A Simple Analytic Model for Estimating Mars Ascent Vehicle Mass and Performance

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The Mars Ascent Vehicle (MAV) is a crucial component in any sample return campaign. In this paper we present a universal model for a two-stage MAV along with the analytic equations and simple parametric relationships necessary to quickly estimate MAV mass and performance. Ascent trajectories can be modeled as two-burn transfers from the surface with appropriate loss estimations for finite burns, steering, and drag. Minimizing lift-off mass is achieved by balancing optimized staging and an optimized path-to-orbit. This model allows designers to quickly find optimized solutions and to see the effects of design choices.

Nomenclature

Az_i	=	inertial azimuth of launch asymptote
g	=	gravitational acceleration of Earth
$GLOM$	=	gross lift-off mass
h	=	orbit altitude
I_{sp}	=	specific impulse
M_0	=	total mass at stage ignition
$M_{1,2}$	=	total mass of stage 1 or 2
MAV	=	Mars ascent vehicle
M_{dry}	=	stage dry mass
M_{fixed}	=	stage dry mass that does not vary with propellant
M_{PL}	=	payload mass
M_{prop}	=	stage propellant mass
OS	=	orbiting sample
r_m	=	radius of Mars
SMF	=	structural mass fraction
T	=	engine thrust
t_b	=	stage burn duration
V_0	=	circular velocity at the surface
V_c	=	circular velocity on orbit
V_{Mars}	=	rotational velocity of the surface
$V_{S,E,Z}$	=	south, east, and zenith components of the launch velocity
ΔV	=	change in velocity
ΔV_{DL}	=	drag losses
ΔV_{GL}	=	gravity losses
ΔV_{SL}	=	steering losses
μ	=	gravitational parameter of Mars
ϕ	=	flight path angle
ϕ_{bo}	=	flight path angle at stage 1 burnout

I. Introduction

There have been dozens of designs for Mars ascent vehicles (MAVs) to return samples over the past four decades.[1,2] One of the most popular designs is that of a two-stage solid rocket, which is often favored because it tends to reduce both mass and complexity.[3] Arriving upon a converged, optimized MAV design complete with all relevant

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masses and performance values generally requires an iterative approach between a team of subsystem engineers and a 3-DOF or 6-DOF numerical optimizer.[4,5] Once a working design is in hand, the computations can be repeated with small variations in order to explore sensitivities.[6] Work has been done to reduce the complexity of the numerical trajectory simulations,[7] but the performance parameters must still be iterated upon with the individual subsystems of the MAV design.

In this paper we simplify the process by treating the ascent problem as a two-burn transfer from the surface of Mars to the desired orbit. Due to the high thrust and short burn times of solid motors, this assumption is quite valid. Typically > 96% of an ascent profile is in coast phase. The total ΔV to orbit can then be computed analytically by making an assumption on the "path-to-orbit". As illustrated by Whitehead,[8] this can vary from a vertical launch to a horizontal, Hohmann-like transfer. This path-to-orbit determines not only the total ΔV, but the relative ΔV needed from each stage as well. Losses due to atmospheric drag, steering, and finite burns are parametrically estimated and added to the ΔV requirements.

Next, we present a universal MAV model which simplifies the model down to masses that vary and masses that are fixed, along with any margin multipliers. This model, combined with thrust and specific impulse (I_{sp}), can be used in the standard rocket equation to determine the mass-optimal ΔV split between the two stages. Minimizing the gross lift-off mass (GLOM) is controlled by balance of total ΔV to orbit and optimal ΔV split between the two stages. Both of these are controlled by one parameter: the path-to-orbit, which we designate by ϕ_{bo}, the flight path angle at stage 1 burn-out.

II. Methods

Ascent profiles on Mars are significantly different than on Earth. Launch vehicles on Earth must provide 9-10 km/s of ΔV through a significantly thicker atmosphere. Optimized ascent profiles typically rise vertically with reduced thrust to minimize drag through the lower atmosphere, throttle back through max-Q, pitch over, and continue burning nearly the entire time to orbit. Ascent modeling requires complex fluid dynamics, aerodynamics, control laws, etc. in addition to the rocket equation.[9] On Mars, however, the ascent problem is much more akin to a two-burn orbital transfer than a complex Earth ascent.[10] This is due to the significantly thinner atmosphere (0.6% of Earth's mean sea level pressure) and reduced ΔV requirement (3.8 – 4.5 km/s). High-thrust solid-rocket motors (SRMs) provide the impulse required in a small fraction of the time it takes to reach orbit, resulting in a long, ballistic coast phase (see Figure 1a).

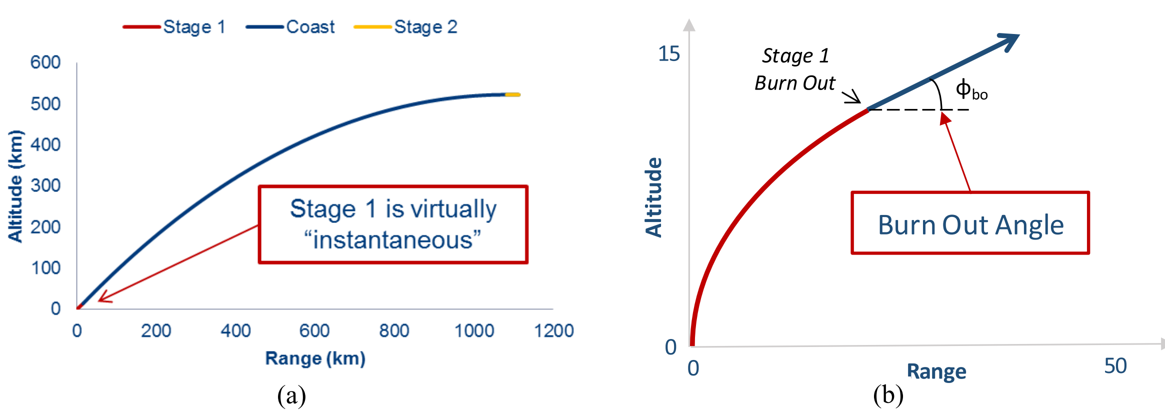

Figure 1. (a) Typical 2-stage ascent profile to a low Mars orbit. *Note that the majority of the ~700 second ascent is spent in coast phase. The maneuvers are so short that they can be modeled as instantaneous with reasonable accuracy*. **(b) Definition of burn out angle, ϕ_{bo}.** *The angle between local horizontal and the velocity vector at stage 1 burn out.*

A. ΔV and the Path-to-Orbit

To first order, the ΔV required to reach Mars orbit can be estimated by calculating the two-burn transfer from the surface of Mars to the desired orbit. In the absence of an atmosphere the optimal would be a Hohmann transfer that launches with a 0° elevation and reaches orbit 180° later. For a 500 km circular orbit this would require 3.79 km/s of

ΔV, ignoring Mars' rotation. Most of the ΔV would be in the 1st burn, leaving only a small 116 m/s burn to circularize. This is denoted as Path A in Figure 2.

To the opposite extreme, it is possible to launch vertically to the desired altitude, then do a 90° turn to reach the circular orbital velocity. This method requires 5.1 km/s of ΔV, with two-thirds of that on the 2nd stage (Path B).[11] Launch angles between 0° and 90° lead to intermediate ΔV requirements along with a varying distribution of the ΔV split, as illustrated in Figure 2. For finite burns the launch angle is often vertical with a rapid pitch-over maneuver to send the MAV on an optimized ascent. The angle, therefore, that determines the path-to-orbit is the flight path angle at the burn out of stage 1, as shown in Figure 1b. This angle is designated ϕ_{bo} and is what determines the total ΔV as well as the ΔV split. This is the primary parameter that is optimized in numerical simulations.

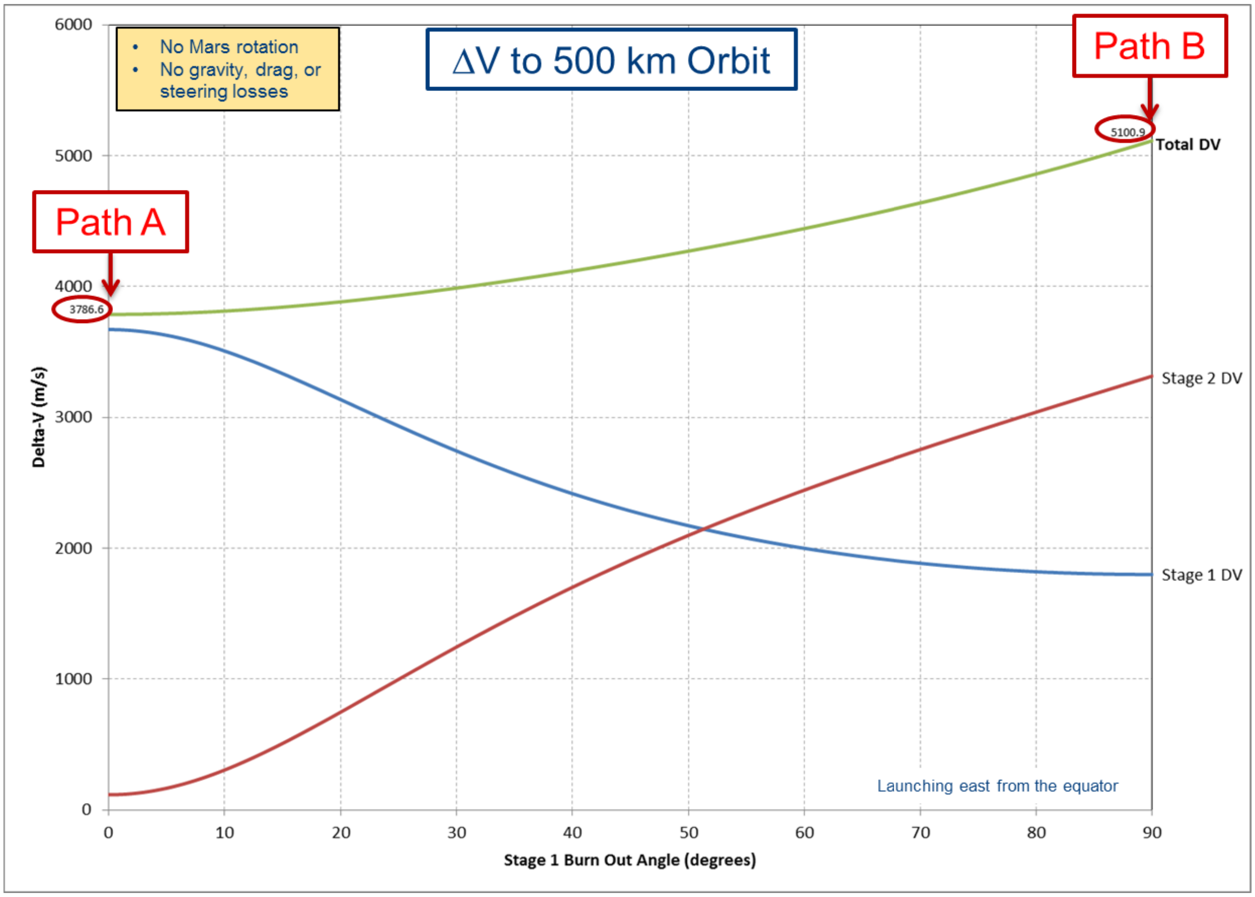

Figure 2. ΔV Requirements for a 500 km Circular Orbit vs. Stage 1 Burn-out Angle. *Without losses, the ideal path to orbit is a horizontal launch Hohmann transfer (path A). Note that low elevation transfers require that the majority of the ΔV be provided by the 1st stage. A vertical launch followed by a 90° circularization burn requires the lease 1st stage ΔV, but the most total (path B).*

B. Calculating ΔV

In order to calculate the ΔV required to get to orbit, we begin by determining the velocity necessary (in an inertial reference frame) on the surface to loft the MAV to the desired apex, h. This is, of course, dependent on ϕ_{bo}.

$$\Delta V_{1,i} = \sqrt{\frac{2(V_c^{\ 2} - V_0^{\ 2})}{[V_c \cos(\phi_{bo})]^2 - 1}} \tag{1}$$

$$\text{where } V_c = \sqrt{\frac{\mu}{r_m + h}} \text{ and } V_0 = \sqrt{\frac{\mu}{r_m}}$$

which are the circular velocities at altitude (h) and at the surface, respectively. r_m is the radius of Mars.

This inertial velocity must then take into account the rotation of Mars. The ΔV required is adjusted by vectorially removing the eastward contribution of Mars' rotation from the first stage ΔV.

$$V_{Mars} = 240.7 * \cos(lat) \text{ m/s} \tag{2}$$

where *lat* is the latitude of the launch site. The addition (or removal) of Mars' contribution is performed in the south-east-zenith (SEZ) reference frame. In this frame V_{Mars} is always due east. To take the most advantage of the rotation it is best to launch from the equator eastward into an equatorial orbit.

The inertial velocity vector is represented in the SEZ frame by eqs. (3)-(5). Note that V_{Mars} is removed from V_E.

$$Vs = -\Delta V_{1,i} \cos(\phi_{bo}) \cos(Az_i) \tag{3}$$

$$V_E = \Delta V_{1,i} \cos(\phi_{bo}) \sin(Az_i) - V_{Mars} \tag{4}$$

$$V_Z = \Delta V_{1,i} \sin(\phi_{bo}) \tag{5}$$

Az_i is the launch azimuth (in the inertial frame) necessary to achieve the desired orbital inclination, i, from the launch latitude. It is calculated by

$$Az_i = \sin^{-1}[\cos(i)\cos(lat)] \tag{6}$$

The magnitude of the impulsive ΔV required on the surface is then given by

$$\Delta V_1^* = \sqrt{{V_S}^2 + V_E^2 + V_Z^2} \tag{7}$$

The asterisk * here represents the idealistic, impulsive ΔV.

The ΔV required to circularize once the MAV has coasted to its apex is simply the difference between the horizontal component of its velocity and the circular velocity at the given altitude, V_c.

$$\Delta V_2 = V_c - \Delta V_1 \frac{r_m}{r_m + h} \cos(\phi_{bo}) \tag{8}$$

For simplicity the equations shown here assume a circular final orbit. In practice any orbit may be targeted by replacing V_c with the velocity required at that point in the orbit. $\Delta V_1^* + \Delta V_2$ is the minimum total ΔV required to achieve a desired orbit for a given ϕ_{bo}.

C. Estimating ΔV Losses

Losses were estimated by comparing idealistic ΔV calculations with those from real numerical simulations. Differences are typically less than 5% and somewhat predictable, thus eliminating the need for high-fidelity models. What is important is to correctly model the trends as functions of the relevant parameters.

Losses can be categorized into three types:

1) Gravity losses
2) Drag losses
3) Steering losses

The second stage burn typically has a zero flight path angle (i.e. horizontal) throughout its duration in addition to occurring above the atmosphere, thus avoiding any appreciable losses listed above. For this model the second stage ΔV is given by eq. (8) without further modification.

Gravity loss is due to the finite nature of the burn. It is a function of the burn time, t_b, and is equivalent to the magnitude of the thrust required to counteract gravity throughout the burn. The burn duration is calculated by

$$t_b = I_{sp} g \frac{M_{prop}}{T} \tag{9}$$

where I_{sp} is the specific impulse, g is Earth's gravitational acceleration, M_{prop} is the mass of the propellant expelled during the burn (see eq. (15)), and T is the magnitude of the thrust. The gravity loss is then given by

$$\Delta V_{GL} = g t_b \sin(\phi_{bo}) \tag{10}$$

where t_b is the duration of the 1st burn and ϕ_{bo} is the flight path angle (ϕ) at stage 1 burnout.† Gravity losses are largest for long burns (low thrust) and near-vertical ascents.

Calculating the actual drag loss would entail having detailed knowledge of MAV geometry, drag coefficients vs. Mach number, atmospheric models, etc. to feed into an optimizer. The good news is that the atmosphere on Mars is quite thin and the drag only represents a small fraction (1-3%) of the total ΔV to get to orbit. Drag loss increases with increasing velocity and lower burn-out angles. Higher thrust to weight ratios cause the MAV to reach higher speeds at lower altitudes where the atmosphere is thicker. Lower burn out angles not only carry larger 1st stage velocities (left-hand side of Figure 2), they also mean traveling at a shallower angle, thus staying in the appreciable atmosphere longer. The drag loss can be approximated by:

$$\Delta V_{DL} = \frac{1}{A}\left[\frac{CSC(\phi_{bo})}{(T/M_0)^B} \right] \tag{11}$$

where M_0 is the initial mass, T/M_0 gives the initial acceleration, and A and B are constants from curve fits. Using A = 2-3 (depends on aerodynamics) and B = 1.1 give good estimates for drag. The cosecant function assures a high penalty for very low elevation launch angles (goes to infinity at $\phi_{bo}=0$).

Steering losses are due to the need to command the vehicle using thrust-vector control in 6 degree-of-freedom simulations. Thrust-velocity misalignment causes the vehicle to pitch and turn throughout the burn – whether it be intentional or unintentional. The loss is proportional to the cosine of the thrust-velocity vector and is numerically integrated throughout the commanding sequence. The integration is quite complex and highly dependent on the MAV design itself. However, steering loss is typically the smallest of the losses (a few 10's of m/s at most) and can be effectively modeled using a linear model:

$$\Delta V_{SL} = C * \Delta\phi + D * \Delta Az \tag{12}$$

where $\Delta\phi = |\phi_{bo} - \phi_{launch}|$ and $\Delta Az = |Az_{desired} - Az_{launch}|$ during the 1st burn. C and D are constants from curve fits. Using $C = D = 0.2$ typically gives reasonable results. $\Delta Az = 0$ is nominal. Steering losses are zero with gravity turn and launch towards the true azimuth.

The total ΔV for stage 1 complete with losses is now given by

$$\Delta V_1 = \Delta V_1^* + \Delta V_{GL} + \Delta V_{DL} + \Delta V_{SL} \tag{13}$$

where ΔV_1^* is the idealistic velocity given in eq. (7).

†The flight path angle is not necessarily constant throughout the 1st stage burn (e.g. vertical launch). However, the pitch-over to the optimal ϕ is typically complete in a few seconds and does not significantly affect gravity losses.

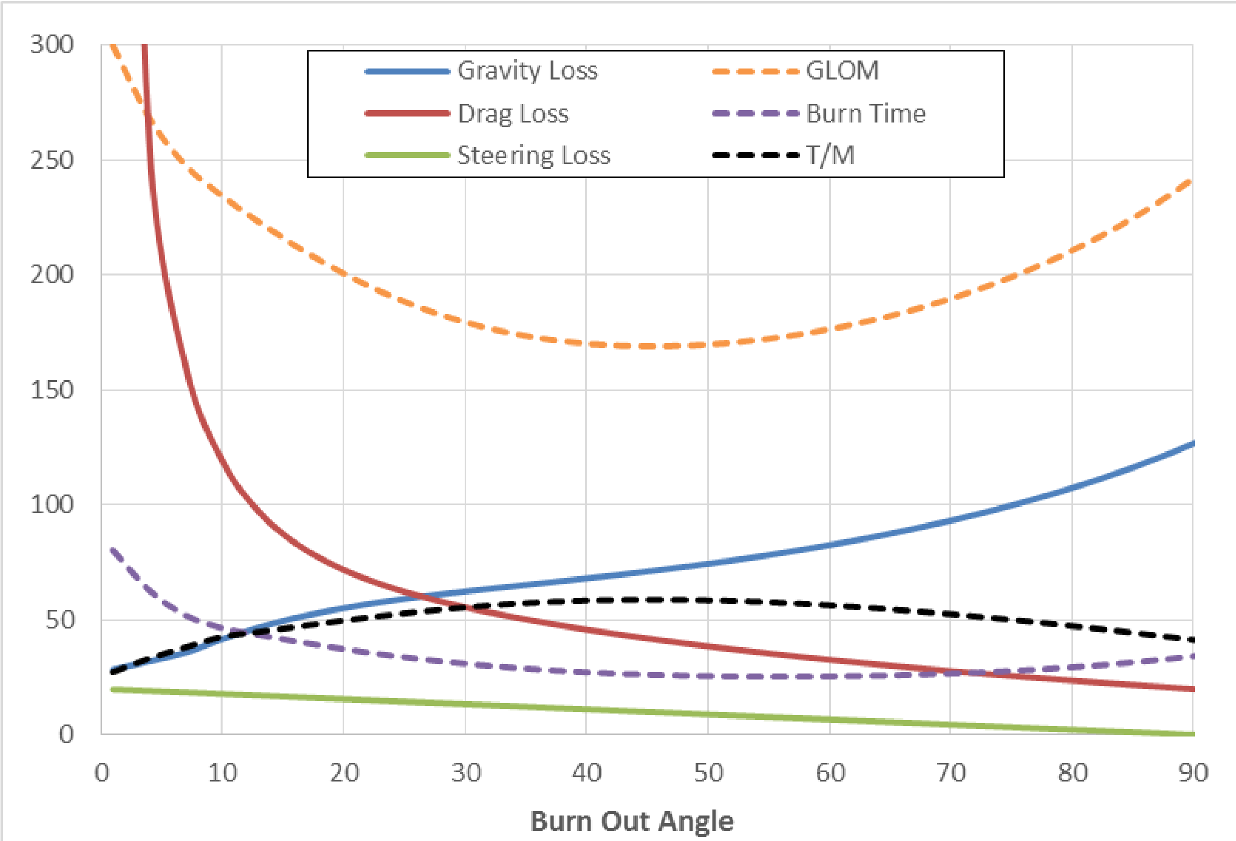

Figure 3. Losses vs. Burn-Out-Angle of Stage 1. *This plot was created from an actual trajectory to a 500 km orbit with all of the losses, ΔV requirements, and masses calculated simultaneously. This is why most of the losses are not linear as the stand-alone equations above would suggest. For this case GLOM is optimized at around ϕ_{bo}= 45°.*

The largest loss for high-thrust solid MAVs is typically due to gravity rather than drag. This is because of Mars' relatively thin atmosphere and typical aerodynamic shape (tall, thin) of most solid MAV designs. What is important to note in Figure 3 is that most losses only contribute 10's of m/s of ΔV, compared to the 4000+ m/s of ΔV required to achieve orbit. This implies that an exact model of each loss mode is not crucial. It is just important to represent the proper trends in the optimization process.

III. The Universal MAV Model

In order to quickly calculate the propellant masses and estimate the total mass of the MAV, we need a simplified model of a representative MAV. Figure 4 shows this universal MAV model which can be used in conjunction with the calculated ΔV's and the ideal rocket equation. Each stage consists of a fixed mass (ACS, telecom, adapters, etc.) and a variable mass that is a function of the total propellant mass carried by the stage (tanks, lines, etc.). In theory this kind of simplistic bookkeeping approach seems easy to do, but in practice it is quite difficult to determine which components will change with varying propellant mass and by what proportion.

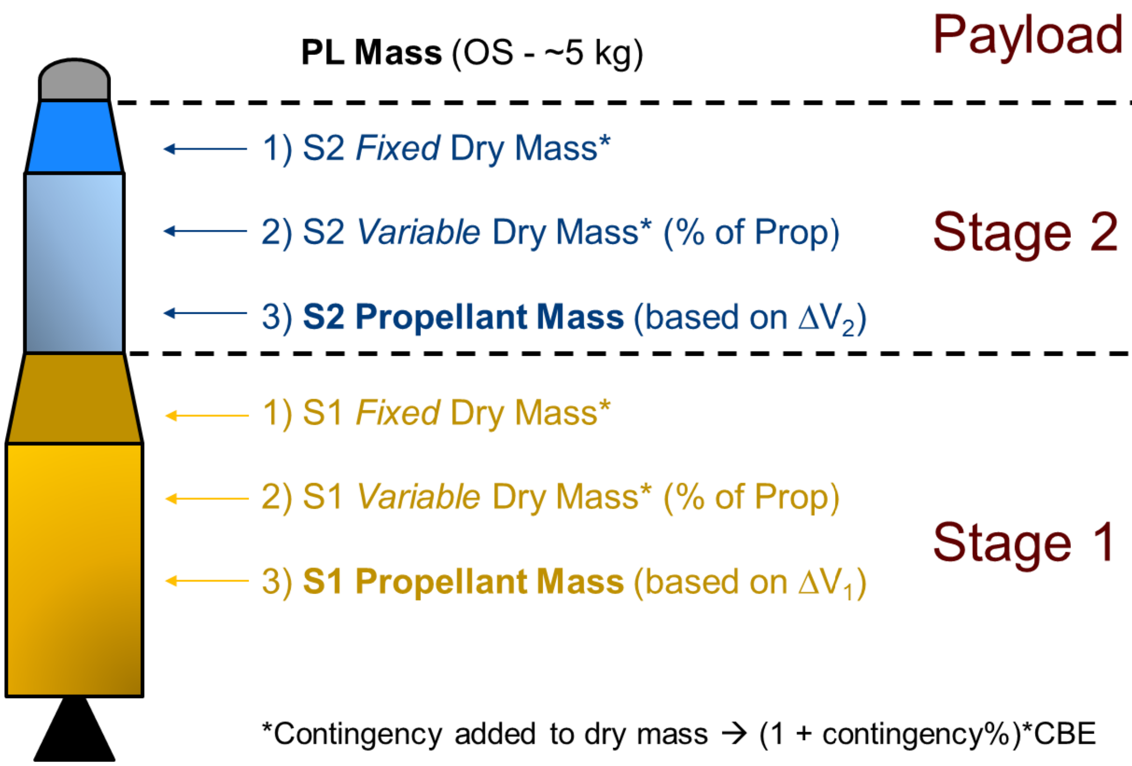

Figure 4. The universal model for 2-stage MAVs. *Each stage consists of a fixed mass and a variable mass, proportional to the propellant load carried. Contingency can also be added to dry masses to meet design guidelines. (CBE = current best estimate, unmargined)*

The dry mass of each stage is a simple function of fixed mass, M_{fixed}, plus a variable mass that scales with the loaded propellant, M_{prop}.

$$M_{dry} = M_{fixed} + SMF * M_{prop} \tag{14}$$

Where *SMF* is the structural mass fraction which is a percentage of the propellant mass. Contingency can be added if desired (omitted here for clarity) by multiplying the dry masses by the required factor. In practice the *SMF* is typically a number between 8-20%. If it is too large (30%+), often the design will fail to converge. In many cases it can be difficult to determine just how the dry mass will grow as propellant is added. In these cases it is reasonable to set *SMF* to 0 and place all of the dry mass, including tanks, into the fixed mass category. As long as the calculated propellant mass does not exceed the capacity of the selected tanks the resulting GLOM will be valid. This model, however, will not work as well for design changes and understanding sensitivities.

From the dry masses and ΔV's the propellant masses can be calculated using the rocket equation:

$$M_{prop} = (M_{dry} + M_{PL})(e^{\Delta V / I_{sp} g} - 1) \tag{15}$$

Where M_{PL} is the payload mass, which for the 1st stage is the entire wet mass of the 2nd stage and for the 2nd stage is the mass of the orbiting sample (OS). Since propellant mass is a function of the dry mass, which in turn is a function of the propellant mass, it is inherently necessary to iterate between eqs. (14) and (15).

The total mass of each stage is simply:

$$M_{stage} = M_{dry} + M_{prop} \tag{16}$$

The gross lift-off mass is then the sum of the stages and payload mass:

$$GLOM = M_1 + M_2 + M_{PL} \tag{17}$$

GLOM is usually the primary figure-of-merit in preliminary MAV designs. Numerical optimizers choose amongst variable parameters so as to minimize this mass. As long as the other constraints (e.g. peak heating, total length, g-loads, orbital accuracy, etc.) are achieved, GLOM can be seen as a surrogate for cost and complexity.

A. Model Implementation in Excel

It was noted that the calculation of dry mass and propellant mass of each stage is iterative. What's more is that the ΔV in eq. (15) for stage one is dependent on the mass calculations because of the drag, steering, and gravity loss models, creating another iteration loop. As we seek to minimize GLOM by varying ϕ_{bo} all of these parameters change and restart the iteration process. Implementing an algorithm for this MAV model would require many nested

Input						Primary Outputs		
Input	**Override**	**Suggested**	**Used**	**Units**	**%**	**Quantity**	**Calculated**	**Units**
Orbit Type	Circular	Circular	Circular			GLOM	194.4	kg
Target Altitude	500	500	500	km		Stage 1		
---------					22%	Burn Out Mass	41.9	kg
Burn 2 Alt		500	500		59%	Propellant	114.9	kg
S1 Burn Out Angle	35	46.5	35	deg	81%	Total	156.8	kg
Inclination	0	0	0	deg		Stage 2		
Latitude	0	0	0	deg	11%	Burn Out Mass	22.2	kg
Launch Elevation	90	90	90	deg	8%	Propellant	15.4	kg
Launch Azimuth	90	90	90	deg	19%	Total	37.6	kg
Stage 1					3%	OS		
Fixed Mass - 1	29.3	22	29.3	kg	63%	ΔV1	2506	m/s
PSMF - 1	0.0%	9%	0.0%		37%	ΔV2	1481	m/s
Contingency	43%	43%	43%	%		ΔVtot	3987	m/s
Propulsion						Time	723.5	sec
Thrust - 1	10000	7600	10000	N		Intermediate Outputs		
Isp - 1	285.7	285	285.7	s		Rmav	3896	km
Stage 2						Vmav	3315	m/s
Fixed Mass - 2	12	6	12	kg		Vmars	240.7	m/s
OS Mass	5	5	5	kg		AZi	90.0	deg
PSMF - 2	0.0%	12.0%	0.0%			ΔV1i	2570	m/s
Contingency	43%	43%	43%	%		VS	0	m/s
Propulsion						VE	1864	m/s
Thrust - 2	4320	4200	4320	N		VZ	1474	m/s
Isp - 2	285.7	285	285.7	s		Gravity Losses	73	m/s
Model Fidelity						Steering Losses	12	m/s
Mars Rotation?	Yes	Yes	Yes			Drag Losses	44	m/s
Drag Loss?	Yes	Yes	Yes			Burn Time 1	32.2	sec
Grav. Loss?	Yes	Yes	Yes			Burn Time 2	10.0	sec
Steering Loss?	Yes	Yes	Yes			T/W 1	51	m/s2
						T/W 2	115	m/s2
Min S1 prop	100.0		61.5			Mass Flow Rate 1	3.6	kg/s
Fixed S2 Total Mas	32.2		14.0			Mass Flow Rate 2	1.5	kg/s
Fixed fuel mass	102	BOA:	51.3			**Masses**		
SSTO	No	51.5	101.9046205	102.10714		S1 Variable Mass	0.0	kg
						S2 Variable Mass	0.0	kg
SENSITIVITIES	**Δ GLOM**					S1 Impulse	322	kN s
+1 kg S1 BO Mass	2.5	kg/kg				S2 Impulse	43	kN s
+1 kg S2 BO Mass	4.2	kg/kg						
+ 10 km altitude	0.75	kg/10 km				Accel S1	12.8	g
+1 S1 Isp	0.60	kg/sec						
+1 S2 Isp	0.17	kg/sec						

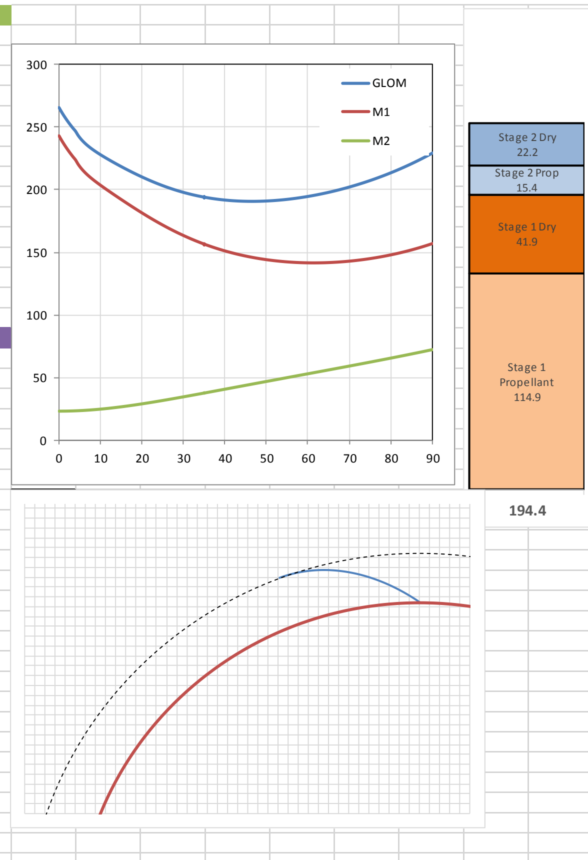

Figure 5. Screen shot of MAV model implementation in Excel. *The yellow cells are user inputs, the green cells are defaults or calculated values, and the blue cells are the values used in the model.*

programming loops to satisfy all of the equations simultaneously. Since the goal of this endeavor was to create a transparent, easy to use estimator of MAV mass and ΔV, we implemented the model using the ubiquitous Microsoft Excel®. Interlinking cells using iterative calculations is easy and straightforward.[‡]

Figure 5 shows a screen shot of how this model is used in Excel. It has suggested values, input overrides, and calculations all shown clearly at a glance. With all of the equations interconnected, it is possible to immediately see the effects of any changes. It also shows gear ratios (sensitivities, lower left), plots of the trajectory, effects of varying burn-out angle, and the mass distribution. On a second worksheet the same equations are repeated for all possible burn-out angles, and the one that minimizes GLOM is suggested on the main page. This "pseudo-optimization" method skirts the need for programming loops and is plenty accurate for this application. Of course, the user can always override the suggested burn-out angle if they wish to constrain ΔV's, propellant masses, etc.

Sometimes it is instructive to manually vary one parameter and watch how another parameter(s) changes. But other times a user may wish to vary a parameter in small steps and capture the outputs in other cells for the purposes of understanding sensitivities and creating plots. This can be done by hand in a few minutes with cut-and-paste to capture the desired information. But in order to do this faster, we wrote a short macro-based VBA script that allows the user to vary *n* input parameters over specified ranges and capture *m* outputs. Figure 7 below shows an example output that was created to visualize the effects varying the 1st stage thrust. The whole process took only a few seconds to set up and execute.

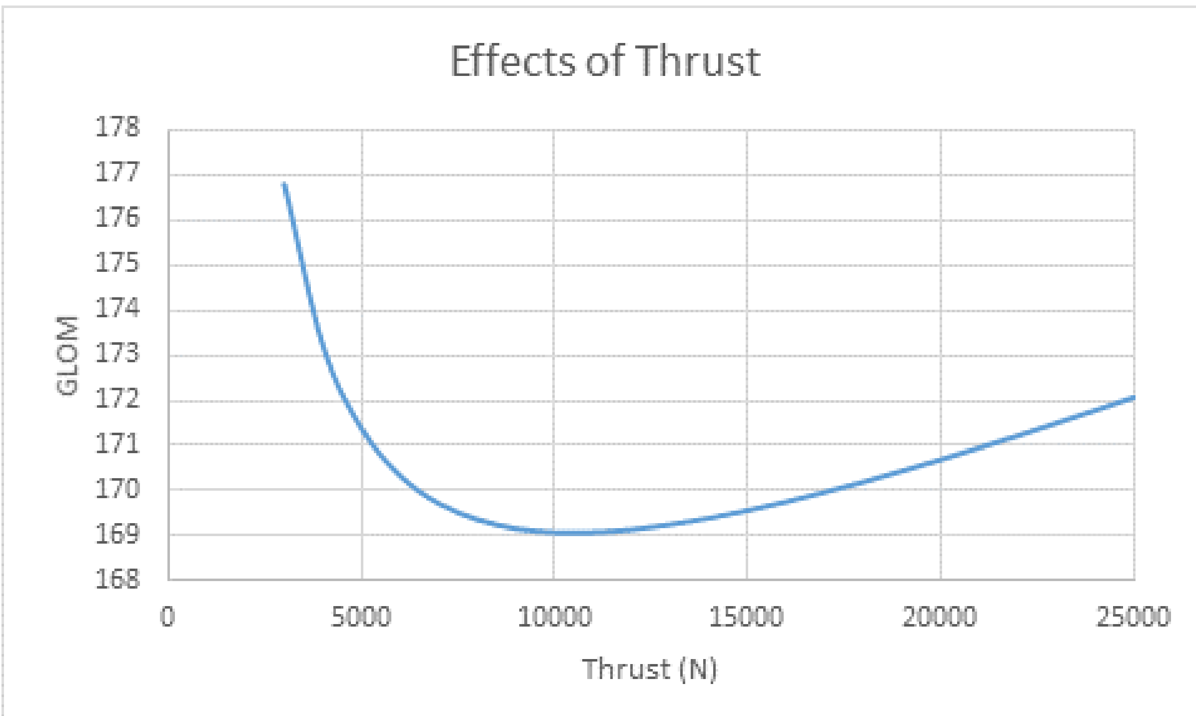

Figure 6. Trade space study: The effects of stage 1 thrust on GLOM. *The trade space explorer tool was used to quickly vary the thrust and capture the new GLOM (in kg). Thrust levels much below 3000 N caused the T/W ratio to be so low so as to allow burn time and gravity losses to go to infinity. High thrust decreases the gravity losses but increases drag.*

B. Optimized Staging

If two stages are identical in their I_{sp} and dry mass fraction (M_{dry}/M_{wet}) then their total mass will be minimized when ΔV is split equally between them. [12] As either of these parameters change, it will become beneficial to shift more of

[‡] It is, however, wise to utilize some precautions so as to not let the iterations grow without bounds by inputting parameters outside the design constraints. This will cause many of the cells to "break" and is sometimes difficult to correct. Save often.

the ΔV to the most efficient stage. Of course, this type of optimized staging can only occur where the total ΔV can be arbitrarily split between the two stages, such as in deep space. In the ascent vehicle problem, total ΔV and the ΔV split is dictated by the path used to achieve orbit, dictated by ϕ_{bo}.

Figure 7 shows the effects of varying the ΔV split for 4200 m/s with two dissimilar stages. In this case the 2nd stage has a higher I_{sp} and lower dry mass fraction, thus making it more efficient to carry a larger portion of the ΔV. This is typically the case for two-stage MAVs, even with similar I_{sp}'s, because some of the hardware (avionics, interstages, etc.) are jettisoned along with the 1st stage, thus creating a more efficient 2nd stage. Placing ~1000 m/s on the 1st stage in this case minimizes the total mass. The minimum is somewhat soft; adding or subtracting 500 m/s from the 1st stage burn only increases the total mass by a few percent.

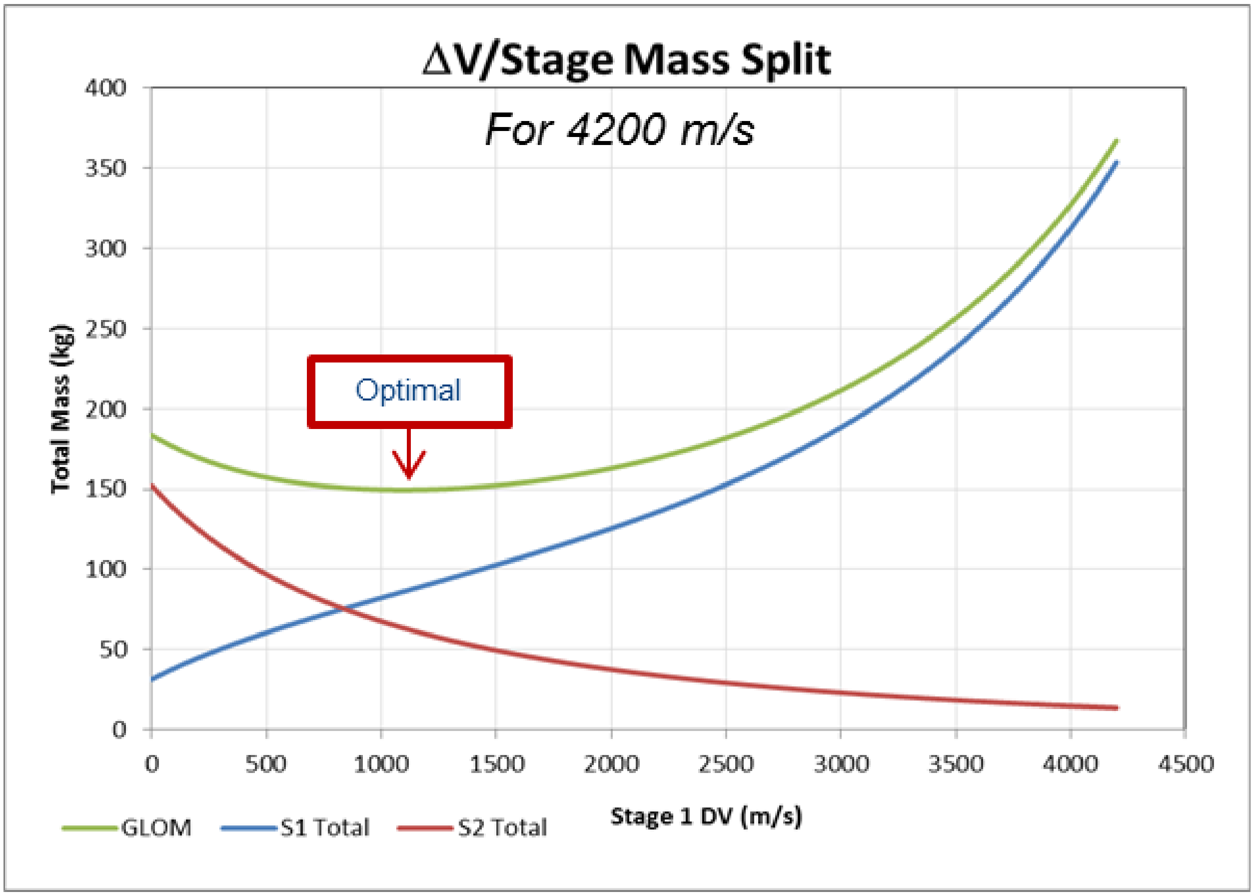

Figure 7. Optimizing the ΔV split between two dissimilar stages. *The green line shows the total mass as a function of the portion of the ΔV provided by stage 1, leaving the rest of the 4200 m/s for stage 2. If the stages had the same I_{sp} and dry mass fraction the minimum would occur at 2100 m/s.*

Minimizing the total ΔV to orbit does not minimize GLOM. As Figure 2 shows, it may require up to 30% less ΔV to launch near horizontal. However, most of the ΔV must be provided by the 1st stage. Figure 7 shows that putting too much ΔV on the 1st stage is highly detrimental – adding 50% or more to the total mass. The balancing act between minimizing total ΔV and optimized staging leads to a minimum total GLOM. Figure 8 shows how varying ϕ_{bo} affects the GLOM for a typical 2-stage solid MAV. For this case GLOM is minimized around 48°. It is also interesting to note that there is a shallow minimum around this angle. Changing ϕ_{bo} by +/-10° only increases GLOM by 4 kg. But, ΔV_1 ranges from 2 – 2.5 km/s, ΔV_2 ranges from 1.5 – 2.3 km/s, and the total ΔV ranges from 4 – 4.3 km/s. This is why some optimizers appear to give quite different results from a different one when in reality they are quite close. It is also useful to be able to "move" ΔV and propellant up and down as required by constraints without affecting GLOM much.

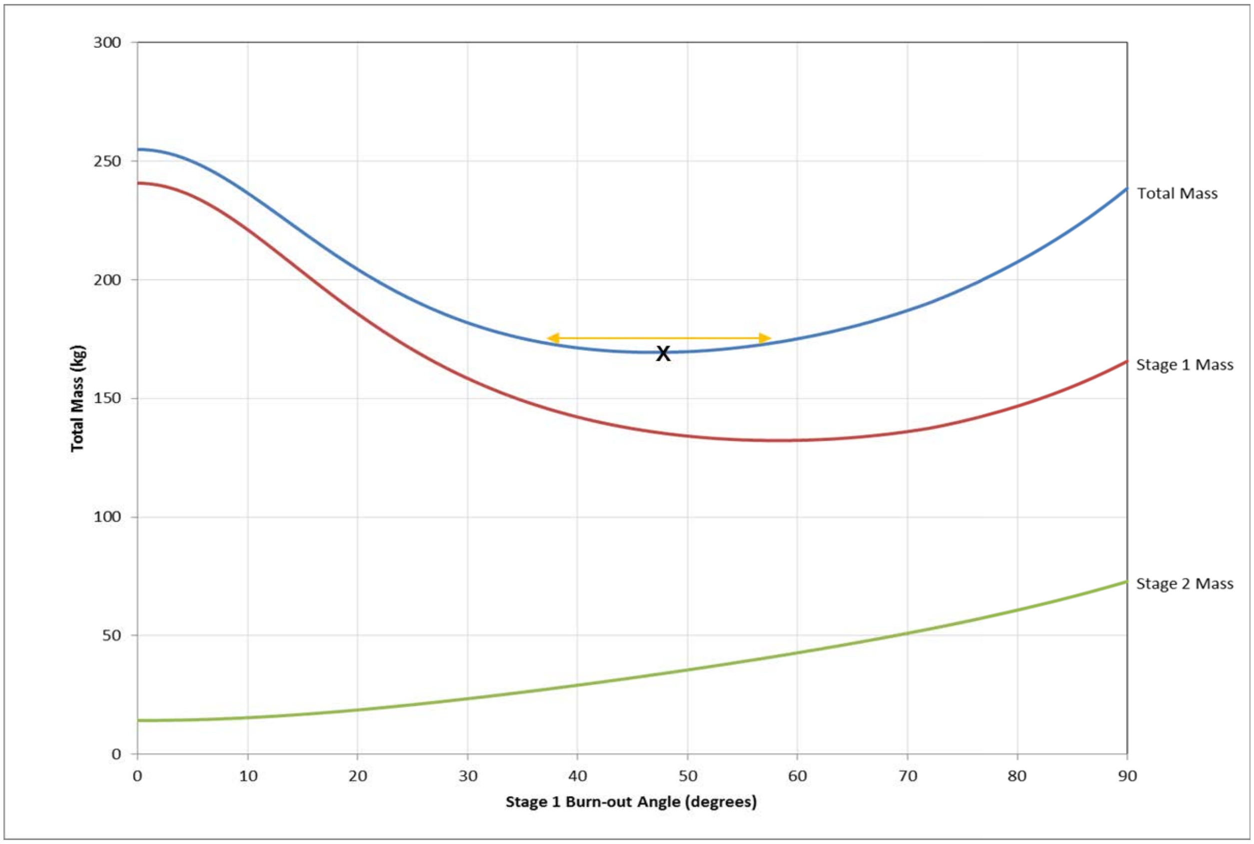

Figure 8. Optimizing the burn-out angle, ϕ_{bo}. *Varying the path-to-orbit changes the total ΔV required as well as the ΔV split between the two stages. Launching to a low angle reduces total ΔV but requires most of it to be provided by the 1st stage, which is suboptimal. These two effects cause the minimum GLOM to occur at some intermediate* ϕ_{bo}, *depending on MAV specific parameters*

The decisions as to whether and where to carry certain pieces of hardware (such as avionics) affects the relative efficiencies of the two stages, and therefore dictates a new optimal path-to-orbit. In this model it is easy to see the effects of design choices on the GLOM as well as individual masses.

C. Single Stage to Orbit

This model can easily be adapted to estimate mass and performance of single-stage-to-orbit (SSTO) MAVs. This is done by zeroing out all of the masses associated with the 1st stage (nothing is jettisoned between burns) and sizing the 2nd stage variable mass off of the combined ΔV of both burns. All SSTO MAVs must have restart capabilities in order to achieve orbit. Since optimal staging is no longer at play, SSTO's typically launch as shallow as possible without accumulating excessive drag. Burn-out angles are typically on the order of 5-15°, which minimizes total ΔV and leaves 100-200 m/s for the 2nd burn. Due to the fact that the ratio of the total ΔV to the characteristic velocity of typical propellants is on the order of 1.2[§], the ratio of propellant to dry mass does not preclude the benefits of a SSTO vehicle. They may be only 10-20% heavier than similar 2-stage vehicles, but they are much more sensitive to assumptions and changes.

IV. Results

This simplified MAV model has proven to be highly accurate and useful for preliminary MAV design. It also serves as a method to compare one design versus another in a common format. Fully optimized MAV designs were

[§] This is the exponent in the rocket equation, making the ratio of wet mass to dry mass $e^{1.2} \approx 3.3$. On Earth the exponent is greater than 2 (> 7:1 wet mass to dry mass), causing true SSTO launch vehicles to be infeasible.

Table 1. Model Comparison with Actual MAV Designs. *GLOM agreement is typically within a few percent of actual values across a wide range of designs and optimization schemes.*

	1	2	3	4	5	6	7	8	9	10	
Type	Solid	Solid	Solid	Solid	Solid	Liquid	Solid	Solid	Solid-Liquid	SSTO	
2nd Stage Guided	Yes	Yes	Yes	No	No	Yes	No	Yes	Yes	n/a	
Optimizer	POST	POST	POST	SNOPT	POST	Unknown	POST	OTIS	SNOPT	SNOPT	
						MODEL INPUTS					
Target Altitude	508	507	500	524	600	514	484	500	390	390	***km***
S1 Burn Out Angle	39	28.2	35	46.2	37.9	28.8	41	31.4	29	6	***deg***
Orbit Inclination	45	45	0	0	0	90	45	45	45	45	***deg***
Launch Latitude	45	45	0	0	0	0	0	45	0	0	***deg***
Stage 1											
Fixed Mass - 1	26.3	24.6	30.5	28.3	22.2	375.5	38.4	30.8	5.7	-	***kg***
PSMF - 1	0.0%	0.0%	0.0%	0.0%	13.0%	0.0%	0.0%	0.0%	23.5%	-	
Contingency	17%	21%	43%	43%	0%	0%	20%	0%	0%	-	***%***
Propulsion											
Thrust - 1	21569	23205	17858	7600	10647	35280	17800	21576	15632	-	***N***
Isp - 1	285.7	297.7	285.7	285.7	283	*300.6*	293	285.7	285	-	***s***
Stage 2											
Fixed Mass - 2	33.6	30.8	29	6.8	3.9	335	10.5	38.4	21.3	44.2	***kg***
OS Mass	5	5	5	3.9	5	20	5	5	5	6	***kg***
PSMF - 2	0.0%	0.0%	0.0%	0.0%	14.0%	0.0%	0.0%	0.0%	18.1%	5.1%	
Contingency	13%	23%	43%	43%	0%	0%	20%	0%	0%	0%	***%***
Propulsion											
Thrust - 2	6319	4052	4724	2850	2475	70560	2600	6318	900	3560	***N***
Isp - 2	285.5	290	285.7	285.5	279	281	293	285.5	236	256	***s***
						MODEL OUTPUTS					
Stage 1											
Burn Out Mass	30.8	29.8	43.6	41.9	31.9	375.5	46.1	30.8	39.0	0.0	***kg***
Propellant	158.1	153.3	176.7	76.4	74.9	1730.9	100.9	158.6	141.8	202.2	***kg***
Total	188.9	183.1	220.3	118.3	106.9	2106.4	147.0	189.4	180.8	202.2	***kg***
Stage 2											
Burn Out Mass	42.9	42.8	46.5	12.4	9.9	355.0	17.6	43.4	30.8	60.7	***kg***
Propellant	32.6	21.3	32.3	15.9	7.3	188.3	14.9	28.3	25.0	4.5	***kg***
Total	75.5	64.2	78.8	28.4	17.3	543.3	32.5	71.7	55.8	65.3	***kg***
ΔV											
ΔV1	2555	2826	2504	2063	2569	3123	2376	2620	2558	3542	***m/s***
ΔV2	1585	1150	1481	2311	1515	1173	1762	1408	1375	181	***m/s***
ΔVtot	4139	3976	3985	4374	4084	4296	4137	4028	3933	3723	***m/s***
Gravity Losses	68	37	63	88	47	281	42	48	58	66	***m/s***
Steering Losses	12	14	12	8	12	12	11	13	13	19	***m/s***
Drag Losses	104	104	52	31	73	12	80	78	67	55	***m/s***
Durations											
Ascent Time	713.5	803.5	731.2	631.5	791.8	837.7	670.7	735.6	698.3	1757.0	***sec***
Burn Time 1	20.5	19.3	27.7	28.2	19.5	144.7	16.3	20.6	25.4	142.6	***sec***
Burn Time 2	14.5	15.0	19.2	10.3	8.1	7.4	16.5	12.6	64.3	3.2	***sec***
Calculated GLOM	264.4	247.3	299.1	146.7	124.1	2649.6	179.5	261.1	236.6	267.4	***kg***
Actual GLOM	**263**	**251**	**302**	**150**	**126**	**2642**	**178.7**	**267.5**	**237.1**	**273**	***kg***
Difference	0.5%	-1.5%	-0.9%	-2.2%	-1.5%	0.3%	0.4%	-2.4%	-0.2%	-2.0%	

collected from both internal and external sources, both historical and current, and adapted to run through the model. Table 1 shows the data of the model inputs and outputs for 10 such designs. They vary from a ~3 ton liquid MAV design from the 1980's (case 6) to modern 2-stage solid propellant unguided designs. The tool estimated the total GLOM remarkably well – within 2% of actual in most cases.

Perhaps the largest drivers of MAV mass in the modern era** are the dry masses of stage 2 and stage 1, in that order. Other parameters like I_{sp}, thrust, orbit altitude, launch latitude, inclination, atmospheric variations, etc. have secondary effects, but do not drive the weight class of the MAV. Figure 9 shows the relationship between stage dry masses and GLOM. It also presents a method to visually classify MAV designs, as well as see the sensitivity to changes in dry mass. This plot assumes an I_{sp} of 285 seconds on both stages and an equatorial 500 km orbit. Burn-out angle is optimized.

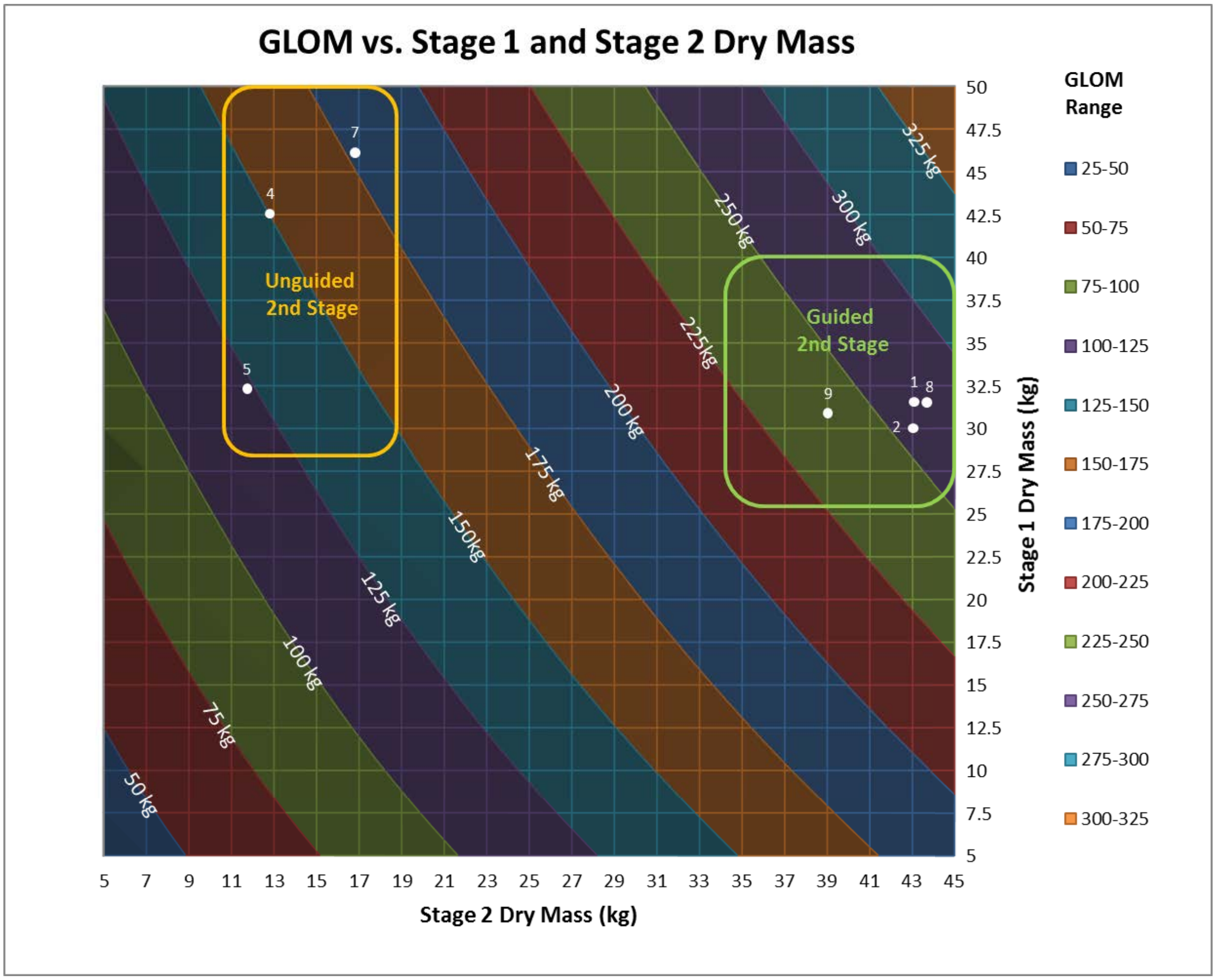

Figure 9. GLOM vs. dry mass look-up chart. *Contours of GLOM vs. total dry mass on stage 1 and stage 2 (includes OS). This plot assumes 500 km circular equatorial orbit and I_{sp} = 285 sec. Cases from Table 1 are shown on the plot. Note that some positions are not exact due to differences in assumptions. The unguided 2nd stage cases have much lower 2nd stage and total masses than the guided cases.*

The lines of iso-mass indicate that the sensitivity to a change in dry mass on the 1st stage is about 2.5:1. On the 2nd stage the sensitivity is greater than 4:1. When the dry mass of the 2nd stage is increased the propellant mass of the 1st stage must also increase, thereby necessitating an increase in 1st stage dry mass as well. Indeed, the GLOM of a 2-stage solid can be approximated by

** In the "modern" era the orbiting sample size is typically a few kilograms rather than the 10's to 100's of kilograms in decades past. MAVs are typically mass constrained as they are part of a multi-element campaign as opposed to an all-in-one approach.

$$GLOM = 2.35 M_{dry,1} + 4.25 M_{dry,2} \tag{18}$$

with a standard error of 3.9 kg over the range shown in Figure 9.

V. Conclusions

Once the basic design decisions of a MAV have been made, the entire ascent trajectory is essentially optimized by one parameter: ϕ_{bo}. Using the analytical equations of orbit transfer, the rocket equation, and some parametric loss models, it is possible to calculate the masses and ΔV's of an optimized MAV ascent trajectory in a user friendly environment such as Excel. What's more is that changes to masses or performance values can be made on the fly and sensitivities can readily be seen and quantified.

The results of this model have been compared to actual MAV designs and numerical simulations with surprising agreement – often to within a few percent. We were able to trace the design process of starting from a fully guided, two-stage solid MAV weighing approximately 300 kg down to a single-string, unguided 2nd stage "mini-MAV" weighing closer to 150 kg. This exercise allows designers to quickly see the efficacy of design decisions in reducing mass. We have also been able to model various other designs from the past decade and draw conclusions on the primary drivers for their mass and performance.

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