

# Deducing Electronic Unit Internal Response during a Vibration Test Using a Lumped Parameter Modeling Approach

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**Aerospace Testing Seminar**



"Navigating the New Normal in Aerospace Testing"

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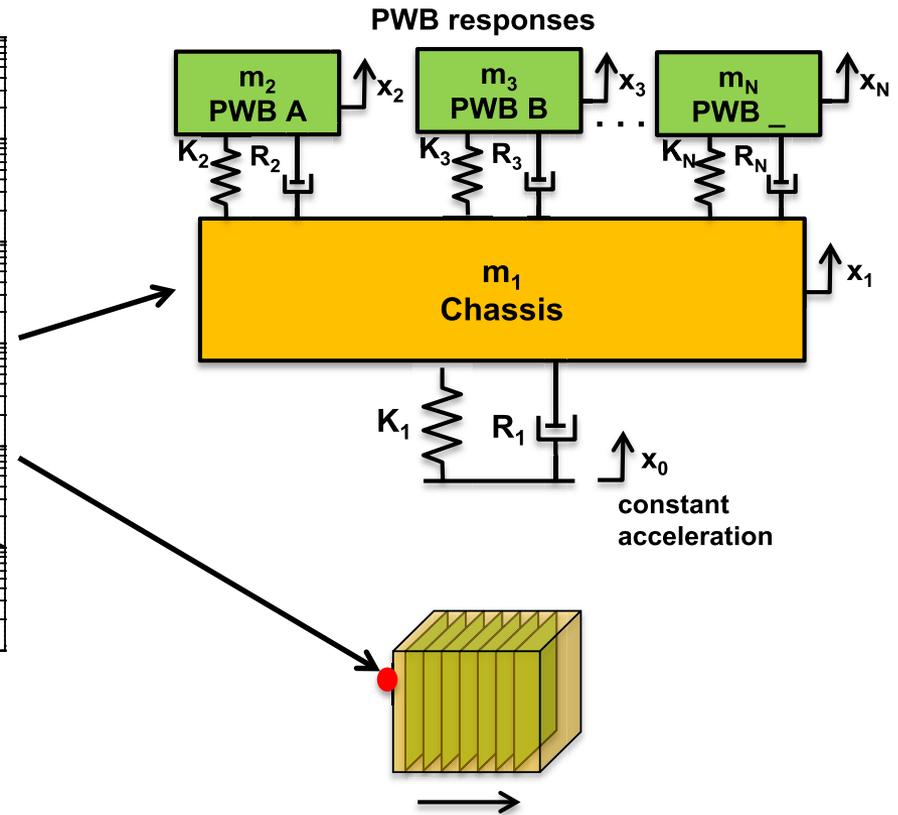
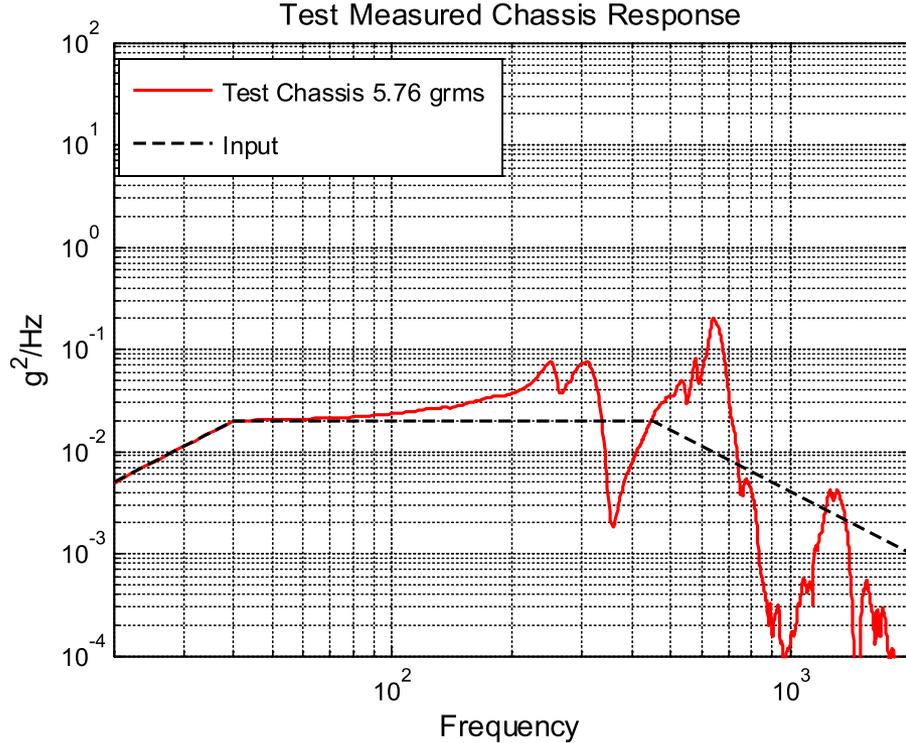
# Overview

- Basic approach using model to deduce internal dynamics from external measurement
- Model development
- Two DOF illustration of parametric influence of model parameters
- Demonstration and correlation with test data
- Conclusions and future development

# Motivation

- Accurately estimate random vibration test response of PWBs when only external measurements are available
  - *Current approximation methods such as use of Miles equation can result in significant over- or under-estimation (do not account for coupled response)*
- During testing – Indirect observability of unit internal PWB dynamics from external test measurement
- After testing – Forensic interpretation of test data
  - *Anomaly resolution*
  - *Design validation*
- Design stage – Quick prototyping of electronics unit dynamics for early design
  - *This approach would supplement but not replace a detailed finite element model (FEM)*

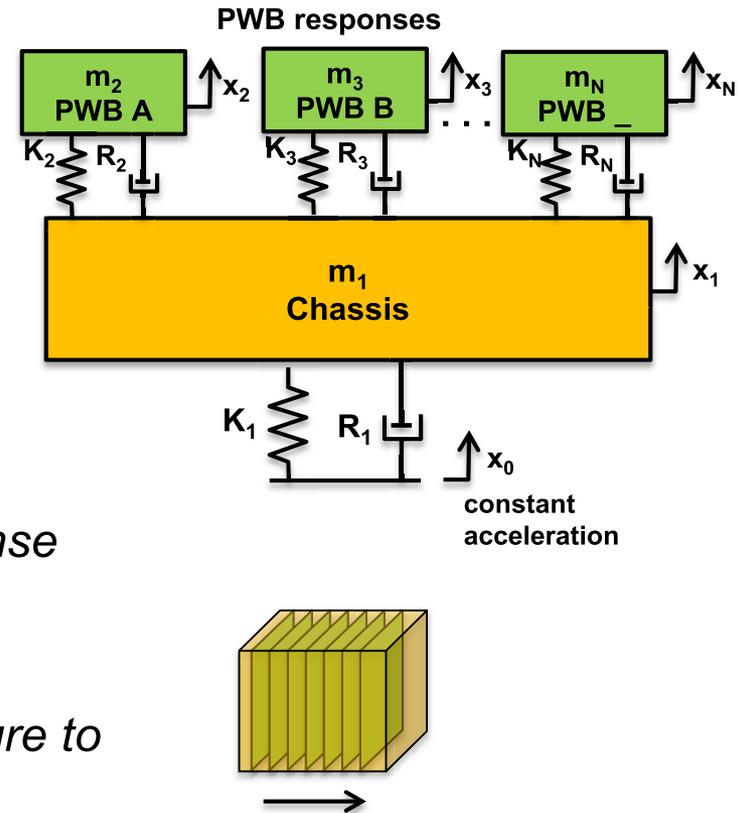
# Basic Approach



- Spectral shape of chassis response is a unique signature from dynamic coupling with mounted oscillators
- Lumped parameter model of chassis and PWBs created by tuning model parameters to approximate chassis test response signature

# Basic Approach

- Assumptions
  - Fixed base excitation normal to PWBs
  - PWBs independently supported by chassis
  - Damping assigned to discrete elements
  - Model elements approximate c.o.g. response of structural elements in fundamental mode
- Limitations
  - Approximates only fundamental modal response of structural element
  - Restricted to linear response
  - Ambiguity in correlating chassis spectral feature to response of specific PWB
  - Methodology still under development
    - Needs more rigorously defined implementation rules and parameters
    - Needs further correlation with test data and finite element models



# Representation of Element Effective and Residual Mass

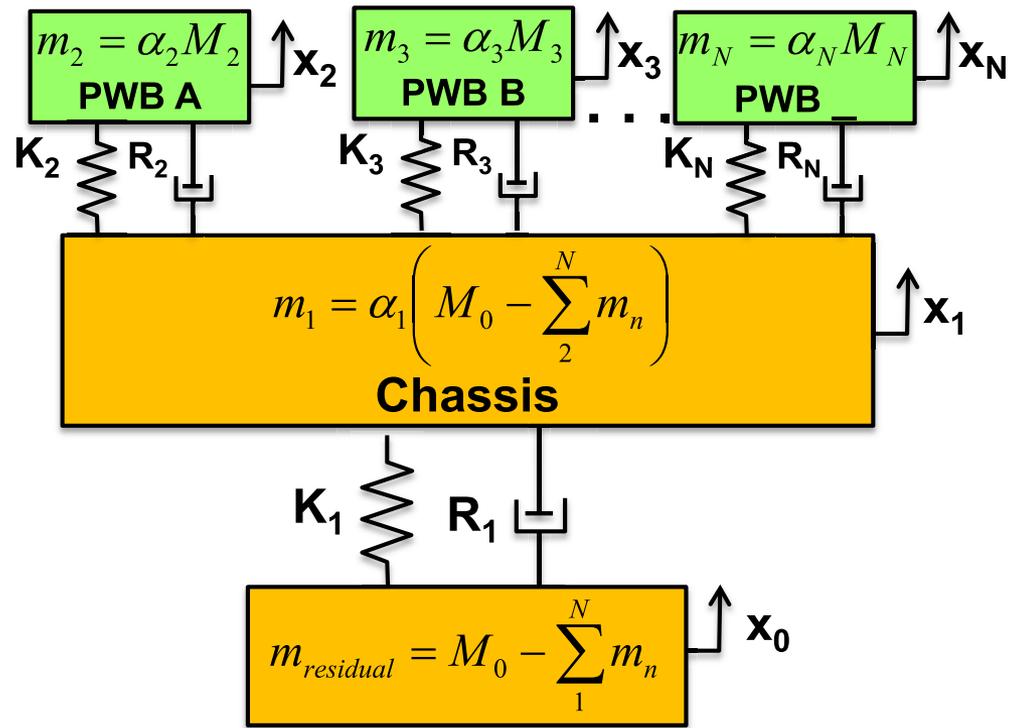
**Element Total mass  $M_n$**

**Element Effective modal mass**  
 $m_n = \alpha_n M_n$

*Dynamically participating in proportion to mode shape*

**Residual mass**  
 $M_{res} = M_{unit} - \sum m_n$

*Moves along with input in rigid body motion*



( $M_0$  = total unit mass)

- $\alpha_n$  = Modal mass participation factor for element  $n$
- Adapted from Bamford, Wada (1971)
  - Differs: model DOF correspond to discrete structural elements (chassis, PWBs) rather than modes

# Model Development: Parameter Assignment

Model DOF	Structural Element	Mass	Effective Modal Mass	Fundamental Frequency	Quality Factor
1	Chassis	$M_1 = M_0 - \sum_{n=2}^N m_n$	$m_1 = \alpha_1 M_1$	$f_1$	$Q_1$
2	PWB 1	$M_2$	$m_2 = \alpha_2 M_2$	$f_2$	$Q_2$
3	PWB 2	$M_3$	$m_3 = \alpha_3 M_3$	$f_3$	$Q_3$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$N$	PWB $N-1$	$M_N$	$m_N = \alpha_N M_N$	$f_N$	$Q_N$

- Model parameters  $m_n$ ,  $f_n$ ,  $Q_n$  initial value assignments based on
  - Available unit information (actual measurements, structural analysis documentation)
  - Best estimates
  - Reasonable values based on experience

# N-dimensional Linear System of Equations

$$\begin{array}{cccccc|c|c}
 1 - a_1(\omega) - \sum_{n=2}^N \gamma_n a_n(\omega) & \gamma_2 a_2(\omega) & \gamma_3 a_3(\omega) & \cdots & \gamma_N a_N(\omega) & z_1(\omega) & -a_1(\omega) \\
 a_2(\omega) & 1 - a_2(\omega) & 0 & \cdots & 0 & z_2(\omega) & 0 \\
 a_3(\omega) & 0 & 1 - a_3(\omega) & \cdots & 0 & z_3(\omega) & 0 \\
 \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
 a_N(\omega) & 0 & 0 & \cdots & 1 - a_N(\omega) & z_N(\omega) & 0
 \end{array} = 0$$

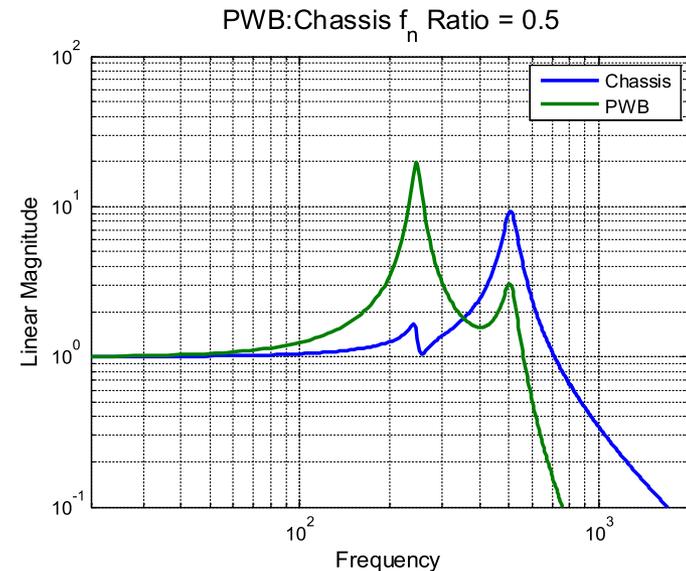
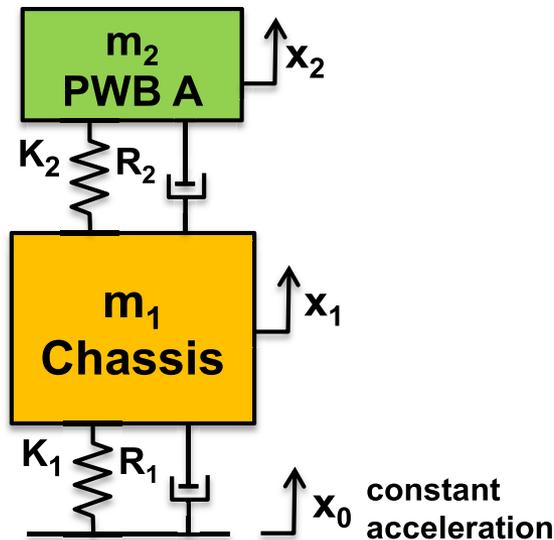
$$a_n(\omega) = j \frac{\omega_N}{\omega Q_N} + \left( \frac{\omega_N}{\omega} \right)^2 \quad \gamma_n = \frac{m_n}{m_1}$$

Solutions  $z_n(\omega)$  = normalized frequency response functions for DOF

# 2 DOF Parametric Studies

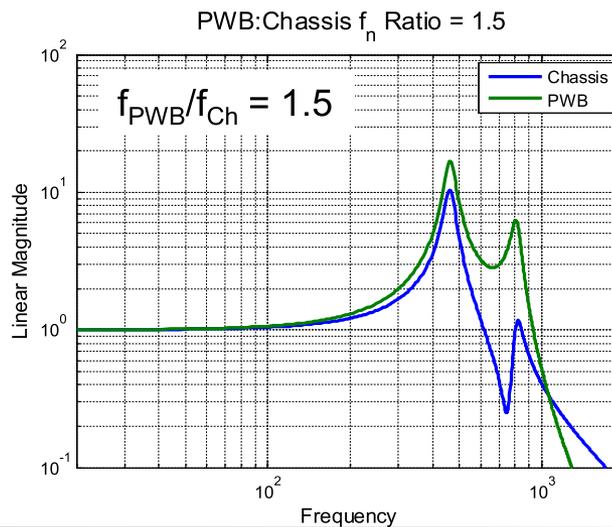
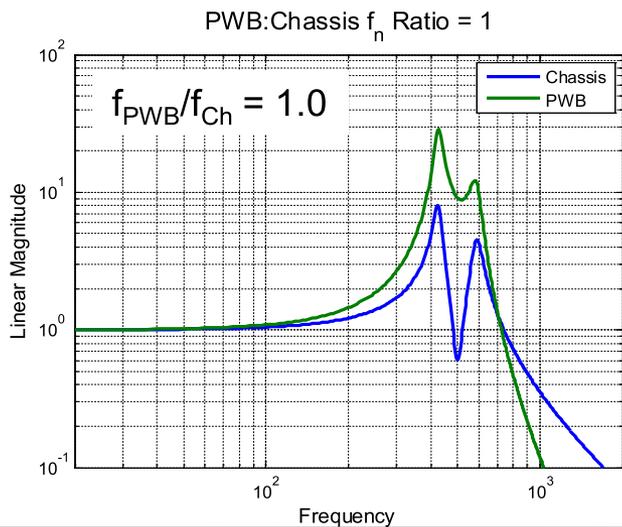
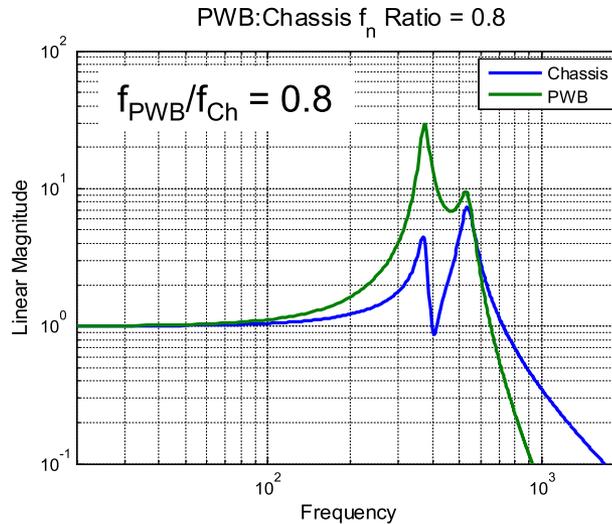
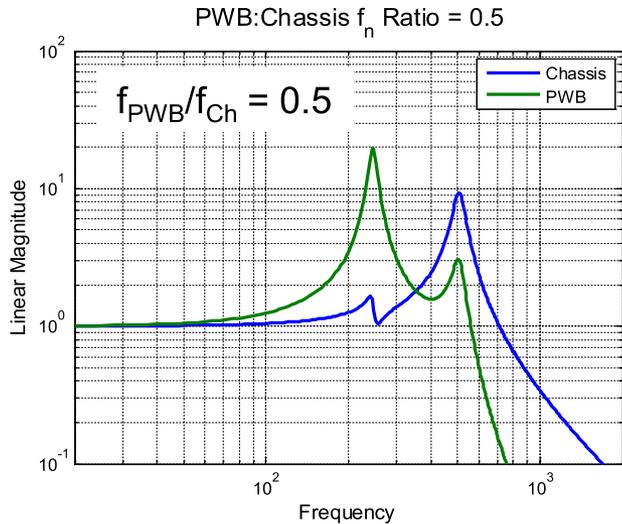
*Illustrate spectral influence of model parameters*

- 2 DOF = Chassis + 1 PWB
- Coupled response studies
  1. Ratio of uncoupled resonance frequencies  $f_2/f_1$  for  $m_2/m_1=1/10$  (small unit)
  2. Ratio of uncoupled resonance frequencies  $f_2/f_1$  for  $m_2/m_1=1/30$  (medium sized unit)
  3. PWB Q value



# Study 1: PWB/Chassis $f_n$ Ratio Spectral Influence

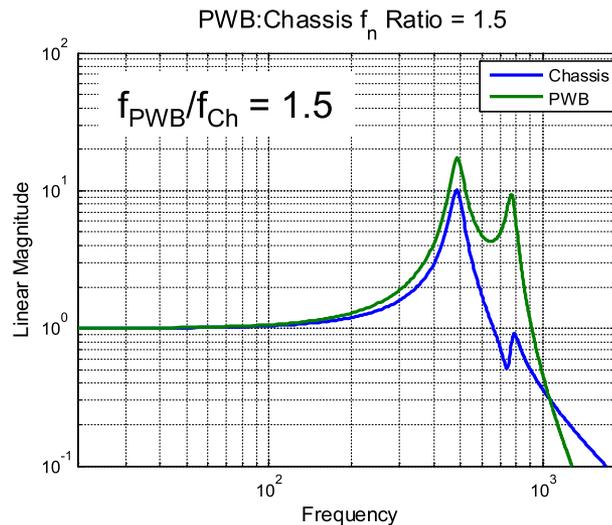
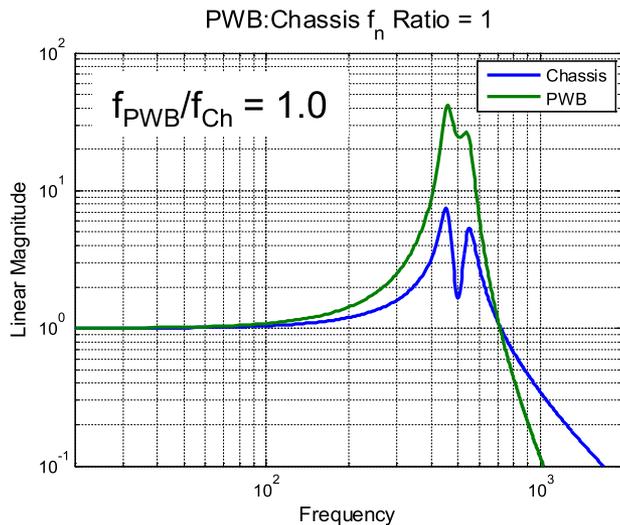
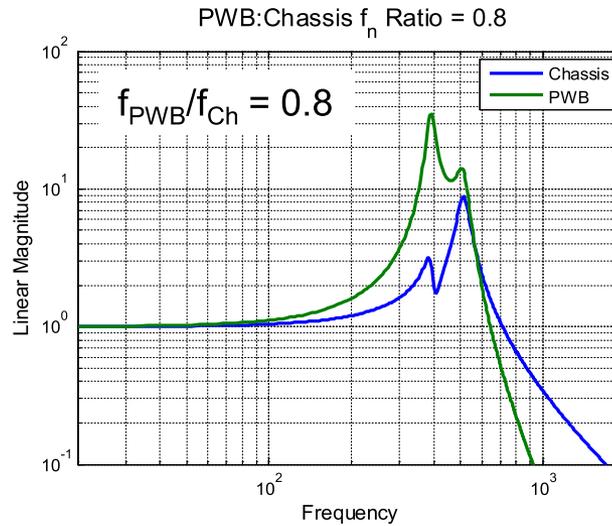
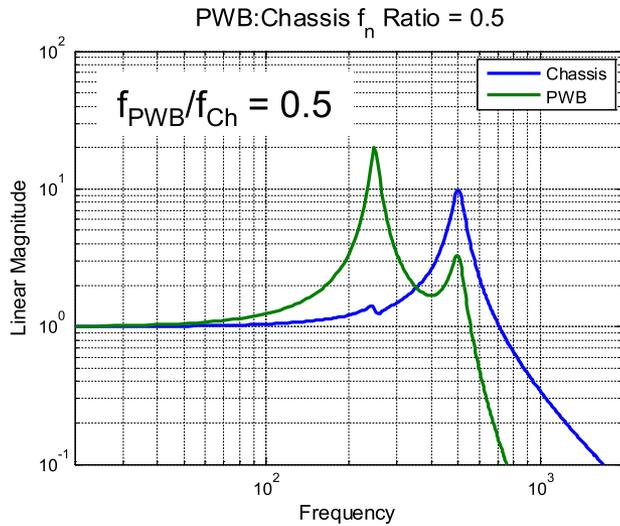
*Relatively small unit – PWB/Chassis mass ratio = 1/10*



PWB Q = 25  
Chassis Q = 10

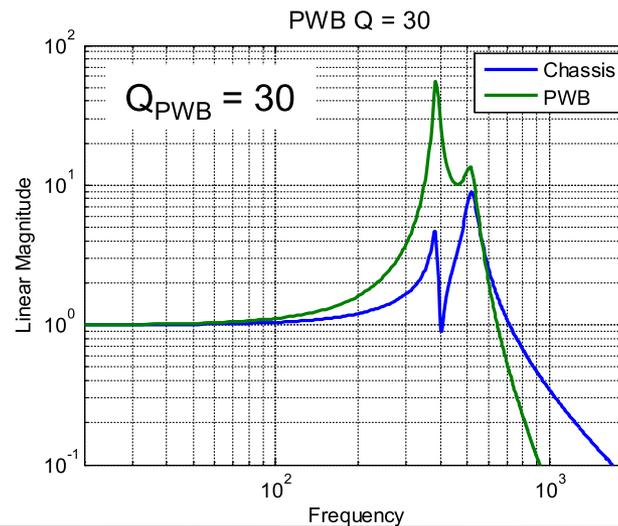
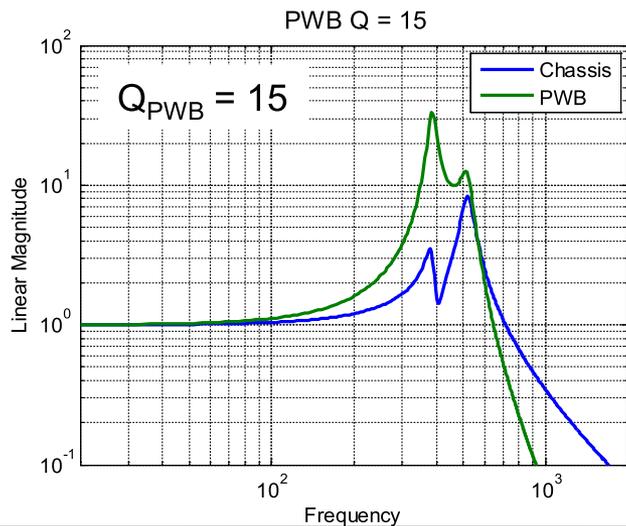
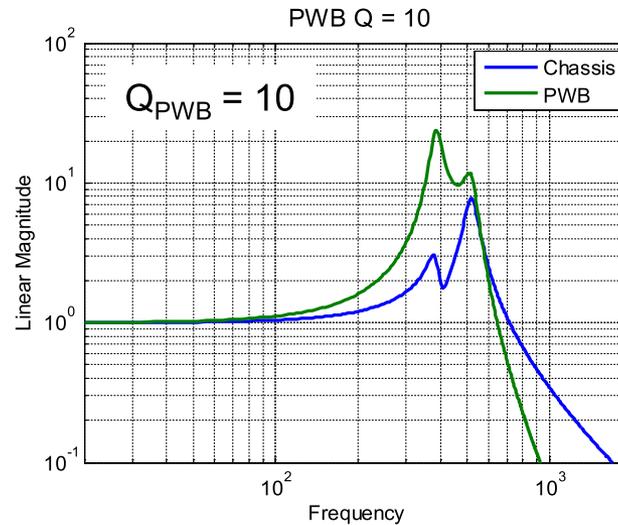
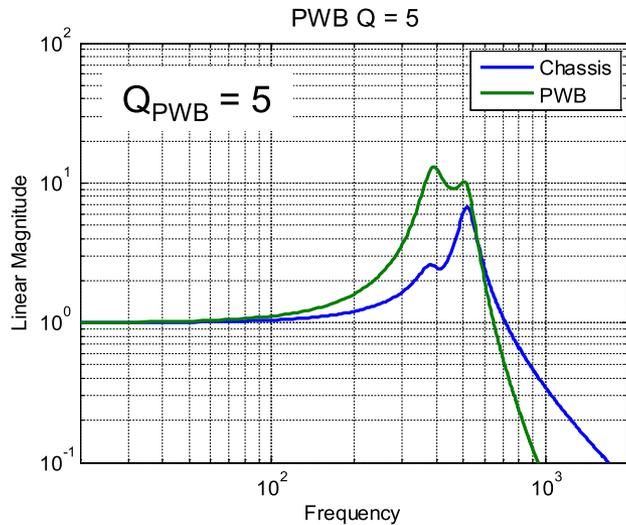
# Study 2: PWB/Chassis $f_n$ Ratio Spectral Influence

Medium-sized unit – PWB/Chassis mass ratio = 1/30



PWB Q = 25  
Chassis Q = 10

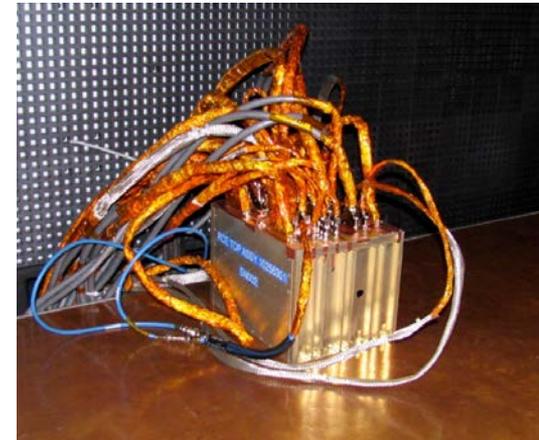
# Study 3: PWB Q Value Spectral Influence



Chassis Q = 10  
 $m_{PWB}/m_{Ch} = 1/10$

# Demonstration Using Test Data

- Test article
  - 28 lb. (12.7 kg) electronics unit with 8 full-span PWBs
  - Random vibration test (force limited) performed in 2008
  - Accelerometer instrumentation
    - 1 PWB (D) near center of board
    - Chassis at approximate c.o.g. height

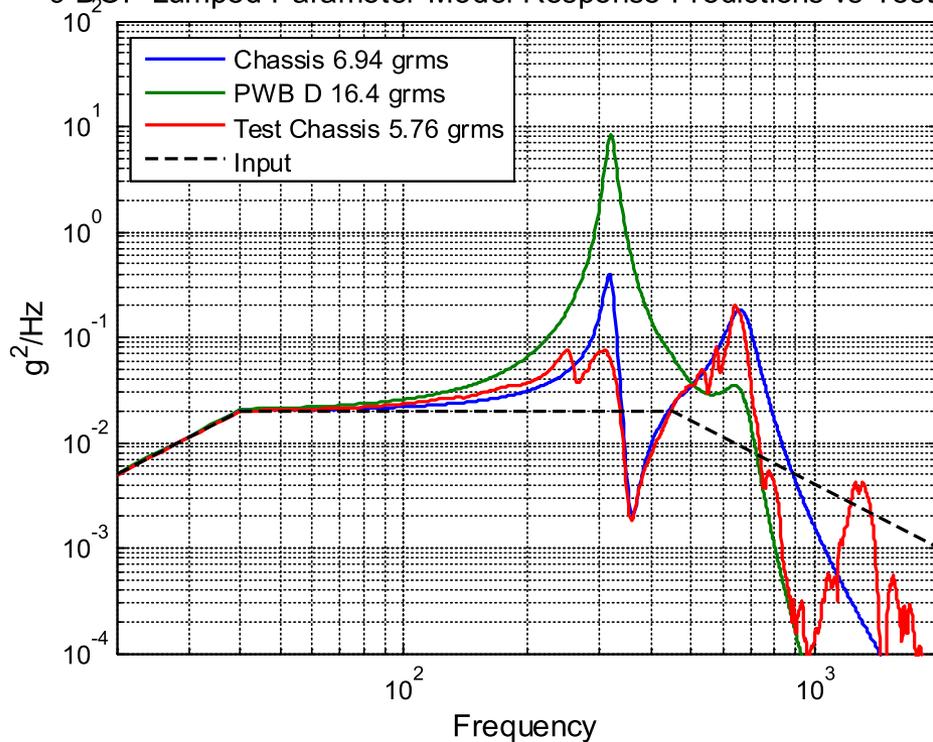


PWB	Frame Weight lbs (kg)	CCA Weight lbs (kg)	PWB Weight lbs (kg)	FEM modal frequency (Hz)
A	1.10 (0.50)	1.70 (0.77)	2.80 (1.27)	218
B	0.94 (0.43)	1.39 (0.63)	2.33 (1.06)	228
C	0.50 (0.23)	1.00 (0.45)	1.50 (0.68)	223
D	0.50 (0.23)	1.00 (0.45)	1.50 (0.68)	213
E (copy of D)	0.50 (0.23)	1.00 (0.45)	1.50 (0.68)	213
F	0.50 (0.23)	1.00 (0.45)	1.50 (0.68)	213
G	0.50 (0.23)	0.90 (0.41)	1.40 (0.64)	228
H (spare)	0.59 (0.27)	0.78 (0.35)	1.36 (0.62)	NA

Data obtained from unit structural analysis documentation

# 9 DOF Model Approximation

9 DOF Lumped Parameter Model Response Predictions vs Test Data



Model Uncoupled Parameters

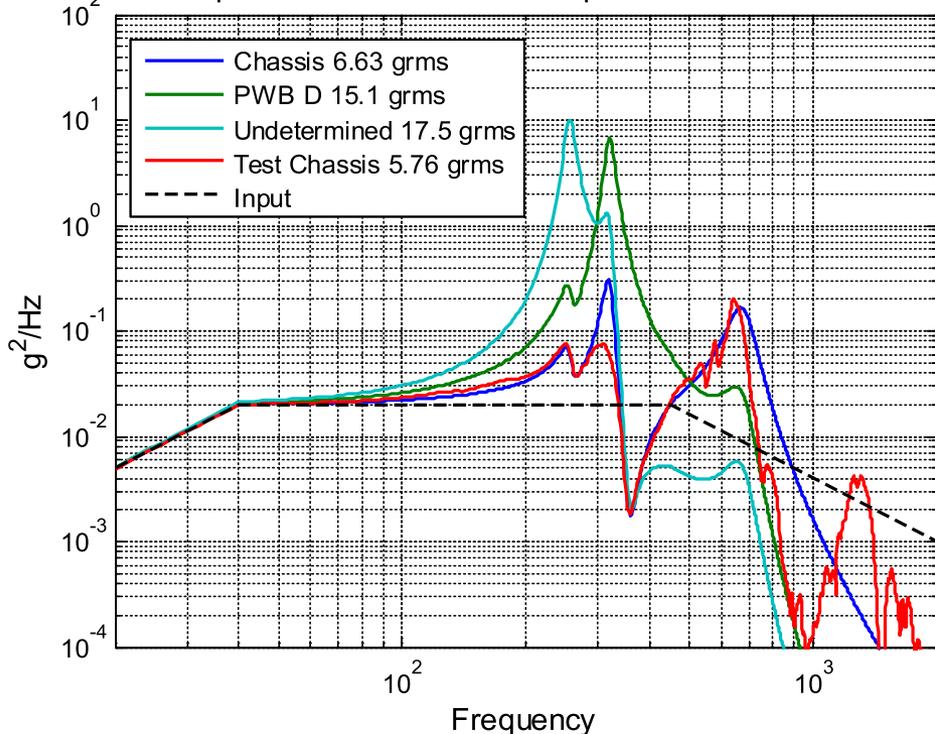
Model Element	Weight lbs (kg)	Mass Partic. Factor $\alpha_n$	Uncoupled Resonance $f_n$ (Hz)	Q
Chassis	*	0.45	595	6.0
PWB A	2.80 (1.27)	0.30	358	15.0
PWB B	2.33 (1.06)	0.30	358	15.0
PWB C	1.50 (0.68)	0.42	358	17.0
PWB D	1.50 (0.68)	0.42	358	17.0
PWB E	1.50 (0.68)	0.42	358	17.0
PWB F	1.50 (0.68)	0.42	358	17.0
PWB G	1.40 (0.64)	0.42	358	17.0
PWB H	1.36 (0.62)	0.42	358	17.0

Chassis response:  
 Measured = 5.76  $g_{rms}$   
 Predicted = 6.94  $g_{rms}$

- Test external chassis response (red)
  - Low level (-6 dB) test run used for model correlation
  - Test data normalized to nominal input spectrum to eliminate spectral noise
- Predicted random vibrate responses =  $|z_n(\omega)|^2 \times$  Input PSD (interpolated)

# 10 DOF Model Approximation

10 DOF Lumped Parameter Model Response Predictions vs Test Data



- Notch in chassis response at 260 Hz predicted with undetermined structural element of 2.5 lbs.,  $Q = 14$

## Model Uncoupled Parameters

Model Element	Weight lbs (kg)	Mass Partic. Factor $\alpha_n$	Uncoupled Resonance $f_n$ (Hz)	Q
Chassis	*	0.45	595	6.0
PWB A	2.80 (1.27)	0.30	358	15.0
PWB B	2.33 (1.06)	0.30	358	15.0
PWB C	1.50 (0.68)	0.42	358	17.0
PWB D	1.50 (0.68)	0.42	358	17.0
PWB E	1.50 (0.68)	0.42	358	17.0
PWB F	1.50 (0.68)	0.42	358	17.0
PWB G	1.40 (0.64)	0.42	358	17.0
PWB H	1.36 (0.62)	0.42	358	17.0
Undetermined	2.50 (1.13)	0.40	260	14.0

Chassis response:

Measured = 5.76  $g_{rms}$

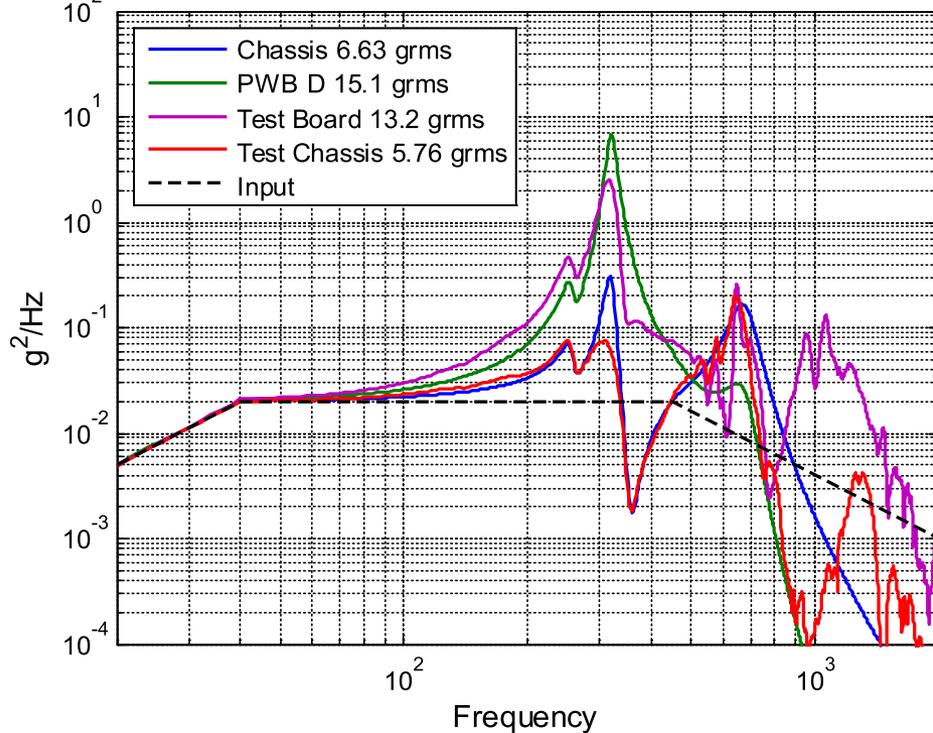
Predicted = 6.63  $g_{rms}$

Predicted PWB D accel. = 15.1  $g_{rms}$

Predicted PWB D rel. defl. = 1.15 mils $_{rms}$  (29.2  $\mu m_{rms}$ )

# PWB Response: 10 DOF Model vs. Test Measurement

10 DOF Lumped Parameter Model Response Predictions vs Test Data



## Model Uncoupled Parameters

Model Element	Weight lbs (kg)	Mass Partic. Factor $\alpha_n$	Uncoupled Resonance $f_n$ (Hz)	Q
Chassis	*	0.45	595	6.0
PWB A	2.80 (1.27)	0.30	358	15.0
PWB B	2.33 (1.06)	0.30	358	15.0
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PWB F	1.50 (0.68)	0.42	358	17.0
PWB G	1.40 (0.64)	0.42	358	17.0
PWB H	1.36 (0.62)	0.42	358	17.0
Undetermined	2.50 (1.13)	0.40	260	14.0

### Chassis response:

Measured = 5.76  $g_{rms}$

Predicted = 6.63  $g_{rms}$

### PWB D response

Measured = 15.1  $g_{rms}$

Predicted = 13.2  $g_{rms}$

# Conclusions

- Lumped parameter modeling is a viable approach for deducing internal board random vibration test responses given only external chassis response measurement
  - *Reasonable correlation of model predictions with test data*
  - *Model can be rapidly developed and tuned to provide quick results*
  - *Due attention must be made to assumptions and limitations*
  - *More development needed to ensure reduce uncertainties in application*
- Empirical alternative to application of crude approximation methods such as application of Miles Equation

# Future Development

- Additional correlation of approach with test data and finite element models
- Incorporate test force measurements along with chassis response as test data observables of internal dynamics
- Explore extension to transient response for application to shock test response predictions for electronic units
  - *Limited to spectral range dominated by structural response (typically <1000Hz)*
- Develop similar modeling approaches for broader range of application

# Acknowledgements

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  - *For collaboration in adapting the lumped element approach to modeling electronic box dynamics*

# Backup



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## Aerospace Testing Seminar

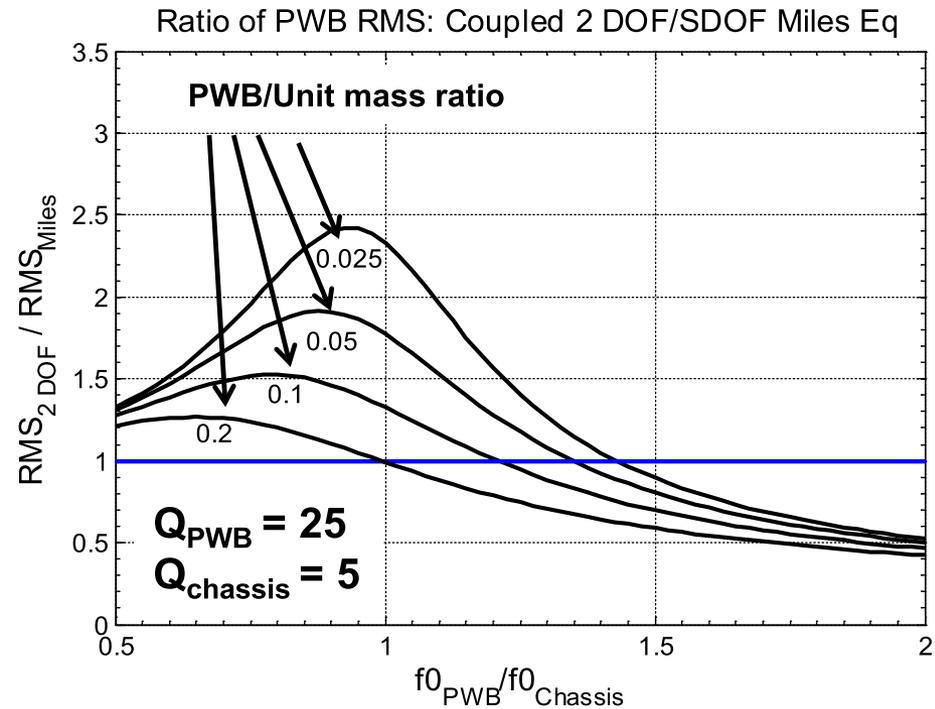
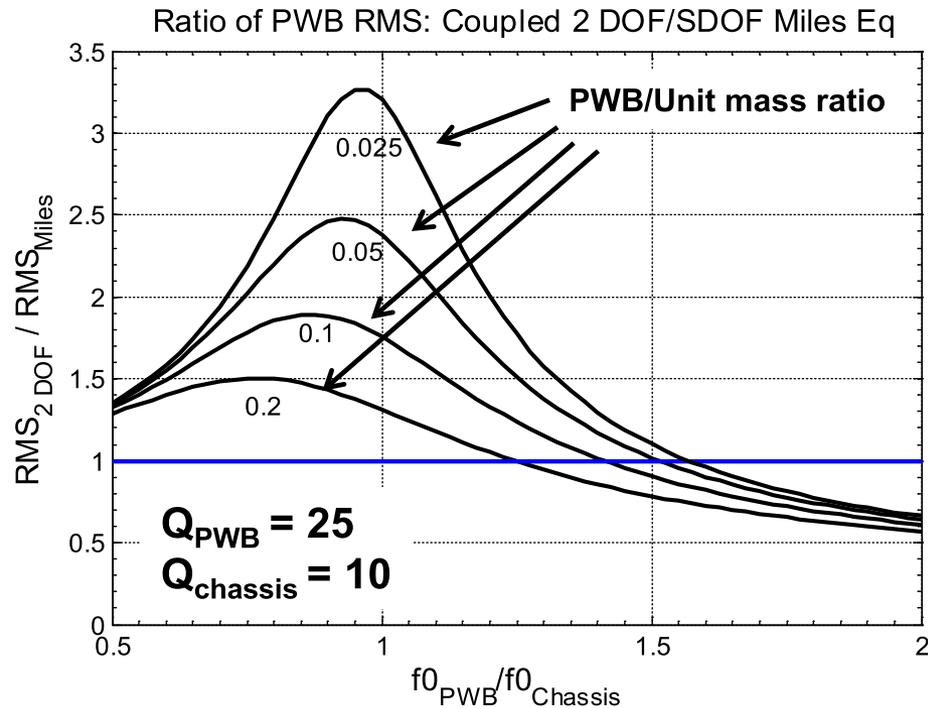
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# RMS: Coupled PWB Response Based on 2 DOF Model



- Coupled relative RMS response of first bending mode (whether g's or mils) can be significantly greater than a simple SDOF Miles Equation would predict
- Degree of amplification dependent on relative mass ratio,  $f0$  ratio,  $Q$