Titan Density Reconstruction Using Radiometric and Cassini Attitude Control Flight Data

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This paper compares three different methods of Titan atmospheric density reconstruction for the Titan 87 Cassini flyby. T87 was a unique flyby that provided independent Doppler radiometric measurements on the ground throughout the flyby including at Titan closest approach. At the same time, the onboard accelerometer provided an independent estimate of atmospheric drag force and density during the flyby. These results are compared with the normal method of reconstructing atmospheric density using thruster on-time and angular momentum accumulation. Differences between the estimates are analyzed and a possible explanation for the differences is evaluated.

Nomenclature

F/D Slope	=	On-board estimate of peak external torque
FSDS	=	Flight Software Dynamic Simulation
HGA	=	High Gain Antenna
INMS	=	Ion and Neutral Mass Spectrometer
RCS	=	Reaction Control System
RWA	=	Reaction Wheel Assembly
TCA	=	Titan Closest Approach

I. Introduction – Cassini Mission to Saturn and Titan

A. Overview of Cassini Mission

The Cassini-Huygens mission is a collaborative effort between the National Aeronautics and Space Administration (NASA), the European Space Agency (ESA), and the Italian Space Agency (ASI) to explore the Saturnian system. Launched in October 1997 onboard the powerful Titan IVB/Centaur, the Cassini-Huygens spacecraft still required multiple gravity assist flybys of Venus, Earth, and Jupiter to achieve the necessary velocity to reach Saturn. On June 30, 2004, Cassini achieved orbital capture at Saturn by firing its main engine for 85 minutes to slow its velocity by about 626 m/s. About 5 months later, the 320 kg ESA-built Huygens probe was successfully deployed from Cassini, and descended into Titan's atmosphere. Following playback of the probe's collected atmospheric data to Earth, Cassini then continued its primary mission of studying the Saturnian system via its suite of 12 remote and direct sensing science instruments.

B. Saturn's Moon Titan

Titan is an intriguing world larger than our own moon with a dense atmosphere and liquid hydrocarbon lakes on its surface. Because Titan is less massive than Earth, its gravity does not hold on to its gaseous atmosphere as tightly as Earth does, so its atmosphere extends roughly ten times farther into space than Earth's atmosphere. At the surface of Titan the atmospheric pressure is approximately 1.5 times greater than that of Earth at sea level with atmospheric density a factor of 4 larger. However, the chemical composition of Titan's atmosphere differs from Earth's; it is

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primarily composed of nitrogen (95%) and methane (5%) with traces of other carbon-rich compounds. High in Titan's atmosphere the Sun's ultraviolet light and high-energy particles coming from Saturn's magnetic field split apart methane and nitrogen molecules. This process produces a rich mixture of organic chemicals. Titan's temperature is so cold (-290 deg Fahrenheit) that methane exists as a solid, liquid and gas, analogous to how water exists in all three phases on Earth. Methane, instead of water, forms Titan's clouds, rivers, and lakes. This complex methane cycle, analogous to Earth's water cycle, makes Titan one of the most interesting places to study because it gives scientists a representation of what Earth might have been like early in our planet's history, before the appearance of life. Studying Titan could provide insight into how Earth's atmosphere has changed through the ages.

II. Titan's Atmosphere and Past and Present Methods of Characterizing it

Titan has a mysterious and rather dynamic atmosphere[1]. One quantitative metric for characterizing Titan's atmosphere is the reconstruction of its atmospheric density from in-situ instrument data and engineering flight data as it varies with altitude and time. Throughout the different phases of the Cassini mission, there have been several independent sources of data collected by Cassini, employed in the reconstruction of Titan's atmosphere as it varies with time and altitude. An early source of atmospheric data came from the insertion of the Huygens probe into Titan's atmosphere. Another source of atmospheric data comes from Cassini's Ion and Neutral Mass Spectrometer (INMS) instrument. Cassini has flown through the upper reaches of Titan's atmosphere (as low as 878 km altitude) several dozen times since January of 2005. During these flybys through Titan's atmosphere, the INMS instrument has provided in-situ measurements of atmospheric composition as well as density.

Another source of atmospheric data can be derived from engineering telemetry of thruster firings during low Titan flybys where spacecraft attitude is controlled by Reaction Control System (RCS) thrusters. A considerable amount of thrusters firings during a low Titan flyby are required to counteract Titan atmospheric torque. Thruster firing telemetry can be used to infer atmospheric torque, and subsequently, reconstruct atmospheric density[1]. We refer to this as the "normal" method because thruster telemetry is played back for all low-Titan flybys. This method uses conservation of angular momentum to infer atmospheric torque.

During a few carefully-planned Titan encounters, real-time radiometric tracking on Earth can provide an independent measurement of drag force from which Titan's atmospheric density can be deduced. Additionally, the onboard accelerometer on Cassini can be used to estimate atmospheric drag force. These measurements are advantageous because drag force estimates do not involve an "aerodynamic moment arm" estimate while thruster torque density reconstruction does require such an estimate.

Real-time radiometric data acquisition requires the Cassini High Gain Antenna (HGA), which is aligned with the –Z spacecraft body axis, to be pointed at Earth throughout Titan Closest Approach (TCA). For science collection purposes, the desired spacecraft attitude is almost always something other than Earth-point, so very few Titan flybys have actually resulted in detailed radiometric measurements. Through November 2014, there have been only two flybys for which such data was obtained, T70 flown June 21, 2010 with TCA altitude at 878 km, and T87 flown on November 13, 2012 with TCA altitude of 973 km.

The radiometric and accelerometer data from T87 was especially valuable because Cassini has performed many Titan encounters with closest approach near this altitude. T87 provides a good "independent" check on the normal method of density reconstruction involving conservation of angular momentum. As this paper will show, some new insight about the accuracy of the normal method can be deduced by comparing the T87 thruster firings with the radiometric and accelerometer data from the same flyby.

III. The Cassini Spacecraft

Cassini is a 3-axis stabilized spacecraft[2] with an 11-m magnetometer boom and three 10-m Radio and Plasma Wave Science antennas (see Fig. 1). Cassini has a body-fixed 4 meter diameter Cassegrain HGA parabolic reflector dish for telecommunications. Uplink and downlink use X-band for commanding, telemetry, and radiometry. Downlink also uses Ka-band for radiometric tracking. On Cassini, radiometric tracking include two-way Doppler and two-way (turnaround) ranging. S-Band was used to receive Huygens probe data during the relay and is used by radioscience throughout the mission.

At launch, the total spacecraft mass was 5560 kg of which 3000 kg was liquid bi-propellant and 132 kg was hydrazine. As of November 2014, the mass of Cassini is about 2250 kg with less than 100 kg of bi-propellant and 47 kg of hydrazine remaining.

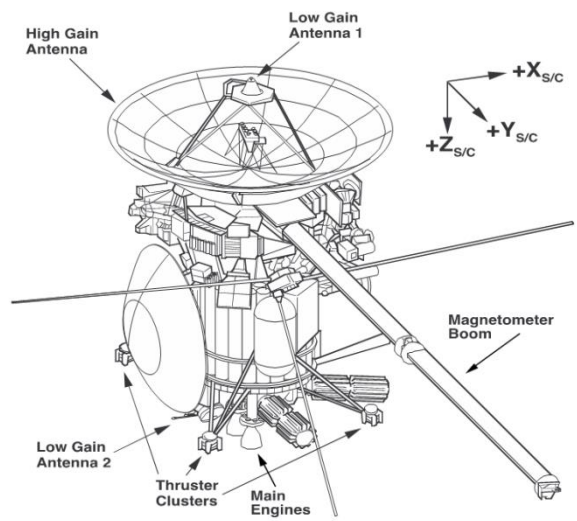

Figure 1. Overview of Cassini Spacecraft

Although Cassini spends most of its time in "RWA control" at Saturn, this paper focuses on low-altitude Titan flybys where the control mode is RCS thruster control. RCS control involves the use of 8 RCS thrusters: 4 of which fire as couples with force in the spacecraft "Y-axis" direction, and 4 thrusters which fire in the –Z direction without couples. These Y and –Z-facing thrusters are depicted in Figure 2 below.

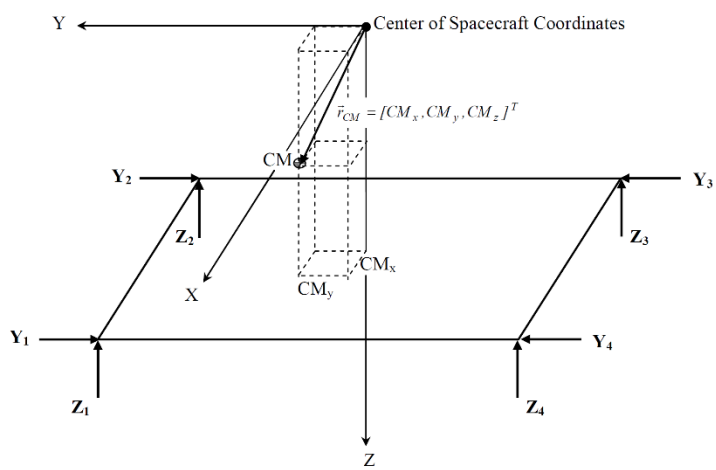

Figure 2. Thruster Configuration of Reaction Control System (RCS) on Cassini

The Y1 and Y3 thrusters fire as a coupled pair with no net delta-V, as do the Y2/Y4 thruster pair. They provide Z-axis rotational control. The 4 Z-facing thrusters pointing along the –Z body axis are used for X and Y axis rotational control (Z1/Z2 for –X torque, Z3/Z4 for +X torque, Z1/Z4 for +Y torque, Z2/Z3 for –Y torque). The Z-thrusters do impart delta-V which must be considered when interpreting accelerometer data. The force of the thrusters varies with hydrazine tank pressure, and was approximately 0.7 N per thruster during T87.

IV. Overview of Density Reconstruction Using Thruster On-Time Reconstruction Method

A. Standard Atmospheric Torque Equation

The normal method of estimating Titan atmospheric density is based on conservation of angular momentum using thruster on-time and additional telemetry[3]. For this method to accurately estimate torque and density versus time and altitude, good estimates of the trajectory, thruster on-times, thruster moment arms, the center of mass and other parameters must be utilized. In particular, a reasonable estimate of where on the spacecraft the "center of pressure" is (the point where the atmospheric force can be treated as a single vector) is part of the calculation.

The normal method using thruster on-times involves equation 1 shown here:

$$T_{Atm} = \frac{1}{2} C_D \rho V^2 A_{Proj} \hat{u}_V \times (\vec{r}_{CP} - \vec{r}_{CM}) \tag{1}$$

Where:

T_{Atm}	=	atmospheric torque in Nm
$\mathrm{C_D}$	=	drag coefficient
ρ	=	atmospheric density in kg/m^3
V	=	magnitude of the spacecraft velocity with respect to Titan in m/s
$\mathrm{A_{Proj}}$	=	spacecraft projected area in m^2
$\hat{\mathrm{u}}_{\mathrm{V}}$	=	unit vector of spacecraft velocity expressed in spacecraft body frame
$\vec{\mathrm{r}}_{\mathrm{CM}}$	=	position vector of the spacecraft's center of mass relative to the origin of the spacecraft coordinate frame
$\vec{\mathrm{r}}_{\mathrm{CP}}$	=	position vector of the spacecraft's center of pressure relative to the origin of the spacecraft coordinate frame

Equation 1 can be expressed as shown in Equation 2. This equation has 3 components and produces density estimates based on each of the three spacecraft body axes, assuming all other parameters are accurately known.

$$\rho = \frac{2T_{Atm}}{C_D V^2 A_{Proj} \hat{u}_V \times (\vec{r}_{CP} - \vec{r}_{CM})} \tag{2}$$

The drag coefficient has been estimated using formulae in Ref. 4. In our work, we assume $\mathrm{C_D} = 2.1 \pm 0.1$. This is a reasonable drag coefficient value when compared with results determined using orbital data of Earth-orbiting satellites.[5] The velocity vector magnitude of the spacecraft relative to Titan is accurately determined by Cassini navigation trajectory reconstruction. It is accurate to about 5 m/s one-sigma. The velocity unit vector in the spacecraft body frame is based on onboard attitude knowledge accurate to about 0.1 mrad. Center of mass is estimated via propellant accounting and ground software and has been confirmed recently by radiometric means.[6] T_{Atm}, $\mathrm{A_{Proj}}$, and $\vec{\mathrm{r}}_{\mathrm{CP}}$, are less accurately known, and are the dominant sources of error in the thruster on-time reconstruction method.

B. Flight Software Dynamic Simulation (FSDS)

The Cassini team has an all-software simulation environment called Flight Software Dynamic Simulation (FSDS), with full environmental dynamics built in[7]. Originally designed for flight software (FSW) testing before launch, it has been maintained as a high-fidelity analysis tool[8]. Currently, the Cassini team uses FSDS for FSW revision testing, sequence simulation, a Delta-V estimate generation which is used by the Navigation team. In addition to these standard team uses, FSDS has also been used to simulate Titan flybys in their early stages of design, to provide mission and science planners with estimates of hydrazine usage, and dynamic flight rule violations. For the purpose of the work presented in this paper, FSDS was used to recreate past Titan flybys with varying parameters, using a posterior knowledge gained after the flybys that the team did not have when originally simulating the flyby.

C. Cassini Projected Area and Center of Pressure

Prior to launch, geometric measurements of the Cassini spacecraft where gathered, to create a computer generated solid model of the Cassini spacecraft in both probe attached and released configuration. From this solid model, a set of projected surface areas that Cassini would present to oncoming flow, at different attitudes was compiled. See Figure 3 for a sample of the areas projected onto a 2-D plane orthogonal to the relative velocity vector.

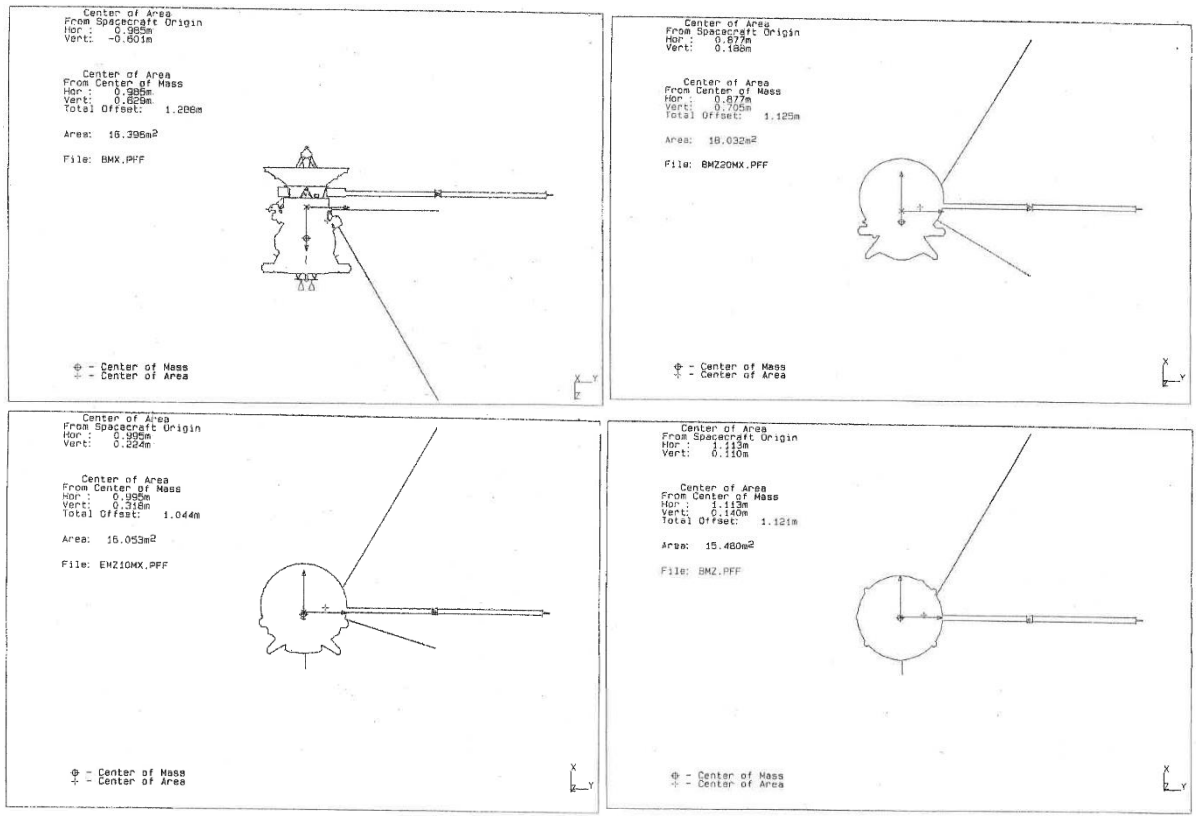

Figure 3. Example Cassini orientations showing estimated projected areas, center of mass, and center of area.

This compilation of projected surface areas was used to create a look-up table that FSDS uses to simulate Cassini's projected areas as it flies through Titan's atmosphere. From the Cassini area projection onto a 2-D plane, perpendicular to the relative velocity vector, the centroid, or geometric center of area can be found. It can be assumed that the centroid of the projected area is the location of the center of pressure of the spacecraft, projected into the same 2-D plane, perpendicular to the relative velocity vector. The assumption that the center of pressure location can be represented as a 2-D point is valid within the context that the CP coordinate parallel to the direction of oncoming flow does not contribute to any aerodynamic torqueing affects, since only the multiplication of force components with moment arms perpendicular to those force components can contribute to the magnitude of a moment. FSDS contains a look-up table that is used to interpolate for the CP at Cassini's attitudes as it flies through Titan's atmosphere. The one remaining unknown in Equation 2 is atmospheric torque. To find that, RCS thruster torque needs to be estimated in order to apply the conservation of angular momentum.

D. Solving for Torque in the Thruster On-Time Reconstruction Method

In most low-Titan flybys, a component of the atmospheric torque is applied around each of the three spacecraft body axes. To reconstruct density, it is often convenient to select the dominant spacecraft axis that contains most of the atmospheric torque. As shown below, the key to torque estimation is the "accumulation" of angular momentum over time. The slope of this curve is expected to be more accurate if the curve itself is "longer" – that is, has a greater total accumulation of angular momentum.

There are two methods that are used to estimate torque: one is a post-flyby ground analysis of telemetry[9]. The other is an estimate of external torque which is calculated onboard to detect thruster leaks[10]. The onboard estimate infers external torque from thruster torque applied but not otherwise accounted for. There are certain simplifying assumptions implicit in the onboard algorithm, but flight experience shows the telemetry "high water marks" of these onboard values are good estimates of the peak Titan atmospheric torques compared to independent ground analysis as described below. Table 1 gives representative peak Titan atmospheric torques from the high water marks of the external torque telemetry (refered to as F/D Slope) for several low-altitude Titan flybys.

Table 1. Peak External Torque (onboard estimate) for selected low altitude Titan flybys

Flyby	TCA Time	TCA Altitude (km)	Prime Instrument	Peak F/D Slope HWM (Nm)			Thruster On-Time Reconstruction Axis
				X-axis	Y-axis	Z-axis	
T16	2006-203T00:25:26	949.9	RADAR	-0.26	-0.76	-1.52	Z-axis
T57	2009-173T18:32:34	955.1	INMS	-0.14	-0.58	-1.32	Z-axis
T70	2010-172T01:27:43	878.1	MAG	1.18	0.1	-0.06	X-axis
T87	2012-318T10:22:08	973.6	NAV/INMS	0.32	-0.24	-0.4	X-axis
T95	2013-287T04:56:27	960.8	RADAR	-0.1	-0.32	-0.8	Z-axis
T100	2014-097T13:41:14	963	INMS	-0.06	-0.2	-0.5	Z-axis
T104	2014-233T08:09:09	964	RADAR	-0.06	-0.24	-0.4	Z-axis
T106	2014-297T02:40:30	1013	RSS	-0.06	-0.02	0.02	X-axis

From Table 1, it can be seen that for a given flyby (T95 is used in this example) the spacecraft experienced most of Titan's atmospheric torque about the spacecraft body Z-axis. In this case, an accumulated angular momentum curve is then generated for the Z-axis. To estimate the thrust of a given Z-firing thruster, a separate ground tool (involving radiometric data and angular momentum conservation during reaction wheel momentum changes) is used to keep an accurate estimate (within 1-2 percent) of each thruster's current force. The individual thruster force along with the known locations of each thruster and their associated moment arms permits estimation of torque generated by each Z-thruster each time they fire. Using the thruster on-time telemetry for each thruster, along with estimates of thrust rise and tail-off times[11], yields a plot of accumulated angular moment for the dominant axis. The accumulated angular momentum curve obtained from raw thruster on-time is noisy, so to smooth it out, two hyperbolic cosine functions are used to curve-fit the angular momentum curve. See Figure 4 for a plot showing the original noisy accumulated angular momentum curve about the Z-axis for T95, and the smoothed curve-fit accumulated angular momentum curve.

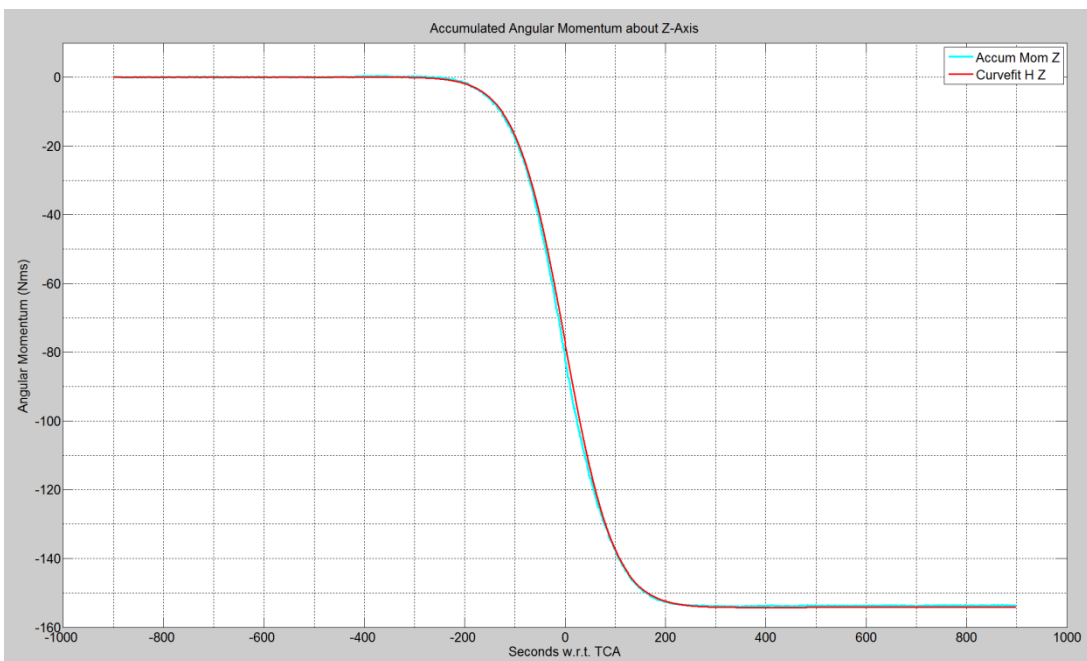

Figure 4. Raw data angular moment curve and curve-fit angular momentum curve for T95 about the Z-axis.

The curve-fitted accumulated angular momentum curve can be differentiated to obtain a smooth curve of atmospheric torque about the dominant axis. See Figure 6 for the atmospheric torque around the dominant Z-axis, for T95.

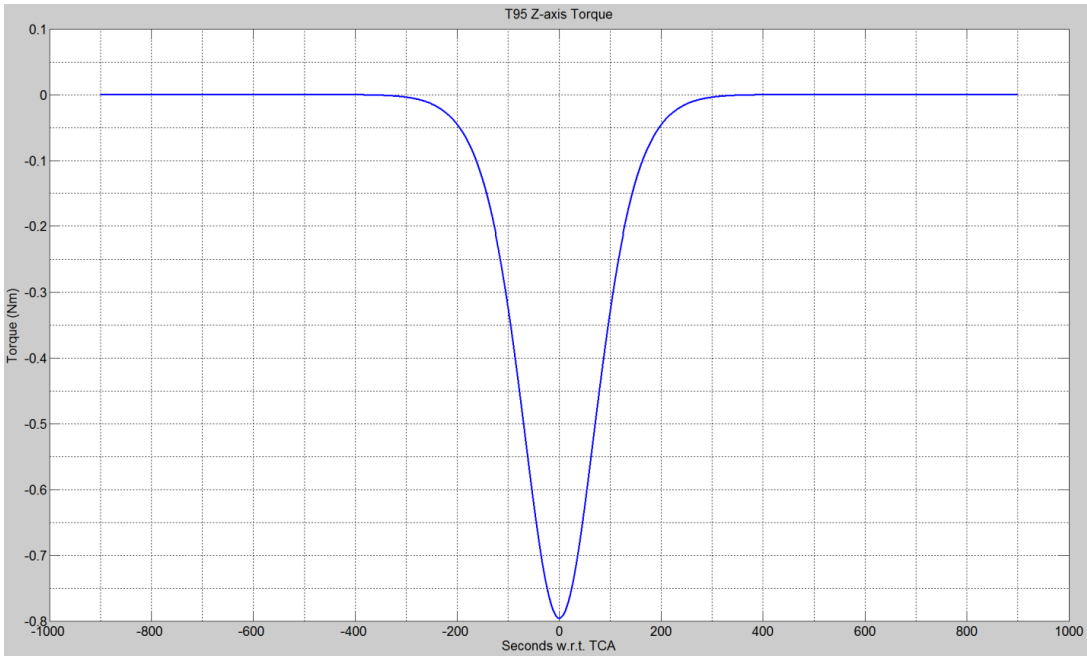

Figure 5. Atmospheric torque about Z-axis derived from curve-fit of accumulated angular momentum

The torque curve from Figure 5 above, provides us the final parameter of Equation 2, which is T_{Atm}. Using Equation 2 and the thruster on-time method, as described above, a profile of atmospheric density can be plotted for the entire T95 flyby as seen in Figure 6 below.

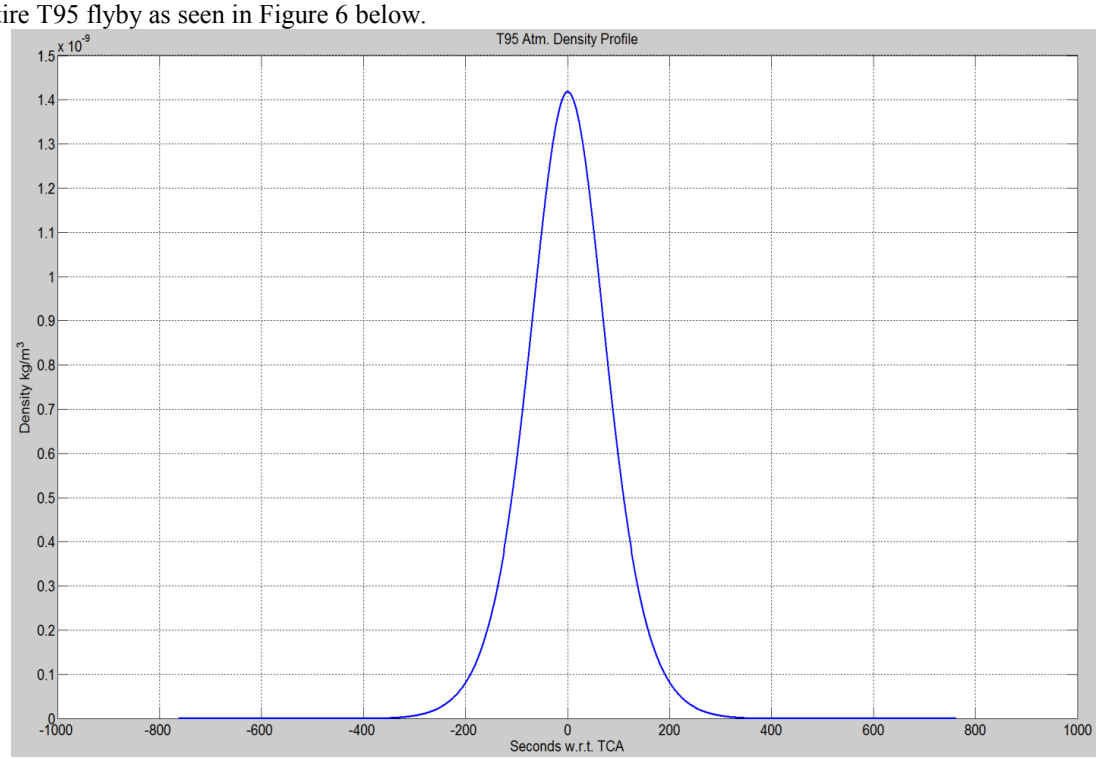

Figure 6. Profile of atmospheric density from T95 thruster on-time reconstruction about Z-axis

Although there are several variables that introduce error in the reconstruction method using thruster on-time, this method has been consistently used to estimate density for all low-altitude Titan flybys for which telemetry is available. Figure 7 shows a compilation of density profiles vs altitude that have been generated over the years using the thruster on-time method.

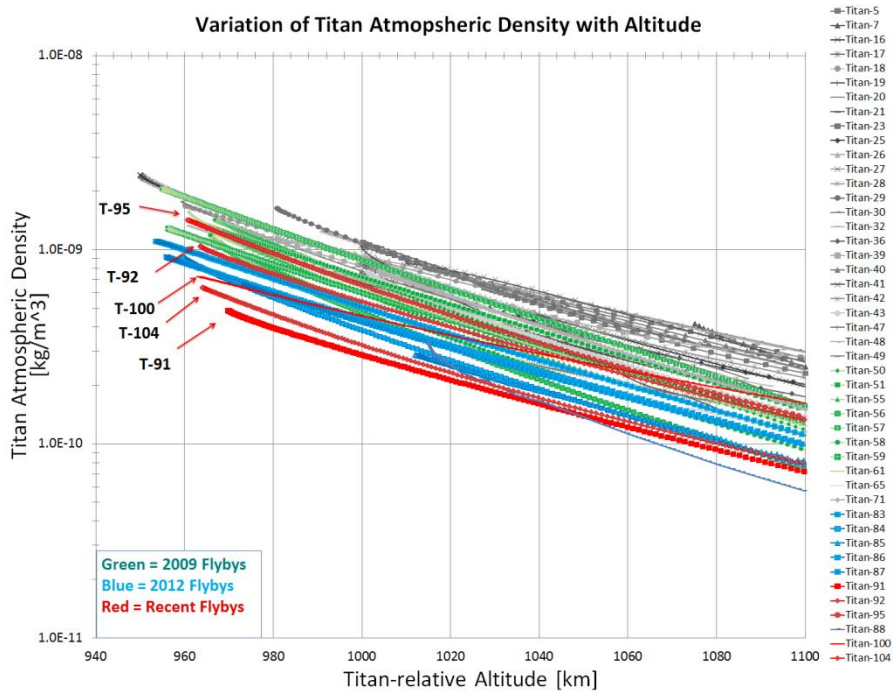

Figure 7. On-going trend of density vs altitude for select low Titan flybys on RCS

The purpose of this paper is to explore a possible improvement to the thruster on-time reconstruction method by adjusting one of the uncertain parameters in Equation 2, specifically, the center of pressure position vector, $\vec{r}_{CP}$.

V. T87 Density Reconstruction: Accelerometer, Radiometric Doppler, and Thruster On-Times

A. Density Reconstruction Using Cassini Accelerometer

The Cassini accelerometer senses acceleration in a single axis. It is mounted in the spacecraft –Z direction and can measure acceleration in either the +Z or –Z directions only. The accelerometer is normally only used during main engine burns. Its sensed acceleration is accumulated in the flight computer and is used to terminate main engine burns when total Delta-V magnitude reaches the desired target. A single accelerometer "count" of Delta-V represents about 0.002 m/s which for Cassini is approximately 4.6 Ns of impulse. This is fine for main engine maneuvers, but is a rather "coarse" signal when used to analyze an aerodynamic force that peaks at roughly half a newton. An additional complication is the presence of RCS thruster firings for attitude control near Titan closest approach. These firings are also sensed by the accelerometer because the Z-axis-firing thrusters are not coupled and thus introduce Z-axis acceleration because all four Z-facing thrusters produce force in the –Z direction. Additionally, the T87 flyby was performed with the High Gain Antenna pointing at Earth (to support radiometric data) which was about 52 degrees away from Cassini's velocity relative to Titan. So the aerodynamic force sensed by the accelerometer was only cos(52°) ≈ 62% of the true aerodynamic force during the flyby. Additionally, the accelerometer "bias" (the number of "counts" output by the accelerometer in the absence of any acceleration) had to be evaluated and removed from the calculations.

The thruster firings in the –Z-direction near T87 closest approach are depicted in Figure 8. This figure is the summation of the four -Z-facing thrusters which are plotted as cumulative on-time over 8-second time period "chunks". The peak "duty cycle" of any single Z-facing thrusters near closest approach was around 28%. A continuously on thruster would have a duty cycle of 100%.

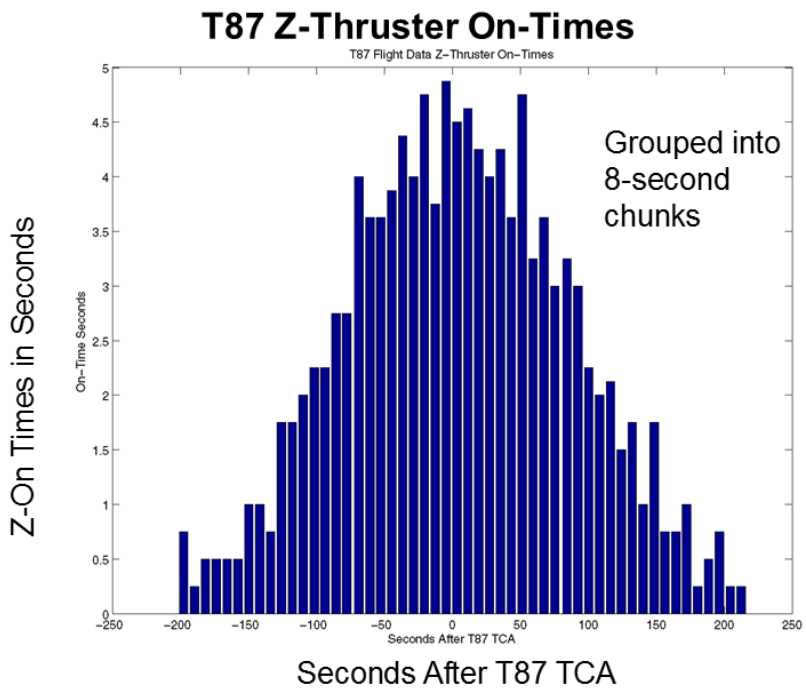

Figure 8. Total on-time for Z-facing thruster during T87 closest approach

The total Z-thruster on-time can be used to estimate a time-history of Z-axis linear impulse due to the thrusters. This is achieved by finding the area under the curve that estimates thrust force versus time for each commanded "pulse" of a thruster, as seen in Figure 9.

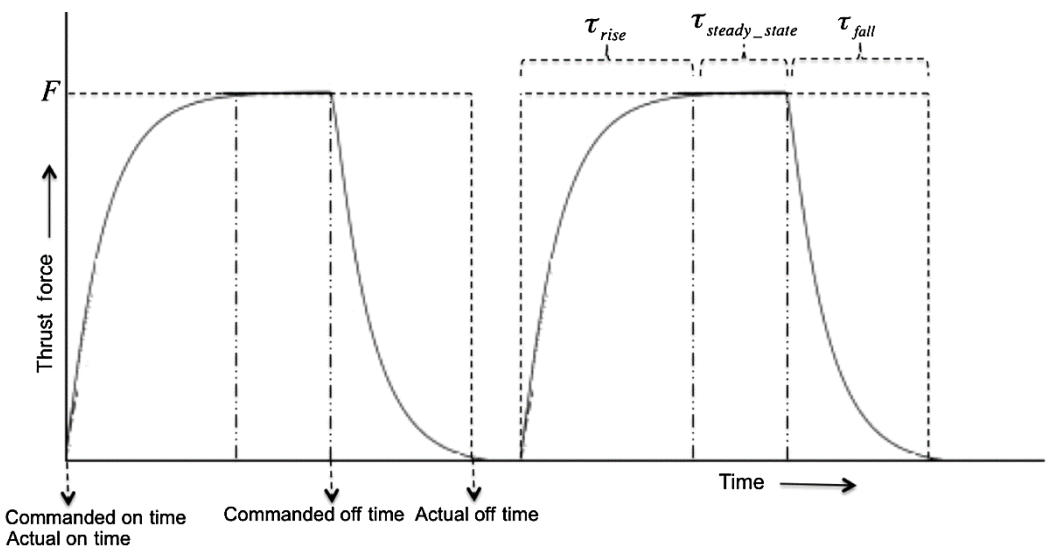

Figure 9. Thruster impulse is the area under the curve, including rise and tail-off times

The cumulative thruster impulse (based on telemetry) during T87 was estimated in this way. The accelerometer sensed ΔV (based on telemetry) is multiplied by the total spacecraft mass and integrated to produce an "accelerometer (ACC) sensed" impulse. By adding them together a reconstructed aerodynamic impulse curve can also be plotted. These three curves are plotted in Figure 10.

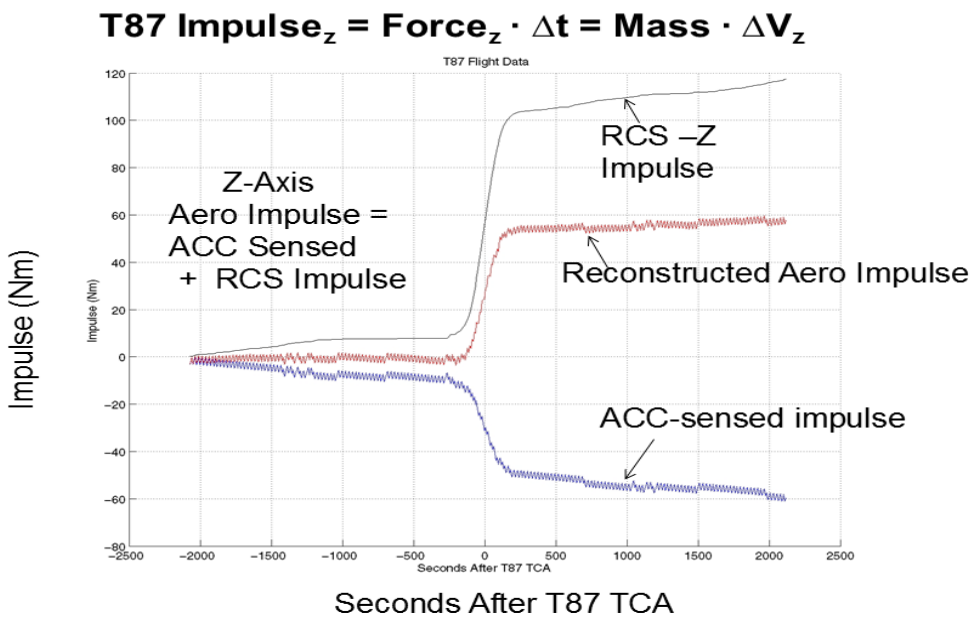

Figure 10. Z-axis estimated linear impulse for thrusters, accelerometer, and derived aerodynamic impulse

The aerodynamic impulse curve is smoothed in Figure 11 and the derivative is taken to estimate aerodynamic force in the spacecraft Z-direction.

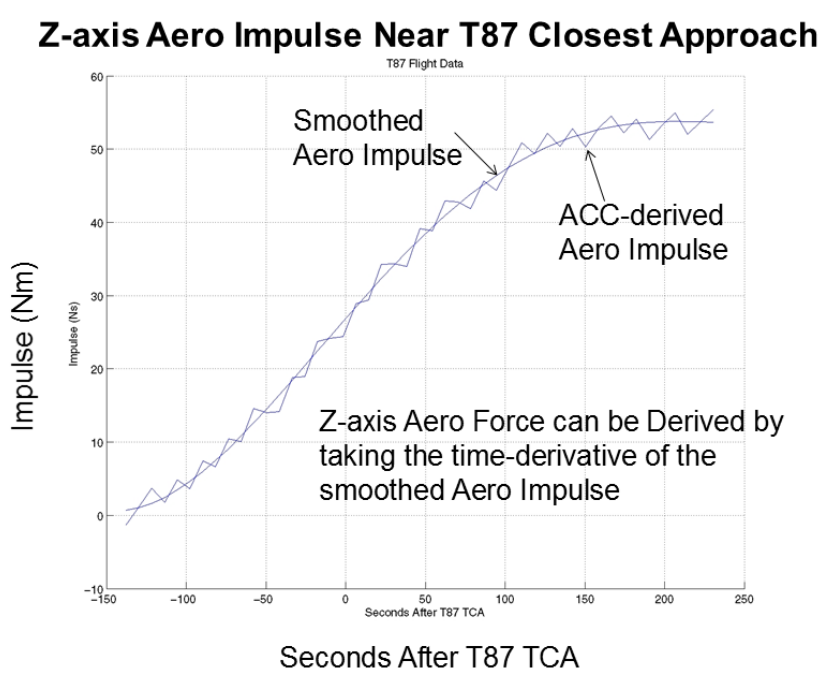

Figure 11. Smoothed aerodynamic Z-axis impulse versus time relative to T87 closest

From the accelerometer-derived drag force, the atmospheric density is estimated versus time in Figure 12.

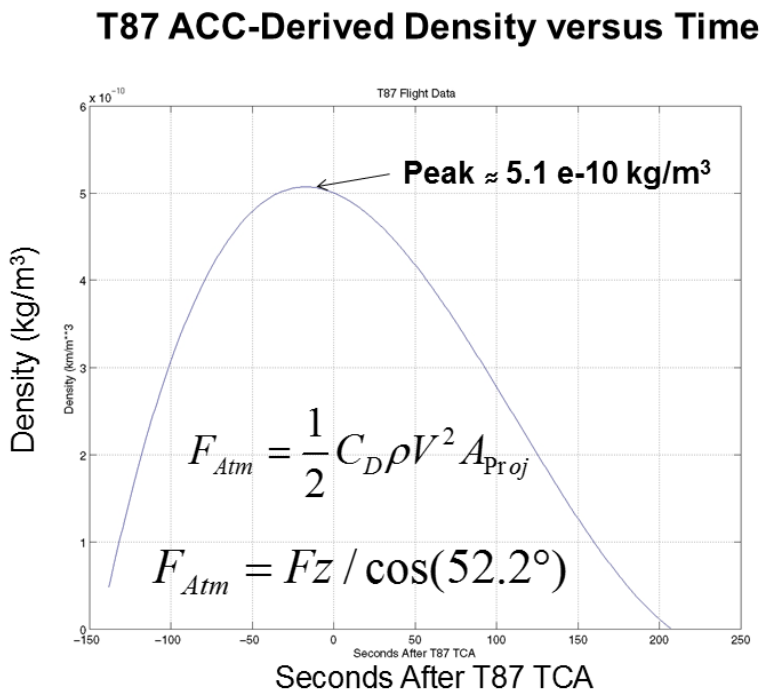

Figure 12. Titan atmospheric density versus time relative to T87 closest approach

The density can then be plotted versus Titan-relative altitude in Figure 13. The peak density is found to be 5.1e-10 kg/m^3. The difference between inbound and outbound could be a real effect or could be an artifact of the uncertainty in estimating the slope of the impulse curve. One piece of evidence that it could be real is seen in Figure 17 below. The radiometric reconstruction of density also shows the inbound density was slightly elevated compared to outbound at around 1000 km.

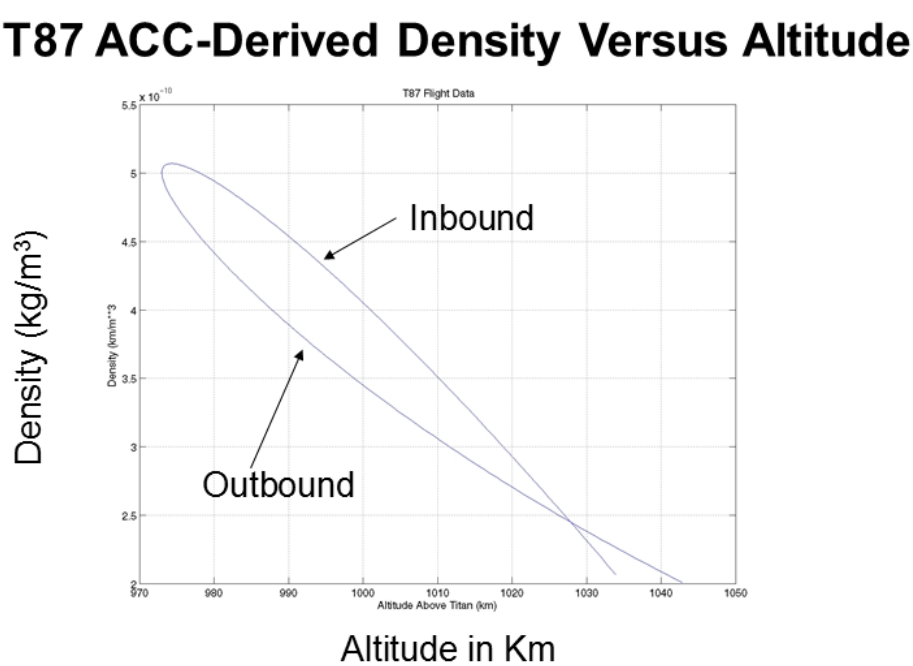

Figure 13. Titan atmospheric density versus Titan-relative altitude during T87

B. Density Reconstruction using Radiometric Doppler

Radiometric estimation of Titan atmospheric density uses the method of orbit determination. Differences between a mathematical representation of the tracking data and the observables are minimized by a filter that uses a least squares method. The spacecraft trajectory is integrated with the equations of motion, where all the forces acting on the spacecraft are modeled. The parameters that define the spacecraft state and the coefficients that define the atmospheric and other forces are variables of the system that can be estimated given an a-priori value and uncertainty. The tracking data is modeled as a function of their parameters and the filter can estimate them by minimizing the residuals between the computed and the observed data.

The real-time Doppler signal during T87 closest approach was measured and results in a time-history of estimated drag acceleration as seen in Figure 14.

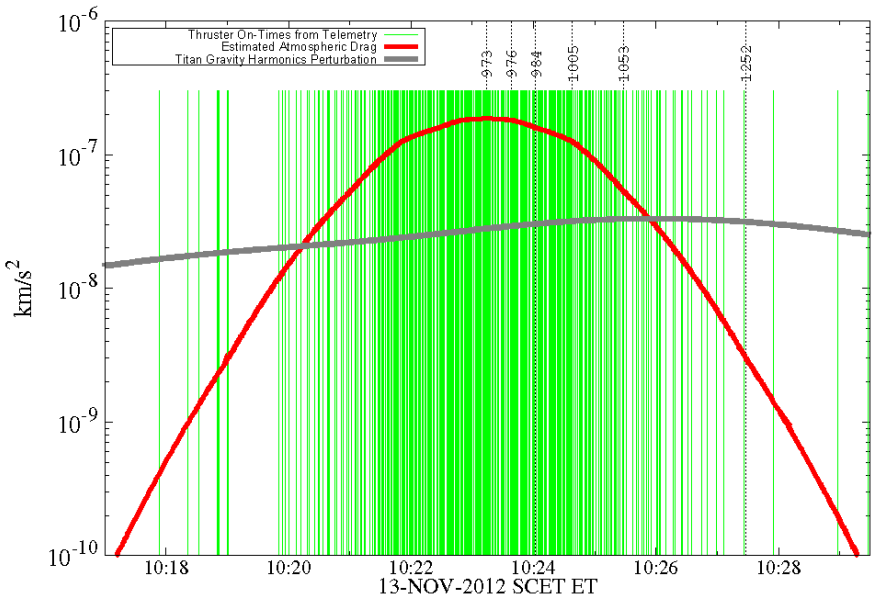

Figure 14. Accelerations during T87 closest approach

For comparison, the acceleration from the Titan gravity harmonics perturbations are also shown. The harmonics signature is quite different compared to the drag and any error in the gravity model will not alias into the density estimates. During T87, thruster firing telemetry was downlinked every second, so this very accurate time-history of thruster activity is not a significant source of error in Titan density estimation. Thruster nominal forces, rise-time, and tail-off time are routinely estimated during the mission, and current estimates of these quantities were used in this analysis.

Figure 15 shows the pre-fit Doppler residuals before the filter estimates any parameter. The blue regions are the RWA/RCS control transitions. The a-priori estimates of the orbital parameters are already well estimated as the residuals prior to the flyby are near zero. The steep slope shows the effect of the atmospheric drag. This figure shows how big the effect of the drag is in comparison to the Doppler noise. Figure 16 shows the post-fit residuals once the atmospheric density has been estimated. Radiometric X-Band to/from the spacecraft (up/down) is plotted along with X-Band up and Ka-Band down. The Ka-down data is slightly more accurate.

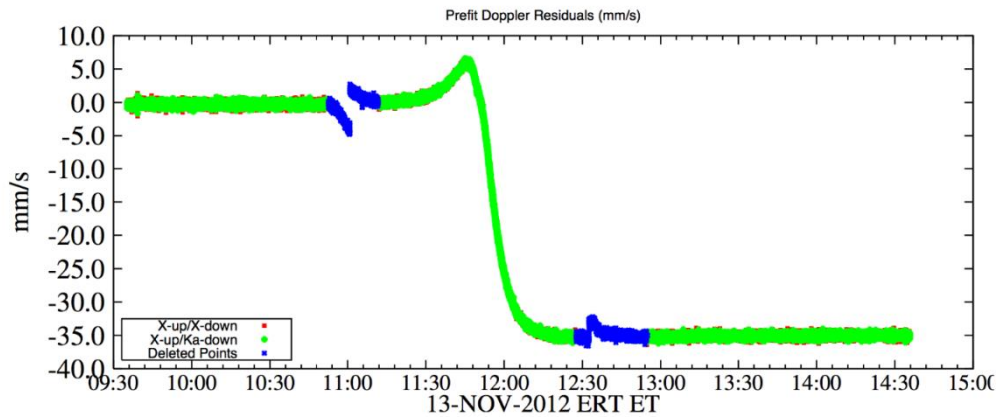

Figure 15. Pre-fit Two-way Doppler around T87 Closest Approach

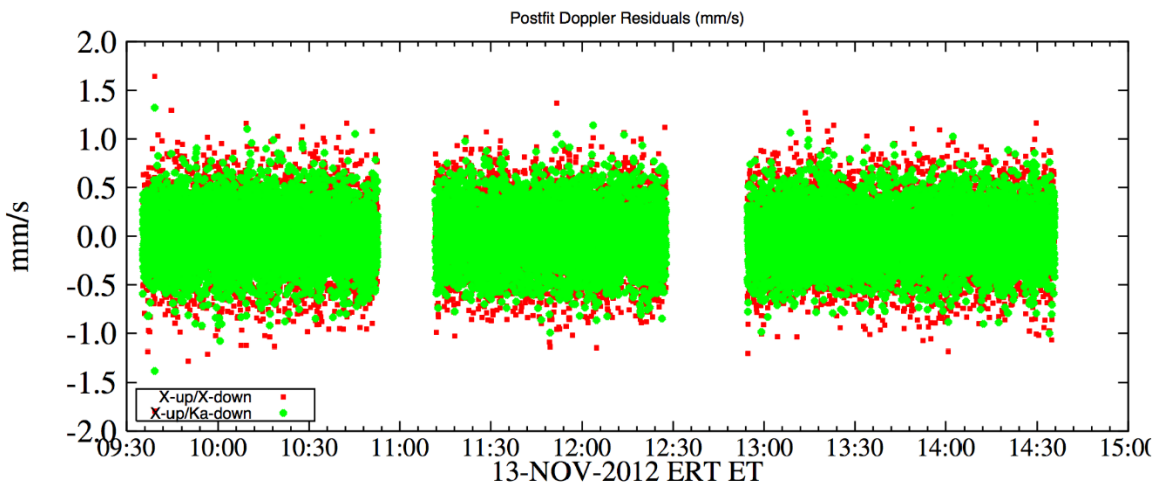

Figure 16. Post-fit Two Doppler around T87 Closest Approach

The atmospheric density derived from Doppler analysis is given in Figure 17 along with 3-sigma error bars. The peak atmospheric Titan density is estimated to be 5.05e-10 km/m3 with a one-sigma uncertainty of about 5 percent.

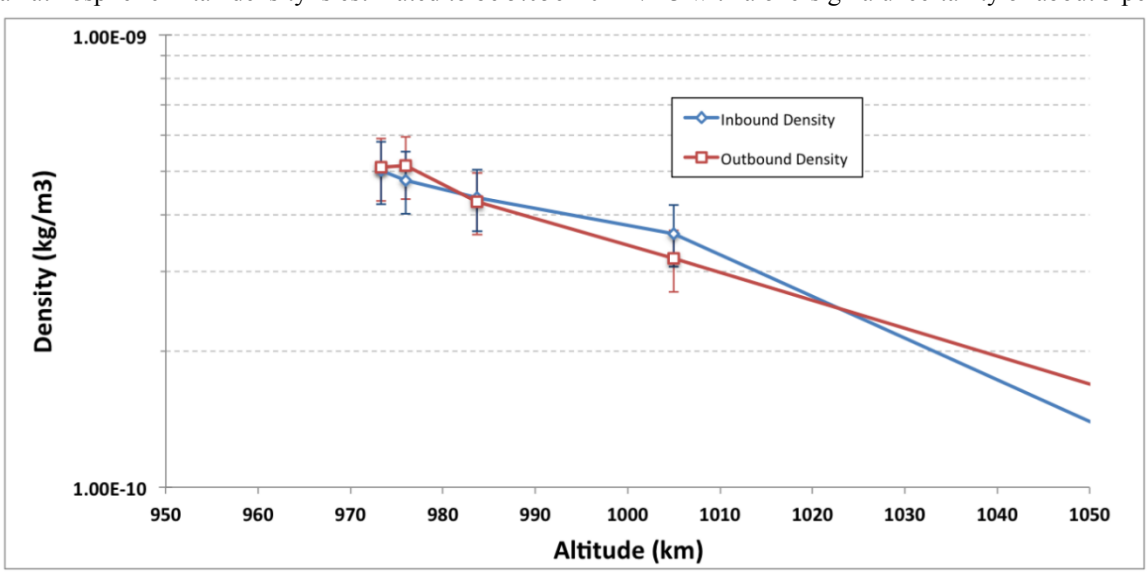

Figure 17. Titan Atmospheric Density versus Altitude Derived from Doppler residuals

The analysis and figures in this section are based on the work of Frederic Pelletier of the Cassini Navigation operations team. Table 2 summarizes the radiometric and accelerometer density reconstruction results for peak atmospheric density during the T87 flyby.

Table 2. Peak Atmospheric Torque during T-87 Flyby Using Radiometric and Onboard Accelerometer Telemetry

Source	Peak Density (kg/m^3)
Radiometric Inbound	5.01e-10
Radiometric Outbound	5.10e-10
Accelerometer	5.10e-10

The above results show very good agreement using two independent methods. The only other flyby that also had these independent sources of information (T70 at 878 km closest approach) also produced peak Titan densities of comparable agreement (radiometric: 4.015e-9, accelerometer: 4.0e-9)

C. Applying the Normal Thruster On-Time Reconstruction for T87

From Table 1, it can be seen that the torques applied by Titan's atmosphere during T87, where more or less distributed evenly among all three spacecraft axis. In light of this, the normal thruster on-time reconstruction method, as described in previous sections, was applied about each spacecraft axis using T87 flight data. Table 3 summarizes the peak atmospheric densities at TCA that resulted from the thruster on-time reconstructions.

Table 3. Summary of normal thruster on-time reconstruction for T87

Reconstruction Axis	Peak Density (kg/m^3)
X-axis	7.27E-10
Y-axis	5.30E-10
Z-axis	6.86E-10

The above results show very good agreement using two independent methods. The only other flyby that also had these independent sources of information (T70 at 878 km closest approach) also produced peak Titan densities of comparable agreement (radiometric: 4.015e-9, accelerometer: 4.0e-9)

VI. Evaluation of the Center of Pressure Location During T87 Flyby

Table 3 shows that all three axes "over-estimate" the peak density at T87, as compared to the radiometric/accelerometer measurements. The density derived from the X-axis is about 43% high, the Y-axis is about 6% high, and the Z-axis is about 35% high. This indicates that the thruster on-time method, at least for T87, may be either over-estimating the torque, or perhaps the aerodynamic moment arm (essentially the center of pressure) may be bigger than is currently assumed. An error in the thruster force and moment arms is not as likely as an error in the center of pressure because radiometric data after RCS maneuver delta-V "burns", as well as RWA momentum biases, provide independent estimates of thruster forces.

Even if good estimates of atmospheric density and torque about all 3 axes exist, the center of pressure vector cannot be uniquely determined using Equation 2. In Section IVC above we describe how center of pressure is estimated using computer generated solid models. An assumption can be made that, of the three spacecraft axes, the spacecraft is more uniformly shaped along the spacecraft X-axis. In Figure 18, the largest appendages, in terms of area, are the magnetometer boom (+Y-direction) and the High Gain Antenna (-Z-direction). So if the X-axis component of the center of pressure is held constant, Equation 2 can be used to solve for the remaining Y and Z components of the center of pressure. This results in a center of pressure that is consistent with the peak torques and density measured during T87 using radiometric and accelerometer reconstructions.

The "baseline" center of pressure in the spacecraft structural frame used in our high-fidelity ground simulation tool (FSDS) and derived from the solid model projection and centroid for T87 closest approach is:

$$\vec{r}_{CP} = [-0.144, 0.873, 0.197] \text{ in meters}$$

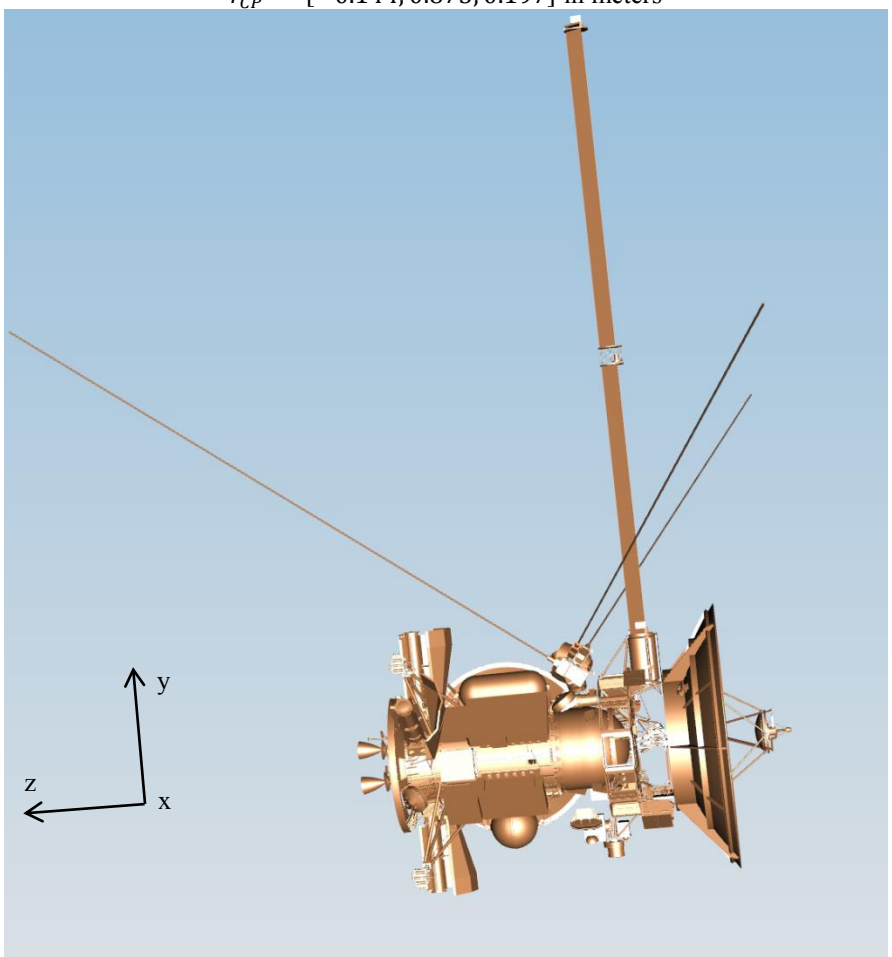

Figure 18. Location of Magnetometer boom and HGA in body axis frame

Using Equation 2, and knowing the components of $\vec{r}_{CM}$, the refined center of pressure for T87 closest approach is found:

$$\vec{r}_{CP_{refined}} = [CP_x, CP_y, CP_z] = [-0.144, 1.179, 0.158] \text{ in meters}$$

Comparing the refined center of pressure with the baseline center of pressure,

$$\vec{r}_{CP} = [-0.144, 0.873, 0.197]$$

indicates that, to match T87 radiometric and accelerometer reconstructions, the center of pressure should be shifted about 30 cm out towards the magnetometer boom and about 4 cm out towards the HGA. Since these are the biggest appendages on Cassini, it is possible that our ground estimates based on solid models may underestimate the effect they have on the center of pressure.

To assess whether this adjustment to the center of pressure at T87 closest approach does a good job of matching the actual torques and thruster usage observed during T87, an update was made to the Flight Software Dynamic Simulation (FSDS) implementation of the center of pressure. The changing direction of the Titan-relative velocity during the flyby leads to a small change over time of the center of pressure. Figure 19 shows this effect for the baseline center of pressure case. This was accounted for and a new T87 FSDS simulation was made using the radiometric-derived density and the refined center of pressure. Figure 20 depicts the revised center of pressure as a function of time.

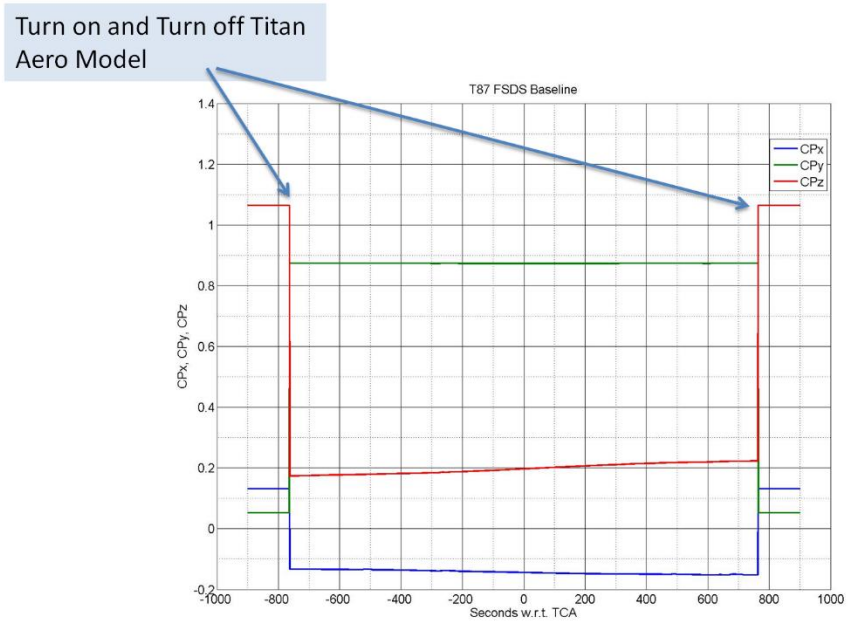

Figure 19. CP Coordinate Profiles of T87 FSDS Baseline (without refined CP)

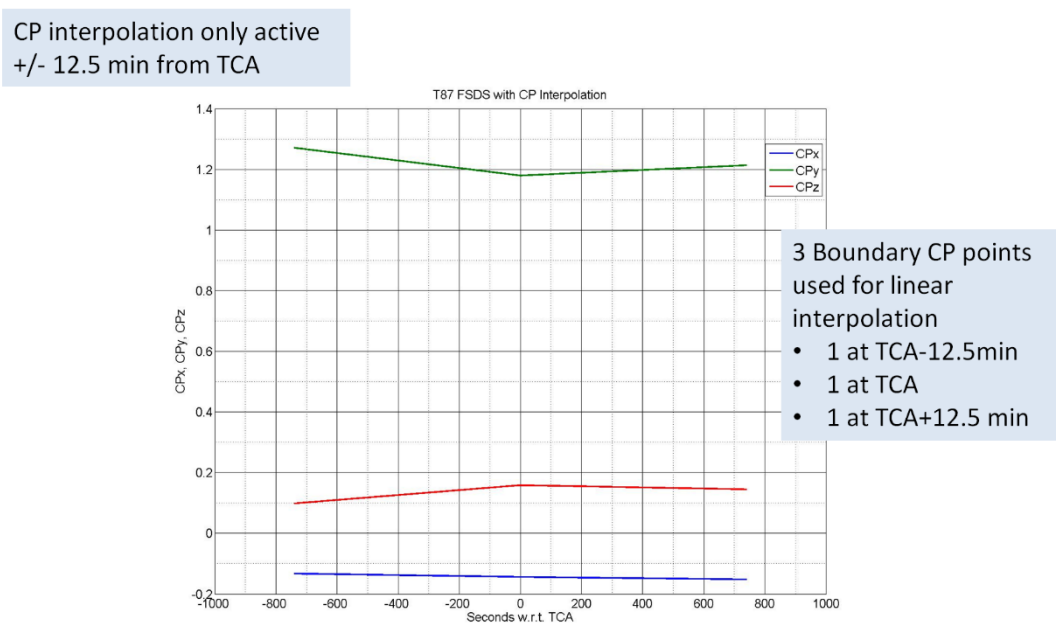

Figure 20. CP Coordinate Profiles of T87 FSDS with Refined CP Interpolation Table

One measure of thruster usage during a flyby is to summarize the peak "duty cycles" of the thrusters. Duty cycles are the percentage that a thruster was actually firing per unit time. Table 4 summaries the peak duty cycles observed during T87 with the refined center of pressure FSDS simulation. For comparison, the pre-flyby predicted "baseline center of pressure " FSDS results are also shown (that prediction had a peak density of 8.5e-10 kg/m^3).

Table 4. T87 Comparison of peak duty cycles at TCA for three sources of data

	FSDS Baseline (%)	Flight Data (%)	FSDS Refined CP (%)
Y1B	0	0	0
Y2B	26.25	20.624	20.0
Y3B	0	0	0
Y4B	26.25	20.624	20.0
Z1B	40.625	28.749	26.875
Z2B	14.375	12.494	12.5
Z3B	0	0	0
Z4B	23.125	13.755	13.125

In addition to the peak flight duty cycles at T87 TCA closely matching the "FSDS refined CP" simulation, it was observed that the flight duty cycle profiles over the whole flyby also matched well with the refined center of pressure. Matching the duty cycle profiles had never been possible with previous FSDS simulations that used the default center of pressure, even when adjusting for the radiometric-derived density profile from T87. Figures 21 through 23 show the flight duty cycle profiles plotted with the "baseline" FSDS pre-flyby estimates and the "interpolated" (that is, refined) center of pressure simulation results.

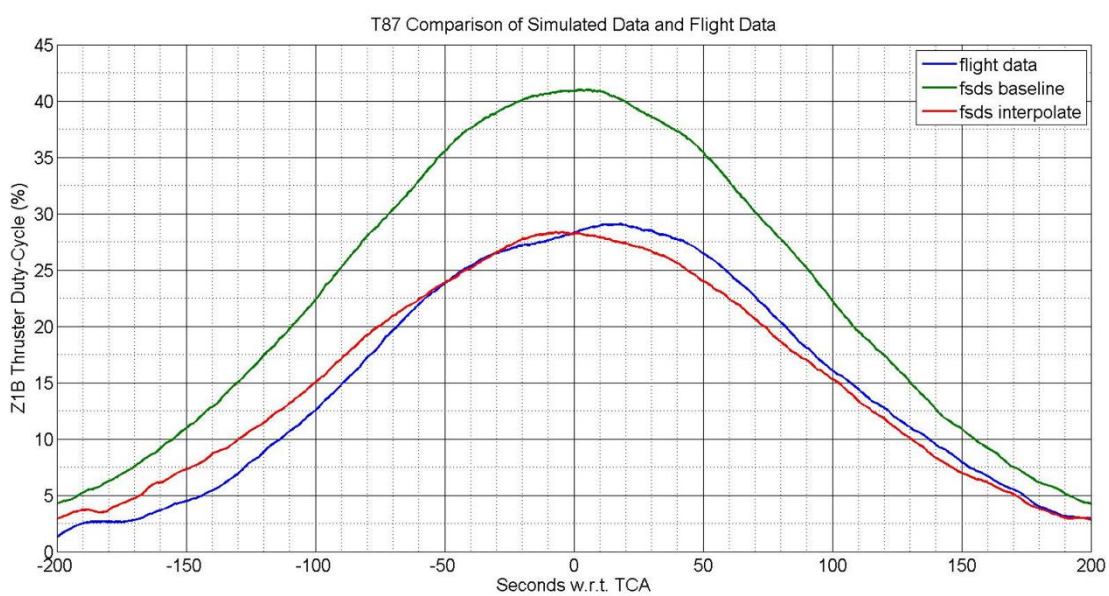

Figure 21. T87 Duty-Cycles for Thrusters Z1B Near Closest Approach

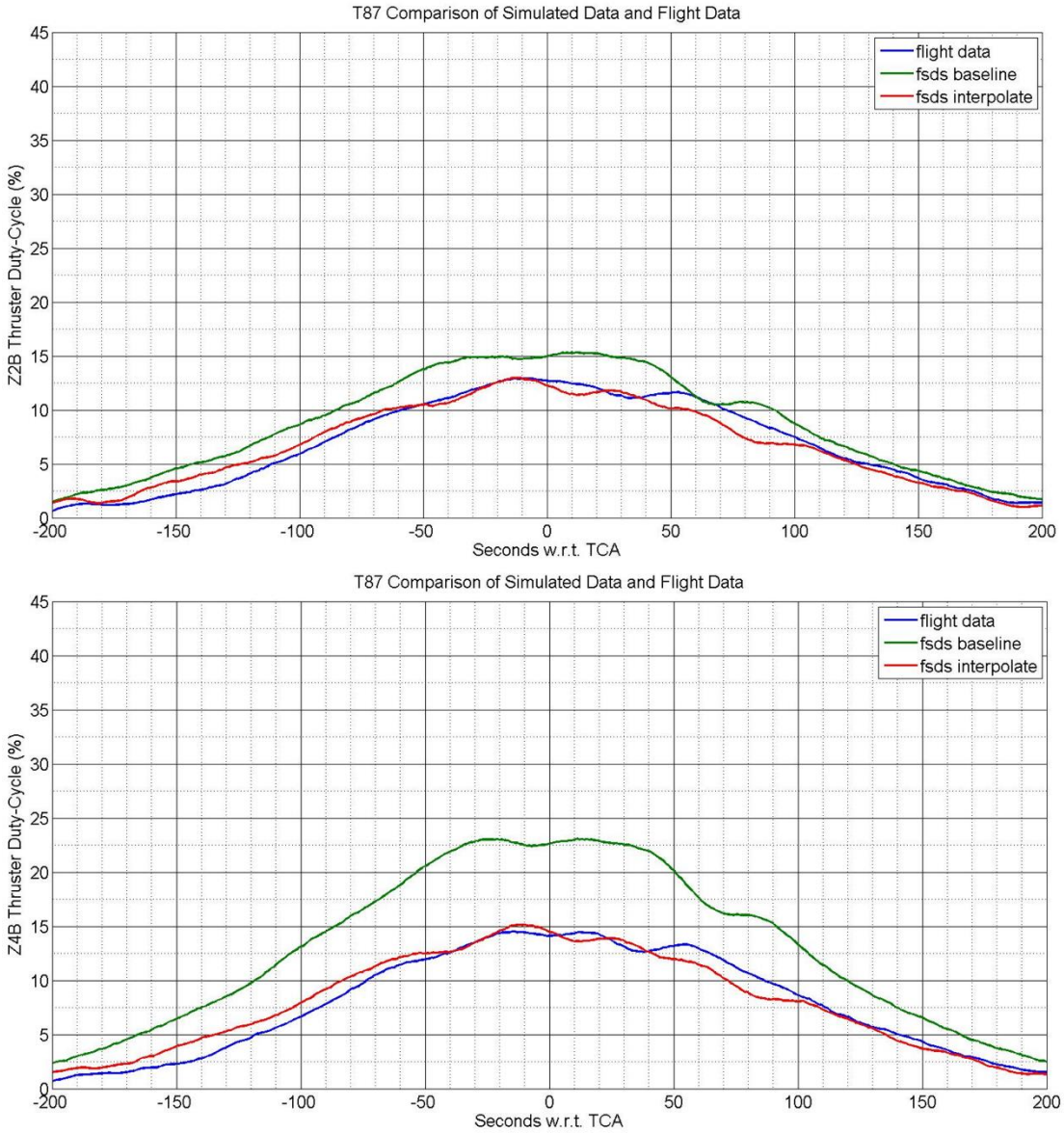

Figure 22. T87 Duty-Cycles for Thrusters Z2B and Z4B Near Closest Approach

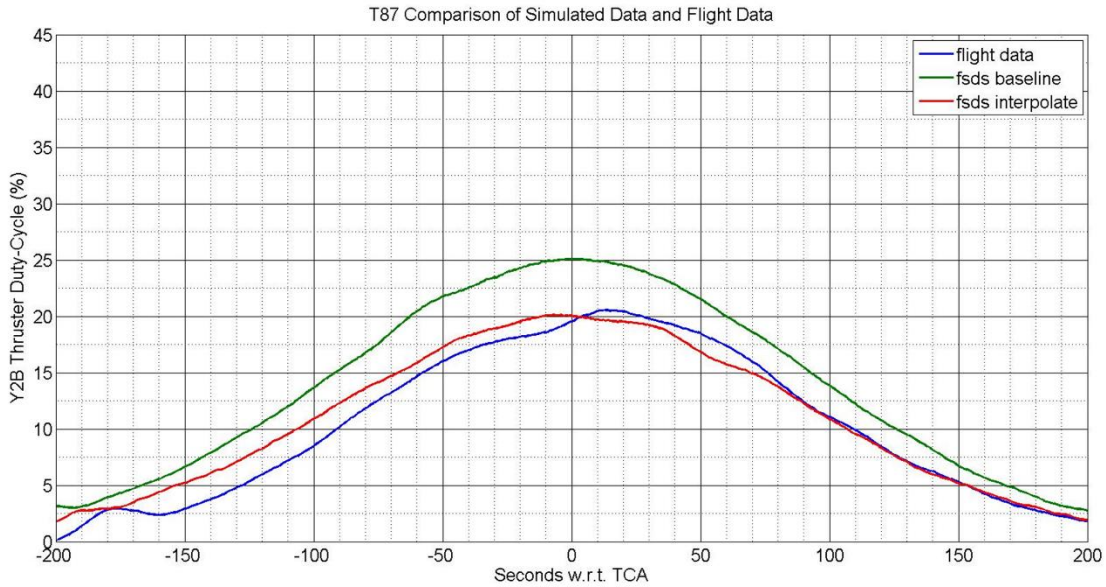

Figure 23. T87 Duty-Cycles for Thrusters Y2B/Y4B Near Closest Approach

VII. Conclusion

The Titan 87 flyby provided a unique opportunity to compare the normal density reconstruction method of angular momentum and torque against radiometric and accelerometer density reconstructions. The results showed generally good agreement, although it is possible that the normal method may overestimate density by roughly 30% at least during T87. One potential reason for this may be the center of pressure may act further away from the center of the spacecraft than our baseline modeling suggests. This would produce the same size torques as actually observed but with a correspondingly lower peak atmospheric density. The results presented in this paper -- refining the location of the center of pressure during T87 -- supports the contention that the default center of pressure used in previous analysis might be underestimating the projected area of components like the High Gain Antenna and the magnetometer boom. By moving the center of pressure out towards the High Gain Antenna and magnetometer boom, the authors of this paper were able reproduce in the FSDS simulation the in-flight thruster duty-cycle profiles that occurred during the T87 flyby.

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