, and P16) are the target points where dispersions are to be controlled below 250 km at the 68th percentile. We use 68%-tile as the measure because these are three-dimensional dispersions and the use of sigma (σ) may cause confusion when interpreting the results from distributions that are not necessarily Gaussian and that 1-σ represents different probability percentiles for different numbers of dimension. For these reasons, we choose the 68%-tile such that there will be no ambiguities and it is approximately equivalent to the 1-σ value if the distributions are more or less one- dimensional and close to being Gaussian in nature. In situations where only scalar quantities such as timing and radial uncertainties are concerned, we use the traditional 1-σ notations for one standard deviation (e.g., in Table 1, to be discussed below).

In operation a maneuver is designed based on the current OD solution that reflects the best knowledge of the spacecraft state (position and velocity). The ΔV (three velocity components) of the maneuver is then solved for iteratively by numerical integrating the trajectory to the target point downstream. Ideally such a maneuver would restore the three out of six state variables of interest back to their reference values at the target point. However, in reality, the OD knowledge and maneuver execution errors will invariably perturb the spacecraft from its intended trajectory and such errors will grow along with the propagation of the trajectory. Furthermore, if there are other events that have uncertainties downstream of the maneuver (e.g., RCS thruster events and atmospheric drag), the propagation errors will be further increased. Therefore, subsequent maneuvers may be needed to readjust the trajectory if the time lapse is long before the target point.

The complexity of modeling such scenarios lies in the intricacy of the various sources of uncertainty at different locations along the trajectory and how they may affect the ΔV search and trajectory propagation. Linear analysis, which relies on covariance mapping based on the unperturbed reference trajectory, could be applicable to the present problem but its validity cannot be easily established without first comparing to the results from a full, non-linear

analysis[1]. For instance, the linear assumptions of small deviations and independence of contributions from various error sources are not obvious here. To accurately model and statistically analyze the trajectory, a self-consistent numerical model that closely simulates the underlying physics of the problem is required. Consequently, the Cassini Navigation Team has developed a Monte Carlo model that incorporates accurate numerical propagation, iterative ΔV search, and direct simulation of random events.

The inputs for the model are: 1) Initial post-T126 Titan flyby dispersion, 2) OD knowledge uncertainty at the time of maneuvers, 3) Cassini specific maneuver execution error model, 4) a set of error distributions for various RCS thruster events, and 5) Uncertainty model for Saturn's atmospheric density. The model imposes no restrictions on the forms of probability distributions used; however, to facilitate exchange of input/output with other navigation software, Gaussian distributions are used for all uncertainty sources, except for the atmospheric density for which a log-norm distribution is adopted.

The Monte Carlo method presented here can be conceptually thought of as a "mission-by-mission" simulation with each sample trajectory represents a possible mission outcome. By repeating the process many times, an ensemble of outcome can then be formed and statistics of interests can be extracted. The mechanics of each single simulation process can be summarized as follows:

1. Sample an atmospheric density profile from the log-norm distribution
2. Sample the initial post-T126 covariance and obtain a perturbed state (position and velocity) of the spacecraft
3. Propagate the perturbed trajectory forward in time
4. At the time of an event, <u>either</u>
    a) it is an RCS thruster event:
        i. sample the RCS velocity error distribution,
        ii. add the sampled errors to the current spacecraft's state,
        iii. continue trajectory propagation.

    <u>or</u>

    b) it is an OTM:
        i. sample the OD knowledge error distribution for this OTM and add on to the best current OD estimate of the spacecraft's state,
        ii. perform an iterative ΔV search to find the impulsive velocity change required to realign the propagating trajectory back to the specified target point downstream,
        iii. add the computed ΔV and the associated execution errors to the current spacecraft's state,
        iv. continue trajectory propagation.

[1] We have performed selective comparisons between the results from linear and non-linear models. Except for areas where linear analyses are intrinsically inapplicable, e.g. non-Gaussian distributions and drag force modeling, linear models did provide adequate first-order approximations.

The above sequence completes one sample simulation for a leg of the trajectory. If more than one leg is required, the ensemble of states (position and velocity) at the end-point of the previous leg will be fed into the start-point of the subsequent leg (the legs are sequential in time).

Since we are trying to only control three target points in the entire trajectory, the 22-orbit mission is divided into three legs within each a single OTM is placed. Note the RCS thruster events (>70) are distributed along orbits; they can be before or after an OTM. The leg division and placement of the OTMs for the three target points in the present case are rather

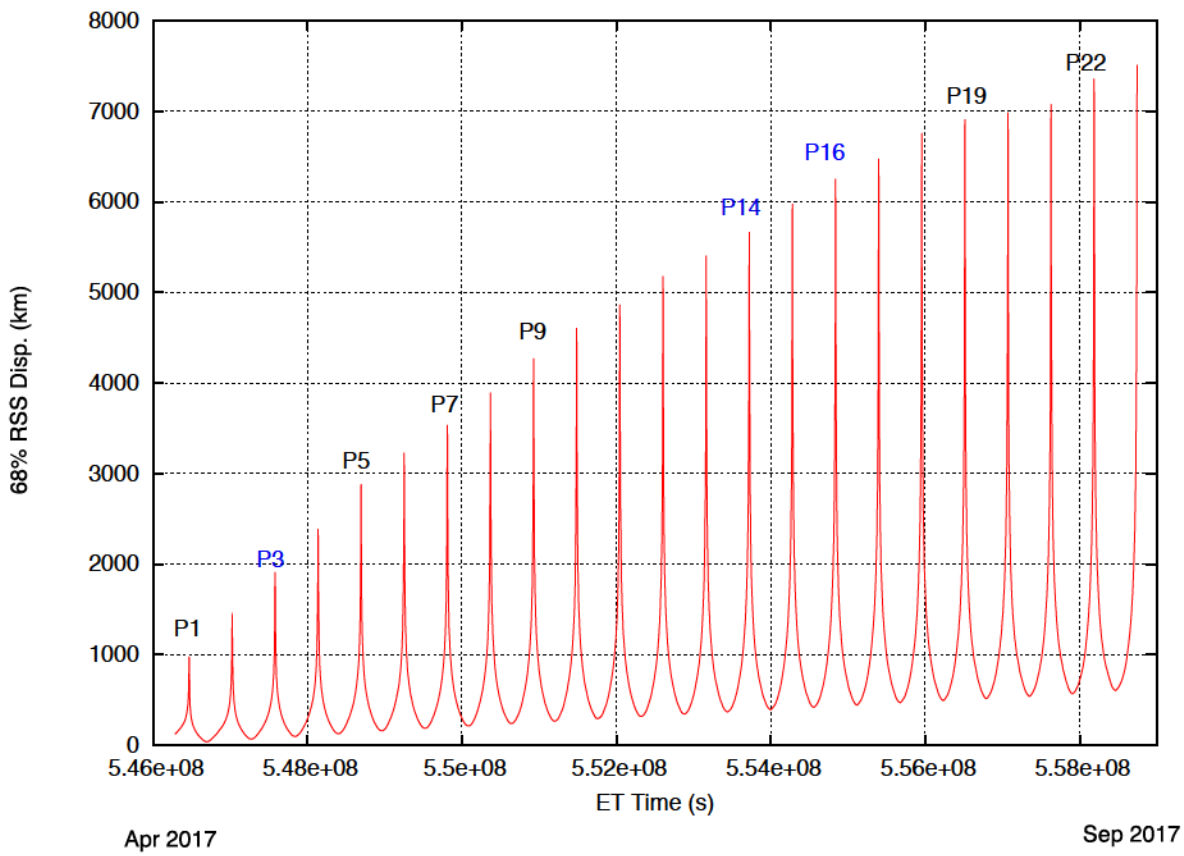

Figure 5: Overall uncontrolled dispersions for the 22 proximal orbits (P1 – P22). Peaks are locations of periapses.

straightforward and intuitive. However, as was initially considered for the mission, if the number of control and target points were to be increased, a more robust design algorithm would be required[2].

[2] In the early phase of the current study when the majority of the 22 orbits were to be controlled, we developed a brute-force design algorithm that evaluated all possible combinations of OTM locations and target points in terms of their ΔV costs and resulting dispersion control. The algorithm made use of linear mapping approximations for simplicity and efficiency. Once OTMs and target points were selected, the present Monte Carlo method was used to complete the analysis.

Figure 5 shows the overall 68% dispersion as a function of time for the whole mission without control. The propagation of trajectory starts shortly after T126 on April 24, 2017, and ends on September 15, 2017 before the spacecraft enters into the denser part of the atmosphere at which point the spacecraft will have been tumbled and become uncontrollable. This simulation shows the statistical results from 1000 sample trajectories. Each trajectory combines dispersion contributions from the initial T126 flyby, subsequent RCS thruster events, and uncertainties in atmospheric drag force. The drag force uncertainties are a combination of that of the atmospheric density, drag coefficient, and frontal area of the spacecraft. As seen, the dispersion can grow up to 8000 km; the desired values of < 250 km at P3, P14, and P16 are obviously not met. Recall that in the previous OD section (Fig. 2) the along-track (timing) component exhibits similar behavior, denoting the agreement, to the first-order, between linear and non-linear models. Table 1 lists the periapsis locations and their timings for the 22 orbits in the mission, for both the reference (unperturbed) trajectory and the perturbed trajectory (uncontrolled). (The controlled case in the last two columns of the table corresponds to results from the selected nominal control strategy, to be discussed below.)

Since P3 is the first location the dispersions at which are to be controlled, it is therefore the obvious choice of target point for the first OTM – OTM470. Because T126 flyby is the largest contributor to the overall dispersion, it serves to place the corrective maneuver as soon as possible after the flyby. The designated location for OTM470 is "24-APR-2017 17:52", roughly 2 days after T-126 flyby, allowing enough time for tracking and preparation for execution. This OTM-target point configuration constitutes the first leg of the mission in our analysis and the results are shown in Fig. 6. As can be seen, the dispersion reduces rapidly from over 1300 km at P1 to around 60 km at P3.

Table 1: Nominal and perturbed values at periapses. The 22 periapses in the mission are labeled in chronological order. The first four columns are periapsis values drawn from the reference trajectory (unperturbed). The next two columns are the timing and radial uncertainties for the uncontrolled case (ref. Fig. 5). The last two columns (shaded green) are for the controlled case.

				Uncontrolled		Controlled	
Periapsis	Date	Time (ET)	Radius (km)	σ_t (s)	σ_r (km)	σ_t (s)	σ_r (km)
P-1	26-Apr-2017	09:04:42	63173	28.8	28.6	40.8	3.9
P-2	02-May-2017	19:43:22	63148	41.9	28.6	20.4	3.9
P-3	09-May-2017	06:17:47	62926	55.8	28.6	2.0	3.9
P-4	15-May-2017	16:46:28	62873	70.0	28.7	2.5	5.3
P-5	22-May-2017	03:15:35	62918	84.1	28.7	2.7	5.3
P-6	28-May-2017	14:27:29	64111	93.8	30.3	2.6	5.4
P-7	04-Jun-2017	01:43:35	64099	103.8	30.3	2.7	5.3
P-8	10-Jun-2017	12:54:23	63623	113.7	30.6	3.0	5.5
P-9	16-Jun-2017	23:56:54	63561	123.4	30.6	3.3	5.4
P-10	23-Jun-2017	10:58:55	63601	133.1	30.6	3.7	5.4
P-11	29-Jun-2017	22:15:32	63930	140.9	31.1	4.2	5.5
P-12	06-Jul-2017	09:36:43	63934	148.6	31.1	4.8	5.5
P-13	12-Jul-2017	20:49:22	63058	155.7	31.2	5.4	5.8
P-14	19-Jul-2017	07:55:58	62995	163.8	31.2	2.5	6.4
P-15	25-Jul-2017	19:00:31	63012	171.8	31.3	1.4	6.4
P-16	01-Aug-2017	06:10:19	63116	179.2	31.5	1.9	6.5
P-17	07-Aug-2017	17:24:21	63138	186.3	31.5	3.8	6.5
P-18	14-Aug-2017	04:24:03	61905	192.2	30.9	5.8	6.9
P-19	20-Aug-2017	15:24:36	61849	197.3	30.9	9.4	6.9
P-20	27-Aug-2017	02:21:34	61821	201.8	31.1	19.0	6.9
P-21	02-Sep-2017	13:19:00	61830	205.8	31.2	36.3	6.9
P-22	09-Sep-2017	00:19:13	61864	210.6	31.2	60.4	6.9

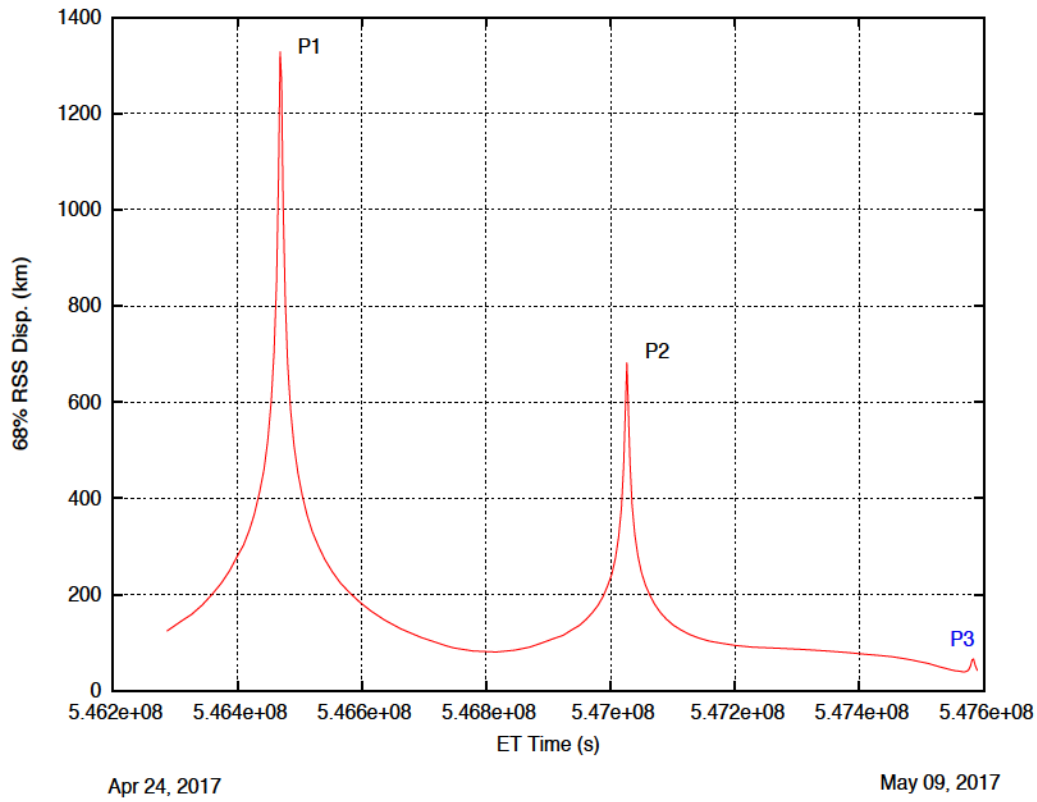

Figure 6: Controlled dispersion between T126 and P3 after OTM470.

Following the same rationale of putting the next maneuver as soon as possible but as least 1 day away from a periapsis location, the second OTM – OTM471 is designated to be at "10-MAY-2017 16:58", roughly 1.5 days after P3. Since the next desired target point is P14, and, intuitively, one would use it as the target point for this leg. However, as have been verified from our preliminary results (not shown), the resulting dispersion is slightly over 200 km at P14. While it will still fulfill the 250-km criterion, it is in general desirable to have more margins. This relatively large growth of dispersion after the first target point at P3 is due to the long propagation time between P3 and P14 within which there are a number of RCS thruster events. Consequently, we have selected P13 instead as the target point for this leg. The idea is to have another corrective maneuver before P14 to clean up these upstream dispersions. The result for this leg is shown in Fig. 7. As can be seen, the dispersion at P13 is already up to nearly 200 km.

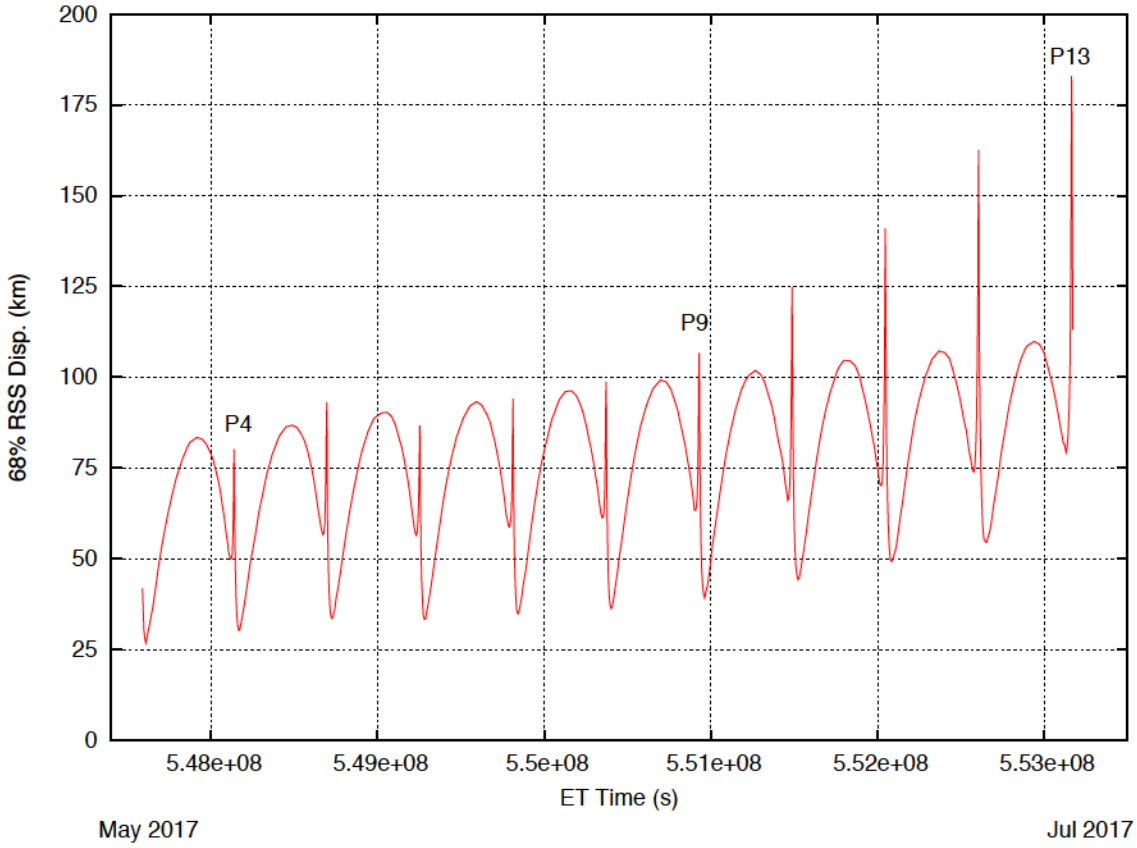

Figure 7: Controlled dispersion between P3 and P13 after OTM471.

The third OTM – OTM472 is designated at "15-JUL-2017 12:21", roughly 2.5 days after P13 (an extra day after the periapsis is added in this case to accommodate DNS tracking schedules). We could set the target point at P14 and add another leg and an OTM to target to P16, the last target point. However, numerical experiments have shown that there is no need for an additional leg and maneuver. We can directly target to P16 in this third leg, and have P14 and P15 under control as well. Therefore, the third leg uses P16 as the target point. Figure 8 shows the result for this leg. As can be seen, the dispersions at P14, P15, and P16 are well-controlled to be less than 100 km. Note in this figure (also in Fig. 7) that dispersion peaks are no longer necessarily at the periapses, but, rather, near the apoapses. This switching of "peaks and troughs" behavior is not uncommon for controlled trajectories, as we have seen throughout the Cassini tour in the last 11 years. When the overall dispersions are being controlled with corrective maneuvers, as in the present example, the peaks of dispersions are not necessary at or near the periapses.

After the last target point P16, no more OTMs are necessary. Therefore, a pure propagation of the trajectory from P16 to the end of the mission follows. Figure 9 combines all three legs and the propagation afterwards to give an overall picture of the controlled dispersions in the mission. As can be seen, the dispersions in the majority portion of the mission are actually below 250 km, with the exception of first two periapses right after T126 flyby and the last few periapses where

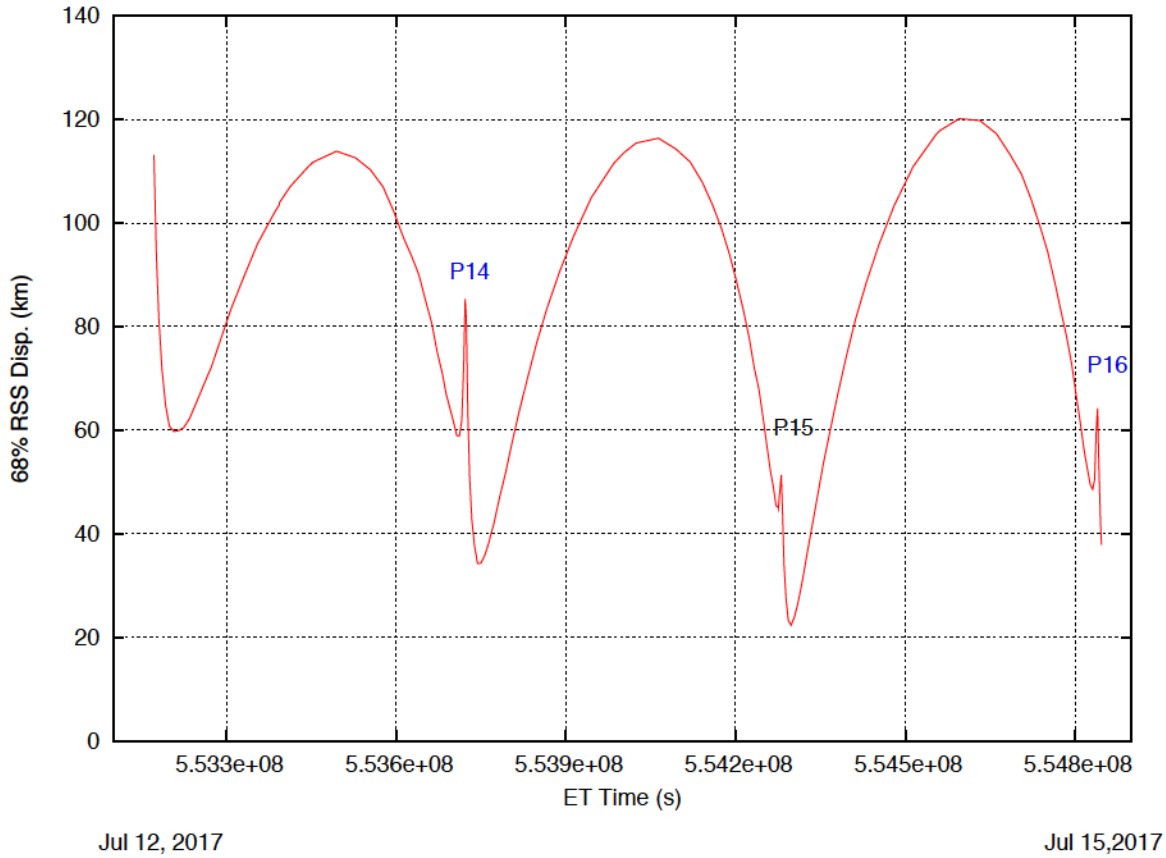

Figure 8: Controlled dispersion between P13 and P16 after OTM472.

the denser part atmosphere is encountered, i.e., perturbations from atmospheric drag increase correspondingly (note lower radii of periapsis near the end of the mission in Table 1). Also included in the figure are the three OTM locations and their respective target point, and a summary of total ΔV usage. Because of the relative large dispersions from the initial T126 flyby, OTM470 has the largest ΔV values, with the following two diminishing. The total 99th percentile usage is 1.94 m/s, estimated to be within the propellant margin at the end of the Cassini mission. The last two columns in Table 1 show the dispersions at periapses expressed in terms of the timing and radial distance for the controlled case. As can be seen, the overall dispersions have been significantly reduced with control. Again, the timing uncertainties are closely related to spatial dispersions shown in Fig. 9 because the dispersions are predominately in the along-track direction. That is, the 68%-tile spatial dispersion roughly equals 1-σ timing dispersion multiplied by the spacecraft's along-track velocity at perispsis (~ 34 km/s for all 22 orbits). The radial dispersions are, as in the uncontrolled case, relatively constant but reduced (1-σ at ~ 4-7 km). The tumble altitude – the altitude at which the atmospheric torque the spacecraft experiences exceeds the spacecraft's attitude control capability – has been estimated to be at a radial distance of ~ 61700 km, which is at least 120 km away from the closest nominal radial distance in the mission (P20; ref. Table 1). Therefore, in the uncontrolled case, the probability of tumbling is beyond 4 sigmas (< 6 x10^{-3} if assumed Gaussian distribution) and, in the controlled case, beyond 17 sigmas, practically a zero probability.

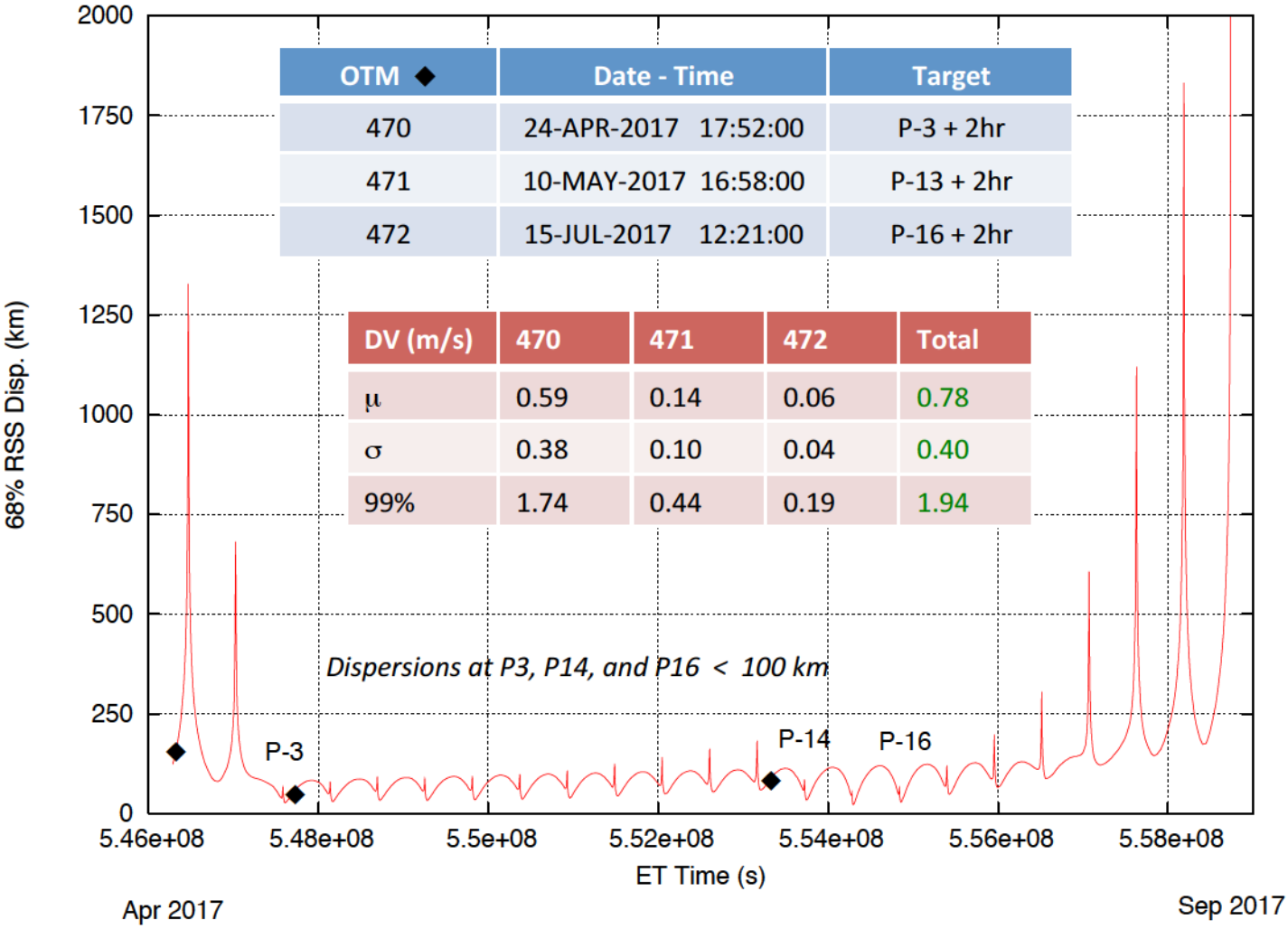

Figure 9: Controlled dispersion with the nominal 3-OTM strategy of the whole mission. Black diamonds denote locations of the 3 OTMs. Upper inset summarizes the OTMs and their respective target points. Lower inset summarizes the ΔV statistics of the 3 OTMs.

4. Conclusion

The Cassini Grand Finale Mission (proximal orbits) will commence with the last targeted Titan T126 flyby on April 22 2017, followed by 22.5 ballistic orbits around Saturn, and end on September 15 2017 with a deep dive into Saturn's atmosphere. To enhance the probability of success for this chapter of the Cassini journey that is expected to provide previously unavailable opportunities for science observations, a high fidelity dispersion control strategy for the trajectory is called for. The primary sources of dispersion are T126 flyby errors, OD knowledge errors, maneuver execution errors, RCS thruster firing errors, and the atmospheric drag force errors. T126 flyby errors, the first to occur and the largest contributor among these, are estimated using OD covariance method. The estimated error values are then used as initial inputs to a direct

simulation Monte Carlo model that simulates the subsequent perturbations and corrective maneuver computations in a self-consistent manner. Here we have reported the nominal maneuver strategy thus found. We have shown that with three strategically placed OTMs and a total ΔV of 1.94 m/s at the 99th percentile level, the dispersions at the three desired target points: P3, P14, and P16 can be controlled to a level of less than 100 km at the 68th percentile level. In fact, dispersions in 16 out of the 22 orbits can be brought below 250 km with this strategy. In addition, because of the relatively small dispersions in the radial direction (in both the controlled and uncontrolled cases), we have demonstrated that, statistically, the spacecraft will be captured by Saturn and that it will not be tumbled by the denser part of the atmosphere before the last, impacting orbit.

Acknowledgements

This research was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration.