UTILIZATION OF THE DEEP SPACE ATOMIC CLOCK FOR EUROPA GRAVITATIONAL TIDE RECOVERY

Jill Seubert; Todd Ely†‡

Estimation of Europa’s gravitational tide can provide strong evidence of the existence of a subsurface liquid ocean. Due to limited close approach tracking data, a Europa flyby mission suffers strong coupling between the gravity solution quality and tracking data quantity and quality. This work explores utilizing Low Gain Antennas with the Deep Space Atomic Clock (DSAC) to provide abundant high accuracy uplink-only radiometric tracking data. DSAC’s performance, expected to exhibit an Allan Deviation of less than 3e-15 at one day, provides long-term stability and accuracy on par with the Deep Space Network ground clocks, enabling one-way radiometric tracking data with accuracy equivalent to that of its two-way counterpart. The feasibility of uplink-only Doppler tracking via the coupling of LGAs and DSAC and the expected Doppler data quality are presented. Violations of the Kalman filter’s linearization assumptions when state perturbations are included in the flyby analysis results in poor determination of the Europa gravitational tide parameters. B-plane targeting constraints are statistically determined, and a solution to the linearization issues via pre-flyby approach orbit determination is proposed and demonstrated.

INTRODUCTION

Originally discovered by Galileo in 1610 as a point of light indistinguishable from Io, the Jovian moon Europa has since revealed itself to be a unique and intriguing world. Mounting evidence from various near and deep spacecraft, including Pioneer 10 and 11, Voyager, Galileo, and the Hubble telescope, suggests that a liquid water ocean is hidden beneath Europa’s icy crust.1 Recent Hubble detections of water plumes near Europa’s south pole has increased the enticement of dedicated exploration of this icy world.2 In order to investigate Europa’s habitability and prepare for follow-on exploratory missions, a potential Europa mission would, among other objectives, definitively confirm the existence of and perhaps characterize this subsurface ocean. Confirmation that a liquid ocean does exist is possible through reconstruction of the Europa gravitational tides. Europa is tidally locked to Jupiter and exhibits a strong longitudinal gravity field profile; measuring the time-varying gravitational tides, dominated by the second-degree harmonics, could differentiate between a solid and liquid body.3,4 A spacecraft in orbit about Europa would return abundant radiometric tracking data to resolve the $k_{22}$ parameter to an order of magnitude greater than that required for resolution of the subsurface ocean. In contrast, a spacecraft in orbit about Jupiter, performing

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periodic flybys of Europa, suffers limited useful tracking data as the gravitational tide sensitivity is confined to roughly an hour about Europa close approach.

Further limiting the accessibility of radiometric tracking data near the Europa close approach, the flyby spacecraft attitude profile required by the science payload or mission operations may not accommodate continuous pointing of the High Gain Antenna (HGA) towards Earth. At deep space distances, HGAs are required for traditional closed-loop two-way communications. Medium gain antennas such as fan beams can provide sufficient gain for two-way open-loop tracking operations, but signal accessibility is hindered by the narrow fields of view (FOV) of such antennas. In contrast, Low Gain Antennas (LGAs), typically included in the baseline spacecraft design to satisfy mission safety requirements, provide sufficient gain for uplink-only open-loop tracking at Jovian distances over nearly hemispherical FOV coverage. The uplink-only Doppler carrier phase measurements may be collected onboard during the flybys and downlinked at a later, non-mission critical time. In such a tracking architecture, abundant tracking data can be collected without imposing attitude requirements on the spacecraft.

One-way radiometric data quality hinges on the accuracy and stability of the onboard frequency source; hence, a high performance clock fit for space, such as the Deep Space Atomic Clock (DSAC), is required to enable one-way data with suitable accuracy for navigation and radio science purposes. DSAC is a NASA Technology Demonstration Mission (TDM) to empirically demonstrate the performance of a low-mass, small mercury ion atomic clock in Low Earth Orbit (LEO) over a period of one year. DSACs stability, as measured by its Allan Deviation (AD), is expected to be less than 3e-15 at one day. Rather than drift over long integration times like a typical ultra stable oscillator (USO), DSAC exhibits white frequency noise, providing long-term stability and accuracy on par with the ground clocks in use at the Deep Space Network (DSN). Such small spacecraft clock error will enable one-way radiometric tracking data with accuracy equivalent to current two-way tracking data. This work investigates the utilization of LGAs coupled with the Deep Space Atomic Clock (DSAC) as applied to recovery of the Europa gravitational tide via a flyby mission. LGA link performance throughout a Jovian moon tour, the expected performance of a deep space version of DSAC, and quality of the uplink-only Doppler signal when coupling DSAC with LGAs are discussed in detail. Additionally, the linearity assumptions of the gravity science sequential Kalman filter are explored, with results demonstrating that additional tracking data is necessary to ensure the filter’s nominal conditions are within the linearity constraints of the filter. A B-plane metric for ensuring that the linearity assumptions are met is statistically determined, and the necessity for pre-flyby approach orbit determination is illustrated.

EUROPA GRAVITATIONAL TIDE ESTIMATION VIA A FLYBY MISSION

Europa is tidally locked to Jupiter and exhibits a strong longitudinal gravity field profile; measuring the time-varying gravitational tides could differentiate between a solid and liquid Europa structure. The presence of a liquid ocean underneath the external ice shell will exhibit significant tidal variations in the gravity field that a rigid body will not produce. In terms of the Love number gravitational tide parameter k, a completely rigid body will exhibit \( k = 0 \), while a body with some liquid component will exhibit \( k > 0 \). While the nominal Europa gravitational tide model used for this work assumes that the second degree Love numbers \( (k_{20}, k_{21}, \text{and} \, k_{22}) \) are equal to 0.2, the assumed values are inconsequential to the radiometric measurement sensitivity to the gravitational tide parameters; the estimation results are therefore valid regardless of whether the nominal model assumes a liquid or solid internal structure. As presented in Reference 4, determination of the sec-
ond degree Love numbers to a 1-σ uncertainty less than 0.05 can provide strong evidence of the existence of a subsurface liquid ocean.

A Europa orbiter mission would provide more than sufficient data to determine the second degree Love numbers to better than \( \sigma = 0.05 \). However, a Jupiter orbiter that performs close flybys of Europa will suffer much more limited tracking data, and thus will experience more difficulty in reducing the Love number uncertainty below the requirement. The Jovian moon tour utilized for this study includes 45 flybys of Europa over a time span of approximately 2.8 years. The Europa flybys include a set of flybys (E17-E24) designed for the gravity science experiment, targeting specific close approach true anomalies, latitudes and longitudes to better estimate the second-degree Love numbers and the strongly-correlated spherical harmonic coefficients. Previous flyby covariance analyses have shown that the \( k_{22} \) Love number can be determined to better than 0.05 (1-σ) given continuous X-band Doppler tracking over a four-hour window centered on each close approach. References 5 and 6 also demonstrated that the \( k_{22} \) solution quality is highly correlated to both the quality and quantity of available tracking data. The one-way X-band tracking configuration proposed in References 5 and 6 achieves a final \( k_{22} \) 1-σ value less than 0.02, maintaining 60% to 75% margin on the Love number requirement. This performance was demonstrated across multiple tour designs, highlighting the robustness to mission redesign, optimization of the trajectory for purposes other than gravity science, and the loss of flyby tracking data.

This level of gravitational tide estimation performance relies upon the ability to obtain abundant, high-quality tracking data over the flyby periods. Two-way Doppler tracking at Jovian distances may be realized via HGA closed-loop tracking or medium gain fan beam open-loop tracking. HGAs are characterized by steep off-boresight loss curves and narrow realizable FOVs (less than 1° half-angle), and medium gain fan beam antennas possess long, narrow FOVs; hence, while both high and medium gain antennas provide sufficient spacecraft antenna gain for two-way Doppler tracking, attitude constraints must be imposed over the flyby period to ensure that the Earth line of sight is within the effective antenna FOV. Such attitude constraints may be in opposition to other spacecraft requirements, such as nadir pointing of the instrument deck. As shown in References 5 and 6, the limited amount of tracking data available via an optimized multiple fan beam configuration barely satisfies the Love number requirement with zero residual margin. A mechanism for providing ample tracking data throughout all flybys, regardless of the spacecraft attitude, is desired in order to satisfy the gravity science requirement without perturbing the spacecraft or mission design. To this end, the concept of utilizing Low Gain Antennas (LGAs) for deep space tracking is explored in the context of a Europa flyby tour.

LOW GAIN ANTENNAS FOR DEEP SPACE TRACKING

Deep space communication system design is typically limited by the great distances the signal must travel and the achievable spacecraft transmit power. Typically, the DSN performs closed-loop two-way radiometric tracking in which the signal is transmitted from a DSN ground station, received and re-transmitted by the spacecraft, and finally received at a DSN ground station. The downlink leg is the limiting design component as spacecraft can transmit at substantially lower power levels than a 34-m or 70-m DSN antenna. This work investigates the possibility of operating in an uplink-only configuration, thus removing the downlink component from the tracking signal. In such a telecommunications configuration, the carrier signal will be transmitted by the DSN and received at the spacecraft, where the tracking data will be stored onboard until downlinked to the Earth at a later non-mission critical time. Removal of the signal transmission from spacecraft to ground al-
allows the telecommunications system constraints on required spacecraft antenna gain to be loosened considerably. Due to the significant transmission power and gain of the DSN antennas, LGAs with typical 5 dBi - 6 dBi gain levels are sufficient to perform open-loop tracking on the uplink only. For mission safety purposes, LGAs are typically mounted on multiple faces of the spacecraft to ensure a communications link with Earth in the event that attitude control is compromised.

In order for the one-way uplink-only radiometric measurement to be as useful for science and navigation purposes as its two-way counterpart, the measurement quality must not be severely degraded by the onboard frequency source, typically an ultra-stable oscillator (USO). When utilizing an uncompensated USO, the short-term noise is driven by the onboard oscillator’s performance, and the signal frequency drift and acceleration must be re-estimated after lengthy tracking data outages. Re-estimation of the signal frequency parameters in turn degrades the trajectory solution quality due to the correlations with the orbital parameters. In order for the uplink-only one-way Doppler tracking configuration proposed here to be viable for science and navigation purposes, space clock accuracy and stability technology advancements such as those introduced by the Deep Space Atomic Clock (DSAC) must be utilized.

LGA Telecommunication System Design

The Deep Space Network (DSN) is capable of making two-way or one-way measurements with a closed-loop receiver when the received signal power-to-noise ratio ($C/N_0$) is as low as 7 dB-Hz, but the signal quality is significantly compromised. To achieve a reasonable thermal noise level (< 0.01 mm/s for X-band), the closed-loop received $C/N_0$ should be limited to $\geq 20 \text{ dB-Hz}$. However, when the received $C/N_0$ is as low as 7 dB-Hz, the measurement quality may be improved by utilizing open-loop recording and post-processing of the carrier radiometric signal. Assuming a margin of 3 dB-Hz, a reasonable open-loop tracking received $C/N_0$ threshold is 10 dB-Hz.

In order to determine whether or not this open-loop threshold may be met throughout the Europa flyby tour, it is necessary to compute received $C/N_0$ values under a realistic telecommunication system design and the trajectory geometric variations. The spacecraft telecommunication system values utilized for this analysis, shown in Table 1, have been sampled from the Cassini spacecraft design unless noted otherwise.

<table>
<thead>
<tr>
<th>Table 1. X-Band Uplink Link Design Parameters</th>
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<tr>
<td>Transmitter Parameters</td>
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<tr>
<td>Transmit Frequency ($f$)</td>
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<td>Transmitter Power ($P_x$)</td>
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<tr>
<td>Transmitt Antenna Gain ($G_x$)</td>
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<td>Wave Guide Loss ($L_{wg}$)</td>
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<td>Transmit Antenna Pointing Loss ($L_{point,x}$)</td>
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<td>Path Parameters</td>
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<tr>
<td>Atmospheric Attenuation Loss ($L_a$)</td>
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<td>Receiver Parameters</td>
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<tr>
<td>Receive Antenna Gain ($G_r$)</td>
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<tr>
<td>Lumped Ckt/Antenna Loss ($L_{ckt}$)</td>
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</table>
The effective isotropically radiated power (EIRP) is computed in dBi as:

\[ EIRP = P_x + G_x + L_{wg} + L_{\text{point},x} \]  

in which \( P_x \) is the DSN transmitter power, \( G_x \) is the DSN transmitter gain, \( L_{wg} \) is the DSN transmitter wave guide loss, and \( L_{\text{point},x} \) is the DSN transmit antenna pointing loss. The values selected for the transmitter represent the 20 kW 34-m beam wave guide DSN antennas. Such antennas are available at each of the Goldstone, Canberra, and Madrid DSN complexes, and are capable of supporting S-band and X-band uplink radiometric tracking. The Goldstone complex also supports Ka-band uplink.

The space loss (\( L_s \)) as the signal propagates across the Earth-to-spacecraft distance \( R \) is modeled as:

\[ L_s = \left( \frac{\lambda}{4\pi R} \right)^2 \]  

where \( \lambda \) is the signal wavelength. The spacecraft received power, \( C \), is computed as:

\[ C = EIRP + L_s + L_a + L_{\text{pol}} + L(\phi)_{\text{point},x} + G_r + L_{\text{ckt}} \]  

in which \( L_a \) is the atmospheric attenuation, \( L_{\text{pol}} \) is the receiver polarization loss, \( L(\phi)_{\text{point},x} \) is the receive antenna pointing loss at off-boresight pointing angle \( \phi \), \( G_r \) is the receive antenna gain, and \( L_{\text{ckt}} \) is the lumped circuit and antenna loss. Typical \( G_r \) values for an LGA are in the range of 5 dBi to 6 dBi. The receiver pointing loss is modeled as a function of off-boresight pointing angle (\( \phi \)), as shown in Figure 1, in accordance with the empirical performance of the Mars Reconnaissance Orbiter’s LGA receiver.

![Figure 1. Pointing loss of the LGA spacecraft receive antenna](image)

Finally, the receiver noise spectral density is computed as:

\[ \text{Finally, the receiver noise spectral density is computed as:} \]
\[ N_0 = k T_s \] (4)

in which \( k \) denotes the Boltzman constant (1.380e-23 J/K) and \( T_s \) denotes the system noise temperature (450.57 K).

The uplink power-to-noise ratio as received by the spacecraft LGA for typical receiver gain values and off-boresight pointing angles of 0° and 75° is shown over the course of the tour in Figure 2. The Europa flyby epochs are denoted by the dashed vertical lines. At 75° off-boresight pointing, 5.5 dBi of receiver antenna gain satisfies the open-loop tracking \( C/N_0 \) threshold for all flybys, demonstrating that open-loop tracking is achievable (with margin) over nearly the full LGA FOV. The results also indicate that approximately 3 dB of margin remains on the closed-loop \( C/N_0 \) threshold when tracking along the antenna boresight. Consulting Figure 1, the closed-loop threshold could be satisfied down to a 35° half-angle FOV. Confining the LGA FOV to a 35° half-angle will impose spacecraft attitude constraints similar to those when using medium gain fan beam antennas. As uplink open-loop tracking is sufficient for the gravity science purposes, it is assumed that the LGA FOV is realizable up to a 75° half-angle. If the telecommunication system design includes one LGA mounted on each of the six spacecraft body-fixed axes, continuous open-loop uplink tracking is feasible throughout the Europa flyby tour.

![Figure 2. Received Signal-to-Noise at spacecraft over flyby mission lifetime. Dashed vertical lines denote Europa flybys.](image)

**Expected DSAC Performance**

The DSAC project is a NASA Technology Demonstration Mission (TDM) that will bridge the gap between ground and space clocks by validating the on-orbit performance of a small, low-mass mercury ion atomic clock with long-term stability and accuracy on par with the clocks in use at the DSN. DSAC is planned to launch in May 2016 as a hosted payload onboard the Surrey Satellite Technology Ltd. Orbital Test Bed spacecraft. The DSAC TDM will validate the performance of the
mercury ion atomic clock in the Low Earth Orbit environment for a duration of one year. DSACs on-orbit stability, as measured by its Allan Deviation (AD), is expected to be less than 3e-15 at one day, with a ground laboratory version of DSAC currently demonstrating an AD of approximately 1e-15 at one day (Figure 3).

DSAC provides a frequency correction capability which allows for an adjustment to be made to the frequency of a USO-generated signal. The signal remains uncorrected for one DSAC control loop cycle (nominally 10 seconds), meaning that the short-term DSAC noise up to 10 seconds is driven by its USO. However, after one control loop cycle the USO signal frequency corrections are effectively implemented, and the corrected signal is dominated by the DSAC white frequency noise, as illustrated in Figure 3. The performance of the mercury ion atomic resonator can be quantified by the white frequency component of the frequency error uncertainty ($\sigma_{xf}$) model presented in Reference 12:

$$\sigma_{xf}^2(\tau) = \frac{h_0}{2\tau}$$

in which DSAC’s white frequency Allan variance parameter, $h_0$, is approximately 1.8e-25. The Doppler integration time used for this analysis is 60 seconds; the 60-second white noise obtained through evaluation of Eq. 5 at $\tau = 60$ seconds is approximately 3.8e-14 in fractional frequency (0.0114 mm/s).

Assuming no DSAC frequency signal compensation, the Cassini-Grade USO whose Allan deviation is shown in Figure 3 exhibits a flicker floor at approximately 1e-13 (0.03 mm/s) up to 100 seconds. However, the short-term Doppler measurement noise alone is not a sufficient metric for comparison of USO-enabled and DSAC-enabled tracking data. Characteristic of USO performance over long integration times, the Cassini-Grade USO frequency error, $x_f$, transitions to a random walk noise process after 100 seconds. The USO frequency error uncertainty over a time interval, $\Delta t$, may be approximated by the white frequency, flicker, and random walk frequency model pre-
sented in Reference 12:

$$\sigma^2_{x_f}(\Delta t) = \frac{h_0}{2\Delta t} + 4h_{-1} + \frac{8}{3}\pi^2 h_{-2}\Delta t$$  \hspace{1cm} (6)$$

Evaluating Eq. 6 for the Allan variance parameters \((h_0, h_{-1}, \text{and } h_{-2})\) corresponding to the Cassini USO performance shown in Fig. 3, the frequency error uncertainty grows to approximately 0.324 mm/s over the four hour Europa flyby pass duration (0.008 Hz for X-band Doppler), more than three times larger than the standard X-band Doppler data weight of 0.1 mm/s. Furthermore, the long-term USO noise characteristics violate the white measurement noise assumption of the Kalman filter, requiring implementation of techniques such as stochastic parameter estimation or pre-whitening of the data. In contrast, DSAC’s frequency error uncertainty decreases as the integration time increases, and is approximately 0.00075 mm/s at the end of the four hour period.

In addition to the random frequency errors, USOs exhibit deterministic frequency drift and discrete discontinuities, typically specified to be within a linear fractional frequency drift level of 1e-10 per day (approximately 30 mm/s per day). Navigation operations processing USO-enabled one-way Doppler data must re-estimate the signal frequency bias and drift following lengthy tracking gaps.

The DSAC TDM has an objective to identify improvements to the demonstration unit (DU) that will be flown in Low Earth Orbit for infusion into future deep space missions and Global Navigation Satellite System (GNSS) applications, such as reducing the size, mass, and power allocations. As of October 2014, the current best estimate size, mass, and average power of the DSAC DU are 29 cm x 26 cm x 23 cm, < 16 kg, and < 52 W; the target mass and average power values for an infusible deep space version of DSAC are < 10 kg and < 30 W, respectively. Tuning of the size, mass, and power can be performed to obtained the most appropriate design for a given application. Though DSAC is quite robust to the radiation, temperature, and magnetic perturbations in LEO, like many instruments DSAC would need to be placed inside a radiation vault if flown in the harsh Jovian radiation environment.

**Doppler Data Quality**

The expected quality of LGA uplink-only Doppler measurements is now computed for a X-band tracking architecture in which DSAC is utilized as the onboard frequency source. A detailed discussion of the Doppler noise model for both X-band and Ka-band has been presented in Reference 6; this section focuses on updates to that model in order to capture the LGA tracking concept. Most notably, the finite SNR (signal-to-noise ratio) noise introduced by the selected telecommunication system is dependent on whether closed-loop or open-loop tracking is implemented.

One-way thermal noise may be approximated as:\textsuperscript{14}

$$\sigma_v = \frac{c}{2\sqrt{2\pi f T}} \sqrt{\frac{B_L}{C/N_0}}$$  \hspace{1cm} (7)$$

in which \(T\) is the Doppler count time, and \(B_L\) is the loop band-width. The open-loop processing SNR may be maximized by setting the open-loop bandwidth to \(\frac{1}{2T}\).\textsuperscript{9} Evaluating Eq. 7 at the open-loop threshold of 10 dB-Hz and the X-band uplink frequency shown in Table 1, the maximum 60-second finite SNR noise for the LGA-enabled uplink-only Doppler measurement is 0.0023 mm/s. As detailed in Reference 6, the two-way finite SNR noise has been computed following the
Cassini HGA two-way closed-loop telecommunication system design as a representative deep space configuration. Other noise sources incorporated in the model include variations due to solar and ionospheric plasma, DSAC (for one-way measurements only), troposphere, Deep Space Transponder (DST, for two-way measurements only), ground antennas, the DSN frequency and timing system (FTS), and ground electronics. X-band Doppler measurement noise is dominated by variations in the plasma; this work utilizes a geometry-based model derived from empirical deep space tracking data:

\[ \theta_p(60\text{s}) = c(1.76\times10^{-14} \sin (SEP)^{-1.98} + 6.25\times10^{-14} \sin (SEP)^{0.06}), \ 0^\circ \leq SEP \leq 90^\circ \]  
\[ \theta_p(60\text{s}) = c(1.76\times10^{-14} + 6.25\times10^{-14}) \sin (SEP)^{1.05}, \ 90^\circ < SEP \leq 170^\circ \]  
\[ \theta_p(60\text{s}) = c(1.27\times10^{-14}), \ 170^\circ < SEP \leq 180^\circ \]  

in which \( SEP \) is the Sun-Earth-probe angle. The model shown in Equations 8 - 10 includes both solar and ionospheric plasma variations, and captures the short-term noise characteristic of the plasma effects. However, plasma noise follows a \( f^{-8/3} \) frequency power spectrum, in contrast to the white noise frequency power spectrum assumed by the Kalman filter \( (f^{-2}) \). As discussed in Reference 16, a scale factor \( (F) \) may be applied to map the plasma noise at relevant long integration times to the integration time of the Doppler data:

\[ F = 0.468 \left( \frac{T_s}{T} \right)^{1/3} \]

in which \( T_s \) represents the dominant underlying time scale of the signal. The dominant time scale for a Europa flyby is hours; as such the solar plasma model presented in Equations 8 - 10 is scaled by 2.7.

The details of all other noise component models are detailed in Reference 6. Assuming the noise components are uncorrelated with each other and over the one-way light time, a total Doppler noise profile is created via the root sum square (RSS) of all individual noise sources. Figures 4 and 5 present the total Doppler noise and all noise components for an HGA two-way X-band telecommunications configuration and an LGA uplink-only X-band design, respectively. Both the two-way and one-way noise profiles demonstrate the dominance of the plasma noise; Figure 5 also shows that the onboard clock noise (0.0114 mm/s) only influences the total Doppler noise when near solar opposition \( (SEP > 170^\circ) \). Two-way X-band Doppler data is typically weighted at 0.1 mm/s, an appropriately conservative value for \( SEP \) angles greater than approximately 32\(^\circ\). In comparison, the one-way X-band uplink-only Doppler data weight is less than 0.1 mm/s for \( SEP \) angles greater than approximately 23\(^\circ\). Rather than apply a fixed data weight across all flybys, this work models the Doppler data weight for each flyby as the RSS total shown in the figures. As the dominant noise is path-dependent and assumed uncorrelated over the light time, the two-way total Doppler noise is approximately \( \sqrt{2} \) times larger than the uplink-only total Doppler noise. However, it must also be considered that two-way data offers twice the measurement sensitivity of one-way data due to the round-trip light time.

**ANALYSIS OF GRAVITATIONAL TIDE ESTIMATION PERFORMANCE**

The ability to recover the gravitational tide parameters given the LGA/DSAC-enabled uplink-only X-band Doppler tracking data is investigated through numerical simulation analysis. A truth
Figure 4. X/X Doppler noise profile throughout flyby mission. Black circles denote the total Doppler noise for each Europa flyby; dashed blue line with circle markers represents the troposphere noise when the nominal model (blue solid line) is mapped to the elevation and season at the primary tracking station.

Figure 5. X-up Doppler noise profile throughout flyby mission. Black circles denote the total Doppler noise for each Europa flyby; dashed blue line with circle markers represents the troposphere noise when the nominal model (blue solid line) is mapped to the elevation and season at the primary tracking station.
trajectory for each flyby is numerically integrated using nominal point mass gravitational models for the Sun, planets, and other Jovian moons. The Europa gravitational tide is nominally defined as \( k_{20} = k_{21} = k_{22} = 0.2 \); only the second-degree Love numbers are non-zero. A 20 x 20 spherical harmonic Europa gravity field is modeled. Simulated one-way Doppler tracking data is generated from the truth trajectory, with the assumption that multiple LGAs provide sufficient coverage such that full tracking coverage is provided over all flybys. The simulated measurements are degraded with Gaussian noise based on the Doppler noise profile shown in Figure 5. Data is nominally simulated for a four-hour window centered on each flyby close approach, a period that covers the measurable sensitivity to Europa’s gravity field.

An Upper-Diagonal sequential Kalman filter is configured to estimate the spacecraft’s Europa-centric inertial position and velocity vectors, constant accelerations in the radial, tangential, and normal (RTN) orbit frame representative of unmodeled accelerations, Europa’s gravitational parameter \( \mu \), second degree Europa gravitational tide Love numbers \( k_{20}, k_{21}, \) and \( k_{22} \), and the zonal and sectoral coefficients of the 20x20 Europa spherical harmonics gravity field. All estimated states are classified as either local or global parameters; local parameters are states that are unique to each flyby, such as the position, velocity, and acceleration terms, while global parameters are states that are constant throughout the simulation, such as Europa gravity field coefficients. All global parameters are estimated as biases, while the local position and velocity states are dynamic parameters. The RTN accelerations are estimated in 8-hour batches as white noise stochastic parameters.

The a priori uncertainties of the Europa gravitational tide Love numbers and strongly correlated second degree spherical harmonic coefficients are sufficiently inflated to represent effectively no a priori information regarding the existence of a sub-surface liquid ocean. The higher degree \( n > 2 \) gravity field coefficient uncertainties are initialized according to the Kaula rule, assuming a Europa radius \( R_e \) of 1565 km and mantle radius \( R_{mantle} \) of 1465 km. The a priori uncertainty of all estimated states is assumed to be uncorrelated. Table 2 summarizes the gravity science filter configuration.

A priori state errors are injected into the filter nominal local and global states at the 1-\( \sigma \) level. The global parameter injected errors remain fixed throughout each realization, while the local parameter injected errors are randomized for each flyby. As a flyby mission produces gravity science data in a discontinuous manner, the information gained from individual flybys must be accumulated to produce a total improvement in gravity field knowledge. On the first flyby, the uncertainty is initialized as shown in Table 2 and a priori error is injected onto all states. On all subsequent flybys, the filter nominal global states and their associated covariance are initialized as the filter solution from the previous flyby, while the filter nominal local parameters are randomized and the covariance initialized by the uncorrelated 1-\( \sigma \) values shown in Table 2. The injected gravitational tide error is constrained such that the filter nominal Love numbers are always positive values, in order to reflect the physical constraints of tidal variations due to a rigid or liquid body.
<table>
<thead>
<tr>
<th>Estimated Parameter</th>
<th>Parameter Type</th>
<th>A Priori Uncertainty</th>
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<tbody>
<tr>
<td>Position (EME2000)</td>
<td>Local</td>
<td>10 km</td>
</tr>
<tr>
<td>Velocity (EME2000)</td>
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<tr>
<td>RTN accelerations</td>
<td>Local</td>
<td>5e-12 km/sec²</td>
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<td>Europa $\mu$</td>
<td>Global</td>
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<td>Europa $k_{20}$, $k_{21}$, $k_{22}$</td>
<td>Global</td>
<td>0.3</td>
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</tbody>
</table>

Europa 20x20 spherical harmonic coefficients: $n = 2$

$$\sigma = 10000 \times \left( \frac{28e5}{n^2} \right) \left( \frac{R_{mantle}}{R_e} \right)^n$$

Europa 20x20 spherical harmonic coefficients: $n > 2$

$$\sigma = \left( \frac{28e5}{n^2} \right) \left( \frac{R_{mantle}}{R_e} \right)^n$$

Initial simulations revealed filter linearization issues when the a priori state errors were injected at the 1-σ level. As an example, Figure 6 presents the $k_{22}$ Love number solution error and 3-σ uncertainty envelope over the course of the flyby mission for several realizations. In each realization, the a priori error on the global states was injected at the full 1-σ level, while the a priori error on the local states was injected at 0%, 1%, 10%, and 100% of the 1-σ level. Gaussian measurement noise was included in each realization in accordance with the uplink-only Doppler noise profile shown in Figure 5. Although only showing a single realization for each set of initial state perturbations, the results demonstrate that the filter performs well when little to no error was injected on the position, velocity, and acceleration local states. However, when the a priori position, velocity, and acceleration error was injected at the full 1-σ level, the reference trajectory may deviate from the truth trajectory such that the linearization assumptions of the sequential Kalman filter are violated, and the estimated $k_{22}$ errors are no longer within the formal 3-σ uncertainty bounds.

![Figure 6. $k_{22}$ solution error and formal 3-σ uncertainty bounds for varying a priori state errors](image.png)
Randomization of the local state initial conditions was utilized to develop constraints to ensure that the filter’s linearization assumptions are not violated. *A priori* errors were injected at 5% and 100% of the 1-σ level for the local and global states, respectively. Over 100 randomized realizations were generated to provide a sufficient number of samples for statistical assessment of the initial conditions. Even with only 5% of the error injected, approximately 3.7% of the flyby realizations violate the 3-σ uncertainty bounds, as is shown in Figure 7.

Statistical analysis of the local initial condition error distributions revealed no distinction between initial conditions that resulted in realizations that did and those that did not violate the 3-σ uncertainty bounds. However, analysis of the filter’s nominal trajectory errors mapped to the Europa close approach clearly highlights the conditions which will drive the filter’s linearization assumptions to be violated. Figure 8 presents the filter nominal trajectory errors in the time of periapsis and the close approach B-plane target vector angular error ($\theta$) for each of the 45 flybys. Each realization is categorized as those for which the estimate error is within the 3-σ bounds (blue) and those for which the estimate error exceeds the 3-σ bounds (red). The second Europa flyby, E2, exhibits a significantly larger spread of the B-plane error than the other 44 flybys. Additionally, the E2 B-plane errors are clearly separated into errors that are within and those that are outside of the filter linearization constraints. At only 100 km altitude, the E2 flyby is the spacecraft’s first extremely close encounter with Europa, and the coupling of large *a priori* uncertainties on the Europa gravity field and the high sensitivity to these parameters has the potential to adjust the filter solution outside of the linear region.

The statistical distribution of the B-plane errors can be utilized to generate B-plane targeting constraints on the gravity science filter’s nominal trajectory. The standard deviations of the sample B-plane parameters B.T, B.R, and time of periapsis errors that do not result in linearization violations are 5.1 km, 5.8 km, and 0.6 seconds, respectively. In order to ensure that a realization will not violate the filter’s linearization assumptions, the gravity science filter’s nominal close encounter B-plane error must be within this 1-σ target space.
APPRAOH ORBIT DETERMINATION

The B-plane uncertainty constraints on the second Europa flyby derived from the statistical results shown in Figure 8 may be satisfied via the addition of pre-flyby approach orbit determination. By processing tracking data for some finite amount of time preceding the flyby, corrections to the initial state at the gravity science filter’s reference epoch may be estimated. The gravity science filter is then initialized with the approach orbit determination state solution, constraining the gravity science nominal trajectory to be within the system’s linear region. A parametric approach was utilized to determine the duration of approach orbit determination tracking data required to satisfy the E2 flyby B-plane constraints. Various durations of continuous pre-flyby tracking data were processed with the filter configuration defined in Table 2, with the data cut off (DCO) set to two hours before the E2 close approach (i.e., the reference epoch of the gravity science filter). The formal state uncertainty at the DCO was then mapped to the close approach B-plane for comparison against the B-plane uncertainty requirements of 5.1 km (B.T), 5.8 km (B.R), and 0.6 seconds (time of periapsis). Figure 9 presents the resulting B-plane 1-σ uncertainties for approach orbit determination tracking durations from 3 hours to 13 hours. The B-plane uncertainty requirements are denoted by the dashed lines. The uncertainty of the three selected B-plane constraint parameters (B.T, B.R, and time of periapsis) are all below the respective requirement given at least 12 hours of approach orbit determination.

The approach orbit determination technique is implemented by separating the data processing for each flyby into two sequential Kalman filters. First, the 12-hour approach orbit determination filter is executed to convergence. The gravity science filter local states are initialized with the solution of the approach orbit determination filter, and the gravity science filter is then executed to convergence. The local parameter errors are injected at the beginning epoch of the approach orbit determination for each flyby, and the global parameter errors for a given realization are universal for all flybys. The approach orbit determination filter is configured as shown in Table 3, with the RTN accelerations estimated as white noise stochastic parameters every 8 hours. The gravity science
filter configuration is initialized as shown in Table 2, and the *global* state solutions and covariances from each flyby used to initialize the next flyby’s *a priori* global conditions.

**Table 3. Approach Orbit Determination Filter Configuration**

<table>
<thead>
<tr>
<th>Estimated Parameter</th>
<th>Parameter Type</th>
<th>A Priori Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position (EME2000)</td>
<td>Local</td>
<td>10 km</td>
</tr>
<tr>
<td>Velocity (EME2000)</td>
<td>Local</td>
<td>1 cm/sec</td>
</tr>
<tr>
<td>RTN accelerations</td>
<td>Local</td>
<td>5e-12 km/sec^2</td>
</tr>
</tbody>
</table>

Numerous realizations were executed to assess the $k_{22}$ estimation performance given the 12 hours of approach orbit determination. The *a priori* errors on all states, *local* and *global*, were injected at the full 1-σ uncertainty levels shown in Tables 2 and 3. The estimated $k_{22}$ error and the formal 3-σ uncertainty bounds for each realization are shown in Figure 10. For reference, the $k_{22}$ uncertainty requirement of 0.05 (1-σ) is marked by the dashed black line. The gravity science requirement is met at the E15 flyby, which is relatively early in the mission and prior to the gravity science specific flybys E17-E24; a sufficient number of flybys remain in the Jovian tour to provide robustness to missed flybys. The final gravity solution is reconstructed to a 1-σ level of approximately 0.01, leaving 80% margin on the requirement. The value at which the $k_{22}$ value becomes negative is denoted by the dashed cyan line. The filter errors appear biased during the early flybys due to the injected error constraint to ensure that the filter nominal Love numbers are always positive. The addition of 12 hours of approach orbit determination tracking prior to each flyby has successfully constrained the gravity science *a priori* state errors such that the Kalman filter linearization assumptions are not violated, and the formal 3-σ uncertainty limits are representative of the estimation errors.

While the addition of 12 hours of approach orbit determination successfully maintains the linearization requirements of the gravity science filter, it holds the potential to place a significant constraint on the spacecraft telecommunication system. If a two-way tracking configuration is uti-
lized, the limited FOVs of an HGA or fan beam antennas would require an Earth-pointing spacecraft attitude profile for the 12 hour duration of approach orbit determination, which may preclude the spacecraft from performing other pre-flyby activities such as instrument check-out and calibration. Conversely, the telecommunication design may take advantage of complete hemispherical coverage of multiple LGAs. In the LGA/DSAC configuration, the high-quality uplink-only Doppler signal may be open-loop recorded onboard with no specific attitude requirements throughout the approach orbit determination campaign and flyby window.

CONCLUSIONS

This work explored the feasibility of utilizing Low Gain Antennas (LGAs) and the Deep space Atomic Clock (DSAC) for uplink-only open-loop Doppler tracking in the context of a Europa flyby mission. Typically baselined into spacecraft design for mission safety purposes, LGAs provide nearly hemispherical coverage and if positioned on each spacecraft body face provide complete tracking coverage regardless of the spacecraft attitude. Link budget analysis of realistic telecommunication system design parameters demonstrated that open-loop tracking is possible (with sufficient margin) for an uplink-only X-band tracking configuration at Jovian distances.

DSAC is imperative for the realization of high-quality one-way Doppler tracking data. In addition to deterministic linear frequency drift and discrete jumps, free-running USOs exhibit frequency error random walk over long integration times, resulting in significant frequency error uncertainty growth over the four hour flyby tracking duration even when utilizing a high-quality USO. In contrast, DSAC’s frequency noise is white for integration times longer than the control loop cycle, and as such the frequency error uncertainty decreases with increasing integration time. The 60-second X-band white Doppler noise model utilized for this work is dominated by path-dependent plasma variations. In general the one-way Doppler data is less noisy than its two-way counterpart by a factor of approximately $\sqrt{2}$, countering the increased measurement sensitivity of two-way Doppler tracking data.
Numerical analysis of the flyby gravity science experiment revealed filter linearization issues when the tracking data is limited to ±2 hours around close approach. Nominal state perturbation at the 1-σ level could produce a trajectory that falls outside of the linear proximity of the truth trajectory, resulting in $k_{22}$ estimate errors outside of the formal 3-σ uncertainty bounds. Constraints on the target space of the second Europa flyby, which is the first low-altitude flyby encountered and thus extremely sensitive to perturbations of the gravity field and very large a priori uncertainties, were statistically determined in terms of close approach B-plane parameters. One solution to constrain the close approach target errors is to perform additional tracking directly prior to the flyby. Parametric covariance analyses of this continuous approach orbit determination technique revealed that a minimum of 12 hours of pre-flyby tracking data is required in order to satisfy the B-plane target constraints. Numerical analysis demonstrated that the inclusion of 12 hours of approach orbit determination maintains the linearization assumptions of the gravity science Kalman filter such that the resulting $k_{22}$ estimation error is well represented by the formal 3-σ uncertainty bounds. The gravity science requirement is met relatively early in the mission at the E15 flyby, leaving sufficient margin in the event of a missed flyby (including the gravity science specific flybys, E17-E24). The final $k_{22}$ solution is known within a 1-σ value of approximately 0.01, leaving 80% margin on the gravity science requirement. The ability to provide 12 hours of pre-flyby tracking data further highlights the flexibility of the LGA/DSAC telecommunication configuration, as additional uplink-only tracking data may be collected with no attitude constraints during what may be a vital pre-flyby check-out period.

ACKNOWLEDGMENTS

This research was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration. The authors would like to acknowledge Brent Buffington and Try Lam of JPL for design of the utilized trajectory, and Bill Folkner and Peter Ilott of JPL for contributions to the telecommunications system parameters.

REFERENCES


