Statistical Ka-Band Link Analysis

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Outline of Talk

• Introduction – Motive of Study
• Review of Link Budget and Statistical Link Analysis
• Weather Availability and Weather Model
• Outline of New Techniques for statistical Ka-band Link Analysis
• Comparison of Current and New Approaches
• Concluding Remarks
Introduction – Motive of Study (1)

• Unlike S- and X- bands, Ka-band link is highly susceptible to weather effects
  ▪ Slow-fading phenomena with long coherence time
  ▪ Range of Ka-band weather loss can be 10+ dB’s
  ▪ Non-linear and non-Gaussian

• Accurate statistical link characterization can
  ▪ Avoid overly pessimistic link design – Waste of dB’s
  ▪ Enable link margin to be expressed in statistically quantifiable term of tolerable risk (e.g. 5% chance of not closing the link for a 95% weather availability)
• An analogy…
  - Link analysis planning for mission data return is analogous to booking passengers on a flight – once the plane takes off, all empty seats are useless
Review of Link Budget/Analysis (1)

- Link analysis is the tabulation of useful signal power and interfering noise power along the signal propagation path from transmitter to receiver
- Signal and noise terms are mathematical abstractions of the performance behavior
  - Deterministic: Transmission power, antenna gains, etc.
  - Statistical: weather effects, AWGN, axial ratio mismatch

\[ y_k = \alpha(x_k + n'_k) + n_k \]

- Receivable
- Signal
- Hot body noise, cosmic noise, etc.
- Equipment noise (AWGN)
- Noise is additive
- Attenuation is multiplicative

\[ R_b \leq \frac{G}{T} \frac{E_{IRP}}{k L_o L_s (\frac{E_b}{N_o})_{Th} M} \]

- Need good understanding on
  - Laws of Physics
  - Mathematics Principles
  - CONOPS
Review of Link Budget/Analysis (2)

- By “summing” up the link terms (in dB), one can come up with an overall signal-to-noise ratio (SNR) estimate to compare with some SNR thresholds to derive the supportable data rate and margin.

\[
\frac{P_r}{N_0} (dB - Hz) = EIRP (dB - W) + G / T (dB - K^{-1}) - k (dB - m^2 kg s^{-2} K^{-1}) - L_s (dB) - L_o (dB)
\]

- Problem with link budget (deterministic approach)
  - Use worst-case numbers
  - Margin policy makes no statistical sense. Why 3 dB?
Statistical link analysis treat each link parameter as a random variable, each associated with a design value, a min value, a max value, and a pdf. This in turn translates into mean and variance, and the resulting SNR is assumed to be Gaussian.
By Lyapunov’s principle, the Gaussian assumption can only be upheld if there is no “dominant term(s)”

But Ka-band weather effects are dominant term(s)
- Ka-band weather loss variance can be 10+ times the sum of other variances combined

Current statistical link analysis use worst-case loss for a given weather availability

New mathematical techniques is needed to accurately perform statistical Ka-band link analysis
Weather Availability and Weather Models (1)

• Weather availability (CD in %) is the long-term statistics of the weather effects that impact the link
  - Ka-band weather effects typically are slow-fading phenomena with long coherence times, affecting a large fraction, if not all of the entire communication session
  - One can regard weather availability as the percentage of communication sessions such that the dB loss due to weather effects is less than a certain value
    - For example, the dB loss due to weather effects for weather availability CD of 90% at 10° elevation angle of the Madrid site is 10 dB or lower (see Figure 2 of paper)
Weather Availability and Weather Models (2)

- Weather effects (esp. Ka-band) include
  - Increase in system noise temperature $T_{\text{sys}}$ due to increase in atmospheric noise temperature $T_{\text{atm}}$
  - Atmospheric attenuation $L_{\text{atm}}$ resulting from the aggregate of oxygen, water vapor, clouds, and rain, etc.
    - ITU models the weather effects individually, double counting the loss
- Relationship between $T_{\text{atm}}$ and $L_{\text{atm}}$ can be found in Section III of paper and 810-005
- Decades of measurements of $L_{\text{atm}}$ at all DSN sites are available to compute $T_{\text{atm}}$ and overall weather loss $L_{\text{tot}}$ (dB) for a given elevation angle $\theta$
Weather Availability and Weather Models (3)

- Example of CDF of $L_{tot}$ of Madrid at 10° elevation

Severe weather loss, probably due to rain

Typical weather loss, gradual
Outline of New Techniques for Statistical Ka-band Link Analysis(1)

- Sum of link parameters $S$ excluding the weather effects has Gaussian distribution

$$N(s, m, \sigma) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(s-m)^2}{2\sigma^2}}$$

$$F(s, m, \sigma) = \int_{\infty}^{s} \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(s-m)^2}{2\sigma^2}} ds$$

- From CDF of $L_{tot}$, compute the discrete probability distribution conditioned on the weather availability CD

$$L_D = \{l_{10}, l_{11}, \cdots l_{CD}\}$$

with probability $\{P_{10}, P_{11}, \cdots P_{CD}\}$, where $l_i$ corresponds to the $i$-th percentile value of the total weather loss $L_{tot}^{dB}(\theta)$

- Complicated convolution $\Rightarrow$ sum of Gaussian

$$X = S + L_D$$

$$P[X \leq x] = \sum_{i=10}^{CD} P_i \times F[x, m + l_i, \sigma]$$
• Solving the above expression, one can compute the SNR value (mean + margin) required to guarantee a certain probability of closing the link
  ▪ For example, the DSN downlink margin policy is $2\sigma$ (in Normal sense), which corresponds to a probability of 0.97725
## Comparison of Current and New Approaches

### Madrid

<table>
<thead>
<tr>
<th>Weather Availability (%)</th>
<th>Current Approach (dB)</th>
<th>New Approach (dB)</th>
<th>Difference (dB)</th>
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</thead>
<tbody>
<tr>
<td>80</td>
<td>13.53</td>
<td>12.65</td>
<td>0.88</td>
</tr>
<tr>
<td>90</td>
<td>16.35</td>
<td>14.66</td>
<td>1.68</td>
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<td>95</td>
<td>20.40</td>
<td>17.72</td>
<td>2.69</td>
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### Goldstone

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<th>Difference (dB)</th>
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<tbody>
<tr>
<td>80</td>
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<td>12.20</td>
<td>0.68</td>
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### Canberra

<table>
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<th>Difference (dB)</th>
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<td>99</td>
<td>23.82</td>
<td>17.79</td>
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</table>
Concluding Remarks

- Link Budget (deterministic link parameters) can be overly conservative for Ka-band link analysis.
- Current statistical link analysis is better, but is still conservative due to the use of worst-case weather loss for a given weather availability.
- Using sound statistical principle the new approach removes the artificial conservatism of the current link analysis approaches and yields a lower and tightly upper bound of the received power required to close the link.
  - Computation is simple also.
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