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A SIMPLIFIED THEORY OF POROUS WALL COOLING

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I. INTRODUCTION AND SUMMARY

The rate of heat transfer between a fluid stream in turbulent flow and a smooth, solid wall is largely controlled by the relatively high resistance of the laminar sublayer next to the wall. Although this laminar layer is extremely thin, heat can be transferred through it only by molecular diffusion. Hence the resistance of this layer is very much greater than for a layer the same thickness farther out in the stream where turbulent exchange is the controlling factor. The thickness of the laminar layer is difficult to define precisely, since there is a gradual transition to the turbulent flow outside, but for the usual scale of many engineering applications almost half the temperature difference between the fluid and the wall occurs in a layer of a few thousands of an inch in thickness.

When the wall is made of porous material and a coolant gas is forced through the wall into the stream, it has been found (Cf. Ref. 1) that a very small flow rate of the coolant is remarkably effective in keeping the wall at a low temperature. The coolant flow rate required is such as to give an average velocity normal to the wall of the order of 1 per cent of the main stream velocity. This flow rate is so low that clearly the injected gas must act as an insulator rather than as a normal coolant. Because of its relatively low velocity, the injected gas can have very little influence on heat convection or momentum transfer in the turbulent stream, and its effect must be confined to the laminar sublayer. The possible influence of the coolant flow on the thickness of the laminar layer will be discussed in Section V.

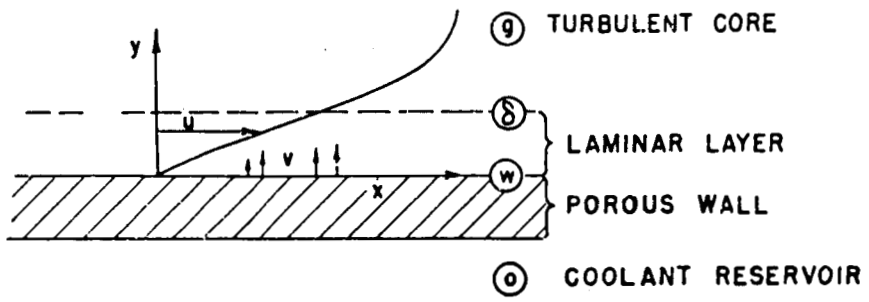
Theoretical studies of boundary layers with flow through the wall have been made recently in Germany but have been confined to the case of the laminar boundary layer in a turbulent-free stream. The interest was directed toward means of reducing skin friction by preventing transition, and most attention was confined to removal rather than to injection of fluid. In engineering applications of porous wall cooling, on the other hand, the cases of turbulent stream flow and the turbulent boundary layer will probably prove most important, but apparently little attention has been given to these problems. On the basis of dimensional analysis, Wheeler (Cf. Ref. 2) was led to the conclusion that the wall temperature is a function of the ratio of coolant mass flow per unit area to main stream mass flow per unit area alone. Measurements on a short length of porous-walled pipe indicate that this ratio is the principal parameter determining the wall temperature.

The analysis given here leads to a method for predicting the temperature of the wall from the friction coefficient for turbulent flow in a pipe. The method is an extension of that used by Prandtl for no coolant flow.

II. VELOCITY DISTRIBUTION IN THE LAMINAR LAYER

The velocity distribution in the laminar sublayer can be determined very easily. Taking x as the coordinate along the wall in the direction of mean flow and y as the distance from the wall, the laminar layer is assumed to extend from the wall to $y = \delta$. Steady flow is assumed and all deriva-

tives with respect to x are zero.* The pores of the wall are supposed sufficiently small and numerous that the velocity normal to the wall is uniform and continuous. The velocity parallel to the wall is u with v normal to the wall. It will be convenient to use subscripts g, δ, w, and o for the stations in the main gas stream, at the outer edge of the laminar layer, at the wall, and in the coolant reservoir, respectively (Cf. sketch below).



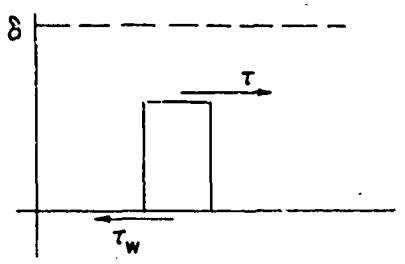
The shear stress τ (Cf. sketch on p. 4) for a surface normal to the y-axis and at a distance y ($<\delta$) from the wall is given by

$$\tau = \mu \frac{du}{dy} \tag{1}$$

where μ is the viscosity coefficient. Consider a cylinder of unit area of cross section with generators normal to the wall and extending from the wall to height y. Since the resultant force on the sides of the cylinder must vanish, the total force on the surfaces is the resultant of the shear

*This latter assumption is equivalent to saying that the thickness of the boundary layer remains constant along the wall, and hence is similar to fully developed turbulent flow in a pipe. The coolant flow is considered sufficiently low so that the average pipe velocity and the pressure gradient along the pipe are not affected appreciably.

forces on the ends and amounts to $\tau - \tau_w$ in the x-direction. The total momentum flux through the sides is zero since there is no variation with x. The flux of x-momentum through the end at y = 0 is zero since u = 0



at the wall, whereas at the upper end the flux of x-momentum outward is ρ vu since ρv is the mass flow rate through the unit surface and u is the velocity in the x-direction. Equating the force in the x-direction to the rate of change of momentum in this direction, and using the relation (1)

$$\tau - \tau_w = \mu \frac{du}{dy} - \mu \left(\frac{du}{dy}\right)_w = \rho vu \tag{2}$$

Since derivatives with respect to x are zero, the continuity equation reduces to

$$\frac{d}{dy}(\rho v) = 0$$

or

$$\rho v = \rho_w v_w = \text{constant} \tag{3}$$

Equation (2) can also be derived from the Navier-Stokes equation for the momentum parallel to the wall, which in this case becomes simply

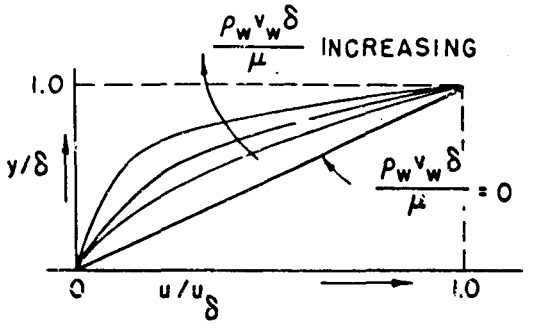
$$\rho v \frac{du}{dy} = \mu \frac{d^2u}{dy^2} \tag{4}$$

and clearly Equation (2) is the first integral of Equation (4). If the

viscosity coefficient μ is assumed constant in the laminar layer, Equation (2) can be integrated to give the velocity u. The most convenient form is

$$\frac{u}{u_\delta} = \frac{e^{\frac{\rho_w v_w y}{\mu}} - 1}{e^{\frac{\rho_w v_w \delta}{\mu}} - 1} \tag{5}$$

where the boundary conditions $u = 0$ at $y = 0$ and $u = u_\delta$ at $y = \delta$ are applied. The shapes of the resulting velocity profiles as functions of y/δ are sketched below.



III. TEMPERATURE DISTRIBUTION IN THE LAMINAR LAYER

The temperature profile in the laminar sublayer can be derived in a manner similar to that used for the velocity profile. The heat flow rate per unit area normal to the y-axis toward the wall is denoted by q and for $y < \delta$

$$q = k \frac{dT}{dy} \tag{6}$$

where k is the conductivity for the fluid in the laminar layer and T is the temperature at a distance y from the wall. The fluid in the laminar sub-

layer can be considered as originating in the coolant reservoir at a temperature T_o. When the coolant fluid arrives at the height y, its temperature has risen to T; hence the heat added per unit mass is $c_{p_w}(T - T_o)$ where c_{p_w} is the average specific heat between T_o and T. Since the mass flow rate per unit area normal to y is $\rho v = \rho_w v_w$, the rate of heat flow per unit area toward the wall is $\rho_w v_w c_{p_w}(T - T_o)$. But this heat flow rate must be q as given by Equation (6). Hence

$$k \frac{dT}{dy} = c_{p_w} \rho_w v_w (T - T_o) \tag{7}$$

This equation can also be derived from the energy equation which, on neglecting minor terms, becomes

$$\rho v c_{p_w} \frac{dT}{dy} = k \frac{d^2T}{dy^2} \tag{8}$$

and clearly Equation (7) is the first integral of Equation (8). If c_{p_w} and k are assumed constant in the laminar layer, the integration of Equation (7) with the additional boundary condition $T = T_w$ at $y = 0$ gives the relation

$$\frac{T - T_o}{T_w - T_o} = e^{\frac{\rho_w v_w c_{p_w} y}{k}} \tag{9}$$

The exponent occurring in this expression differs from the exponent in the expression (5) for the velocity u by a factor of the Prandtl number $\sigma = (c_{p_w} \mu)/k$. Hence Equation (9) can be written in the form

$$\frac{T - T_o}{T_w - T_o} = e^{\sigma \frac{\rho_w v_w y}{\mu}} \tag{10}$$

To compare the temperature profile with the velocity profile, Equation

(10) can be used to derive the relation

$$\frac{T - T_w}{T_\delta - T_w} = \frac{e^{\sigma \frac{\rho_w v_w y}{\mu}} - 1}{e^{\sigma \frac{\rho_w v_w \delta}{\mu}} - 1} \tag{11}$$

and it is seen that the temperature and velocity profiles are of similar shapes and furthermore, if $\sigma = 1$, are of identical shapes.

IV. JOINING OF LAMINAR AND TURBULENT REGIONS

Outside the laminar layer (i.e., $y > \delta$), the turbulent fluctuations in the stream cause diffusion of material, heat, and momentum in a manner somewhat similar to molecular diffusion but with very much higher rates. In regions where the velocity gradient of the mean speed is high, the rate of transport of any property across the stream can be taken as roughly proportional to the velocity gradient. Reynolds used this approach to obtain a relation between momentum transfer and heat transfer across a turbulent stream. This relation can be expressed in the form

$$\frac{q}{\rho\, c_p (T_m - T)(u_m - u)} = \frac{\tau}{\rho (u_m - u)^2} \tag{12}$$

where q is the heat-transfer rate per unit area toward the wall; τ is the shearing stress per unit area; u_m and T_m are velocity and temperature of the main stream flow; and u and T, the velocity and temperature at the point where q and τ are measured. (Cf. p. 649 of Ref. 3 for a complete discussion.)

Although there is actually no sudden change from laminar to turbulent

flow at $y = \delta$, this point will be identified as the point where the turbulent diffusion rate becomes appreciable and Equation (12) will be applied to the entire region $y \geqslant \delta$. Hence at $y = \delta$ the relation between heat flow rate and shearing stress is

$$\frac{q_\delta}{c_p(T_g - T_\delta)} = \frac{\tau_s}{(u_g - u_\delta)} \tag{13}$$

where $(\ )_g$ refers to the main stream far outside the laminar layer and $(\ )_\delta$ refers to the station $y = \delta$. In this case c_p is assumed to remain fairly constant between the main stream and the laminar layer.

From the relations derived in Section III, putting $y = \delta$, expressions for T_δ, q_δ, and τ_δ can be found. These are

$$\left.\begin{aligned} T_\delta &= T_o + (T_w - T_o)\, e^{\sigma \frac{\rho_w v_w \delta}{\mu}} \\ q_\delta &= k\left(\frac{dT}{dy}\right)_\delta = c_{p_w}\rho_w v_w (T_w - T_o)\, e^{\sigma \frac{\rho_w v_w \delta}{\mu}} \\ \tau_\delta &= \mu\left(\frac{du}{dy}\right)_\delta = u_\delta \rho_w v_w \frac{1}{1 - e^{\frac{-\rho_w v_w \delta}{\mu}}} \end{aligned}\right\} \tag{14}$$

Substituting for these three quantities in Equation (13), an expression for T_w is found in the form

$$\frac{T_w - T_o}{T_g - T_o} = \frac{e^{-\sigma \frac{\rho_w v_w \delta}{\mu}}}{1 + \frac{c_p}{c_{p_w}}\left(\frac{u_g}{u_\delta} - 1\right)\left(1 - e^{\frac{-\rho_w v_w \delta}{\mu}}\right)} \tag{15}$$

The quantities δ and u_δ are still unknown. Some method of evaluating these two quantities must be found if Equation (15) is to be useful.

V. EVALUATION OF δ AND u_δ

At the present time it is not known how the boundary layer thickness δ varies with the velocity v_w normal to the wall. However, since in engineering applications the range of most interest for the ratio of v_w/u_m is from 0.005 to 0.020, it seems plausible to assume that the thickness of the laminar layer and the velocity at the outer edge of the layer are not affected appreciably by the low velocity normal to the wall. The thickness of the laminar layer has been measured in smooth pipes and found to satisfy the relation

$$\frac{\delta\, u_\tau}{\nu} = y^* \tag{16}$$

where $u_\tau^2 = \tau_o/\rho$ (τ_o is the shearing stress at the wall) and y^* is a constant. Prandtl has taken $y^* = 5.6$ after examination of the velocity profile measured close to a wall.

It will be assumed that the flow in the turbulent core is not affected by the velocity normal to the wall, and hence that the shearing stress τ and the velocity u_δ at the edge of the core are the same as for flow in a smooth pipe. This is a rather bold assumption and will almost certainly require modification. The only excuse for making it here is that no experimental information is available at present. It is the simplest postulate which reduces to the correct form when the velocity normal to the wall is zero.

For the Reynolds number range $5000 < Re < 200{,}000$ the friction coefficient C_F for smooth pipes satisfies the empirical relation

$$C_F = 0.046/Re^{0.2} \tag{17}$$

where $C_F = \tau_o/\frac{1}{2}\rho\bar{u}^2$ and $\bar{u}$ is the average velocity. Identifying $\bar{u}$ with u_g the following expressions are obtained

$$\left.\begin{aligned} u_\tau &= \sqrt{\tau_o/\rho} = u_g\sqrt{C_F/2} \\ \delta &= \frac{\nu y^*}{u_g}\sqrt{2/C_F} \\ u_\delta/u_g &= \frac{\tau_o\delta}{u_g\mu} = y^*\sqrt{C_F/2} \end{aligned}\right\} \tag{18}$$

where $y^* = 5.6$ and C_F is given by Equation (17) with the Reynolds number $Re = Du_g/\nu$ based on the flow in the turbulent core. Here D is the pipe diameter.

Substituting the expressions (18) in Equation (15) for the wall temperature

$$\frac{T_w - T_o}{T_g - T_o} = \frac{e^{-\sigma\frac{\rho_w v_w}{\rho_g u_g}y^*\sqrt{C_F/2}}}{1 + \frac{c_p}{c_{p_w}}\left(\frac{1}{y^*} + \sqrt{\frac{2}{C_F}} - 1\right)\left(1 - e^{\frac{-\rho_w v_w}{\rho_g u_g}y^*\sqrt{C_F/2}}\right)} \tag{19}$$

In this expression some effort has been made to keep the properties of the gas in the turbulent core separate from those in the laminar layer, but the process has not been entirely consistent. The expression would be expected to be correct for the limiting case of very small v_w and very small temperature drop $T_g - T_o$. The application to cases where the temperature drop to the wall is large so that physical properties of the gas change across the laminar sublayer or to cases where the injected gas is quite different from the main stream gas may not be satisfactory.

VI. COMPARISON WITH EXPERIMENT

Duwez has measured the wall temperature in a porous tube for several flow rates and temperatures of coolant and main flow (Cf. Ref. 3). The experimental results are shown in Figures 1 through 4. The tests were made with very short specimens $1\frac{1}{2}$ inches in length and 1 inch in diameter. In the Figures, $Q = \rho_w v_w$ and $W = \rho_g u_g$ in terms of the notation used in the calculations of Section V. The ratio $(T_g - T_w)/(T_g - T_o)$ is plotted rather than the expression given in Equation (19), which is $1 - (T_g - T_w)/(T_g - T_o)$. Four different materials were used in the porous specimens: (a) mullite, a refractory material with very low thermal conductivity, (b) stainless steel and (c) nickel with approximately equal conductivities, and (d) copper with very high conductivity. The main stream gases were products of combustion of gasoline burned in air, and the coolant gas was nitrogen. Further details can be found in Reference 1. The Reynolds numbers based on the main stream flow ranged from 30,000 to 100,000.

The expression for the temperature ratio determining the theoretical wall temperature can be derived from Equation (19) in the form

$$\frac{T_g - T_w}{T_g - T_o} = 1 - \frac{e^{-0.85\sigma(Q/W)Re^{-0.1}}}{1 + \left(1.18\,Re^{0.1} - 1\right)\left(1 - e^{-0.85(Q/W)Re^{-0.1}}\right)} \tag{20}$$

where $c_{p_w} = c_p'$, $y^* = 5.6$, and $C_F = 0.046\,Re^{-0.2}$. For nitrogen $\sigma = 0.74$. The temperature ratio has been evaluated as a function of Q/W for the two extreme Reynolds numbers in the experiments. In Figures 1 to 4, inclusive, the upper solid line is for $Re = 1.0 \times 10^5$ and the lower line for $Re = 0.3 \times 10^5$.

On comparing the theoretical curves with the experimental results, it is seen that the shapes of the curves are correct and the order of magnitude of the wall temperatures is correct. For the mullite specimen (Cf. Fig. 1) the theory is remarkably good at the smaller coolant flows but deviates somewhat at larger flows. This deviation may be due to the larger coolant flows affecting the thickness of the laminar sublayer, whereas in the theory it is assumed that there is no effect. The wall temperatures measured in the stainless-steel and nickel specimens (Cf. Figs. 2 and 3) are appreciably higher, and for the copper specimen (Cf. Fig. 4) much higher than the theory predicts. It is believed that this discrepancy is largely a result of temperature variations along the length of the specimen both in the laminar sublayer and the specimen itself.

In the theoretical treatment, it was assumed that temperatures and velocities were functions of distance from the wall only. Actually there is very likely an "inlet length" for the porous material where the temperature distribution changes from that typical of flow in an uncooled pipe to the final distribution for the porous wall cooling. This inlet length will vary depending on the conductivity of the wall material and will be large for a material of high conductivity. The theory assumes that there is no temperature gradient and hence no heat conducted along the wall. In the experiments, the wall temperatures were measured near the downstream end of the specimen, about $1\frac{1}{2}$ diameters from the upstream end. Further experiments with much greater ratios of length to diameter

are required in order to determine whether or not the theory fits better at stations farther downstream.

In the analysis it has been assumed that the coolant flow normal to the wall is uniform. Actually, the coolant is injected from pores as a series of jets and it would be expected that the theory would apply only to cases where the pores are very small and densely packed. Further analysis on this point is required.

VII. CONCLUSIONS

In conclusion the following remarks concerning the investigation can be made:

1. A simple theory for the process of porous wall cooling where the coolant gas has the same physical properties as the main stream gas is presented. The only empirical data required involve the well-established fluid mechanical laws for turbulent flow in smooth pipes. The theory predicts the wall temperature remarkably well when there is no temperature gradient along the wall.
2. Further refinements of the analysis should be made to take account of the introduction of coolant as jets from a finite number of pores rather than as uniform flow from the wall, and to estimate the influence of temperature gradient along the wall for porous materials of high conductivity.

4. Measurements of main stream velocity profiles and laminar sublayer thicknesses as affected by rate of coolant flow through the porous walls are required. The measurements can be made first for isothermal conditions but should be extended as far as possible into the laminar sublayer; they should be made also for coolant gases differing in physical properties from the main stream gas.

5. Theories of heat transfer through films in which the physical properties (density, viscosity, conductivity) vary across the film can be really satisfactory only after a method for predicting the thickness of the laminar sublayer under these conditions is developed. At present the stability conditions for the laminar sublayer are not known. The solution of this problem is important for all applications in which large temperature differences occur.

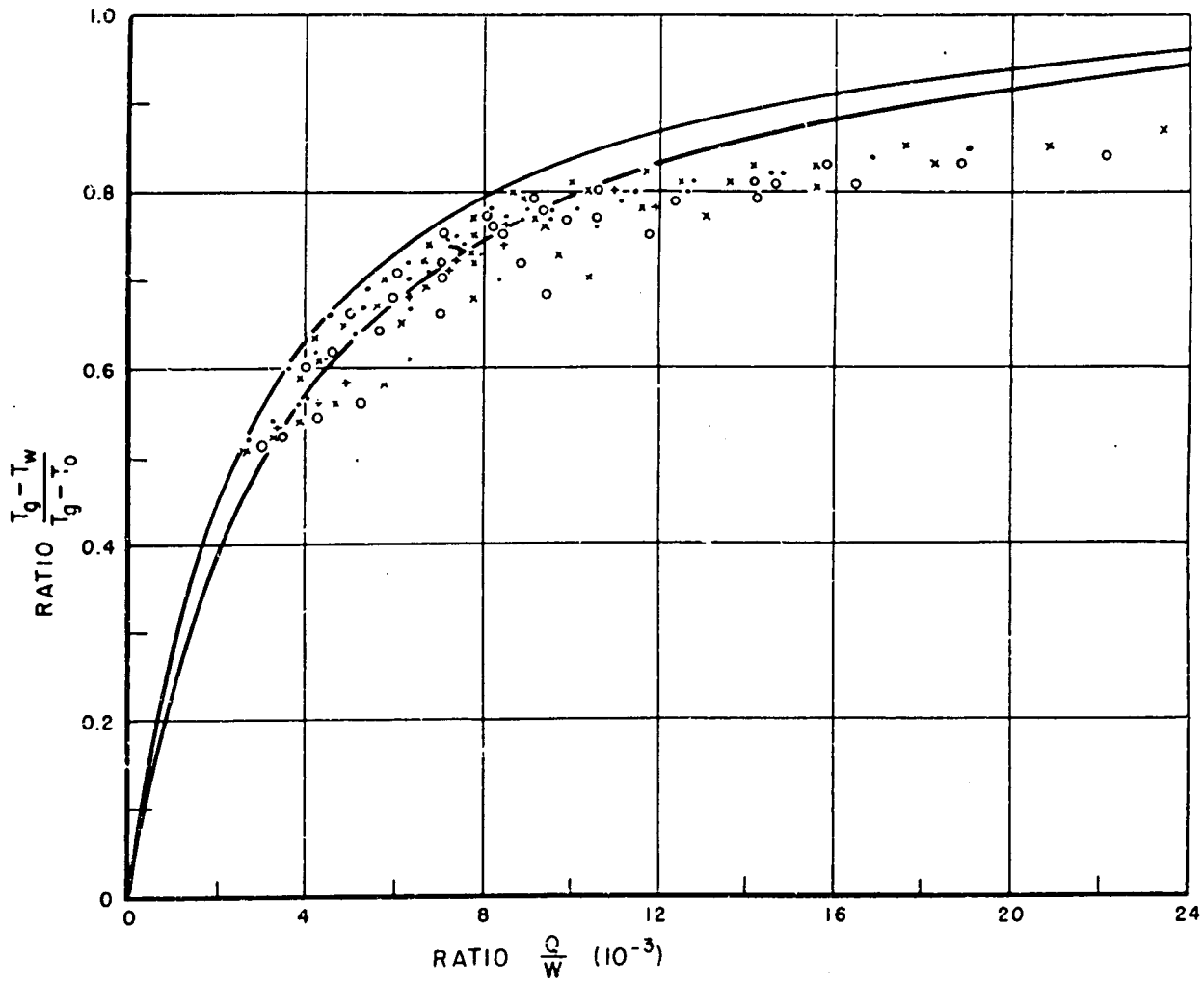


Figure 1. Ratio $(T_g - T_w)/(T_g - T_o)$ vs Ratio Q/W for Porous Mullite Specimen

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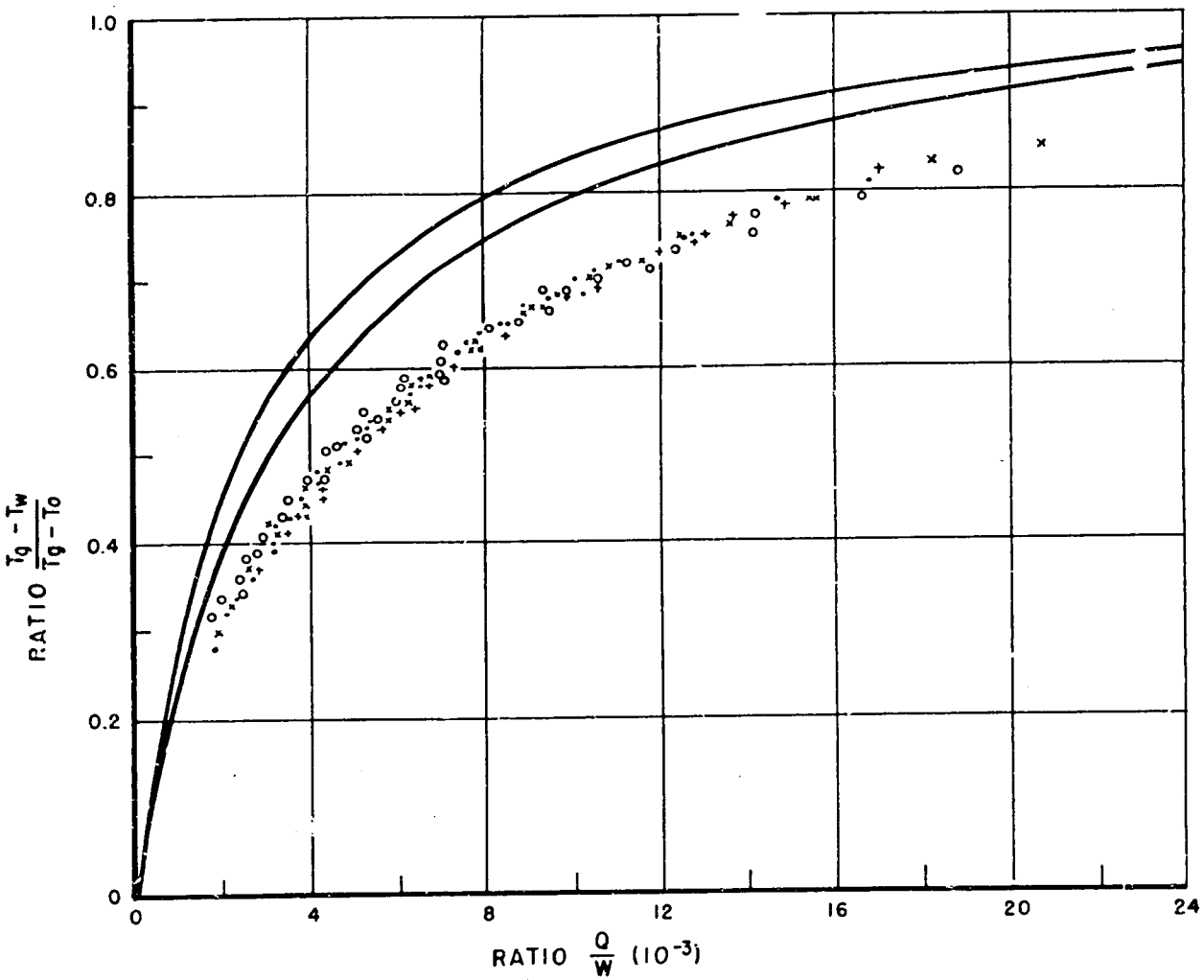

Figure 2. Ratio $(T_g - T_w)/(T_g - T_o)$ vs Ratio Q/W for Porous Stainless-Steel Specimen

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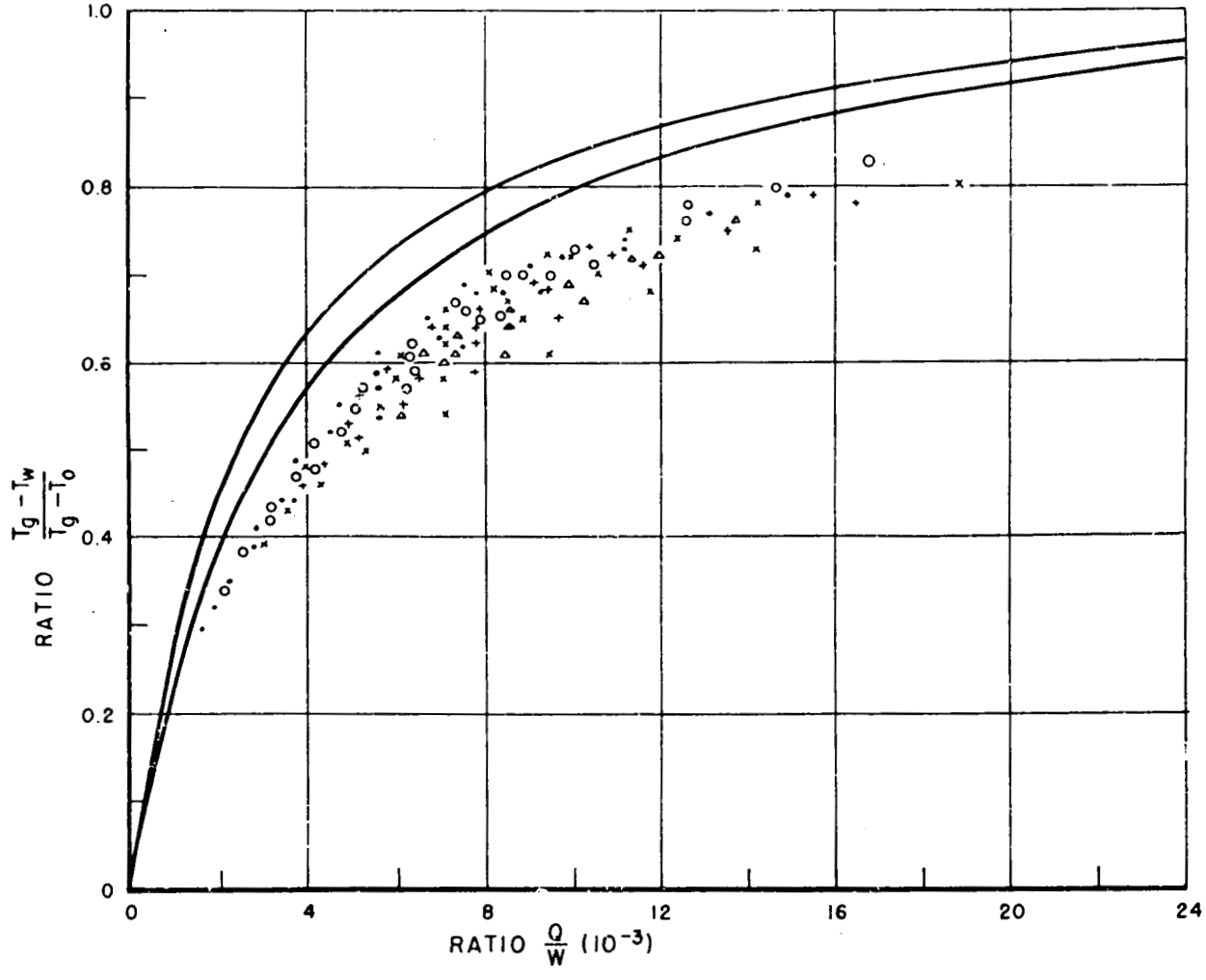

Figure 3. Ratio $(T_g - T_w)/(T_g - T_o)$ vs Ratio Q/W for Porous Nickel Specimen

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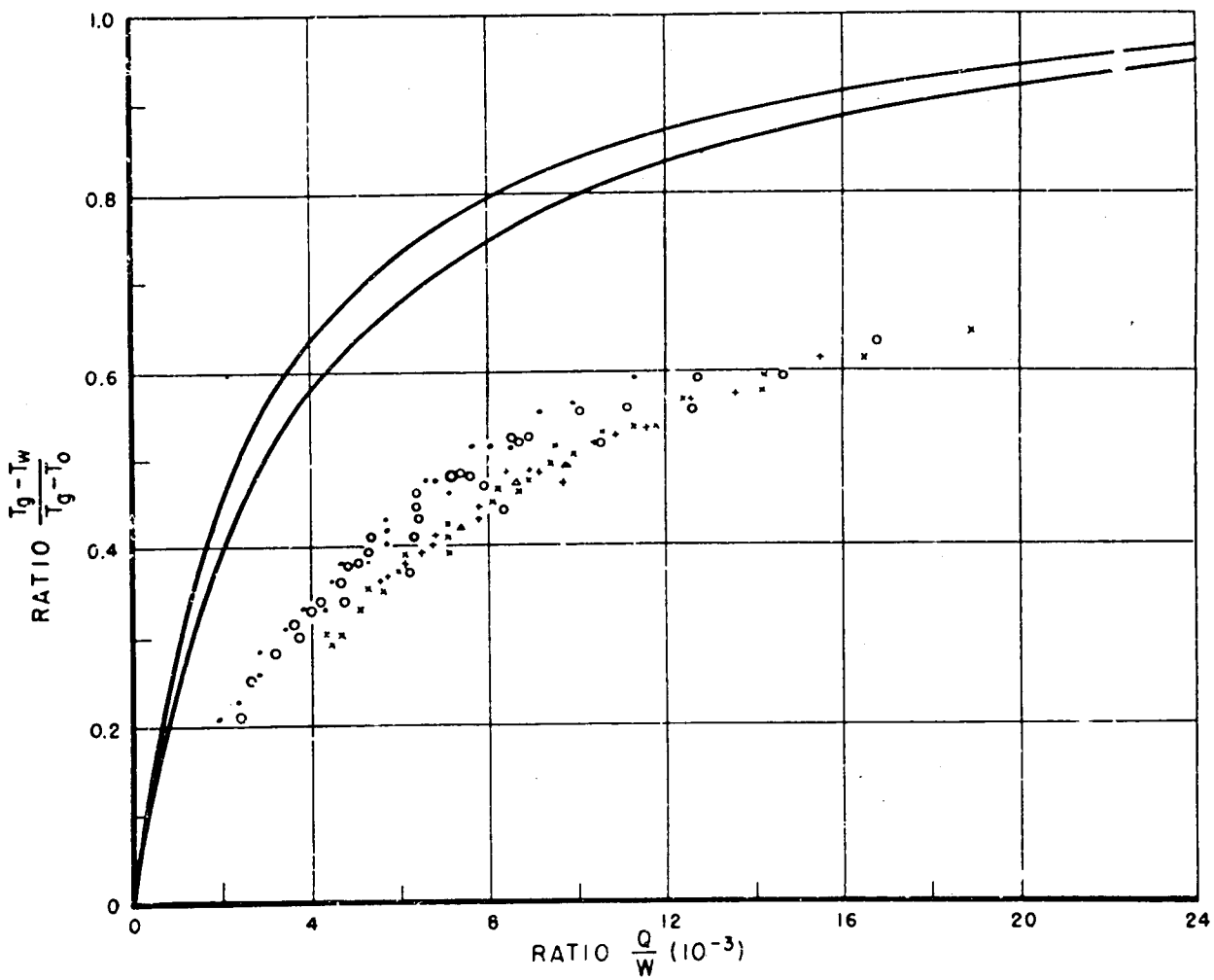

Figure 4. Ratio $(T_g - T_w)/(T_g - T_o)$ vs Ratio Q/W for Porous Copper Specimen

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